Scientia Iranica, Vol. 7, No. 1, pp 69–74
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SCIENTIA
IRANICA

Research Note

Measuring Small Displacements by Using a Capacitive Transducer System

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In this paper, application of a capacitive sensor system in recording small displacements is discussed. Through using a simple transducer and a monitoring circuit, change in capacitance has been detected as a function of the capacitor air-gap distance. There is a linear relation between gap distance and output of a sensor for a wide gap range of 0.5 to 5 mm. The reported system provides means for measuring displacements up to 5 mm with a resolution better than 250 nm. By decreasing the dynamic range of the transducer to 250 μm, a small displacement of the order of 100 nm is detectable. With the present system, a minimum displacement of about 200 nm (200 ppm) for the gap distance of 1 mm is obtained. However, through changing the sensor design, the measuring circuit becomes capable of recording smaller distances of the order of 10 nm at this operating range.

INTRODUCTION

A wide variety of capacitive transducers and measuring circuits has been reported in the literature [1]. Some of these studies have focused on the transducer design in order to modify edge effects, stability and sensitivity [2,3]. Several attempts have also been made for designing different techniques for measurements [4,5].

In practice, special attention has been paid to the differential capacitive sensor, since it can be an accurate and sensitive means for measuring displacements over a wide range. However, in precise applications, the displacements to be measured are often in the range of 0.1-100 μm and require a resolution better than 1 in 10^7. Satisfying this point and some environmental and structural stability in design calls for the development of still better measuring instruments. In this work, a precise sensor system and a readout circuit have been constructed that provide means for measuring such small displacements.

Using a simple capacitive sensor and a readout circuit, change in capacitance has been monitored as a function of the capacitor air-gap distance which provides good means for measuring small displacements. There is a linear relation between the air-gap distance and the output of the sensor. The reported system provides a precise way for measuring displacements in the range of 200 nm to 1000 μm with a high resolution.

The reported system can be used as a reliable distance probe for small distance gauging which is required for practical applications. This type of measurement could be beneficial in sensing distances for a manipulator in robotic systems.

THEORY

To develop the theory, the basic formulas obtained for the measuring system, shown in Figure 1, are considered. In this method, the capacitance change is converted to a phase difference and the output voltage can be written as:

$$V_0 = \frac{C_0 B}{C_T + C_0 + C_x}\sin(\omega t + \pi - \psi) + \frac{C_x A}{C_T + C_0 + C_x}\sin(\omega t), \tag{1}$$

where C_T, C_0 and C_x (transducer capacitance) are shown in Figure 1. A and B define the amplification of the op-amps and $(\pi - \psi)$ is phase difference produced between the two input signals. A novel scheme for producing two signals with a 180 degree phase difference is given in [6], where the frequency consideration in producing such signals is discussed. Using a sinusoidal oscillator, two signals such as A sin (ωt) and B sin $(\omega t + \pi - \psi)$ are produced. The phase angle ψ is

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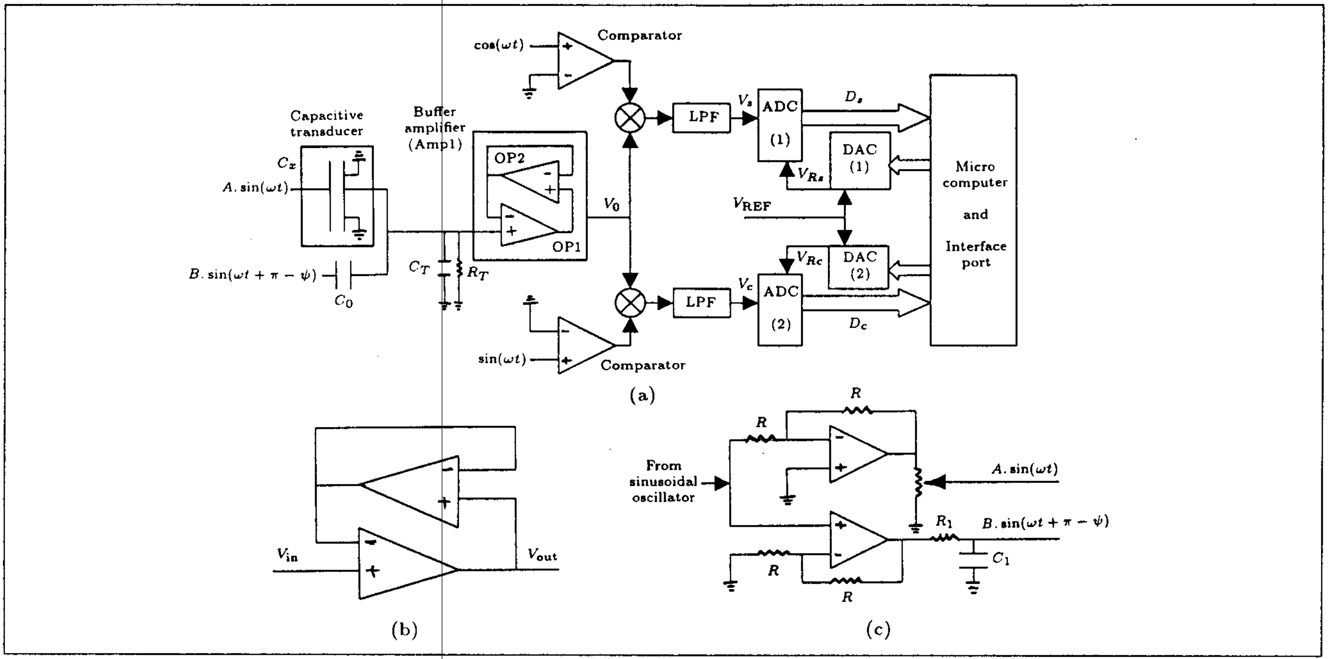

Figure 1. (a) Block diagram of the readout circuit and monitoring system. (b) Buffer amplifier. (c) Circuit for producing two signals with a phase difference of $(\pi - \psi)$.

equal to $\tan^{-1}(R_1C_1\omega)$, where R_1 and C_1 are shown in Figure 1c, and ω is the operating frequency. This frequency for the two matched op-amps is 10 kHz which causes negligible error in producing the two signals with a definite phase difference.

Following analysis given in [7], the detected output can be written as:

$$V_{\text{out}} = \frac{V_c}{V_s} = \frac{AC_x}{C_0B\sin\psi} - \cot\psi, \tag{2}$$

where V_c and V_s are the output voltages of the two low-pass fillers as shown in Figure 1a. Considering Equation 2, there is a linear relation between the output voltage and C_x.

The value of C_x, in general, can be the sum of the different terms such as:

$$C_x = C_{x0} + C_{x1} + C_{st}, \tag{3}$$

where C_{x0} is the capacity of the transducer at the initial gap distance X_0, C_{x1} is the capacitance change due to a displacement X_1, from X_0, and C_{st} is the total stray capacitance affecting the transducer. By substituting C_x from Equation 3 into Equation 2, the following is obtained:

$$V_{\text{out}} = \frac{AC_{x0}}{C_oB\sin\psi} + \frac{AC_{x1}}{C_0B\sin\psi} - \cot\psi + \frac{C_{st}A}{C_0B\sin\psi}. \tag{4}$$

Now, if one neglects the effect of the stray capacitance in the last term of Equation 4 and under the condition:

$$C_{x0}A = C_0B\cos\psi, \tag{5}$$

then Equation 4 can be rearranged into a new form:

$$\frac{V_c}{V_s} = \frac{C_{x1}}{C_{x0}}\cot\psi. \tag{6}$$

If the normalized capacitance ratio C_{x1}/C_{x0} is considered, then the following can be written:

$$\frac{C_{x1}}{C_{x0}} = \frac{V_c}{V_s}\tan\psi\ , \tag{7}$$

and the normalized digitized output signal becomes:

$$D_{\text{out}} = \frac{C_{x1}}{C_{x0}} = \frac{D_c}{D_s}\frac{\tan\psi}{(V_{Rs}/V_{Rc})}, \tag{8}$$

where V_{Rs} and V_{Rc} are two reference voltages for the ADCs; D_s and D_c are the digitized output of the corresponding ADCs which are shown in Figure 1a.

In this measuring method, any changes in the value of C_{x1} can be detected. By exchanging the position of C_{x0} and C_0 in the circuit (see Figure 1), the calibration line from Equation 7 becomes:

$$D_{\text{out}} = \frac{X_1}{X_0} = \frac{D_c}{D_s}\frac{N_c}{N_s}\tan\psi, \tag{9}$$

where N_c and N_s are the digital input numbers of the two DACs (N_c/N_s is equal to V_{Rc}/V_{Rs}) and X_0 is the distance at which the zero adjustment is made (a condition that Equation 5 is satisfied and the effects of A, B, and C_0 on measurement values are eliminated). Here, $1/X_0$ defines the slope of the calibration line for the sensor.

Following analysis given in the previous report [7], the recorded output number is directly related to the capacitance changes. This capacitance change can be obtained from a simple relation such as:

$$D = \frac{C_{x0}}{C_{x1}}, \tag{10}$$

where C_{x0} is the capacitance of the sensor and C_{x1} is the capacitance change due to a gap variation of X_1 from initial X_0 value. By considering a parallel plate capacitor, the recorded number is related to the displacement X_1 by:

$$X_1 = DX_0, \tag{11}$$

where X_1 is the displacement to be measured and X_0 is the initial gap distance in which the zero adjustment (i.e., $D = 0$) is made for the transducer.

According to Equation 11, there is a linear relation between the displacement X_1 and the readout number, D. The slope of this line is given by X_0 value which also determines the dynamic range of the capacitive transducer. The interesting point of this method of detection is that the zero adjustment can be easily applied to any given gap distance with reasonable accuracy. Therefore, the value of X_1 is determined precisely by knowing X_0 value and recording D values, accordingly.

Details of the resolution for the readout circuit can be found in a previous work [7] that introduced a new method of measuring a very small capacitance change by converting the resulting capacitance change into a phase factor. The resolution of the readout circuit can be represented by:

$$\Delta D = \frac{\Delta C_{x1}}{C_{x0}} = \frac{\tan\psi}{n \times 2^{m-2}}, \tag{12}$$

where parameters ψ, n, m and C_{x0} control this resolution. By substituting C_{x0} in terms of other related parameters, the resolution for displacement can be written as:

$$\Delta X_1 = X_0 \frac{\tan\psi}{n \times 2^{m-2}}. \tag{13}$$

Hence, ΔX_1 is directly related to X_0 (the initial distance that zero adjustment is made) and the value of phase factor ψ, however, is inversely related to parameters n and m. The quantity n is related to the voltage ratio of the two multipliers used in the circuit design and m is the bit number for the ADC. For the given values of $X_0 = 1$ mm, $\psi = 3.7°$, $n = 5$ and $m = 12$, the minimum detectable displacement for the readout circuit from Equation 13 is about 12.5 nm.

However, the overall resolution of the reported system is limited by the mechanical drive system that includes the stepper motor and the micrometer screw. The resolution of the drive system is controlled by the number of motor steps and screw pitch which is about 625 nm/step for the present system.

The operating range of the measuring circuit mainly depends on the bit number of the ADCs used in digitizing the output and also to the C_0 (a reference capacitor parallel to C_x) and the phase factor ψ. The dynamic range of the capacitor transducer depends upon the C_{x0}. Since C_{x0} is related to the gap distance, X_0, the scanning distance of the proposed system is mainly determined by this value. In practice, increasing X_0 value would increase the dynamic range, however, causes a decrease in the sensitivity of the system. Therefore, for effective application of the system, one has to compromise in selecting the proper values for these two parameters.

EXPERIMENTAL RESULTS

The block diagram of the apparatus used in this study is presented in Figure 2. The main components are: a capacitive sensor, a stepper motor, a drive circuit, a capacitance measuring circut and a PC which is interfaced with the readout and also controls the stepper drive via an interface card. For constructing the capacitive sensor, different electrode plates were made from the printed circuit boards or machined from metallic rods. The main body consists of two circular electrodes with an air-gap. The upper electrode plate has a diameter of 50 mm and the lower electrode consists of an inner electrode plate (25 mm diameter) as well as a Kelvin guard-ring (50 mm diameter). The nominal capacity of this transducer for 1 mm air-gap is about 4.3 pF. There are three adjusting screws in order to keep the electrodes in parallel and this point is checked with a vernier.

In this arrangement, one of the capacitor plates is fixed in position while the other plate is moved in a parallel way via a coupler which connects the motor shaft to the plate holder. It is, therefore, possible to

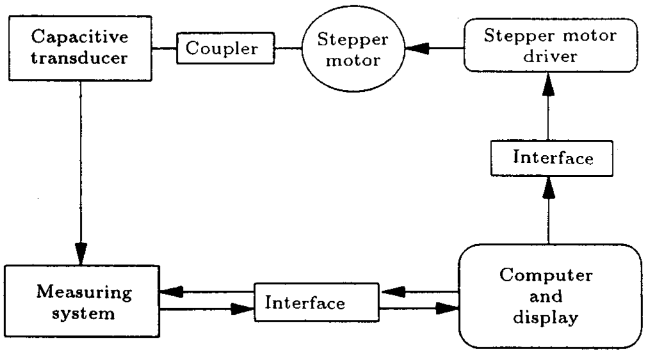

Figure 2. Block diagram of the apparatus used in this work.

scan the air-gap distance precisely by the computer controlled stepping motor drive system.

Displacement measurements can be conducted by counting the number of scanning steps and considering the pitch of micrometer screw in calculations. It is also possible to compute displacement value X_1 from Equation 11 by measuring the initial air-gap value X_0 and by monitoring the output number, D. In practice, two methods of measurements have been tested and the results are reported.

For data acquisition, at a given initial gap distance, the zero adjustment is made and through scanning the stepper motor by a few steps the output was displayed on a PC monitor. In the first experiment, the readout number was monitored and the displacement, X_1, for the initial gap distance of $X_0 = 0.5$ mm was computed as shown in Figure 3. As can be seen, there is a very good agreement between the result obtained by measuring the capacitance changes and that of expected values from counting the number of steps.

Figure 3 shows the variation of displacement with readout number for the inital gap value of $X_0 = 3$ mm. By considering Figures 3 and 4, it is noticed that, as expected, there is a good linear relation between displacement and readout number. However, the slope of this line is determined by the X_0 value, as described in the theoretical section.

Comparing Figures 3 and 4 demonstrates that the slope deviation is more noticeable in Figure 4, while the initial gap distance X_0 has been increased. To show this important point in Figure 5, a similar variation of displacement with respect to readout number has been illustrated for the initial gap distance of $X_0 = 5$ mm. The results of Figure 5 clearly indicate that by increasing the initial gap distance, a noticable difference is observed which suggests that the operation of such a system for smaller gap distances promisesa higher accuracy in results.

The variation of the resolution with the initial gap distance, X_0, is shown in Figure 6. As discussed in the previous section, parameters such as ψ, n, m and X_0 determine the resolution of the transducer. As a matter of fact X_0 and, as a result, C_{x0} has the major effect in defining the resolution of the capacitive sensor, which is

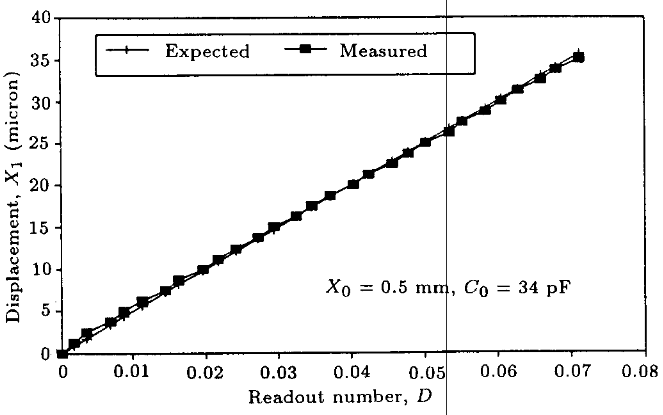

Figure 3. Variation of displacement with respect to readout number for $X_0 = 0.5$ mm.

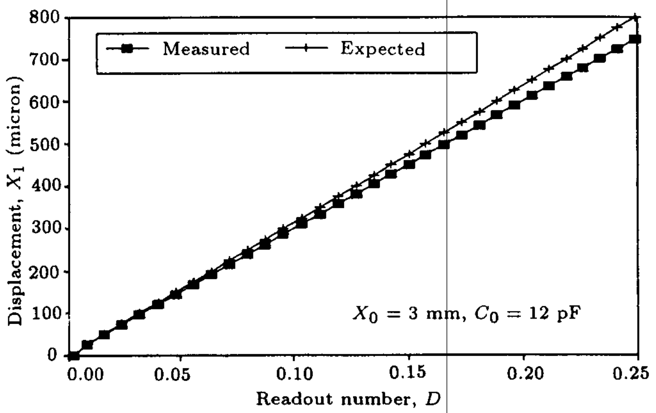

Figure 4. Variation of displacement as a function of readout number for $X_0 = 3$ mm.

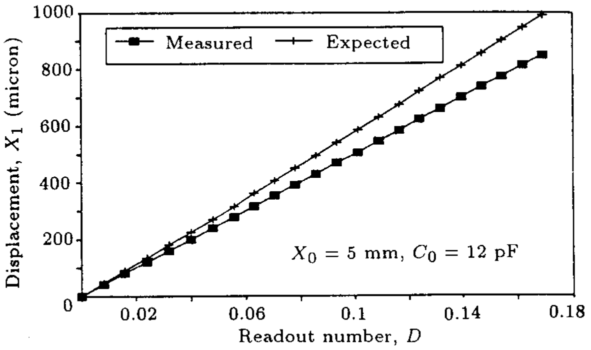

Figure 5. Displacement variation as a function of readout number for $X_0 = 5$ mm.

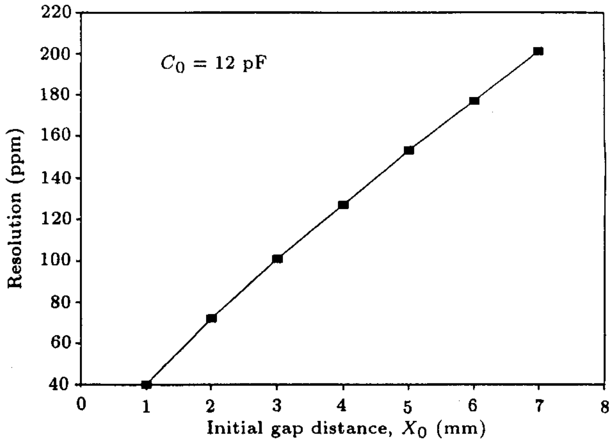

Figure 6. Resolution variation as a function of the initial gap distance.

displayed in Figure 6. As shown in Figure 6, for a given value of $C_0 = 12$ pF, the resolution of the measuring system can be changed from 40 ppm ($X_0 = 1$ mm) to 200 ppm ($X_0 = 7$ mm).

It must be pointed out that because of practical limitations in capacitive sensor design, the actual overall resolution of the transducer system was found to be a little lower than the expected theoretical values. The minimum resolvable capacitance change for the case of the present system was found to be 200 fF which corresponds to a displacement of 200 nm and a resolution of 200 ppm. This number should be compared with the estimated value of 40 ppm for $X_0 = 1$ mm which is higher by a factor of 5.

Although decreasing the initial gap distance, X_0, increases the resolution and improves the performance of the transducer in terms of slope deviation; however, the dynamic range of the system will be considerably reduced. Figure 7 shows the variation of this parameter as a function of the initial gap distance from 1 mm to 7 mm. The operating range of the reported sensor for 1mm is about 250 μm, while it increases to a value of about 4500 μm for X_0 value of 7 mm. Considering the obtained result, one must optimize the transducer according to required sensitivity, resolution and dynamic range.

In order to show the deviation of the displacement measurement with that of directly counting the number of scanning steps in Figure 8, this deviation is presented with displacement at a given X_0 value of 1 mm. As can be seen in Figure 8, there is a reasonable agreement between the results of the two methods. For a scanning range of about 110 μm, the maximum difference is found to be about 0.75 μm. Considering Figure 8, it is also noticed that the maximum deviation is for the beginning of the scan which is in the range of 0-30 μm.

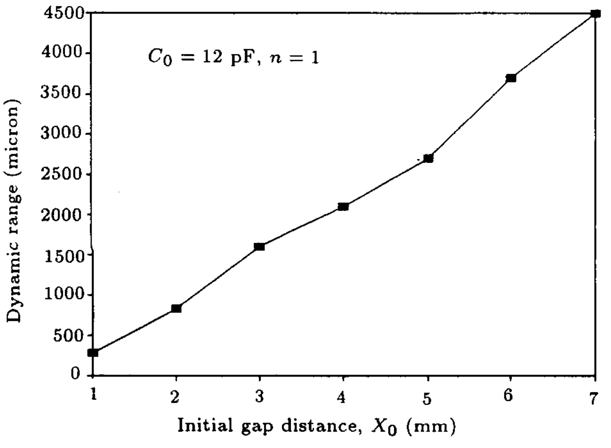

Figure 7. Dynamic range as a function of the initial gap distance.

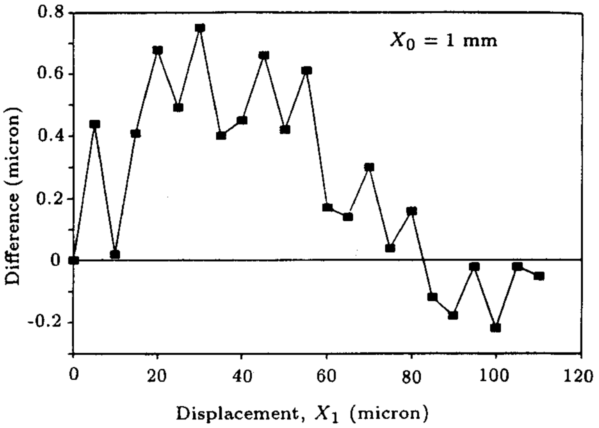

Figure 8. Difference in the displacement results of capacitive measurements and that of direct counting scan steps.

It is predicted that the obtained difference is because of the probable error in the starting stage of the stepper motor drive system. As noticed in other experiments, at the beginning of the drive there is a possibility of missing one or two steps which might cause the probable error in displacement reading and computations.

In order to check the repeatability of the transducer system in Figure 9, the percentage of the deviation obtained for the two successive measurements is demonstrated at X_0 value of 1 mm. As can be seen, the maximum deviation of about 1.4% between the two runs belongs to the beginning of the scan. In the range of 20 - 110 μm, there is a good agreement between the two results which indicates that the reproducibility of the system is very promising. However, the deviation observed at the beginning of the run can be explained by the fact that the error is caused by the mechanical drive system as discussed previously for the comparison of the two methods of displacement determination.

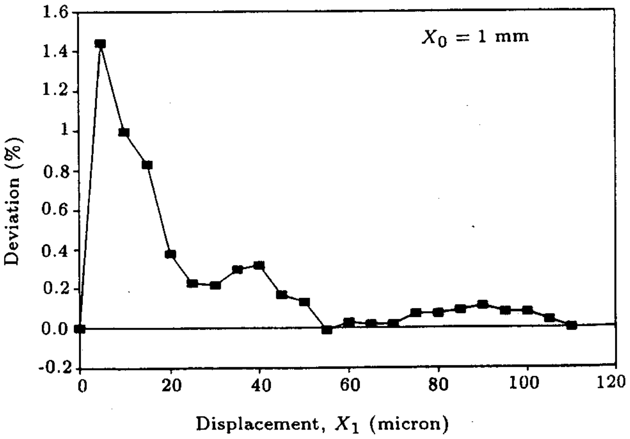

Figure 9. Deviation in two successive runs to show the repeatability of the transducer.

CONCLUSIONS

The reported system provides a good method of displacement recording by measuring the capacitance changes of a capacitive sensor. This transducer has been used successfully to detect displacements in the range of 500 nm to 5 mm with a resolution better than 250 nm. By decreasing the operating range of the transducer to 250 μm, smaller displacements of the order of 100 nm can be detected.

The major results of this study can be classified as follows:

1. Provision of a precise method for distance gauging by means of recording the capacitance changes,
2. There is a linear relation between displacement and readout number for a wide range of 500 nm to 5 mm,
3. Because of high sensitivity, resolution and repeatability of the method, it is suitable for distance probing,
4. There is a good agreement between the two methods of displacement recording, capacitance change recording and the direct method of counting the number of scanning steps,
5. The final output is digitized which can be processed and analysed by a PC. The scanning drive system is also computer controlled and these features make the method suitable for use in automated and robotic applications,
6. The proposed method is superior to the other conventional methods for measuring small displacements. It is comparable with optical techniques in terms of sensitivity, resolution and operating range [8-11].

ACKNOWLEDGMENT

This research project was supported in part by Sharif University of Technology Research Organization. The author is, therefore, grateful for the grant provided by the Research Council. He is also thankful to A. Ashrafi for the assistance given during the experimental measurements.

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