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Numerical Method for Determination of Pavement Deflection from Moving Load Measurements

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A numerical method for processing and correction of pavement deflection measurements recorded by moving load deflectometers is presented. The method takes into account some peculiarities inherent to this kind of device, such as multiple load system, displacement of the reference plane and initial deflection at the beginning of measurements. Results of the method evaluation are drawn from validation tests, using known theoretical solutions and absolute deflection measurements. Application of the method to different pavement structures and measuring devices demonstrates an improvement of measurement results through processing the required corrections.

INTRODUCTION

The deflection bowl of structures is used in rational methods of pavement design or rehabilitation for the calibration of theoretical models as well as for the evaluation of the mechanical parameters of structures, such as layers moduli and thickness. Different equipment for measuring pavement deflection has been devised by Bonnot [1]. Devices differ in loading system or measurement technological principle. Lacroix deflectograph [2] and Curviameter [3] are high output equipment for deflection testing under a moving load, the former being used in France as well as many other countries throughout the world.

The study of deflection bowls under the above-mentioned moving load devices presents some peculiarities which require accurate analyses and interpretation. Firstly, device loading conditions are rather complicated because of the multiple load system of a four-wheel vehicle. Each wheel applies a load which results in a cumulative effect on the measurements. Secondly, when using Lacroix deflectograph and Benkelman beam for rigid pavements, displacement of the measurement reference plane can occur. Finally, most devices assume a negligible deflection value at the farthest abscissa of the recorded curve instead of the true initial deflection.

These effects have been considered partially or totally by different authors [4-6]. In a previous work [7], a general numerical method for measurements processing, which takes into account the multiple loads system and the possible displacement of the measurement reference plane, has been proposed.

As a continuation of that work, the effect of the initial deflection will be introduced and different results from validation tests of the measurements processing method will be presented.

PRESENTATION OF THE DEFLECTION PROCESSING METHOD

Lacroix deflectograph principle is shown in Figure 1. Loads $P_j (j = 1 \text{ to } 4)$ of the vehicle wheels are applied at points M_j. Points $A_i (i = 1 \text{ to } 2)$ correspond to the sensors and $A_k (k = 3 \text{ to } 5)$ to the footings of the measurement beam; moreover, the latter ones define the measurement reference plane. During the measure, the vehicle moves towards direction V, while the measurement beam is kept at a fixed position. Abscissa x is the variable longitudinal distance between the sensors and the rear axle.

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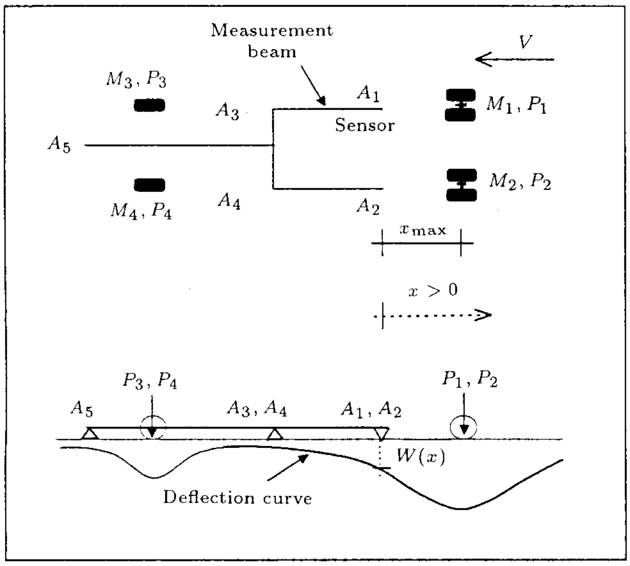


Figure 1. Lacroix deflectograph principle. Position at the beginning of the measurements.

The principle of the processing method is illustrated in the flowchart of Figure 2. Assuming horizontal homogeneity of the pavement structure, the deflection bowl under a unique reference load of P_0 is axi-symmetrical. Its expression can thus be chosen as a function $W_0(r,p)$ of the radial abscissa r, dependent on a set p of unknown parameters which must be determined.

From the transfer function of the measuring device, the relative deflection $W_i^r(x)$ (i.e., image of the deflection $W_0(r)$ as seen by sensor i of the deflectograph) can be estimated as a function of $W_0(r)$, according to the following expression:

$$W_i^r(x,p) = \sum_{j=1}^{4} L_j W_0\big(r_{ij}(x)\big) + \sum_{k=3}^{5} B_{ik} \sum_{j=1}^{4} \left[L_j W_0\big(r_{kj}(x)\big)\right] - W_f \qquad i = 1,2 \, , \tag{1}$$

with:

$L_j = P_j/P_0$	load coefficient ($j = 1$ to 4),
B_{ik}	displacement of sensor i due to a unit displacement of footing k,
x	longitudinal distance between sensor A_i and load $P_i (i = 1,2)$,
$r_{ij}(x)$	distance between A_i and P_j,
$W_f = W_i^r(x_{\max})$	initial deflection.

Relation 1 is valid with the hypothesis of a linear elastic behavior of the structure and due to symmetry, the summations do not depend on subscript i which can be omitted.

From the measured deflection $W^m(x_\alpha)$ at a given set of N abscissas $x_\alpha(\alpha = 1$ to $N)$, the minimization of the difference E,

$$E = \sum_{\alpha=1}^{N} \omega_\alpha \left[W^m(x_\alpha) - W^r(x_\alpha, p)\right]^2 \, , \tag{2}$$

using a nonlinear least squares fitting method, results in computing the unknown parameters p_i of $W_0(x)$. The relevancy of the optimization is quantified with a coefficient of correlation ρ defined by:

$$\rho = 1 - \sqrt{\frac{E}{V}} \, , \tag{3}$$

with:

$$V = \sum_{\alpha=1}^{N} \omega_\alpha \left[W^m(x_\alpha) - \overline{W}^m\right]^2 \, ,$$

$W^m(x_\alpha)$	measured deflection at abscissas x_α,
$\overline{W}^m$	average measured deflection,
ω_α	arbitrary relative weight at point x_α; by default: $\omega_\alpha = 1 (\alpha = 1$ to $N)$.

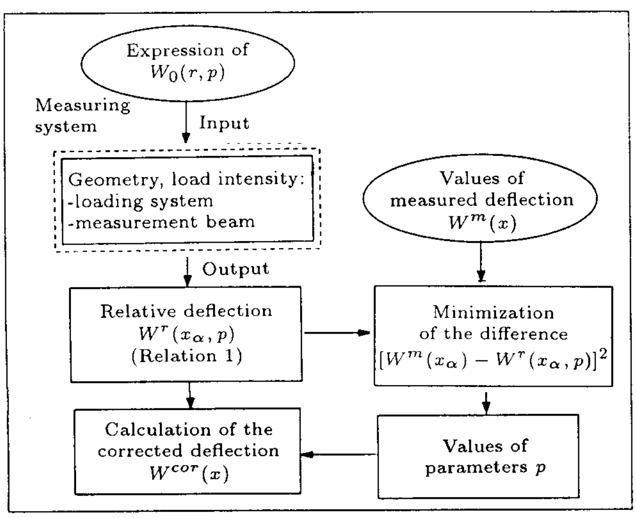
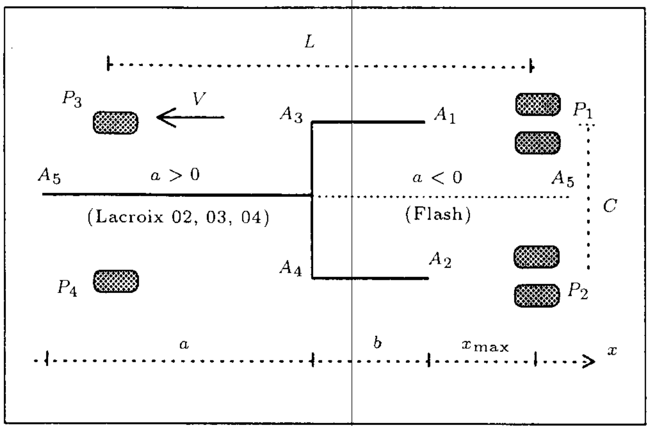


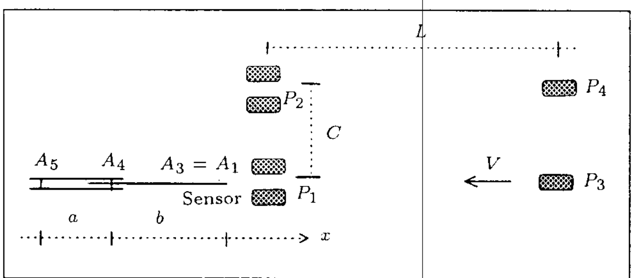
Figure 2. Flowchart of the procedure for calculation of parameters p_i.

The algorithm used for the fitting process is the Gauss-Newton method implemented in Matlab software. By minimizing the above difference, E, as an objective function, this approach converges rapidly to the optimal parameters (p).



Lacroix Type	02	03	04	Flash
L (m)	4.80	6.75	7.75	4.80
a (m)	1.375	2.15	4.25	-3.75
b (m)	1.385	2.15	2.55	1.616
c (m)	1.83	1.86	1.86	1.793
x_{max}(m)	1.15	1.45	1.45	1.50
P_3, P_4(kN)	27.09	22.65	22.65	27.09
P_1, P_2(kN)	64.03	64.03	64.03	64.03
P_0(kN)	65			

Figure 3. Geometrical and loading characteristics of different types of Lacroix deflectograph.



L (m)	a (m)	b (m)	c (m)	P_1, P_2 (kN)	P_3, P_4 (kN)
4.80	1.25	2.50	1.83	65	33

Figure 4. Example of Benkelman beam geometry (Loire Atlantique type, Autret 1972).

Finally, the corrected deflection $W^{cor}(x)$ (i.e., an approximation of the absolute deflection under multiple load due to the deflectograph) is equal to:

$$W_i^{cor}(x) = \sum_{j=1}^{4} L_j W_0\left(r_{ij}(x)\right); \quad i = 1, 2 \,. \tag{4}$$

The above procedure can also be applied to measurements processed with Benkelman beam. Figures 3 and 4 and Tables 1 to 4 provide the specific parameters, B_{ik} and r_{ij}, corresponding to different devices.

Table 1. Values of B_{ik} coefficients for Lacroix deflectograph.

B_{ik}		Footings $k = 3$	$k = 4$	$k = 5$
Sensors	$i = 1$	$-\left(1 + \frac{b}{2a}\right)$	$-\frac{b}{2a}$	$\frac{b}{a}$
	$i = 2$	$-\frac{b}{2a}$	$-\left(1 + \frac{b}{2a}\right)$	$\frac{b}{a}$

There is no theoretical restriction concerning the mathematical expression of function $W_0(r, p)$, but the number m of unknown parameters p must be less than the number N of measuring points. Practically, for numerical considerations, it is recommended to use a limited number of p parameters. As an illustration, Martinez and Jouve have chosen the following three-parameter expression for $W_0(r, p)$ [8] derived from [9]:

$$W_0(r, p) = \frac{d}{\left[1 + \frac{1}{(2n+1)}\left(\frac{r}{\lambda}\right)^2\right]^n} \,, \tag{5}$$

Table 2. Values of $r_{ij}(x)$ and $r_{kj}(x)$ coefficients for Lacroix deflectograph.

$r_{ij}(x), r_{kj}(x)$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	$\lvert x \rvert$	$\sqrt{x^2 + c^2}$	$\lvert L - x \rvert$	$\sqrt{(L - x)^2 + c^2}$
$i = 2$	$\sqrt{x^2 + c^2}$	$\lvert x \rvert$	$\sqrt{(L - x)^2 + c^2}$	$\lvert L - x \rvert$
$k = 3$	$\lvert x + b \rvert$	$\sqrt{(x + b)^2 + c^2}$	$\lvert L - x - b \rvert$	$\sqrt{(L - x - b)^2 + c^2}$
$k = 4$	$\sqrt{(x + b)^2 + c^2}$	$\lvert x + b \rvert$	$\sqrt{(L - x - b)^2 + c^2}$	$\lvert L - x - b \rvert$
$k = 5$	$\sqrt{(x + a + b)^2 + \frac{c^2}{4}}$	$\sqrt{(x + a + b)^2 + \frac{c^2}{4}}$	$\sqrt{(L - x - a - b)^2 + \frac{c^2}{4}}$	$\sqrt{(L - x - a - b)^2 + \frac{c^2}{4}}$

Table 3. Values of B_{ik} coefficients for Benkelman beam.

B_{ik}	Footings		
	$k = 3$	$k = 4$	$k = 5$
$i = 1$ or $(i = 3)^*$	1	$-\left(1 + \frac{b}{a}\right)$	$\frac{b}{a}$

$^*A_1 = A_3$ is a unique sensor in the case of Benkelman beam.

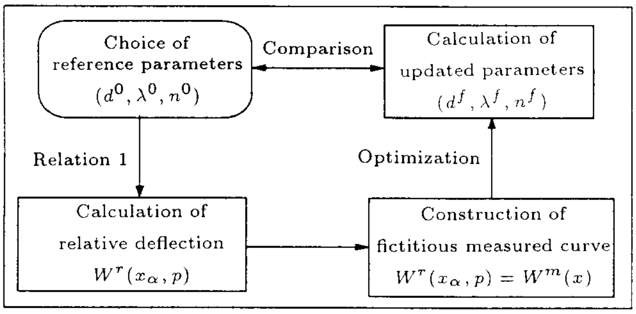


Figure 5. Validation of the processing method.

with:

$p_1 = d$ maximum deflection,
$p_2 = \lambda$ Abscissa of the point of inflection of the deflection curve,
$p_3 = n$ exponent.

Other more sophisticated analytical expressions of the deflection bowl could be chosen which directly depend on layers moduli, such as in [10,11].

VALIDATION OF THE PROCESSING METHOD

The proposed correction method is subjected to a double validation: a strictly numerical validation from known solutions and a more general validation from absolute deflection measurements.

Numerical Validation from Known (d, λ, n) Parameters Solution

The processing method is first verified through known solutions. Starting from known parameters (d^0, λ^0, n^0), the next three stages are followed (Figure 5).

Stage 1: Choice of Reference Parameters

Sets of reference parameters $p^0 = (d^0, \lambda^0, n^0)$ are chosen within realistic ranges, which give absolute deflections, $W_0(r, p^0)$, satisfying Expression 5.

Stage 2: Construction of the Fictitious Measured Curve W^m

The relative deflection $W^r(x, p^0)$ is calculated from $W_0(r, p^0)$, according to Expression 1. Then, a fictitious measured curve $W^m(x)$ is considered:

$$W^m(x) = W^r(x, p^0). \tag{6}$$

Stage 3: Calculation of the Computed Parameters

According to the above numerical procedure, different initial values of parameters (d^i, λ^i, n^i) are considered and final updated parameters (d^f, λ^f, n^f) are computed using Equation 2, which are compared with the original ones used in the first stage as reference values.

Furthermore, in order to test the numerical stability of the iterative solution (i.e., sensibility to initial parameters), this stage is repeated three times with different initial values and the results are compared.

The results (Table 5 and Figure 6) show that the updated values do not depend on the initial values and that they are, in each case, equal to the reference

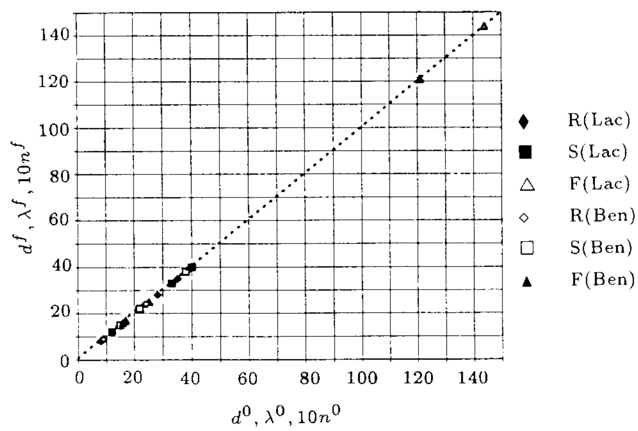


Figure 6. Numerical validation of the method from known solutions (see Table 5). Updated values (d^f, λ^f, n^f) versus reference values (d^0, λ^0, n^0).

Table 4. Values of $r_{ij}(x)$ and $r_{kj}(x)$ coefficients for Benkelman beam.

$r_{ij}(x), r_{kj}(x)$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$ or $(i = 3)$	$\lvert x \rvert$	$\sqrt{x^2 + c^2}$	$\lvert L + x \rvert$	$\sqrt{(L + x)^2 + c^2}$
$k = 4$	$\lvert x + b \rvert$	$\sqrt{(x + b)^2 + c^2}$	$\lvert x + b + L \rvert$	$\sqrt{(x + b + L)^2 + c^2}$
$k = 5$	$\lvert x + b + a \rvert$	$\sqrt{(x + a + b)^2 + c^2}$	$\lvert x + b + a + L \rvert$	$\sqrt{(x + a + b + L)^2 + c^2}$

Table 5. Numerical validation of the method from known solutions.

Devices	Structures	Parameters	Reference Values (d^0, λ^0, n^0)	Initial Values (d^i, λ^i, n^i)			Updated Values (d^f, λ^f, n^f)		
				Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
Lacroix	Rigid (R)	$d(m10^{-05})$	28	10	21	29	28	28	28
		$\lambda(m10^{-02})$	35	40	20	15	35	35	35
		n	0.8	2	0.8	1.5	0.8	0.8	0.8
		ρ	-	-	-	-	1.00	1.00	1.00
	Semirigid (S)	$d(m10^{-05})$	40	55	62	23	40	40	40
		$\lambda(m10^{-02})$	33	41	15	48	33	33	33
		n	1.2	0.7	2.5	1.1	1.2	1.2	1.2
		ρ	-	-	-	-	1.00	1.00	1.00
	Flexible (F)	$d(m10^{-05})$	144	52	71	40	144	144	144
		$\lambda(m10^{-02})$	25	25	62	11	25	25	25
		n	1.5	2.5	1.2	3	1.5	1.5	1.5
		ρ	-	-	-	-	1.00	1.00	1.00
Benkelman Beam	Rigid (R)	$d(m10^{-05})$	24	17	12	35	24	24	24
		$\lambda(m10^{-02})$	39	15	31	22	39	39	39
		n	0.9	1.5	1.9	2.1	0.9	0.9	0.9
		ρ	-	-	-	-	1.00	1.00	1.00
	Semirigid (S)	$d(m10^{-05})$	38	40	21	55	38	38	38
		$\lambda(m10^{-02})$	22	31	70	11	22	22	22
		n	1.5	3	1.9	0.9	1.5	1.5	1.5
		ρ	-	-	-	-	1.00	1.00	1.00
	Flexible (F)	$d(m10^{-05})$	121	99	143	118	121	121	121
		$\lambda(m10^{-02})$	17	33	48	52	17	17	17
		n	1.6	2.8	1.1	0.8	1.6	1.6	1.6
		ρ	-	-	-	-	1.00	1.00	1.00

values. This leads to a positive conclusion regarding the stability and relevancy of the procedure from a numerical point of view.

Validation from Absolute Deflection Measurements

The presented procedure is also validated by applying it to the absolute deflections measured with a displacement transducer anchored at a 6-meter depth. This procedure is conducted as follows:

a) The parameters $p = (d, \lambda, n)$ of the function $W_0(r,p)$ are determined from the measured values $W^{abs}(x_\alpha)$ by minimization of the difference:

$$\sum_{\alpha=1}^{N} \left[W^{abs}(x_\alpha) - W_i^{fit}(x_\alpha, p)\right]^2 \quad i = 1,2 \ , \tag{7}$$

with:

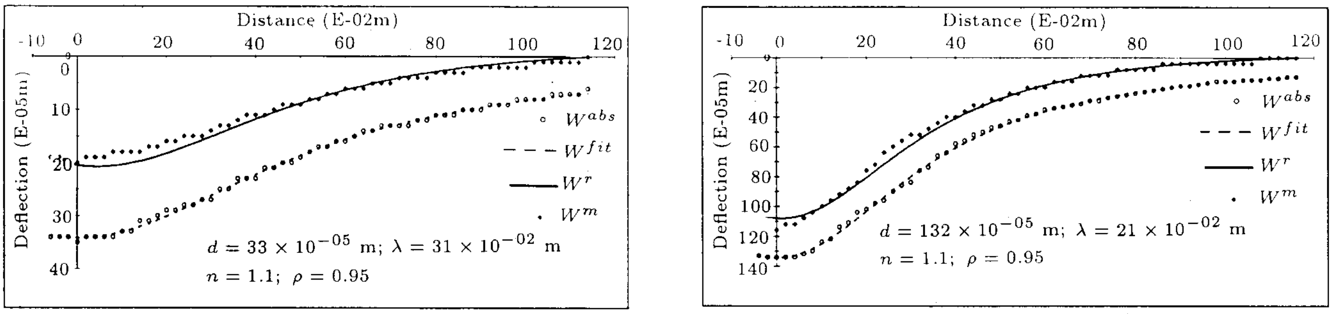
N: number of the recorded values,
$W^{fit}(x_\alpha,p)$: fitted values of the absolute deflection,

$$W_i^{fit}(x_\alpha, p) = \sum_{j=1}^{4} \frac{P_j}{P_0} W_0 \left[r_{ij}(x_\alpha), p\right] \quad i = 1,2 \ . \tag{8}$$

b) The relative deflection $W^r(x)$ is computed from $W_0(r,p)$ using Expression 1. This deflection is then compared with the measured deflection $W^m(x)$.

Figure 7 and Table 6 demonstrate the results of this validation from the measurements processed on two structures (one rigid, another flexible), both with an anchored transducer and a 02 Lacroix deflectograph, provided by the Laboratoire Central des Ponts et Chaussées (LCPC). Significant differences are first observed between Lacroix and absolute measurements, specially on the stiffer structure, which needs a correction procedure.

From the high values of the coefficients of correlation ($\rho = 0.95$), it can be deduced that the



a) Full depth Bituminous structure.

b) Flexible structure.

Figure 7. Validation of the processing method from absolute measurements (W^{abs}) and comparison with 02 deflectograph measured deflection (W^m).

Table 6. Comparison of computed and measured deflections on two pavements; Rigid Structure (RS) and Flexible Structure (FS).

Parameters	Validation $\text{Min}[(W^{abs} - W^{fit})^2]$		Application $\text{Min}[(W^m - W^r)^2]$	
	RS	***FS***	***RS***	***FS***
$d(m10^{-05})$	33	132	31	136
$\lambda(m10^{-02})$	31	21	33	19
n	1.1	1.1	1.1	1
ρ	0.95	0.95	0.93	0.95
$W_f(m10^{-05})$	6	12	7	12
$d^{fit}(m10^{-05})$	34	134	-	-
$d^{abs}(m10^{-05})$	35	134	35	134
$d^{cor}(m10^{-05})$	-	-	34	138
$d^r(m10^{-05})$	20	108	18	118
$d^m(m10^{-05})$	20	116	20	116
ξ	-	-	0.93	1.22

choice of Expression 5 for function W_0 is appropriate here for both rigid and flexible structures. Moreover, the agreement between the curves $W^r(x)$ and $W^m(x)$ is excellent, leading to a conclusion regarding the relevancy of the hypothesis (homogeneity, linearity) underlying transfer Function 1.

APPLICATION EXAMPLES OF THE CORRECTION METHOD

According to the flowchart of Figure 2, the correction method is applied to measurements recorded with a 02 Lacroix deflectograph on the two mentioned structures. The corrected deflections are computed and compared with the absolute deflections (Figure 8 and Table 6). The results provide satisfactory values of the correlation coefficients ($\rho = 0.93$ and 0.95) which means a good agreement between the Lacroix measured deflection W^m and the corresponding computed one W^r.

For both structures, the correction improves the measurements as the corrected curve W^{cor} is closer to the absolute deflection W^{abs} than the Lacroix measurements W^m. The effect of the correction can be quantified by the parameter ξ:

$$\xi = \frac{d^{cor} - d^m}{d^{abs} - d^m} . \tag{9}$$

Values of ξ between 0 and 2 signify an improvement of the measurements by the correction and values out of this range mean a negative effect.

The computed values of parameter ξ, reported in Table 6 appear to be close to 1, which indicates optimal efficiency of the correction.

CONCLUSIONS

The proposed method of processing the measurements under moving load devices allows consideration of the multiple loads system and possible displacement of the measurement reference plane, as well as the effect of an initial deflection.

The method computes the parameters of the absolute deflection curve, whose analytical expression is given, under a fixed reference plane and unique load. This general and quite simple method can be used for on-board processing of the measurements for any deflection measurement device under moving loads.

The examples considered here have validated the method and shown that the correction improves the measurement results significantly.

Application of the method to other structures would further prove the generality and efficiency of the processing and correction procedures.

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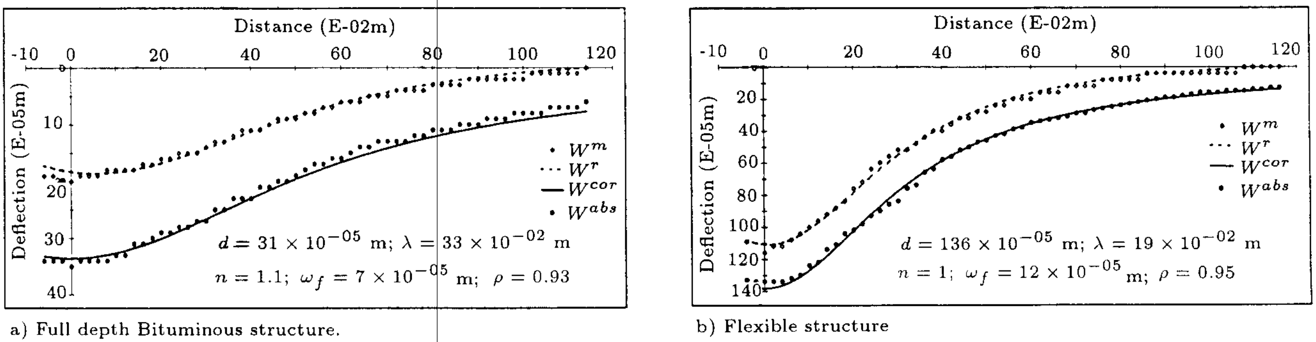

Figure 8. Comparison of measured (W^m), relative (W^r), corrected (W^{cor}) and absolute (W^{abs}) deflections produced by 02 Lacroix deflectograph.

thors are grateful to this institution for authorizing the publication of the absolute and Lacroix measurement results.

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