Scientia Iranica, Vol. 7, No. 1, pp 32–40

SCIENTIA IRANICA

Analysis of Gain Dynamics in EDFA and Transient Response of Optical Fiber Inverter Using a Simplified Model

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In this paper, a simplified model for a multi-wavelength erbium-doped fiber amplifier and an optical fiber inverter is introduced. Through averaging over the length of the active medium, the balance system of equations is considerably simplified. Consequently, a system of ordinary differential (instead of partial differential) equations is obtained. Gain dynamics of an all-optical stabilized multi-wavelength erbium-doped fiber amplifier (EDFA) and transient response of an optical fiber inverter are analyzed, using the solution of the system of ordinary differential equations. Theoretical conclusions are in satisfactory agreement with the published experimental results.

INTRODUCTION

Cross saturation phenomenon and its induced problems in multi-wavelength Er-doped fiber amplifiers and also in some optical switching devices, from both theoretical and experimental points of view were studied by several researchers [1-14]. In some optical devices, such as optical fiber flip-flops, operation is based on the cross saturation effect of the active medium. Wavelength Division Multiplexing (WDM) amplifiers mainly operate in saturation or near saturation regimes; therefore, the total output of amplified signals is nearly constant. Moreover, any changes in the deriving network or in the number of active channels and individual channel input signal level would induce a noticeable change in other channel output power levels. This "cross talk" problem is one of the barriers for the application of Er-Doped Fiber Amplifiers (EDFA) in WDM networks. Many attempts for limiting the variation of output powers have been made [1-4], including the optical feedback loop insertion. However, to understand the basic dynamics of the problem many models for transient response of the amplifier are introduced [8-14].

C.R. Giles et al. have modeled EDFA systems using propagation and rate equations of a two-level laser homogeneous medium [10]. In that model, the term $\frac{1}{v_k}\frac{\partial p_k}{\partial t}$ is neglected in comparison with the term $\frac{\partial p_k}{\partial z}$, where p_k is the channel power of wavelength λ_k which travels with the speed of v_k through the optical fiber amplifier (this neglecting is suitable for analyzing open loop optical fiber amplifiers and also the steady state operation of the optical fiber networks). Starting from a system of Partial Differential (PD) governing equations, they reach to a single implicit equation. Sun et al. [8] have simplified the PD governing equations into a single Ordinary Differential (OD) equation for gain dynamics in EDFA with an arbitrary number of signal channels, power levels and propagation directions in an open loop architecture or in steady state regimes. Fatehi used the Giles model to analyze the steady state behavior of EDFAs with a wavelength-selective optical feedback which operates as a low speed all-optical fiber inverter. As stated above, one of the schemes for preventing the unwanted power excursions in EDFAs is gain clamping by optical feedback loop [1-3]. In optical networks with optical feedback loop, the speed of variation in the mode power is fast and the term $\frac{1}{v_k}\frac{\partial p_k}{\partial t}$ cannot be neglected in comparison to the term $\frac{\partial p_k}{\partial z}$ and, therefore, must be considered in the propagation equations. The system of partial differential equations of Giles has been solved by Luo et al. [1] and the relaxation oscillation has been demonstrated. Their conclusions are in satisfactory agreement with the experimental results.

In this paper, Giles system of partial differential equation with an additional term $\frac{1}{v}\frac{\partial p_k}{\partial t}$ in the left

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hand side of the propagation equations is used in a multi-wavelength EDFA with an optical fiber feedback loop operating with an arbitrary number of inputs. Consequently, a simple model for characterizing the time dependent gain and power dynamics is introduced in the form of a system of ordinary differential equations. Then, using this fast computing model, spiking behaviors in the transient response of Fatehi's inverter, observed experimentally, are described [15]. The relaxation oscillation of a gain stabilized multi-wavelength amplifier is also described, for which the results are in good agreement with direct numerical solution of Giles partial differential equations and experimental conclusions.

PROPAGATION AND RATE EQUATIONS

In an erbium-doped fiber amplifier, the $^4I_{11/2}-^4I_{15/2}$ transition corresponds to the 980-nm pump band and $^4I_{13/2}-^4I_{15/2}$ to the 1520-1570 nm signal band and the resonant pumping in the 1460-1500 nm band. Other pump bands and the potential for more complex phenomena, such as pump Excited-State Absorption (ESA), are associated with other energy levels of Er^{3+}. Negligible ESA occurs for 980-nm or 1480-nm pump amplifiers [10]. In a glass host, erbium ion is subjected to electric fields, known as crystal fields, due to the surrendering atoms in the host lattice. This causes a Stark splitting of erbium ion orbital and site to site variations of the field due to the amorphous nature of glass [7]. In this paper, the inhomogeneous broadening is neglected and so its effects such as spectral hole burning cannot be interpreted by this model. The discussion is limited here to the case of a two-level homogeneous system with an arbitrary number of inputs. The rate and propagation equation of Giles can be written as follows [10]:

$$\frac{\partial \bar{n}_2}{\partial t}=\sum_k \frac{p_k\sigma_{ak}\Gamma_k\bar{n}_1}{h\nu_k A}-\sum_k \frac{p_k\sigma_{ek}\Gamma_k\bar{n}_2}{h\nu_k A}-\frac{\bar{n}_2}{\tau}\ , \tag{1}$$

$$\frac{\partial p_k(z)}{\partial z}=u_k(\alpha_k+g_k)\frac{\bar{n}_2}{n_t}p_k(z)+u_k g_k^* m h\nu_k\Delta\nu_k\frac{\bar{n}_2}{n_t}-u_k(\alpha_k+l_k)p_k$$

$$k=1,\cdots,m+2\ . \tag{2}$$

Here $l_k=l(\lambda_k)$, where $l(\lambda)$ is the spectrum of the intrinsic loss of optical fiber, τ is the life-time of the upper laser level, A is the effective cross-sectional area of the active optical fiber, the wavelength number k extends up to the total number of operating wavelength $m+2$ and h is Plank constant. The population of the upper laser level $\bar{n}_2$, the population of the lower laser level $\bar{n}_1$ and erbium density n_t are related by $\bar{n}_1+\bar{n}_2=n_t$.

The measured loss $\alpha(\lambda)$ and gain $g(\lambda)$ spectrum are:

$$\alpha(\lambda)=\sigma_a(\lambda)\Gamma(\lambda)n_t\ , \tag{3}$$

$$g(\lambda)=\sigma_e(\lambda)\Gamma(\lambda)n_t\ , \tag{4}$$

where $\Gamma(\lambda)$ is the overlap integral between the optical mode and erbium ion which is called the confinement coefficient. The absorption and emission cross-sections are $\sigma_a(\lambda)$ and $\sigma_e(\lambda)$, respectively. For a beam entering at $z=0$, u_k is 1, while for beams entering at $z=l$, $u_k=-1$. In this paper, the notation y_k is used instead of $y(\lambda_k)$ for variables such as σ_a, σ_e, etc.

Population of laser levels are normalized to the local ion density n_t($N_2=\bar{n}_2/n_t$) and, by ignoring the Amplification of Spontaneous Emission (ASE), it is obtained that:

$$\frac{\partial N_2}{\partial t}=\sum_k\frac{\Gamma_k}{h\nu_k A}[\sigma_{ak}-(\sigma_{ak}+\sigma_{ek})N_2]p_k-N_2/\tau, \tag{5}$$

$$\frac{\partial p_k}{\partial z}=u_k[(\alpha_k+g_k)N_2-(\alpha_k+l_l)]p_k,$$

$$k=1,\cdots,m+2\ . \tag{6}$$

Now, for considering the variation of mode power with respect to time, propagation of optical power in the wavelength λ_k from the point z to $z+\Delta z$, requiring the time $\Delta t=\Delta z/v_k$, is considered:

$$\Delta p_k=p_k(z+\Delta z,t+\Delta t)-p_k(z,t)\ . \tag{7}$$

Increase in the optical power is due to two sources; a variation with distance and a variation with time. The first source leads to the term $\frac{\partial p_k}{\partial z}\Delta z$ in the expansion of Δp_k while the second contributes to the term $\frac{\partial p_k}{\partial t}\frac{\partial t}{\partial z}\Delta z=u_k\frac{1}{v_k}\frac{\partial p_k}{\partial t}\Delta z$. Thus:

$$\Delta p_k=\left(\frac{\partial p_k}{\partial z}+\frac{u_k}{v_k}\frac{\partial p_k}{\partial t}\right)\Delta z\ . \tag{8}$$

Now Equation 6 can be rewritten as:

$$\frac{\partial p_k}{\partial z}+\frac{u_k}{v_k}\frac{\partial p_k}{\partial t}=u_k[(\alpha_k+g_k)N_2-(\alpha_k+l_k)]p_k\ ,$$

$$k=1,\cdots,m+2\ . \tag{9}$$

For an open loop system, the second term on the left hand side of Equation 9 in comparison with $\frac{\partial p_k}{\partial z}$ is negligible, while for transient time of a closed loop system, it is of the order of $\left(N(t)/N_{th}-1\right)/\tau_l$, where $1/\tau_l$ is the cavity decay rate at oscillating wavelength λ_l and $\frac{N(t)}{Nth}$ is the instantaneous ratio of laser gain to cavity loss [16] which is not ignorable. The two

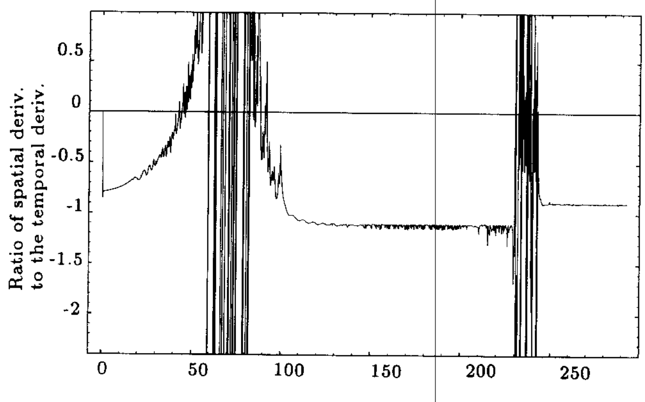


Figure 1. Variation of $\frac{\partial p_l}{\partial z}$ with respect to $\frac{1}{v}\frac{\partial p_l}{\partial t}$ in the switching transient time of an optical fiber inverter.

terms of the LHS of Equation 9, which are computed as the solution of Equations 5 and 9 for a closed loop amplifier, are compared in Figure 1 and it is shown that neither of the two terms are ignorable. When $\frac{1}{v_k}\frac{\partial p_k}{\partial t}$ $k = 1, \cdots, m+2$ terms are neglected in the propagation equations, relaxation oscillations in the closed loop system, which are observed experimentally, could not be described by Giles system of partial differential equations. Hence, in transient response analysis of the closed loop network, the propagation equations must be considered in the form of Equation 9. Coupling terms on the right hand side of Equation 5 can be arranged into three groups: pump, laser and amplified signals. Amplification process of the third group is represented by $p_k(z,t) = exp(G_k(z,t))q_k(t)$, where $G_k(z,t)$ is the gain function till the position z and time t and $q_k(t)$ is the input power at wavelength λ_k $(k = 1, \cdots, m+2)$; therefore, Equation 9 for the amplifying signals is rewritten in terms of the gain G_k as follows:

$$\frac{\partial G_k}{\partial z} + \frac{u_k}{v_k}\frac{\partial G_k}{\partial t} = u_k[(\alpha_k + g_k)N_2 - (\alpha_k + l_k)] - \frac{u_k}{v_k}\frac{\dot{q}_k(t)}{q_k(t)}, \quad k = 1, \cdots, m \ , \tag{10}$$

where $\frac{dq_k}{dt}$ is denoted by $\dot{q}_k$.

Propagation equations for the laser power p_l and the pump power p_p are as follows ($u_l = 1$):

$$\frac{\partial p_l}{\partial z} + \frac{1}{v_l}\frac{\partial p_l}{\partial t} = [(\alpha_l + g_l)N_2 - (\alpha_l + l_l)]p_l \ , \tag{11}$$

$$u_p\frac{\partial p_p}{\partial z} = -(\alpha_p + l_p)p_p \ , \tag{12}$$

where l and p indices are used for the oscillating and pumping wavelengths, respectively, and the rate equation is:

$$\frac{\partial N_2}{\partial t} = -\frac{N_2}{\tau} + \frac{\Gamma_l}{h\nu_l A}[\sigma_{al} - (\sigma_{al} + \sigma_{el})N_2]p_l + \frac{\Gamma_p}{h\nu_p A}\sigma_{ap}(1 - N_2)p_p + \sum_{k=1}^{m}\frac{\Gamma_k}{h\nu_k A}[\sigma_{ak} - (\sigma_{ak} + \sigma_{ek})N_2]e^{G_k(z,t)}q_k(t) \ , \tag{13}$$

Now another simplification can be applied. This simplification is based on averaging the normalized upper laser level population density as well as the optical powers and gains along the length of the active medium (more precisely, over the active length (L) of the fiber amplifier). Through integrating Equations 10 to 13 over the length of the active medium and using the $<>$ notation for averaged functions ($< f > (t) = \frac{u_k}{L}\int_0^L f(z,t)dz$):

$$\bar{G}_k + \frac{L}{v_k}\frac{\partial < G_k >}{\partial t} = L(g_k + \alpha_k) < N_2 > - L(\alpha_k + l_k) - \frac{L}{v_k}\frac{\dot{q}_k}{q_k} \tag{14}$$

For a beam entering at $z = 0$, $\bar{G}_k = G_k(L,t)$, while for beams entering at $z = L$, $\bar{G}_k = G_k(0,t)$.

$$p_l(l,t) - p_l(0,t) + \frac{L}{v_l}\frac{\partial < p_l >}{\partial t} = L(g_l + \alpha_l) < N_2 p_l > - L(\alpha_l + l_l) < p_l > \ , \tag{15}$$

$$u_p(p_p(L,t) - p_p(0,t)) = -L(\alpha_p + l_p) < p_p > \ , \tag{16}$$

$$\frac{\partial < N_2 >}{\partial t} = -\frac{< N_2 >}{\tau} + \frac{\Gamma_l}{h\nu_l A}[\sigma_{al} < p_l > - (\sigma_{al} + \sigma_{el}) < N_2 p_l >] + \frac{\Gamma_p}{h\nu_p A}\sigma_{ak}(1 - < N_2 p_p >) + \sum_k \frac{\Gamma_k}{h\nu_k A}[\sigma_{ak} < e^{G_k(z,t)} > - (\sigma_{ak} + \sigma_{ek}) < N_2 e^{G_k(z,t)} >]q_k(t) \ . \tag{17}$$

It has been shown in [17] that the following relations are very effective in a real situation: $< N_2 p_k > \simeq < N_2 >< p_k >$ and $< G_k^n > \simeq < G_k >^n$. Also it can be easily seen that the laser power emerging from the resonator, $p_{l,out}$ is given by:

$$p_{l,out}(t) = p_l(l,t) - p_l(0,t) \ . \tag{18}$$

Therefore, using the coefficient of overall losses γ_l, laser output power may be written in the form of $p_{l,out} =$

$L\gamma_l < p_l >$, where γ_l is approximately $(1 - f_l)/L$ and f_l is the feedback rate at oscillation wavelength.

Using Equation 18 in Equations 15 and 17, it is obtained that:

$$\frac{d < p_l >}{dt} = v_l[(g_l+\alpha_l) < N_2 > -(\alpha_l+l_l+\gamma_l)] < p_l > , \quad (19)$$

$$\frac{d < N_2 >}{dt} = -\frac{< N_2 >}{\tau} + \sum_{k=1}^{m} \frac{\Gamma_k}{h\nu_k A} [\sigma_{ak} - (\sigma_{ak} + \sigma_{ek}) < N_2 >] e^{< G_k(z,t) >} q_k(t) + \frac{\Gamma_l}{h\nu_l A}[\sigma_{al} - (\sigma_{al} + \sigma_{el}) < N_2 >] < p_l > + \frac{\Gamma_p}{h\nu_p A}\sigma_{ak} < p_p > \ . \quad (20)$$

At the first gain, $G_k(z, t)$ can be approximated by the first term of Tailor expansion series with respect to z, i.e.,

$$G_k(z, t) = g(t)z \ . \quad (21)$$

In this case, its average function is given by $< G_k(z, t) > = \frac{1}{2}\bar{G}_k$. In a more general case, where $G_k(z, t)$ is separable i.e., $G_k(z, t) = g(t)f(z)$, $< G_k(z, t) > = \eta_k G_k(l, t)$, where $\eta_k = u_k \int_0^L f(z)dz/ (Lf(\mu_k)), \mu_k = L$ for $u_k = 1$ and $\mu_k = 0$ for $u_k = -1$. Generally, the system of partial differential Equations 10 to 13 is solved numerically for a WDM optical fiber amplifier and η_k versus time is approximately constant and is plotted in Figure 2. Hence, Equation 14 may be written in the form:

$$\frac{d\bar{G}_k}{dt} = -\frac{v_k}{\eta_k L}\bar{G}_k + \frac{v_k}{\eta_k L}(g_k + \alpha_k) < N_2 > - \frac{v_k}{\eta_k L}(\alpha_k + l_k) - \frac{1}{\eta_k L}\frac{\dot{q}_k}{q_k} \quad k = 1, 2, \cdots, m \ . \quad (22)$$

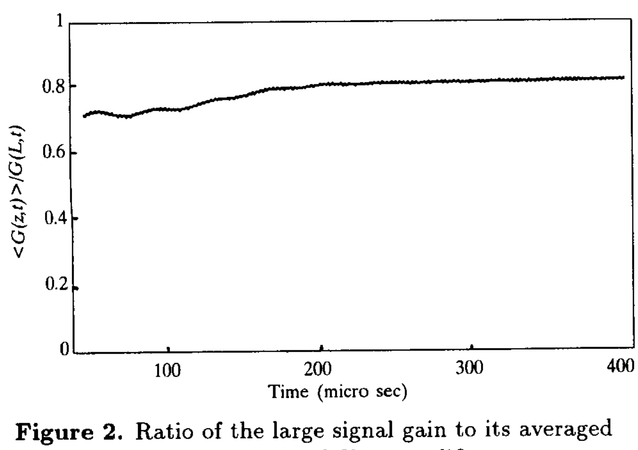


Figure 2. Ratio of the large signal gain to its averaged value versus time, in a WDM fiber amplifier.

The system of ordinary differential Equations 19, 20 and 22 is called averaged balance equations. From now on, only the averaged balance equations are used and angle brackets are omitted. While considering time functions appearing in the balance equations, it is assumed that averaging over space has already been carried out. Taking this into consideration and introducing the following notations:

$$a_k = \frac{\Gamma_k}{h\nu_k A}\sigma_{ak} \ , \quad (23)$$

$$b_k = \frac{v_k}{\eta_k L}(\alpha_k + l_k) \ , \quad (24)$$

$$M_k = \frac{\sigma_{ak}}{\sigma_{ak} + \sigma_{ek}} \ , \quad (25)$$

$$N_{k,th} = \frac{\alpha_k + l_k}{L(\alpha_k + g_k)} \quad k = 1, \cdots, m \ , \quad (26)$$

the system of averaged balance equations may be rewritten in the following form:

$$\frac{dN_2}{dt} = -\frac{N_2}{\tau} + \sum a_k(1 - \frac{N_2}{M_k})e^{\eta_k \bar{G}_k} q_k(t) + a_l(1 - \frac{N_2}{M_l})p_l + a_p(1 - N_2)p_p \ , \quad (27)$$

$$\frac{dp_l}{dt} = -\frac{1}{\tau_l}(1 - \frac{N_2}{N_{th}})p_l \ , \quad (28)$$

$$\frac{d\bar{G}_k}{dt} = -\frac{\bar{G}_k}{\tau_k} - b_k(1 - \frac{N_2}{N_{k,th}}) - \frac{1}{\eta_k L}\frac{\dot{q}_k}{q_k} \quad k = 1, \cdots, m \ , \quad (29)$$

where N_{th} is the population inversion threshold for laser oscillation, τ_l is the laser photon lifetime and τ_k is the photon lifetime of an amplifying signal, given by the following relations:

$$N_{th} = \frac{\alpha_l + l_l + \gamma_l}{g_l + \alpha_l}, \quad \tau_l = \frac{1}{v_l}\frac{1}{\alpha_l + l_l + \gamma_l},$$

$$\tau_k = \eta_k \frac{L}{v_k} \quad k = 1, \cdots, m \ . \quad (30)$$

Let the state variable vector X be $X^T = (N_2, p_l, \bar{G}_1, \cdots, \bar{G}_m)$ and the RHS of Equations 27 to 29 be denoted by $F(X, q, \dot{q})$, where $q = (q_1(t), \cdots, q_m(t))$ is the input variable vector. Then, the averaged balance equations are written in the standard form:

$$\frac{dX}{dt} = F(X, q, \dot{q}) \ , \quad (31)$$

where $X \in \mathbb{R}^{m+2}$ and $F : \mathbb{R}^{2m+2} \to \mathbb{R}^{m+2}$.

By the above averaging over the length of the active medium, a considerable simplification of the

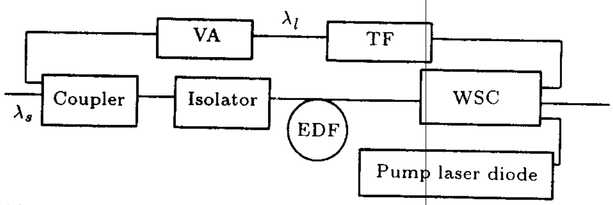


Figure 3. Scheme of an all-optical fiber inverter, VA (Variable Attenuator), TF (Tunable Filter) and WSC (Wavelength Selective Coupler).

balance system of equations is made. Instead of a system of partial differential equations, a system of ordinary differential equations is now obtained.

The optical inverter and the stabilized all-optical multi-wavelength amplifiers are essentially a ring laser with a narrow bandpass filter in the feedback loop (see Figure 3); therefore, the pump and the controller amplifying signal time constant are small. An approximation can be made by neglecting these time constants in Equations 27 and 29. Furthermore, the variation of the input signal power in Equation 29 and the output power of pump in Equation 27 can be neglected. Now, the average balance equations are used to describe the relaxation oscillation and spiking behavior in the transient response of the surviving channels of an all-optical stabilized multi-channel Er-doped fiber amplifier and an optical inverter, respectively.

RELAXATION OSCILLATION IN THE ALL-OPTICAL STABILIZED MULTI-CHANNEL ER-DOPED FIBER AMPLIFIER

Energy oscillation between the excited state of erbium ions of the active medium and photons of the optical cavity is damped through the optical fiber losses and laser power output coupling. This phenomenon causes a relaxation oscillation in the population inversion and the laser output power of the optically stabilized multi-channel Er-doped fiber amplifiers. Relaxation oscillation in the population inversion of the active medium is transferred to gain and output power of the surviving channels. This phenomenon is studied using the averaged balance Equations 27 to 29.

Now, the system of ODE (Equation 31) in the vicinity of stationary values X_{st} is considered. For this purpose,

$$X(t) = X_{st} + x(t), \tag{32}$$

and it is assumed that:

$$x_i(t) << X_{i,st} \quad i = 1, \cdots, m+2 \; . \tag{33}$$

The value of the components of X_{st} are obtained by putting $dX/dt = 0$ in Equation 31:

$$N_{2,st} = N_{th} \; , \tag{34}$$

$$\bar{G}_{k,st} = b_k \tau_k (1 - \frac{N_{th}}{N_{k,th}}) \; , \tag{35}$$

$$p_{l,st} = \frac{1}{a_l(1 - \frac{N_{th}}{M_l})} \left[\frac{N_{th}}{\tau} - a_p(1 - N_2)p_p - \sum_{k=1}^{m} a_k (1 - \frac{N_{2,st}}{M_k}) e^{\eta_k \bar{G}_{k,st}} q_k \right] . \tag{36}$$

As a consequence of substituting Equation 32 into Equation 31, using Equation 36 and taking Equation 33 into account (nonlinear terms are neglected), the system of equations is linearized, i.e., is converted from a nonlinear to a linear system:

$$\dot{x} = Ax, \tag{37}$$

when A is Jacobean matrix of F ($A = \frac{\partial F}{\partial X}|_{X_{st}}$), written in the following form:

$$A = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & \cdots & A_{1,m+2} \\ A_{21} & A_{22} & 0 & 0 & 0 & \cdots & 0 \\ A_{31} & 0 & A_{33} & 0 & 0 & \cdots & 0 \\ A_{41} & 0 & 0 & A_{44} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{m+2,1} & 0 & 0 & 0 & 0 & \cdots & A_{m+2,m+2} \end{pmatrix} \tag{38}$$

where,

$$A_{11} = -\frac{1}{\tau} - \sum \frac{a_k}{M_k} e^{\eta_k \bar{G}_{k,st}} q_k - \frac{a_l}{M_l} p_{l,st} - a_p p_p,$$

$$A_{12} = a_l (1 - \frac{N_{th}}{M_l}),$$

$$A_{1,i} = a_i \eta_i (1 - \frac{N_{th}}{M_i}) e^{\eta_i \bar{G}_{i,st}} q_i \quad 3 \leq i,$$

$$A_{21} = \frac{1}{\tau_l N_{th}} p_{l,st},$$

$$A_{22} = 0, A_{i,1} = \frac{b_i}{N_{i,th}} \quad 3 \leq i, \text{ and } A_{ii} = \frac{-1}{\tau_i} \quad 3 \leq i.$$

From elementary theory of ordinary differential equations, the eigenvalues of A are the natural frequencies of the linearized system, i.e., the real and imaginary parts of eigenvalues are damping coefficients and angular frequencies, respectively. Eigenvalues corresponding to the amplifying signals are of the order of $\frac{v_k}{l}$, which are approximately the inverse of the time constant of energy discharge of each mode and are negligible. Hence, it is enough to obtain the eigenvalues corresponding to the interaction of laser oscillation photon number and population inversion of erbium ions. Consider the eigenvalues of the 3 × 3 real matrix A (calculations have been done for a one-input system). One of the eigenvalues is real ($p_k \ll p_l$) $\lambda_3 = A_{33} = -\frac{1}{\tau_1}$ and under the condition:

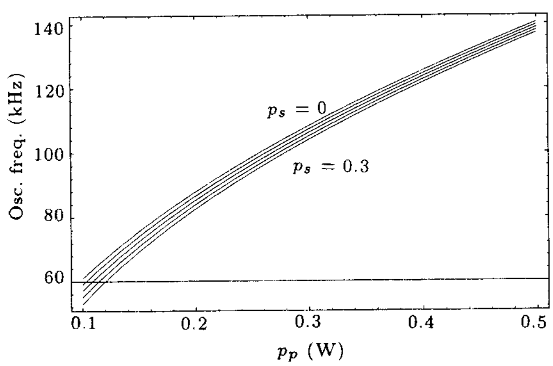
$$\Delta < 0 ,$$

the other two eigenvalues are complex conjugate ($\lambda_1 = \alpha + i\omega$ and $\lambda_2 = \lambda_1^*$). The damping coefficient (α) and the frequency of oscillation (ω) are given by the following relation:

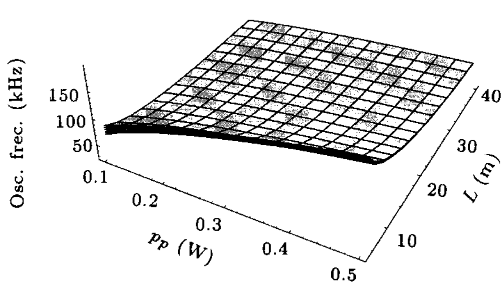
$$\alpha = A_{11}/2 ,$$

$$\omega = \sqrt{-A_{11}^2 - 4A_{21}A_{12}/2} . \qquad (39)$$

Variation of oscillation frequency (ω) versus pump power (p_p), length of the active fiber, and input signal power (p_s) are plotted in Figure 4. It is shown in Relation 39 and Figure 4 that the frequency ω is an increasing function of the total power. On the other hand, increasing the input signal power decreases the power of the compensating signal (laser) (see Relation 36) and, hence, decreases the frequency of the relaxation oscillation as shown in the figure. The conclusions are also in good agreement with the experimental results [1]. In an eight-channel WDM network, the relaxation oscillation in surviving channel output power is demonstrated (Figure 5a). In Figure 5b, timing of lost/restored channels is shown. These results have considerable compatibility with experimental results [1] in cases where operating wavelengths are close together and the Spectral Hole Burning (SHB) effects are negligible. As a check point, it can be seen from Figure 5b, that greater oscillation frequency occurs when channels are turned off. As found, power excursions in an all-optical stabilized EDFA have two contributions [7]: a steady state contribution owing to the Spectral Hole Burning (SHB) and a dynamic contribution owing to the relaxation oscillations in the laser. These effects are shown to be small but will be compounded in large networks of concatenated EDFAs in which laser AGC is employed in each EDFA and the performance of surviving channels may be impaired. In the homogeneous model, the residual power excursions related to spectral hole burning due to inhomogeneity

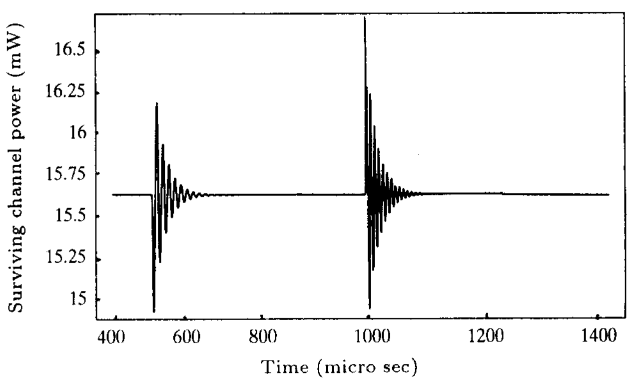


(a)

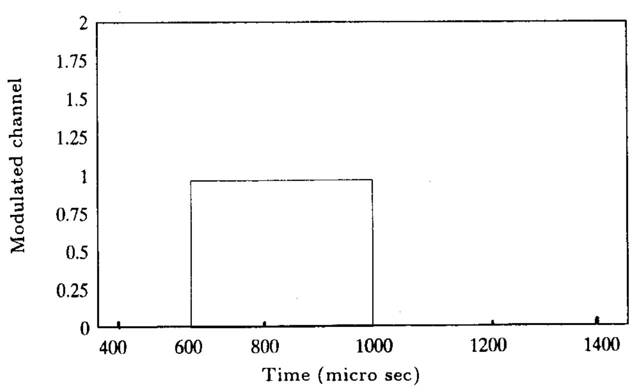


(b)

Figure 4. Relaxation oscillation frequency of laser signal in an optical fiber inverter: a) Versus pump power, b) Versus the length of the active fiber and pump power. (In both (a) and (b), dependency on the input signal power, p_s, is also shown.)

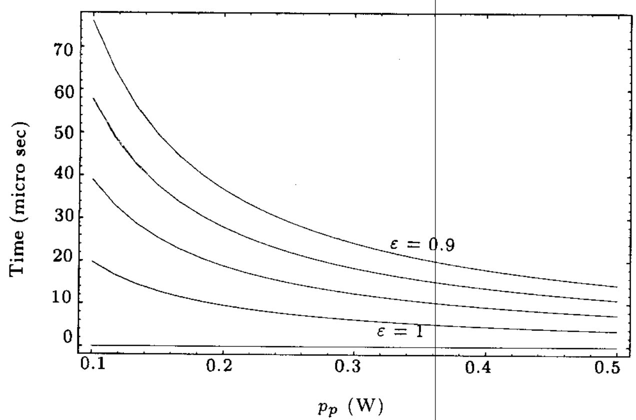


(a)

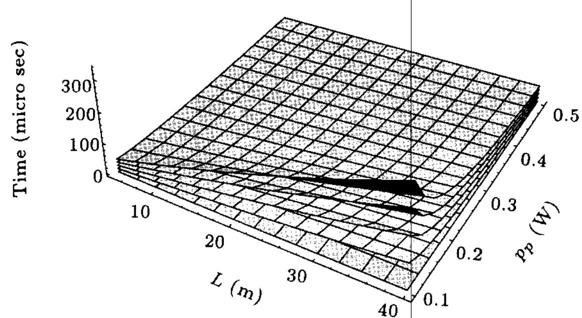


(b)

Figure 5. Transient response of the output power of a WDM optical fiber amplifier: a) Output power of the surviving channel, b) Modulated channel corresponding to 7 lost/restored channels of 8.



(a)



(b)

Figure 8. The time between switching off the input controlling signal and beginning of the laser generation (t_{th}): a) Versus pump power, b) Versus the length of the active medium and pump power. In both cases, dependency is shown for different values of ϵ.

Fatehi (Lucent Tech., USA) and careful revision of Dr. M. Rahnama (Kerman University, Iran). The financial support provided by International Center of Science, High Technology, and Environmental Sciences (ICST, Kerman, Iran) is also gratefully acknowledged.

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