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Research Note

The Effect of Geometric Configuration on Hydrodynamic Characteristics Through Wavy Cross-Sectional Channels

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In this paper, finite Fourier transform is used to obtain fully developed hydrodynamic characteristics of Newtonian fluids flowing inside channels with wavy cross sections. The velocity profiles and product of friction factor and Reynolds number are obtained for different geometry configurations. The results indicate that the influence of the geometric configuration of the duct (channel height, vertical distance of two wavy plates and number of waves) on the fluid flow characteristics are significant.

INTRODUCTION

Channels with non-circular cross sections have become increasingly important since these channels are being employed as compact heat exchangers. Laminar Newtonian flow and heat transfer in circular and non-circular cross sectional ducts have been surveyed extensively by Shah and London [1] and Shah and Bhatti [2]. Trupp and Lau [3] numerically determined the average Nusselt number of Newtonian fluids flowing through circular sector ducts with isothermal walls. Lei and Trupp [4,5] have analyzed the flow and heat transfer characteristics of fully developed laminar flow inside circular sector cross sectional channels. Ben-Ali et al. [6] employed the finite difference numerical method to solve the momentum and energy equations of Newtonian flow through circular sector ducts. Soliman et al. [7] used a linearization technique and the finite difference method to solve laminar flow problems in the entrance region of circular sector channels. Etemad [8,9,10] investigated the effect of sharp corners of the channel on flow and heat transfer behaviors. Recently Etemad and Bakhtiari [11] have obtained general equations for calculating pressure drop and Nusselt number using an equivalent diameter, the number of imaginary circles inside geometry and the diameter of surrounded circles.

The objective of the present analytical study is to investigate the characteristics of fully developed laminar flow in channels with wavy cross sections. The hydrodynamic characteristics are obtained for a wide range of geometric parameters. The cross sections with wavy parallel plates are used in different industries, e.g., power plants as air preheaters.

PROBLEM STATEMENTS

The flow domain and the coordinate system are shown in Figures 1 and 2 for ducts with cross section of one and two waves. For laminar Newtonian steady and fully developed flow subject to constant fluid properties, the governing momentum equation is:

$$\mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\partial p}{\partial x}=0,$$

$$\frac{\partial p}{\partial x}=\frac{\Delta p}{L}=c, \tag{1}$$

where, μ, u, p, L and c are viscosity, axial velocity, static pressure, length of channel and a constant, respectively. In order to render the dimensionless requisite governing equations, the following parameters are defined:

$$Y=\frac{y}{H},\ Z=\frac{z}{H},\ U=\frac{u}{\frac{1}{\mu}\left(-\frac{dp}{dx}\right)H^2/32}. \tag{2}$$

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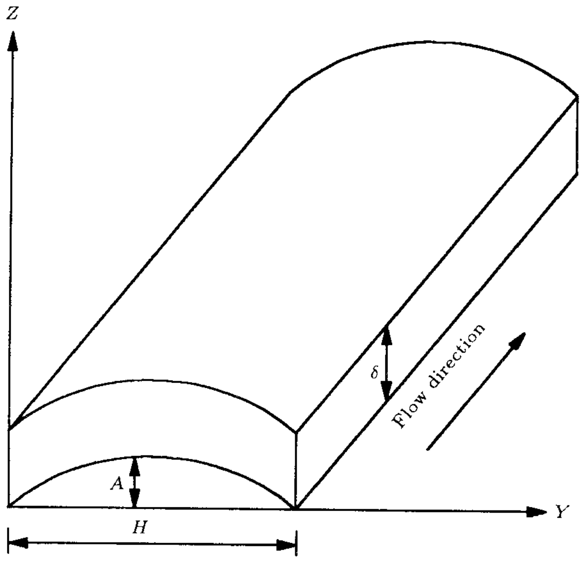

Figure 1. Geometry and coordinate system for one-wave cross section.

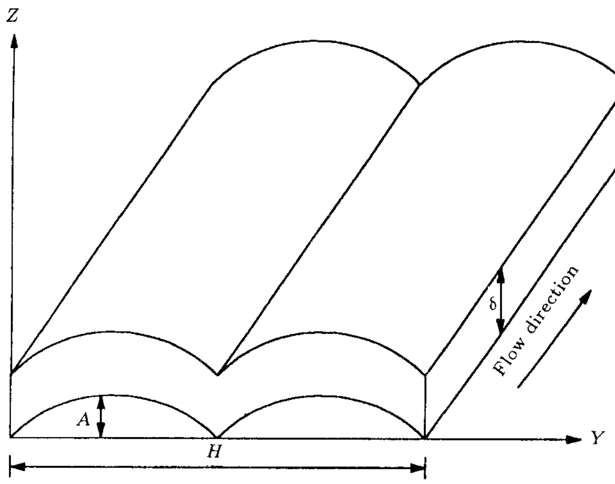

Figure 2. Geometry and coordinate system for two-wave cross section.

Introducing these parameters, the momentum differential equation and related boundary conditions can be expressed in dimensionless form as:

$$\frac{\partial^2 U}{\partial Y^2}+\frac{\partial^2 U}{\partial Z^2}=-32,$$
$$Y=0,\quad U=0,\quad Y=1,\quad U=0,$$
$$Z=\alpha\sin(k\pi Y),\quad U=0,$$
$$Z=\alpha\sin(k\pi Y)+\beta,\quad U=0, \tag{3}$$

where $\alpha=\frac{A}{H}$ and $\beta=\frac{\delta}{H}$.

A solution for dimensionless momentum equation with appropriate boundary conditions (Equation 3) is developed using the finite Fourier transform, resulting in:

$$\begin{aligned} U=&\sum_{n=1}^{\infty}\frac{32\times 2[(-1)^n-1]}{\lambda_n^3 e^{\alpha\lambda_n \sin k\pi Y}}\\ &\sin\lambda_n Y\frac{e^{\lambda_n z}-e^{2\alpha\lambda_n\sin k\pi Y-\lambda_n Z}}{e^{\lambda_n\beta}-e^{-\lambda_n\beta}}\\ &+\frac{e^{\lambda_n Z}-e^{2\alpha\lambda_n\sin k\pi Y+2\lambda_n\beta-\lambda_n Z}}{1-e^{2\lambda_n\beta}}\\ &-16Y^2+16Y, \end{aligned}\tag{4}$$

where $\lambda_n = n\pi$. The product of Fanning friction factor and Reynolds number can be obtained from the following equation:

$$f.Re=\frac{16D_h^2}{H^2\bar{U}},\quad \bar{U}\frac{\int_0^1\int_{\alpha\sin(k\pi Y)}^{\alpha\sin(k\pi Y)+\beta}UdA}{\int_0^1\int_{\alpha\sin(k\pi Y)}^{\alpha\sin(k\pi Y)+\beta}dA}, \tag{5}$$

where $D_h=\frac{4S}{P}$. S and P are cross section and perimeter of the channel given by:

$$S=\int_0^H(A\sin k\pi Y+\delta)dy-\int_0^H(A\sin k\pi Y),$$
$$P=2\int_0^H(\sqrt{1+z'^2})dy+2\delta, \tag{6}$$

z' is the derivative of sinusoidal function.

RESULTS AND DISCUSSION

Table 1 contains the product of friction factor and Reynolds number ($f.Re$) obtained in the present investigation and the available results in the literature for $\alpha=0$ (rectangular ducts). The comparison shows very good agreement which supports the high accuracy of the present study. Figures 3 to 5 illustrate $f.Re$ versus β for different values of α when the number of wave (K) is 1, 5 and 10 respectively. Based on the results, increasing α at constant β causes a decrease in the value of $f.Re$. For example when $\beta=1$, increasing α from 0 to 1 results in reduction of $f.Re$ from 14.227 to 5.210, from 14.227 to 0.462 and from 14.227 to 0.128 for $K=1$, $K=5$ and $K=10$, respectively. Therefore, changing α has a strong effect

Table 1. Comparison of the product of friction factor and Reynolds number obtained in the present investigation and the available results in the literature.

	$f.Re$		
Rectangular Ducts	$\beta=0.2$	$\beta=0.5$	$\beta=1.0$
Shah and London [1]	19.074	15.577	14.220
Etemad [11]	19.077	15.570	14.234
Present Investigation	19.070	15.548	14.227

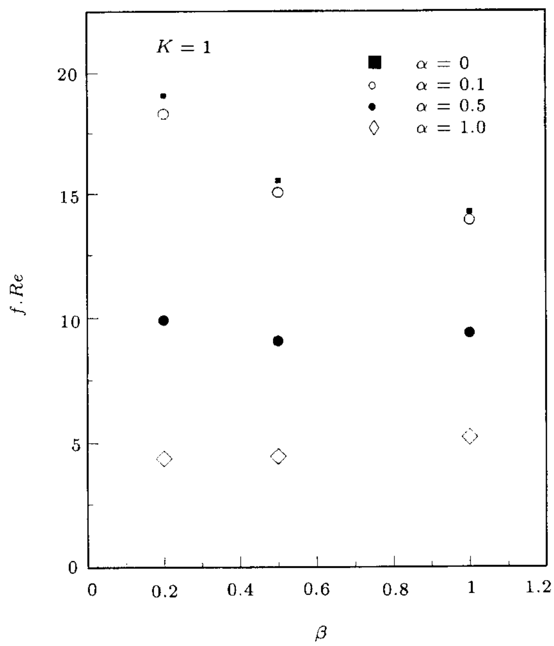

Figure 3. The product of friction factor and Reynolds number ($f.Re$) for $K = 1$ and different values of α and β.

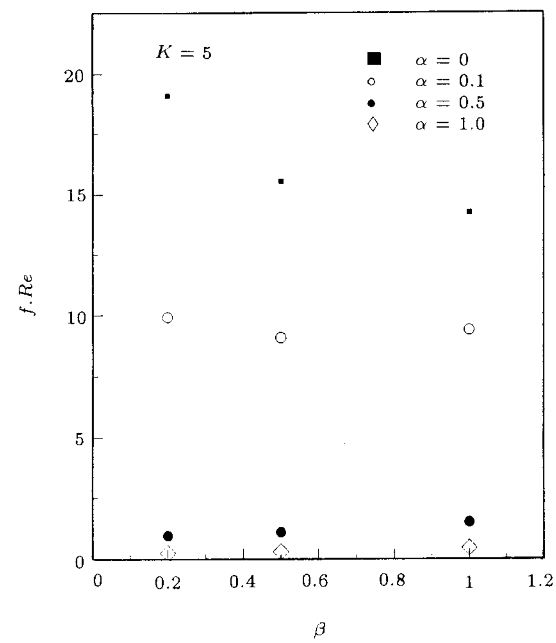

Figure 4. The product of friction factor and Reynolds number ($f.Re$) for $K = 5$ and different values of α and β.

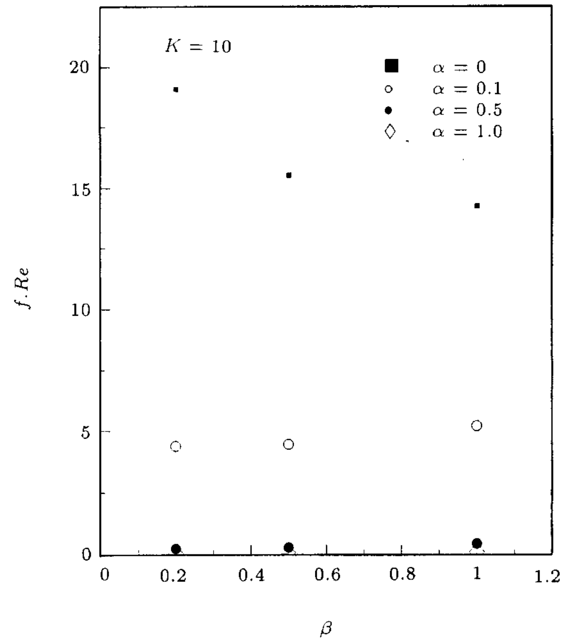

Figure 5. The product of friction factor and Reynolds number ($f.Re$) for $K = 10$ and different values of α and β.

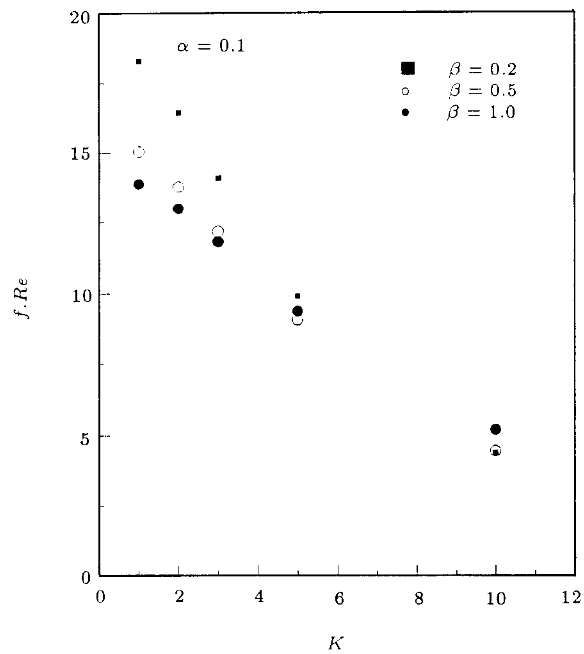

Figure 6. The product of friction factor and Reynolds number ($f.Re$) for $\alpha = 0.1$ and different values of K and β.

on hydrodynamic characteristics of the channel with higher K values.

Furthermore, varying β at a constant α has an appreciable effect on the friction factor. For single wave cross section, when $\alpha < 0.5$, decreasing β increases $f.Re$ but for $\alpha > 0.5$ the inverse effect is observed. The corresponding value of α for $K = 5$ is 0.1 (Figure 4).

Figures 6 to 8 show $f.Re$ versus the number of waves (K) for different values of β and $\alpha = 0.1$ to $\alpha = 1$. The effect of K on flow behavior is noticeable. Increasing K causes a reduction of the product of friction factor and Reynolds number for all values of $\alpha = 0.1$ to $\alpha = 1$. For example, increasing the number of waves from one to ten for $\beta = 1$ and $\alpha = 1$ decreases the product of friction factor and Reynolds number from 5.120 to 0.128.

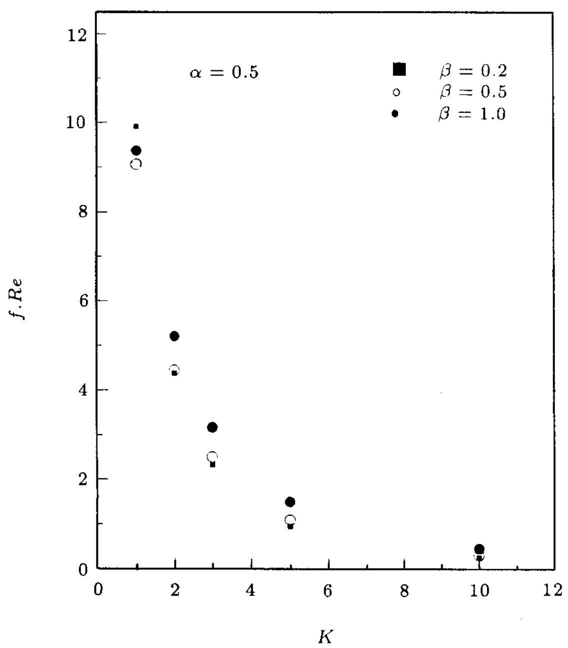

Figure 7. The product of friction factor and Reynolds number ($f.Re$) for $\alpha = 0.5$ and different values of K and β.

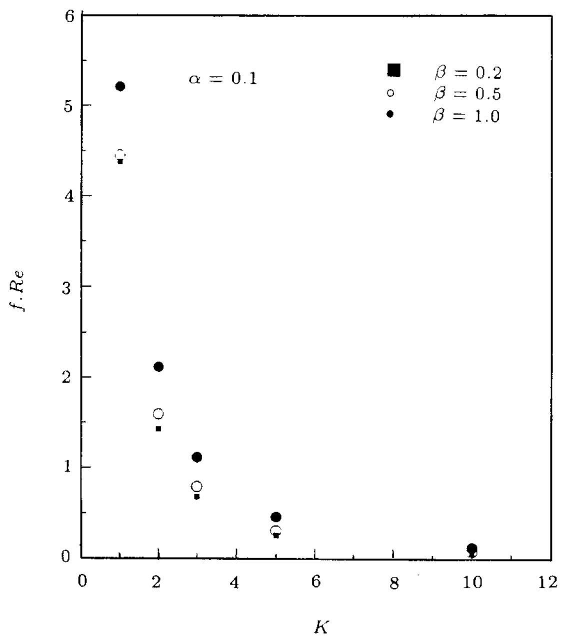

Figure 8. The product of friction factor and Reynolds number ($f.Re$) for $\alpha = 1.0$ and different values of K and β.

CONCLUSIONS

Finite Fourier transform is used to solve the dimensionless momentum equation for steady laminar flow under hydrodynamically developed conditions of Newtonian fluids passing through channels with two vertical and two wavy sinusoidal parallel plates. In the analysis, the effects of geometry variables such as α (dimensionless height of the channel), β (dimensionless vertical distance between two wavy plates) and K (number of waves at cross section) on hydrodynamic characteristics were considered. Results indicate the strong effect of geometry configuration on the product of friction factor and Reynolds number. When α and β are constant increasing the number of waves (K) decreases the product of friction factor and Reynolds number. Also, for specific K and constant β increasing α results in reduction of $f.Re$.

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