Scientia Iranica, Vol. 14, No. 3, pp 230–235

SCIENTIA
IRANICA

A Simulated Annealing Approach for Product Mix Decisions

S.K. Chaharsooghi* **and N. Jafari**[1]

The identification and determination of products and their quantities, according to available resources, is an important matter in manufacturing plants, the answer to which is achieved by referring to objectives. This problem is often called the "product mix" decision, which is one of the applications of the Theory of Constraints (TOC). As the number of resources and products in the "product mix" decision, increase, solving the problem becomes much more complex. Therefore, optimization methods are limited to small-scale problem instances and heuristic methods are introduced for large-scale problems. In this research study, while reviewing previous related works, Simulated Annealing (SA) is used to solve the product mix problem. Furthermore, the effect of parameter changes on accuracy and computing time is investigated. The results of SA, in comparison with other heuristic results, show that the efficiency of SA in solving product mix problems is greater than other meta-heuristic methods.

INTRODUCTION

TOC is a management philosophy, which defines three important performance indicators. These are throughput, inventory and operating expenses [1]. TOC focuses on the constraints in a system and employs five steps for continuous improvement. The product mix problem is one of the TOC applications that maximizes the profit by involving the determination and identification of the quantity of products to be produced.

Several studies have been carried out to solve the product mix problem or to improve previously introduced approaches to this problem. These studies may be categorized in three groups.

Research work in the first group has tried to solve product mix through exact approaches. Patterson formulated the product mix decision as a linear programming problem [2].

The second group involves heuristic methods. TOC, which has been employed to solve the product mix, has been criticized by some researchers. Plenert has shown the deficiency of TOC encountering multiple constraints [3]. Lee and Plenert have described that when a new product is added to the problem, the TOC heuristic could create a non-optimal product mix [4].

Fredendall and Lee introduced the Revised TOC heuristic to compensate for the shortcomings of the original TOC heuristic [5].

Low [6] and Spoedo et al. [6], by using numerical examples, have illustrated that TOC leads to a more profitable product mix than Activity-Based Costing (ABC). Kee [6], using a similar example, has illustrated that an activity-based model, integrating the cost and capacity of the production activities, outperforms the TOC. Bakke and Helberg [6] have examined the complementary nature of TOC and ABC. They have then suggested that TOC is appropriate for the short run, while ABC is appropriate for long-term decisions. Kee and Schmit have modeled the selection of a product mix with the TOC and an ABC model [6].

The above reviewed approaches are limited to solving small-scale problems, while most real-life problems are large-scale.

Finally, the third group involves meta-heuristic approaches, which can handle large-scale problems in reasonable computational time. Onwubolu has employed a Tabu search-based algorithm for the TOC product mix decision [1]. A genetic algorithm has also been applied by Onwubolu and Mutingi to solve the product mix problem [7,8].

This paper employs the SA approach for product mix decision. In the next sections, the product

*. *Corresponding Author, Department of Industrial Engineering, Tarbiat Modarres University, Tehran, I.R. Iran.*
1. *Department of Industrial Engineering, Tarbiat Modarres University, Tehran, I.R. Iran.*

mix problem is described, then, the SA approach is explained. Computational results are also presented, along with the conclusions of this research work.

PRODUCT MIX PROBLEM

The product mix problem determines the types of product and their corresponding amounts, which can be produced toward maximizing profit. The product mix decision formulation is, as follows:

$$\text{Maximize } z = \sum_{j=1}^{n} C_j X_j, \tag{1}$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} X_j \leq b_i, \qquad i = 1, 2, \cdots, m, \tag{2}$$

$$0 \leq X_j \leq d_j, \quad j = 1, 2, \cdots, n, \quad X_j \text{ is integer}, \tag{3}$$

where:

$C_j =$ the throughput of the product type, j,
$a_{ij} =$ the needed amount of resource i to produce product type, j,
$x_j =$ the decision variable representing the quantity of the product type, j,
$b_i =$ the capacity of resource i,
$d_j =$ the demand for product type, j,
$m =$ the number of resources,
$n =$ the number of product mix types.

SIMULATED ANNEALING APPROACH

Simulated Annealing was proposed by Metropolis et al. in 1953 and was applied to combinatorial problems by Kirkpatrick et al. in 1983 [9]. Its concept is based on an analogy between the physical annealing process of solids and the solution process of combinatorial optimization problems.

SA explores the solution space from one solution (S) to its neighborhood (S') to reach a good solution, which can optimize the objective function, F (maximizing or minimizing). Whenever $F(S')$ is better than $F(S)$, S' is accepted. Otherwise, it is treated probabilistically. The probability of acceptance is evaluated by a parameter called "Temperature". The probabilistic function is as the following equation:

$$P(\Delta F) = e^{\ \ \Delta F/T}, \tag{4}$$

where, T is the current temperature and ΔF is the resulting change in objective function through a displacement from the current solution to the new one. By decreasing T, the probability for the acceptance of bad solutions is reduced.

At each temperature, a set of solutions is generated. Whenever there is no remarkable improvement in the objective function, the equilibrium is met and the temperature will be decreased. The above procedure is continued to meet the stopping criterion and then it is said that the problem is frozen.

According to related studies, the solution performance or quality is sensitive to certain control parameters, as follows: Method of generating initial solution and neighborhood space, initial temperature, temperature tuning, equilibrium test and frozen test.

To define the parameters of the approach, one can use information from related previous studies. In the case where there is not any related research, parameters can be clarified through trial and error. First, one or some problems are considered, then, these problems will be solved several times. During each time, all the parameters are fixed except for one and the objective function is calculated. After repeating this process, the amount of objective function is chosen and, therefore, the related parameters will be assigned to the approach. This process is called "set parametering", which is also used in this paper.

Initial Solution and Neighborhood Space

There are two ways to generate both an initial solution and neighborhood space, randomly or structurally. In this research, the quantity of product j, x_j, is equated to the demand of product j, d_j. Then by using a "feasibility mechanism", an initial feasible solution is found, considering resource capacity and demand constraints.

To generate the neighborhood space, a random pair-wise exchange is used. The feasibility mechanism is then applied, as follows:

Step 1 Compare the amount of product $j(x_j)$ with its demand (d_j). If $x_j > d_j$, then, generate a random number between $(x_j \quad d_j)$ and x_j. By subtracting the random number from x_j, the new amount of product j is produced. Go to Step 2;

If $x_j < d_j$, then, generate a random number between 0 and $(d_j \quad x_j)$. By adding the random number to x_j, the new amount of product j is produced. Go to Step 2.

Step 2 Calculate the amount of each resource, used to produce products.

$$\text{Load}_i = \sum_{j=1}^{n} a_{ij} x_j \qquad i = 1, 2, \ldots, m. \tag{5}$$

Step 3 If $\text{load}_i > b_i$, then, choose the product with the least profit (c_j). This product must consume

$b_i(a_{ij} \neq 0)$. Reduce x_j till load$_i$ <= b_i. Again, go to Step 2.

If load$_i$ <= b_i, then, go to END.

For example, the current feasible solution for a 4-product problem is (18, 20, 10, 50). It means that $x_1 = 8$, $x_2 = 20$, $x_3 = 10$ and $x_4 = 50$. Two products are chosen randomly and exchanged.

$$(18, \mathbf{20}, 10, \mathbf{50}) \rightarrow (18, 50, 10, 20).$$

The demands of the products (d_j) are 22, 30, 15 and 60. It means that $x_2 > d_2(50 > 30)$ and $x_4 < d_4(20 < 60)$. Therefore, two random numbers are generated as Step 1 of the feasibility mechanism: $n_2 = 22$ and $n_4 = 17$.

Finally, the new amounts of the second and fourth products are: $x_2 = 50 \quad 22 = 28$ and $x_4 = 20 + 17 = 37$.

After running Steps 2 and 3, the feasible neighborhood solution is achieved.

Initial Temperature

The number of iterations during the SA algorithm partly depend on the initial temperature. There are several approaches to determine this control parameter in the literature. In this research, a fixed number of "450" is chosen for the initial temperature. It is achieved through set parametering.

Temperature Tuning

The process of reducing the temperature schedules the freezing process in SA. There are two kinds of procedure for tuning the temperature parameter. One is to lower the temperature function and the other is based on the information obtained from previous trails. In this research, a geometric function is used for tuning the temperature:

$$T_{r+1} = \alpha T_r, \tag{6}$$

T_r is the temperature in stage r and α is a coefficient for lowering temperature. In this research, a value of 0.95 seemed appropriate for α through set parametering.

Equilibrium Test

When it is not possible to improve the objective function, the problem is equilibrated to the current temperature. Therefore, the temperature must be reduced and, then, the annealing process will be continued at the next temperature.

One of the equilibrium tests is a certain number of exchanges at each temperature. This number of exchanges or duration is called an epoch. In some studies, the epoch is the number of all the exchanges taking place at a temperature but, in others, the epoch is the number of accepted exchanges.

Besides the epoch, another procedure for reaching equilibrium, which was employed by Wilhelm and Ward, is used in SA for the product mix problem. After the execution of each epoch, the value of the objective function ($f_i(T_r)$) and its means are calculated. The equilibrium test, according to Equation 7, is then conducted. If steady state is reached, then, the temperature will be lowered. Otherwise, another epoch at the current temperature will be started and so on.

$$\frac{|\overline{f}_e(T_r) \quad \overline{f}_g(T_r)|}{\overline{f}_g(T_r)} \leq \varepsilon_1, \tag{7}$$

$$\overline{f}_e = \frac{\sum_{i=1}^{e} f_i(T_r)}{e}, \tag{8}$$

where $\overline{f}_e(T_r)$ is the mean value of the objective function during the last epoch at T_r; $\overline{f}_g(T_r)$ is the mean value of $\overline{f}_e(T_r)$ for all epochs at T_r and e is the number of exchanges in each epoch. ε_1 is a fixed number between 0 and 1 which controls the equilibrium test. As a result of set parametering in this research, ε_1 is equated to 0.3.

FROZEN TEST

There are many types of stopping criterion for SA in the literature, such as reaching the final temperature and the total number of accepted exchanges in the annealing process, etc.

In this research, for applying SA to the product mix problem, two criteria have been employed. One reaches the final temperature. The second is presented in Equation 9:

$$T_f = 0.1\ T_0, \tag{9}$$

$$V(T_r) = \frac{1}{n} \sum_{i=1}^{n} f_i^2(T_r) \quad \overline{f}^2(T_r), \tag{10}$$

$$S = \frac{V(T_r)}{T_r * [\overline{f}_0(T_0) \quad \overline{f}(T_r)]}, \tag{11}$$

$$S \leq \varepsilon_2, \tag{12}$$

where $\overline{f}(T_r)$ is the mean value of the objective function of the accepted exchanges done in T_r and ε_1 is a fixed positive number less than one, which is equal to 0.3 in this research.

SIMULATED ANNEALING ALGORITHM FOR A PRODUCT MIX PROBLEM

The SA for a product mix problem is developed in this section. Its flowchart is illustrated in Figure 1.

Step 0 Input the following parameters and issues:

- e length of epoch,
- M maximum number of accepted exchanges at the temperatures: ε_1, ε_2 and T_0,
- n number of products,
- m number of resources,
- C profit of each product,
- A the amount of each resource needed for each product,
- B capacity of resources,
- D demand of each product.

Step 1 Calculate the initial feasible solution (X_0) and final temperature (T_f);

a. Set the number of annealing processes equal to zero ($r = 0$);

b. Set the number of accepted solutions (exchanges) at a temperature equal to zero ($t = 0$);

c. Set the number of accepted solutions (exchanges) at an epoch equal to zero ($ch = 0$);

d. Set the current temperature $T_r = T_0$.

Step 2 Calculate the total profit of the initial product mix, ($f_0(T_r)$). Set the temporal solution $X^* = X_0$ and the temporal maximum profit $\mathbf{Z} = (f_0(T_r))$;

Step 3 Generate the new product mix (X_J) in the neighborhood of the current product mix (X_I), according to the previous section;

Step 4 Calculate the resulting change in profit Δf:

$$\Delta f(T_r) = f_I(T_r) \quad f_J(T_r), \tag{13}$$

If $\Delta f(T_r) <= 0$, then, go to Step 6;

Step 5 Generate a random number $Y \sim U(0,1)$. Calculate $P(\Delta f) = \exp[\ \Delta f(T_r)/T_r]$. If $P(\Delta f) >= Y$, then, go to Step 6, otherwise, go to Step 3;

Step 6 Accept the new product mix and $ch = ch + 1$. If $f_J(T_r) > \mathbf{Z}$, then, $\mathbf{Z} = f_J(T_r)$ and $X^* = X_J$. If $ch < e$, then, go to Step 3, otherwise, go to the next step;

Step 7 $t = t + ch$ and $ch = 0$, then, check the equilibrium test procedure,

a) If the number of accepted solutions at $T_r(t)$ are more than the maximum number of accepted exchanges at temperature (m), then, go to Step 8;

b) If equilibrium in Relation 7 is met, then, go to the next step, otherwise, go to Step 3.

Step 8 Frozen test: Calculate $\mathbf{S}$ and $V(T_r)$ by Equations 10 and 11; $t = 0$;

a) If $T_r <= T_f$, then, go to Step 10, otherwise, go to Step 8b;

b) If $S <= \varepsilon_2$, then, go to Step 10, otherwise, go to the next step.

Step 9 Calculate the new temperature $T_{r+1} = \alpha T_r$ and $r = r + 1$ and go to Step 3;

Step 10 Write the final solution ($\mathbf{Z}, X^*$);

Step 11 Stop.

COMPUTATIONAL RESULTS

To compare the SA-based heuristic with other existing methods, two sets of problem type were used. One of these types includes those problems excerpted from literature and research papers in the field of solving the product mix problem. These are categorized as small size problems. The other includes randomly generated problems by Onwubolu and Mutingi [7,8]. These problems are large scale. The computer program of the SA algorithm described in the previous section was written in Q-Basic and ran on a Pentium PC, which has characteristics almost similar to the PCs used to run GA and TS.

The comparison results for both small scale and large scale sizes are presented in Tables 1 and 2.

The comparison between results obtained with the SA-based heuristic, original TOC, revised TOC and ILP methods shows that SA can find the optimal solution in half of the cases and compares well with the cited methods in five problems (5, 6, 7, 9 and 10). For large-scale problems, SA is compared with TS and GA meta-heuristic methods. Table 2 shows total throughput and CPU times. Except for the problem number 1, the solutions obtained by SA are better than those of TS and GA.

CONCLUSION

There are several published methods which have solved the product mix problem, which are, however, mostly devoted to small-scale problems. Furthermore, some of them can take a tremendous amount of computational time. In reality, in firms, both the number of products

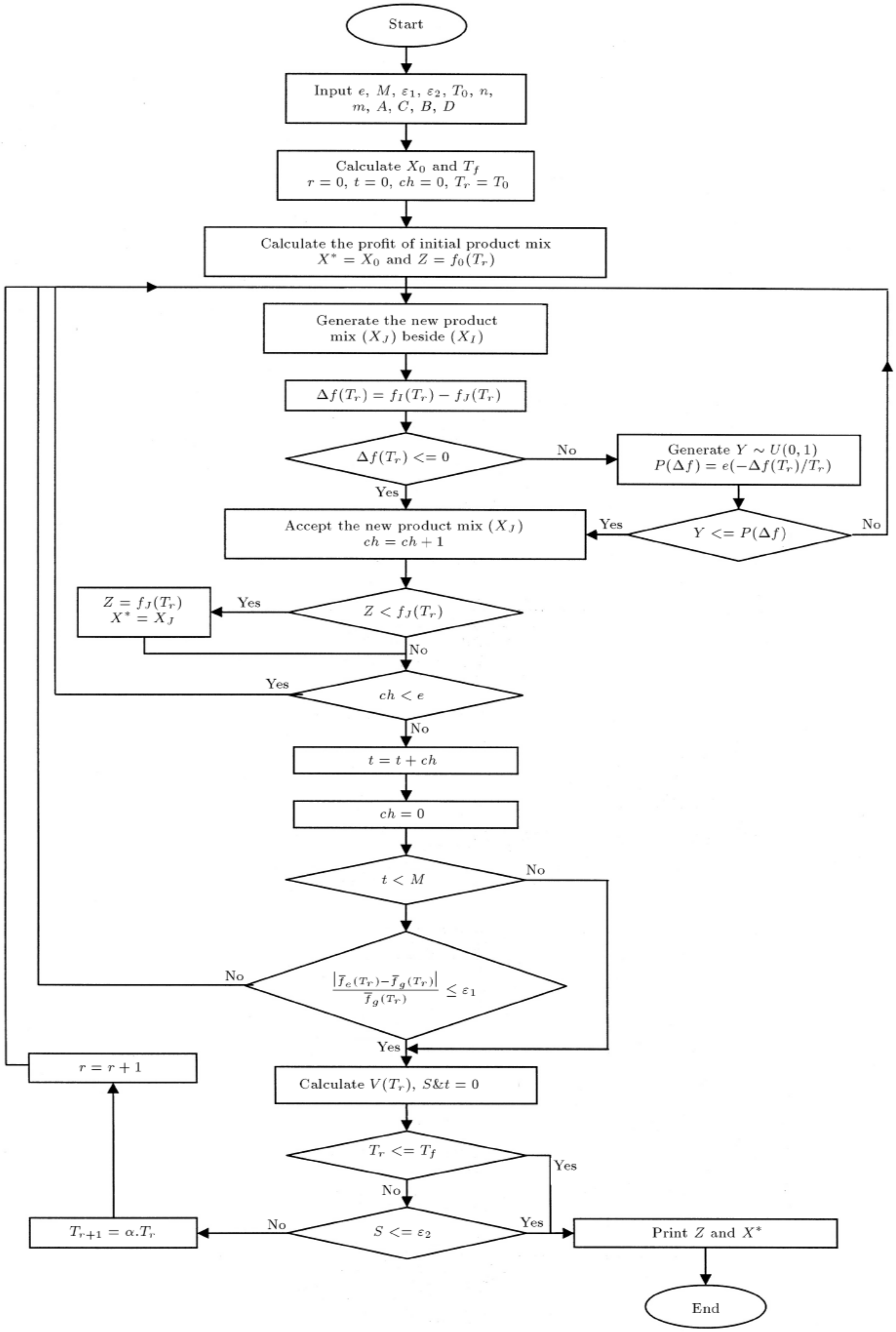

Figure 1. Flowchart of SA for product mix problem.

Table 1. Result of small-scale problems.

No.	Problem		TOC	R-TOC	ILP	TS	GA	SA
1	Plenert, 1993 [3]	(4 × 2)	4860	4875	4875	4875	4875	4875
2	Lubbe and Finch, 1992 [10]	(4 × 2)	6300	–	6300	–	6300	6300
3	Lee and Plenert, 1993 [4]	(4 × 3)	6310	6444	6444	6420	6420	6444
4	Lee and Plenert, 1993 [4]	(4 × 3)	6460	6775	6775	6715	6750	6775
5	Lubbe and Finch, 1992 [10]	(4 × 4)	6600	6600	6600	6660	6568	6529
6	Lubbe and Finch, 1992 [10]	(4 × 4)	9110	9110	9110	8955	9002	9074
7	Patterson, 1992 [2]	(4 × 5)	8872	8872	9050	8392	8888	8698
8	Plenert, 1993 [3]	(5 × 4)	14450 N.F.	14370	14370	14165	14370	14370
9	Fredendall & lea, 1997 [5]	(6 × 5)	2325	2230	2230	2221	–	2229
10	Hsu and Chung, 1998 [7]	(7 × 4)	14100 N.F.	–	11860	–	11860	11840

Table 2. Result of large-scale problems.

No.	No. of Products	No. of Resources	TS		GA		SA	
			Objective Function	Time (s)	Objective Function	Time (s)	Objective Function	Time (s)
1	10	5	7300	0.05	8868	0.72	7982	0.5
2	10	10	12987	1.76	13642	1.10	13648	0.5
3	15	10	12633	2.31	13014	6.37	13063	0.8
4	15	25	31971	32.68	29221	3.62	34650	0.9
5	20	25	36837	41.25	39794	3.95	40365	1.7
6	20	50	70335	66.13	79378	6.59	83203	2.5

to be produced and the number of machines to be used are high.

Meta heuristic methods are able to solve large-scale problems. Therefore, the Tabu-Search (TS) method and the Genetic Algorithm (GA) have been used to solve product mix problems. In this paper, the Simulated Annealing (SA) method has been employed for both small-scale and large-scale product mix problems. The results of SA, in most cases, are as good as those of TS and GA. Also, in some cases, the SA results were better. Future studies can be done on using uncertainty for modeling the product mix. Furthermore, the discovery of other methods for finding initial solutions may be useful.

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