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Research Note

Two-Objective Stacking Sequence Optimization of a Cylindrical Shell Using Genetic Algorithm

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In this paper, the stacking sequence optimization of a laminated cylindrical shell has been studied for obtaining maximum natural frequency and buckling stress, simultaneously. An anisotropic cylindrical shell has a finite length with simply supported conditions at both ends. Three-dimensional elasticity approaches are used for free vibration analysis and the buckling stress function is based on the theory of classic shells. A genetic algorithm is used for optimization and, regarding the two-objective problem, a Pareto-optimally curve is used to help determine the best way to simultaneously satisfy all objectives. Finally, numerical results are presented for the optimization of a six-layer cylindrical shell.

INTRODUCTION

Laminated cylindrical shells are widely used in industries as structural elements and their vibration characteristics are important, in view of the current interest in designing with composite materials. The greatest advantage of laminated composite materials, in addition to high strength to weight properties, is that they provide designers with the ability to tailor the directional strengths and stiffnesses of the material to the given loading environment of the structure. Therefore, laminated composite constructions offer many opportunities for engineers and designers to optimize structures for a particular, or even multiple, tasks. The problem is often formulated as a continuous optimization problem, with the thickness and orientation of the plies as the design variable [1], but for most particular problems, laminate thicknesses are fixed and orientations are limited to a set of angles, so the design problem becomes a stacking sequence optimization. Haftka and Walsh solved the stacking sequence problem for buckling load maximization [2]. The nonlinear problem, resulting from using ply thicknesses as design variables, is linearized by using ply- orientation identity variables and then solved using a branch and bound algorithm. Nagendra solved a similar problem with the additional strain constraints, once again introducing non-linearities to the problem. This problem was solved by using a sequence of linear integer programming techniques [3]. The design space usually contains many local extrema, even singular ones and, also, many near optima designs may exist. Thus, there is a need for optimization techniques that can identify multiple and singular exterma. Optimization methods based on Genetic Algorithms (GA), have been applied to structural problems [4]. In the area of composite structural design, a GA is used to optimize the stacking sequence of laminated plates for buckling loads [5], to design stiffened composite cylindrical shells against buckling [6] and optimized tailoring problems [7]. A GA is the probabilistic optimization method that works on the population of design [8]. In genetic algorithm approaches, the solution is called a chromosome or string. A genetic search requires a population of chromosomes, each representing a combination of features from the set of features. In recent years, genetic algorithms have been successfully applied to large, non-convex, integer programming problems [9,10]. Thus, it was obvious that genetic algorithms would be well suited for the design and optimization of laminated composite plates. Early works include Callahan, who used genetic algorithms for the stacking sequence optimization of composite plates [11]. Negendra did extensive work with genetic algorithms and stiffened composite panels [12,13]. The optimization problem utilized a single objective frequency of a composite laminated cylindrical shell. For multi-constraint prob-

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lems, since the best single objective usually doesn't imply that the other objective(s) is simultaneously optimized, the concept of Pareto-optimality is often used to help determine the best way to simultaneously satisfy all objectives to the greatest extent possible. In this case, each string represents an alternative stacking sequence of a laminated cylindrical shell. A genetic algorithm is devised for designing composite laminates comprised of one material only. The genetic algorithm procedure, which is referred to as GA-I, will be used for maximization of the lowest natural frequency of a laminated cylindrical shell with strength and ply contiguity constraints.

Shakeri et al. studied the stacking sequence optimization of a laminated cylindrical shell, based on natural frequency [14]. They used the genetic algorithm procedure for maximization of the lowest natural frequency with strength and ply contiguity constraints. Grosset et al. considered the optimization of a composite laminated plate made from two materials [15]. The multi-objective optimization problem requires the construction of a Pareto trade-off curve. This paper uses genetic algorithms to optimize the natural frequency and buckling load simultaneously. For this purpose, by solving a series of optimization problems combining the two objective functions, the Pareto set is generated by optimizing a combination of the two objectives and, finally, the best stacking sequence is obtained. The free vibration solution is based on Shakeri et al. [16] and, for buckling load, Tasi's work [17] is used. In the end, numerical results are presented for a six layer composite cylindrical shell.

PROBLEM FORMULATION

A laminated composite hollow cylindrical shell is considered of length L with MN constituent monoclinic laminate. The mean radius and the thickness of layers are denoted by R and h, respectively. The material axis of any orthotropic layer is not necessarily aligned in the x and θ directions, hence, the constitutive equations of a layer are, as follows:

$$\left\{\begin{matrix}\sigma_x\\ \sigma_\theta\\ \sigma_r\\ \sigma_{r\theta}\\ \sigma_{xr}\\ \sigma_{x\theta}\end{matrix}\right\} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16}\\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26}\\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36}\\ 0 & 0 & 0 & C_{44} & C_{45} & 0\\ 0 & 0 & 0 & C_{45} & C_{55} & 0\\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66}\end{bmatrix} \left\{\begin{matrix}\varepsilon_x\\ \varepsilon_\theta\\ \varepsilon_r\\ \gamma_{r\theta}\\ \gamma_{xr}\\ \gamma_{x\theta}\end{matrix}\right\}, \qquad (1)$$

The equations of motion in the absence of body forces are:

$$\sigma_{ij,j} = \rho \ddot{u}_i. \qquad (2)$$

The strain-displacement relations are written as:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \qquad (3)$$

By combining Equations 1 with 3 for axisymmetric loading and substituting into Equation 2, one can obtain the governing equations in terms of the displacement components (u_r, u_θ, u_x). The coefficients of the obtained governing equations are a function of variable r, which makes the solution formidable. To circumvent this difficulty, the following changes of variable are used:

$$\eta_k = \frac{\xi_k}{R_K}, \frac{1}{r} = \frac{1}{R_k}(1 \quad \eta_k),$$

$$\frac{1}{r^2} = \frac{1}{R_k^2}(1 \quad 2\eta_k). \qquad (4)$$

The local coordinate is used, as $\xi_k = r \quad R_k$. Each layer is assumed to be thin ($\xi_k/R_k << 1$). By substituting Equation 4 into the equations of motion, the equations of motion, with constant coefficients for each layer, are obtained:

$$C_{33}^k \frac{\partial^2 u_r}{\partial \eta^2} + C_{33}^k \frac{\partial u_r}{\partial \eta} + C_{55}^k R_k^2 \frac{\partial^2 u_r}{\partial x^2} \quad C_{22}^k \frac{u_r}{r^2}$$

$$+ R_k \Bigg[(C_{36} \quad C_{45} \quad C_{26}) \frac{\partial u_0}{\partial x} + (C_{36} + C_{45}) \frac{\partial^2 u_\theta}{\partial \eta \partial x}$$

$$+ (C_{13}^k \quad C_{12}^k) \frac{\partial u_x}{r \partial x} + (C_{13}^k + C_{55}^k) \frac{\partial^2 u_x}{\partial x \partial \eta} \Bigg]$$

$$= \rho^k R_k^2 \frac{\partial^2 u_r}{\partial t^2},$$

$$R_k \left[(2C_{45}^k + C_{26}^k) \frac{\partial u_r}{\partial x} + (C_{45}^k + C_{36}^k) \frac{\partial^2 u_r}{\partial x \partial \eta} \right]$$

$$+ C_{44}^k \frac{\partial^2 u_0}{\partial \eta^2} + C_{44}^k \frac{\partial u_0}{\partial \eta} \quad C_{44}^k u_0 + R_k^2 C_{66} \frac{\partial^2 u_0}{\partial x^2}$$

$$+ C_{45}^k \frac{\partial^2 u_x}{\partial \eta^2} + 2C_{45}^k R_k \frac{\partial u_x}{r \partial x} + R_k^2 C_{16}^k \frac{\partial^2 u_x}{\partial x^2}$$

$$= \rho^k R_k^2 \frac{\partial^2 u_0}{\partial t^2},$$

$$R_k\left[(C_{11}^k + C_{55}^k)\frac{\partial u_r}{\partial x} + (C_{13}^k + C_{55}^k)\frac{\partial^2 u_r}{\partial x \partial \eta}\right]$$
$$+ C_{45}^k\frac{\partial^2 u_0}{\partial \eta^2} + C_{16}^k\frac{\partial^2 u_0}{\partial \eta^2} + C_{11}^k\frac{\partial^2 u_x}{\partial x^2}$$
$$+ C_{55}^k\left(\frac{\partial^2 u_x}{\partial \eta^2} + \frac{\partial u_x}{\partial \eta}\right) = \rho^k R_k^2\frac{\partial^2 u_x}{\partial t^2}. \tag{5}$$

For simply supported conditions, one has:

$$\sigma_r(\eta, x, t) = 0, \qquad \eta = \frac{h_1}{2R_1},$$
$$\tau_{xr}(\eta, x, t) = \tau_{r\theta}(\eta, x, t) = 0,$$
$$\eta = \frac{h_1}{2R_1}, \qquad \frac{h_M}{2R_M}. \tag{6}$$

To study the free vibrations, the inner and outer surfaces are traction free:

$$\sigma_r = \tau_{r\theta} = \tau_{xr} = 0. \tag{7}$$

Moreover, the conditions of continuity of displacements and inter-laminar stresses should be imposed on the solution. These conditions are, as follows:

$$u_i^k\left(x, \quad \frac{h_k}{2R_k}\right) = u_r^{k+1}\left(x, \quad \frac{h_{k+1}}{2R_{k+1}}\right),$$
$$\sigma_i^k\left(x, \quad \frac{h_k}{2R_k}\right) = \sigma_r^{k+1}\left(x, \quad \frac{h_{k+1}}{2R_{k+1}}\right), \quad i = r, x, \theta,$$
$$\tau_{r\theta}^k\left(x, \quad \frac{h_k}{2R_k}\right) = \tau_{r\theta}^{k+1}\left(x, \quad \frac{h_{k+1}}{2R_{k+1}}\right). \tag{8}$$

SUCCESSIVE APPROXIMATE SOLUTION

The solutions to Equations 5, which identically satisfy the boundary conditions on the two ends, are considered, as follows:

$$u_r = A_r(\eta_k)\cos(p_m x)\exp(i\omega t),$$
$$u_\theta = A_\theta(\eta_k)\sin(p_m x)\exp(i\omega t),$$
$$u_x = A_x(\eta_k)\sin(p_m x)\exp(i\omega t), \tag{9}$$

where $p_m = m\pi/L$.

The substitution of Equations 9 into Equations 5 yields a system of homogeneous ordinary differential equations, the solutions to which are, as follows:

$$A_r(\eta_k) = u_r^* e^{\lambda\eta},$$
$$A_\theta(\eta_k) = u_\theta^* e^{\lambda\eta},$$
$$A_x(\eta_k) = u_x^* e^{\lambda\eta}. \tag{10}$$

Upon inserting Solution 10, one arrives at a system of homogeneous algebraic equations, which may be written in matrix form, as follows:

$$[A]\{U\} = 0,$$

where:

$$\{U\} = \{u_r^*, u_\theta^*, u_x^*\}. \tag{11}$$

The condition for Equation 11 to have a nontrivial solution is that the determinant of matrix A should vanish. This leads to the following sixth order algebraic equation:

$$A'\lambda^6 + B'\lambda^5 + C'\lambda^4 + D'\lambda^3 + E'\lambda^2 + F'\lambda + G' = 0. \tag{12}$$

The mode shapes may be obtained, which are a function of natural frequency. By substituting the roots of Equation 12 into Equation 11, one finally obtains the displacement components, as follows:

$$u_r = \sum\sum K_{mj} e^{\lambda\eta}\cos(P_m x)\exp(i\omega t),$$
$$u_\theta = \sum\sum K_{mj} e^{\lambda\eta}\sin(P_m x)\exp(i\omega t),$$
$$u_x = \sum\sum Q_{mj} e^{\lambda\eta}\sin(P_m x)\exp(i\omega t). \tag{13}$$

Substituting Equations 13 into the traction free conditions (Equation 7) and the continuity requirements (Equations 8), leads to a system of 6MN (MN is the number of layers) homogeneous algebraic equations, which are represented in the following matrix form:

$$[H]\{K\} = 0, \tag{14}$$

where $\{K\}$ are the mode shapes. The components of $[H]$, which are a 6MN*6MN matrix, are a function of natural frequency. From Equation 14 one has:

$$|H| = 0. \tag{15}$$

Equations 12 and 15 should be solved simultaneously by a successive approximate procedure to obtain the first few natural frequencies. To obtain the buckling stress function, one proceeds as follows:

$$[\varepsilon] = [a].[N] \quad [b]^T.[\kappa],$$
$$[M] = [b].[N] \quad [d].[\kappa], \tag{16}$$

where $[N]$ and $[M]$ are resultant force and moment components, respectively, $[\kappa]$ is curvature matrix and:

$$[a] = [A \ ^1], \qquad [b] = [B][a], \qquad [d] = [D] \quad [b][B], \tag{17}$$

$[A]$, $[B]$ and $[D]$ are obtained from the stiffness matrix of each layer, as follows:

$$[A]=\int[C]dz, \qquad [B]=\int[C]z\ dz,$$

$$[D]=\int[C]z^2dz. \tag{18}$$

Finally, the buckling load, based on the Donnel approach, is resulted, as follows [17]:

$$N_{xx}= \left(\frac{a_{22}}{a_{11}}\right)^{0.5}\left(\frac{1}{\mu}\right)^{2}\left(\frac{d_{22}}{R^2}\right)$$
$$\left\{n^2\phi_1+\frac{N_{\theta\theta}R^2}{d_{22}}+\frac{(n^2\phi_3+R\mu^2(a_{22}d_{22})\quad^{0.5})^2}{n^2\phi_2}\right\}. \tag{19}$$

Complete calculations for obtaining the buckling load have been described in [17]. For this relation, the coefficients description is, as follows:

$$\mu=(\lambda^2/n^2)(a_{22}/a_{11})^{0.5},$$

$$\phi_1=\gamma\mu^4+2\beta\gamma^{0.5}\mu^2+1,$$

$$\phi_2=\mu^4+2\alpha\mu^2+1,$$

$$\phi_3=\zeta\mu^4+2\eta\mu^2+\varsigma,$$

$$\alpha=(a_{12}+0.5a_{66})/(a_{11}+a_{22})^{0.5},$$

$$\beta=(d_{12}+2d_{66})/(d_{11}+d_{22})^{0.5},$$

$$\gamma=(d_{11}a_{11})/(a_{22}a_{22}),$$

$$\lambda=b_{21}/(a_{11}d_{22})^{0.5},$$

$$\eta=(0.5(b_{11}+b_{22})\quad b_{66})/(a_{22}d_{22})^{0.5},$$

$$\varsigma=(b_{12}/a_{22})(a_{11}/d_{22})^{0.5}. \tag{20}$$

GENETIC ALGORITHM

A GA is a guided random technique that works on a population of design. An initial population of a genetic string, with randomly chosen genes, is carried first. The size of the population used in this paper remains constant throughout the genetic optimization. Various genetic operators are applied at given probabilities to generate new laminates. In order to form successive generations, parents are chosen from the current population, based on their fitness. Parent selection is accomplished, using a roulette wheel concept. This method of selection differs from other evolutionary algorithms because it gives every member of the population a chance to become a parent. Before parent selection can begin, all laminates must be ranked from best to worst, according to the value of each laminate's objective function. Children are created by combining a portion of each parent's genetic string in an operation called a one-point crossover. To determine the crossover point, a uniformly distributed random number is chosen and then multiplied by one less than the maximum number of non-empty genes in the two parents. The integer ceiling value of this product determines the crossover point. The gene string is then split at the same point in both parents. The left piece from parent 1 and the right piece from parent 2 are combined to form a child laminate. After a child is created, the operations of adding, deleting or mutating genes occur with small probabilities. These operators make up genetic mutation. When adding a ply stack, a uniform random number is chosen to determine the orientation. To delete a ply stack, a random number is chosen and the corresponding stack is removed from the stacking sequence by replacing it with a 0 gene. The laminate is then re-stacked, so that all empty plies are pushed to the outer edge of the laminate. The ply swap operator is implemented by randomly selecting two genes in the string and switching their positions. Ply swap can be effective for problems where certain parts of the laminate stacking sequence get set up faster than others. For example, if the optimal stacking sequence for the outer section of the laminate has been determined first, the ply swap operator may help the GA determine the optimal orientations for the inner part of the laminate by swapping plies from each section. The genetic algorithm procedure, referred to as GA-I (genetic algorithm for designing composite laminates comprised of one material only) is used. For the GA-I algorithm, one string of genes is used to represent one half of a symmetrical laminated composite cylindrical shell. The length of the gene string is kept fixed throughout the optimization process. Each gene in the string is represented by an integer value between 0 and 10 and determines whether the ply stack location is empty or occupied with a 3-ply stack, which may be oriented at any angle between 0 and 90, in increments of 10. The fitness calculation usually involves function values that are determined from separate analysis subroutines or packages. Next, the crossover mutation and ply swap operators are applied to create child designs, which are, helpfully, better suited to their environment than their parents. The child population is then analyzed and ranked. To complete the generation cycle, a selection scheme is implemented that determines which laminates from the child and parent populations will be placed in the next generation. One generation after another is created until some stopping criterion is met.

MULTI-OBJECTIVE OPTIMIZATION

As mentioned earlier, optimization is carried out for both buckling strength and natural frequency simultaneously. For this purpose, a genetic algorithm is used for optimization. To obtain a Pareto set of designs, the influence of buckling load and natural frequency on the overall fitness function of a shell configuration is adjusted from one extreme to the other by varying the weight factor accordingly.

$$F = \text{Fitness function} = \alpha(\text{Buckling load}/100) + (1 \quad \alpha)(\text{Frequency}/100). \tag{21}$$

This allows the general configuration of the genetic algorithm to be maintained, since the stiffness of each laminate design is still based on a single value that is comprised of both buckling load and natural frequency.

RESULTS AND DISCUSSION

It is assumed that laminates are constructed from graphite-epoxy with the properties given in Table 1. Numerical results are presented for a six-layer cylindrical shell with $L/R = 10$ and three different values, $h/R = 0.1$, 0.05, 0.02. A genetic algorithm is used to optimize the stacking sequence of layers. The population size is six and the present selection is accomplished using a roulette wheel. An elitist method (EL) ranks the child population and present population of the laminates separately.

In order to construct the Pareto front, the weighting factor, α, is varied from 0.0 to 1.0 and the composite objective function, (F), is maximized using a genetic algorithm. The optimum designs, obtained as their frequency and buckling load, are summarized in Table 2 for a thin composite cylindrical shell. As observed from this table, for thin cylindrical shells, natural frequency values are very close for different values of the weighting factor and, thus, the buckling stress is the determining parameter for the optimization of thin composite cylindrical shells.

Tables 3 and 4 show optimized frequency and buckling load for h/R ratios higher than similar ones given in Table 2. As noticed, natural frequency has more effect on the optimization. Therefore, buckling stress and natural frequency are of similar importance for the ratios $h/R = 0.05$ and $h/R = 0.1$.

Convergency of the stiffness function has been shown in Figures 1 to 4 for $h/R = 0.1$ and different

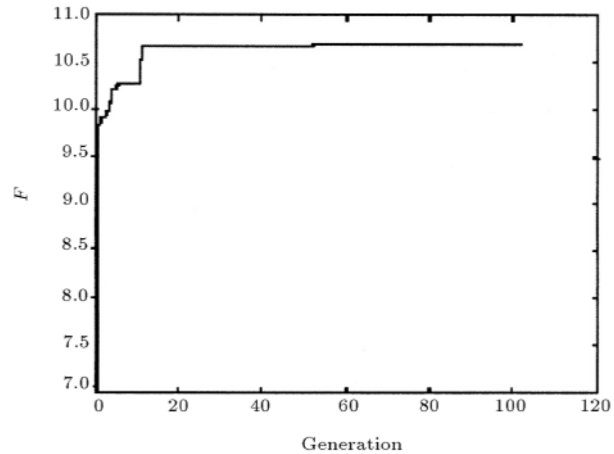

Figure 1. Convergency of F for $\alpha = 0$.

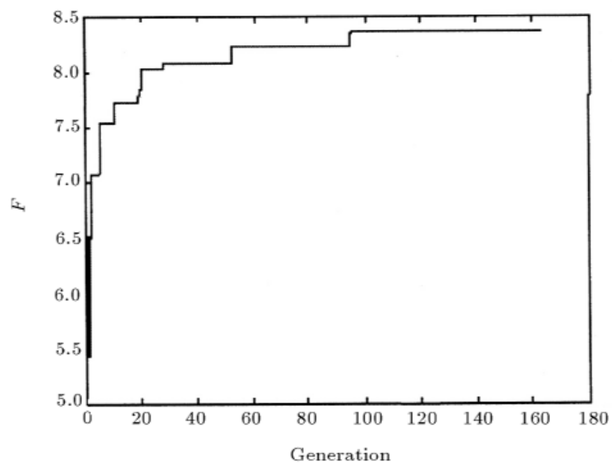

Figure 2. Convergency of F for $\alpha = 0.4$.

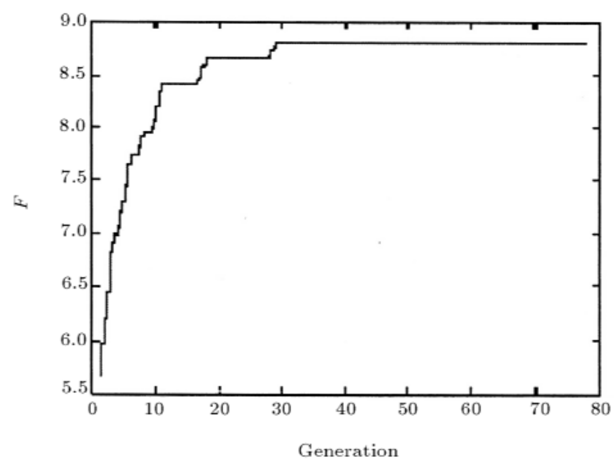

Figure 3. Convergency of F for $\alpha = 0.6$.

Table 1. Properties of the composite material.

Density [kg/m^3]	E_{11} [pa]	E_{22} [pa]	v_{12}	v_{21}	G_{12}/E_{11}	G_{23}/E_{11}
1408	84.9 E9	$E_{11}/40$	0.25	0.32	0.6	0.5

Table 2. Optimized Frequency and buckling load with $h/R = 0.02$.

Fitness Function	Buckling Stress (MPa)	Natural Frequency (Hz)	Generation	α
14550	585	14550	102	0
11370	649	14050	94	0.2
8978	621	14530	78	0.4
6172	587	14517	163	0.6
3465	700	14526	102	0.8
770	770	12834	144	1.0
Stacking Sequence				α
[55, 15, -35, 35, -55, -15]				0
[45, -45, 5, -45, 45, -5]				0.2
[-55, 15, 55, 35, -35, -15]				0.4
[50, -15, 15, -25, -50, 25]				0.6
[-65, 40, -15, 65, -40, 15]				0.8
[70, -40, 40, -25, 25, -70]				1.0

Table 3. Optimized Frequency and buckling load with $h/R = 0.05$.

Fitness Function	Buckling Stress (MPa)	Natural Frequency (Hz)	Generation	α
13967	517	13967	101	0
8965	626	11050	129	0.2
6783	569	10926	87	0.4
4732	551	11003	100	0.6
2678	631	10867	79	0.8
648	648	7014	100	1.0
Stacking Sequence				α
[-75, 70, -20, 75, -70, 20]				0
[-60, 20, 80, 60, -80, -20]				0.2
[65, -15, -80, 80, 15, -65]				0.4
[-70, 20, 70, 80, -20, -80]				0.6
[65, -15, -65, -80, 80,15]				0.8
[55, -50, 50, -55, -50, 50]				1.0

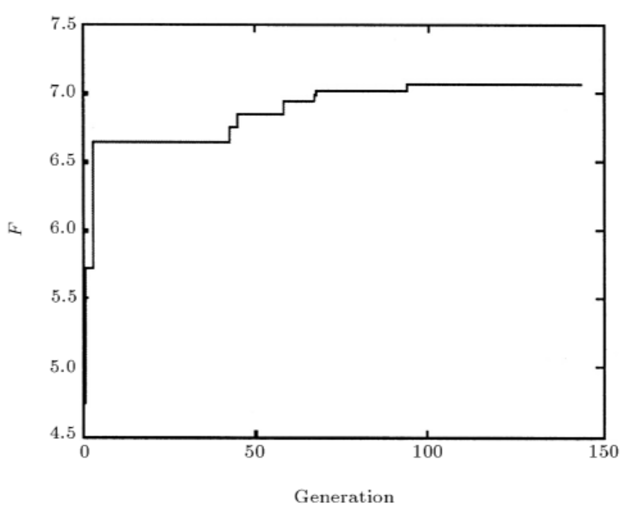

Figure 4. Convergency of F for $\alpha = 1$.

weighting functions. As observed, the convergency speed is faster and, also, the stiffness function increases when α decreases. The search has been stopped after a minimum of 50 generations, with no improvement in the fitness function.

The effective parameters, on increasing the convergency performance, including the size of the first population, parent population and different genetic operators, have been studied by the authors in detail [14]. Figure 5 shows the Pareto front obtained for $\alpha = 0.6$ and $h/R = 0.1$. The Pareto trade-off curve can be used to help the designer determine the optimal configuration for this problem. The final choice of the best design will depend on additional information that will enable him to assign priorities to the two objectives. There is no single best design.

Depending on the application that is considered,

Table 4. Optimized Frequency and buckling load with $h/R = 0.1$.

Fitness Function	Buckling Stress (MPa)	Natural Frequency (Hz)	Generation	α
10.676	369.86	10676	102	0
10.078	619.16	11050	94	0.2
8.7892	666.55	10205	78	0.4
8.3639	657.32	11050	163	0.6
7.4243	706.23	8872	102	0.8
7.0623	706.23	8872	144	1.0
Stacking Sequence				α
[85 5 -85 -5 -50 50]				0
[-70 25 70 -75 75 -25]				0.2
[55 -10 -85 -55 10 85]				0.4
[-65 15 70 65 -70 -15]				0.6
[30 -85 -30 -25 85 25]				0.8
[30 -85 -30 -25 85 25]				1.0

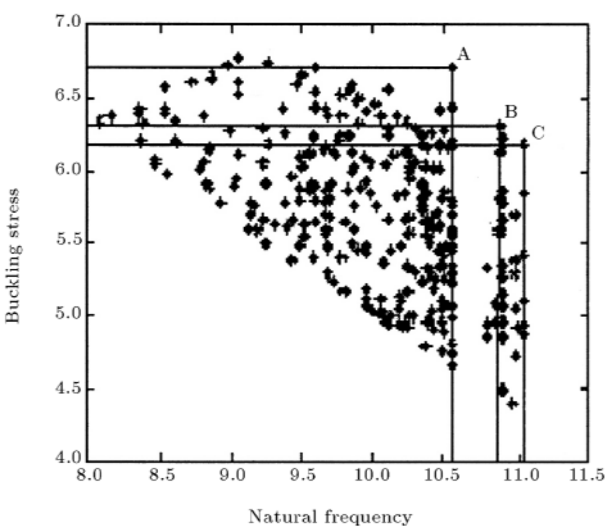

Figure 5. Pareto set for natural frequency and buckling load optimization.

Table 5. Three different optimal configurations.

Optimized Point	Stacking Sequence
A	[-65 15 65 65 -65 -15]
B	[-70 15 65 70 -65 -15]
C	[65 -15 -65 -75 15 75]

the choice will be different. For example, for $\alpha = 0.6$ in Tables 3 and 4, natural frequency and buckling load are of similar importance. Finally, three different optimal configurations have been chosen from Figure 5 and brought into Table 5. These points are A, B and C, corresponding to $\alpha = 0.65$, 0.3, 0.2. As mentioned earlier, the final choice depends on additional information.

CONCLUSION

A stacking sequence optimization with GA is done to maximize the natural frequency and buckling strength of a six-layer cylindrical shell, simultaneously. It is concluded that, for thin shells, buckling strength is the determining parameter for optimization while, for thick ones, both natural frequency and buckling strength are of the same importance for optimization. It is also concluded that the convergency speed is faster and, also, the stiffness function higher when α decreases. Pareto trade-off can be used to help designers determine optimal configuration. The final choice of the best design will depend on additional information that will enable the designer to assign priorities to the two objectives. However, there is no single best design.

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