Scientia Iranica, Vol. 8, No. 2, pp 149–157

SCIENTIA
IRANICA

Research Note

Thermoelastic Buckling of Orthotropic Plates Based on Higher-Order Displacement Field

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In this paper, nonlinear higher-order strain-displacement relations for thin orthotropic plates are considered and substituted into the potential energy function for thermoelastic loadings. Euler equations are, then, applied to the functional of energy and the general thermoelastic equilibrium equations of thin orthotropic plate are obtained. The stability equations are, consequently, derived through the second variation of the potential energy function. The thermal loadings include the uniform temperature rise, axial temperature difference and the gradient temperature through the thickness. The thermoelastic buckling of a thin plate under these thermal loadings are investigated.

INTRODUCTION

Chen et al. [1] have studied thermal buckling of laminated composite rectangular plates based on a first-order displacement field, subjected to uniform temperature change. Displacement equations of equilibrium are used and Galerkin method is employed to determine the critical buckling temperature. Thermal buckling behavior of laminated plates subjected to a non-uniform temperature field, based on first-order displacement, is investigated using the finite-element method [2].

Tauchert has considered thermal buckling behaviors of rectangular antisymmetric angle-ply laminates and based on a first-order displacement field, has obtained the buckling thermal loads of a plate subjected to uniform temperature rise [3-5]. He derived exact solutions for the buckling temperature of simply supported thin [3] and thick [4,5] perfect plates. Chang and Leu [6] have also studied thermal buckling analysis of antisymmetric angle-ply laminates based on a higher-order displacement field. A higher-order deformation theory, which accounts for transverse shear and transverse normal strains, is derived for the thermal buckling analysis of antisymmetric angle-ply simply supported laminates subjected to uniform temperature rise. In their paper, linear strain-displacement relations were assumed to derive the equilibrium equations. To introduce the higher-order theory, they considered higher-order displacements of up to third order for u and v and second order for w.

In this article, the equilibrium and stability equations are obtained and employed to compute critical thermoelastic buckling loads of thin rectangular orthotropic plates. Equilibrium equations are obtained using nonlinear strain-displacement relations and stability equations are found through the force-summation method, based on the linearized strain-displacement relations. To introduce the higher-order theory, displacement components u and v are approximated by third order polynomials and lateral deflection w is approximated by just one term. Thus, while the transverse shear strains are nonzero, the normal lateral strain and stress are assumed to be zero. The eigenvalue solutions of the stability equations based on the pre-assumed displacement fields are obtained to present the thermal buckling loads. In addition to the thermal buckling load of the uniform temperature rise given by Chang and Leu [6], the gradient through the thickness thermal buckling load and the one edge direction buckling temperature difference are also derived and presented.

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ANALYSIS

For the purpose of analyzing, a thin plate of thickness h is considered. The general strain-displacement relations are [7]:

$$\begin{aligned}
\epsilon_{xx} &= u_{,x} + \frac{1}{2}w_{,x}^2, \\
\epsilon_{yy} &= v_{,y} + \frac{1}{2}w_{,y}^2, \\
\epsilon_{xy} &= u_{,y} + v_{,x} + w_{,x}w_{,y}, \\
\epsilon_{xz} &= u_{,z} + w_{,x} + w_{,x}'w_{,z}, \\
\epsilon_{yz} &= v_{,z} + w_{,y} + w_{,y}w_{,z},
\end{aligned} \tag{1}$$

where ϵ_{xx} and ϵ_{yy} are the normal strains and ϵ_{xy}, ϵ_{xz} and ϵ_{yz} are the shear strains, respectively. The indices x, y and z refer to the coordinate system. Also u, v and w are displacements in x, y and z directions, respectively and (,) indicates partial derivative.

Now, the following higher-order displacements are assumed [8]:

$$\begin{aligned}
u(x,y,z) &= u_0(x,y) + z\phi_0(x,y) + z^2\phi_0'(x,y) \\
&\quad + z^3\phi_0''(x,y), \\
v(x,y,z) &= v_0(x,y) + z\psi_0(x,y) + z^2\psi_0'(x,y) \\
&\quad + z^3\psi_0''(x,y), \\
w(x,y,z) &= w_0(x,y),
\end{aligned} \tag{2}$$

where u_0, v_0 and w_0 denote the displacements of a point (x,y) on the midplane and ϕ_0 and ψ_0 are the rotations of normals to midplane about the y and x axes, respectively. The functions $\acute{\phi}_0, \phi_0'', \acute{\psi}_0$ and ψ_0'' are determined using the conditions that transverse shear stresses, σ_{xz} and σ_{yz} vanish on the plate top and bottom surfaces. These are equivalent to $\epsilon_{xz} = \epsilon_{yz} = 0$ at $z = \pm\frac{h}{2}$, which yields [8]:

$$\begin{aligned}
\acute{\phi}_0 &= \acute{\psi}_0 = 0, \\
\phi_0'' &= \frac{-4}{3h^2}(\phi_0 + w_{0,x}), \\
\psi_0'' &= \frac{-4}{3h^2}(\psi_0 + w_{0,y}).
\end{aligned} \tag{3}$$

Using Conditions 3, Equation 2 is reduced to [8]:

$$\begin{aligned}
u &= u_0 + z\phi_0 - \frac{4z^3}{3h^2}(\phi_0 + w_{0,x}), \\
v &= v_0 + z\psi_0 - \frac{4z^3}{3h^2}(v_1 + w_{0,y}), \\
w &= w_0.
\end{aligned} \tag{4}$$

The strains associated with the displacements in Equation 2 are [8]:

$$\begin{pmatrix}\epsilon_{xx}\\ \epsilon_{yy}\\ \epsilon_{xy}\end{pmatrix} = \begin{pmatrix}\epsilon_{xx}^0\\ \epsilon_{yy}^0\\ \epsilon_{xy}^0\end{pmatrix} + z\begin{pmatrix}k_{xx}^0\\ k_{yy}^0\\ k_{xy}^0\end{pmatrix} + z^3\begin{pmatrix}k_{xx}^2\\ k_{yy}^2\\ k_{xy}^2\end{pmatrix},$$

$$\begin{pmatrix}\epsilon_{xz}\\ \epsilon_{yz}\end{pmatrix} = \begin{pmatrix}\epsilon_{xz}^0\\ \epsilon_{yz}^0\end{pmatrix} + z^2\begin{pmatrix}k_{xz}^1\\ k_{yz}^1\end{pmatrix}, \tag{5}$$

where:

$$\begin{pmatrix}k_{xx}^0\\ k_{yy}^0\\ k_{xy}^0\end{pmatrix} = \begin{pmatrix}\phi_{0,x}\\ \psi_{0,y}\\ \phi_{0,y} + \psi_{0,x}\end{pmatrix},$$

$$\begin{pmatrix}k_{xx}^2\\ k_{yy}^2\\ k_{xy}^2\end{pmatrix} = \begin{pmatrix}\frac{-4}{3h^2}(\phi_{0,x} + w_{0,xx})\\ \frac{-4}{3h^2}(\psi_{0,y} + w_{0,yy})\\ \frac{-4}{3h^2}(\phi_{0,y} + \psi_{0,x} + 2w_{0,xy})\end{pmatrix},$$

$$\begin{pmatrix}k_{xz}^1\\ k_{yz}^1\end{pmatrix} = \begin{pmatrix}\frac{-4}{h^2}(\phi_0 + w_{0,x})\\ \frac{-4}{h^2}(\psi_0 + w_{0,y})\end{pmatrix},$$

$$\begin{pmatrix}\epsilon_{xx}^0\\ \epsilon_{yy}^0\\ \epsilon_{xy}^0\end{pmatrix} = \begin{pmatrix}u_{0,x} + \frac{1}{2}w_{0,x}^2\\ v_{0,y} + \frac{1}{2}w_{0,y}^2\\ u_{0,y} + v_{0,x} + w_{0,x}w_{0,y}\end{pmatrix},$$

$$\begin{pmatrix}\epsilon_{xz}^0\\ \epsilon_{yz}^0\end{pmatrix} = \begin{pmatrix}\phi_0 + w_{0,x}\\ \psi_0 + w_{0,y}\end{pmatrix}. \tag{6}$$

The constitutive equations may be expressed in terms of stress and strain in the plate coordinates as [8]:

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{pmatrix} = \begin{pmatrix}\acute{Q}_{11} & \acute{Q}_{12} & \acute{Q}_{16}\\ \acute{Q}_{12} & \acute{Q}_{22} & \acute{Q}_{26}\\ \acute{Q}_{16} & \acute{Q}_{26} & \acute{Q}_{66}\end{pmatrix}\left(\begin{pmatrix}\epsilon_{xx}^0\\ \epsilon_{yy}^0\\ \epsilon_{xy}^0\end{pmatrix}\right.$$

$$\left. + z\begin{pmatrix}k_{xx}^0\\ k_{yy}^0\\ k_{xy}^0\end{pmatrix} + z^3\begin{pmatrix}k_{xx}^2\\ k_{yy}^2\\ k_{xy}^2\end{pmatrix} - T\begin{pmatrix}\alpha_{xx}\\ \alpha_{yy}\\ \alpha_{xy}\end{pmatrix}\right),$$

$$\begin{pmatrix}\sigma_{xz}\\ \sigma_{yz}\end{pmatrix} = \begin{pmatrix}\acute{Q}_{44} & \acute{Q}_{45}\\ \acute{Q}_{45} & \acute{Q}_{55}\end{pmatrix}\left(\begin{pmatrix}\epsilon_{xz}^0\\ \epsilon_{yz}^0\end{pmatrix}\right.$$

$$\left. + z^2\begin{pmatrix}k_{xz}^1\\ k_{yz}^1\end{pmatrix} - T\begin{pmatrix}\alpha_{xz}\\ \alpha_{yz}\end{pmatrix}\right), \tag{7}$$

in which T is the absolute temperature and α_{ij} are linear thermal expansion coefficients. $\acute{Q}_{ij}$ are the plane-stress-reduced elastic constants in the material axes of the layer [8]. The stress resultants

$N_{ij}, M_{ij}, P_{ij}, Q_{ij}$, and R_{ij} are [8]:

$$\left(\begin{pmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{pmatrix}, \begin{pmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{pmatrix}, \begin{pmatrix} P_{xx} \\ P_{yy} \\ P_{xy} \end{pmatrix} \right) == \int_{\frac{-h}{2}}^{\frac{h}{2}} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} (1, z, z^3) dz,$$

$$\left(\begin{pmatrix} Q_{xz} \\ Q_{yz} \end{pmatrix}, \begin{pmatrix} R_{xz} \\ R_{yz} \end{pmatrix} \right) = \int_{\frac{-h}{2}}^{\frac{h}{2}} \begin{pmatrix} \sigma_{xz} \\ \sigma_{yz} \end{pmatrix} (1, z^2) dz. \qquad (8)$$

Substituting Equation 7 into Equation 8, the stress resultants are obtained as [8]:

$$\begin{pmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xx} \\ \epsilon^0_{yy} \\ \epsilon^0_{xy} \end{pmatrix} + \begin{pmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{pmatrix} \begin{pmatrix} k^0_{xx} \\ k^0_{yy} \\ k^0_{xy} \end{pmatrix} + \begin{pmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{pmatrix} \begin{pmatrix} k^2_{xx} \\ k^2_{yy} \\ k^2_{xy} \end{pmatrix} - \begin{pmatrix} N^T_{xx} \\ N^T_{yy} \\ N^T_{xy} \end{pmatrix},$$

$$\begin{pmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xx} \\ \epsilon^0_{yy} \\ \epsilon^0_{xy} \end{pmatrix} + \begin{pmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{pmatrix} \begin{pmatrix} k^0_{xx} \\ k^0_{yy} \\ k^0_{xy} \end{pmatrix} + \begin{pmatrix} F_{11} & F_{12} & F_{16} \\ F_{12} & F_{22} & F_{26} \\ F_{16} & F_{26} & F_{66} \end{pmatrix} \begin{pmatrix} k^2_{xx} \\ k^2_{yy} \\ k^2_{xy} \end{pmatrix} - \begin{pmatrix} M^T_{xx} \\ M^T_{yy} \\ M^T_{xy} \end{pmatrix},$$

$$\begin{pmatrix} P_{xx} \\ P_{yy} \\ P_{xy} \end{pmatrix} = \begin{pmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xx} \\ \epsilon^0_{yy} \\ \epsilon^0_{xy} \end{pmatrix} + \begin{pmatrix} F_{11} & F_{12} & F_{16} \\ F_{12} & F_{22} & F_{26} \\ F_{16} & F_{26} & F_{66} \end{pmatrix} \begin{pmatrix} k^0_{xx} \\ k^0_{yy} \\ k^0_{xy} \end{pmatrix} + \begin{pmatrix} H_{11} & H_{12} & H_{16} \\ H_{12} & H_{22} & H_{26} \\ H_{16} & H_{26} & H_{66} \end{pmatrix} \begin{pmatrix} k^2_{xx} \\ k^2_{yy} \\ k^2_{xy} \end{pmatrix} - \begin{pmatrix} P^T_{xx} \\ P^T_{yy} \\ P^T_{xy} \end{pmatrix},$$

$$\begin{pmatrix} Q_{xz} \\ Q_{yz} \end{pmatrix} = \begin{pmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xz} \\ \epsilon^0_{yz} \end{pmatrix} + \begin{pmatrix} D_{44} & D_{45} \\ D_{45} & D_{55} \end{pmatrix} \begin{pmatrix} k^1_{xz} \\ k^1_{yz} \end{pmatrix} - \begin{pmatrix} Q^T_{xz} \\ Q^T_{yz} \end{pmatrix},$$

$$\begin{pmatrix} R_{xz} \\ R_{yz} \end{pmatrix} = \begin{pmatrix} D_{44} & D_{45} \\ D_{45} & D_{55} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xz} \\ \epsilon^0_{yz} \end{pmatrix} + \begin{pmatrix} F_{44} & F_{45} \\ D_{45} & D_{55} \end{pmatrix} \begin{pmatrix} k^1_{xz} \\ k^1_{yz} \end{pmatrix} - \begin{pmatrix} R^T_{xz} \\ R^T_{yz} \end{pmatrix}, \qquad (9)$$

where $A_{kl}, B_{kl}, \ldots$ are the plate stiffnesses defined by [8]:

$$(A_{kl}, B_{kl}, D_{kl}, E_{kl}, F_{kl}, H_{kl}) = \int_{\frac{-h}{2}}^{\frac{h}{2}} \acute{Q}_{kl}(1, z, z^2, z^3, z^4, z^6) dz$$

$$k, l = 1, 2, 6 \,,$$

$$(A_{kl}, D_{kl}, F_{kl}) = \int_{\frac{-h}{2}}^{\frac{h}{2}} \acute{Q}_{kl}(1, z^2, z^4) dz \qquad k, l = 4, 5 \,. \qquad (10)$$

The total potential energy function is [9]:

$$U = \frac{1}{2} \int \int \int \{\sigma\}^T (\{\epsilon\} - T\{\alpha\}) dx dy dz. \qquad (11)$$

Upon substitution of Equations 5 and 7 into Equation 11, the total potential energy function of an orthotropic plate is obtained as:

$$\begin{aligned} U = & \frac{1}{2} \int \int \int_{\frac{-h}{2}}^{\frac{h}{2}} (\{\epsilon^0_{ij}\}^T + z\{k^0_{ij}\}^T + z^3\{k^2_{ij}\}^T \\ & - T\{\alpha_{ij}\}^T)_{i,j=x,y} [\acute{Q}]^T (\{\epsilon^0_{ij}\} \\ & + z\{k^0_{ij}\} + z^3\{k^2_{ij}\} - T\{\alpha_{ij}\})_{i,j=x,y} dx dy dz \\ & + \frac{1}{2} \int \int \int_{\frac{-h}{2}}^{\frac{h}{2}} (\{\epsilon^0_{ij}\}^T + z^2\{k^1_{ij}\}^T \\ & - T\{\alpha_{ij}\}^T)_{i=x,y;j=z} [\acute{Q}]^T (\{\epsilon^0_{ij}\} \\ & + z^2\{k^1_{ij}\} - T\{\alpha_{ij}\})_{i=x,y;j=z} dx dy dz. \end{aligned} \qquad (12)$$

Substituting the left-hand side of Equations 6 through 11 into 12 and using Euler equations [7], the equilibrium equations for general thin orthotropic rectangular plate are obtained as [8]:

$$N_{xx,x} + N_{xy,y} = 0,$$

$$N_{xy,x} + N_{yy,y} = 0,$$

$$\begin{aligned} &Q_{xz,x} + Q_{yz,y} - \frac{4}{h^2}(R_{xz,x} + R_{yz,y}) \\ &\quad + \frac{4}{3h^2}(P_{xx,xx} + 2P_{xy,xy} + P_{yy,yy}) \\ &\quad + (w_{0,x}N_{xx})_{,x} + (w_{0,y}N_{xy})_{,x} \\ &\quad + (w_{0,y}N_{yy})_{,y} + (w_{0,x}N_{xy})_{,y} = 0, \end{aligned}$$

$$\begin{aligned} &M_{xx,x} + M_{xy,y} - Q_{xz} + \frac{4}{h^2}R_{xz} \\ &\quad - \frac{4}{3h^2}(P_{xx,x} + P_{xy,y}) = 0, \end{aligned}$$

$$\begin{aligned} &M_{xy,x} + M_{yy,y} - Q_{yz} + \frac{4}{h^2}R_{yz} \\ &\quad - \frac{4}{3h^2}(P_{xy,x} + P_{yy,y}) = 0. \end{aligned} \tag{13}$$

Stability equations of thin orthotropic rectangular plates are derived using the force summation method. Now, it is assumed that the state of equilibrium of a general orthotropic rectangular plate under load is defined in terms of the displacement components $\bar{u_0}$, $\bar{v_0}$, $\bar{w_0}$, ϕ_0 and $\bar{\psi_0}$. The displacement components of a neighboring state of stable equilibrium differs by u_1, v_1, w_1, ϕ_1, ψ_1 with respect to the equilibrium position. Thus, the total displacements of a neighboring state are:

$$u_0 = \bar{u_0} + u_1,$$

$$v_0 = \bar{v_0} + v_1,$$

$$w_0 = \bar{w_0} + w_1,$$

$$\phi_0 = \bar{\phi_0} + \phi_1,$$

$$\psi_0 = \bar{\psi_0} + \psi_1. \tag{14}$$

Similarly, the stress resultant components of a neighboring state may be related to the state of equilibrium as:

$$N_{ij} = N_{ij0} + \Delta N_{ij},$$

$$M_{ij} = M_{ij0} + \Delta M_{ij},$$

$$P_{ij} = P_{ij0} + \Delta P_{ij},$$

$$Q_{ij} = Q_{ij0} + \Delta Q_{ij},$$

$$R_{ij} = R_{ij0} + \Delta R_{ij}. \tag{15}$$

Now, stability equations might be obtained by substituting Equations 14 and 15 into 13, in which nonlinear terms with subscript (1) are ignored due to their small values compared with the linear terms. The remaining terms form the stability equations of a thin orthotropic rectangular plate under general load are as follows [6]:

$$N_{xx1,x} + N_{xy1,y} = 0,$$

$$N_{xy1,x} + N_{yy1,y} = 0,$$

$$\begin{aligned} &Q_{xz1,x} + Q_{yz1,y} - \frac{4}{h^2}(R_{xz1,x} + R_{yz1,y}) \\ &\quad + \frac{4}{3h^2}(P_{xx1,xx} + 2P_{xy1,xy} + P_{yy1,yy}) \\ &\quad + (w_{1,x}N_{xx0})_{,x} + (w_{1,y}N_{xy0})_{,x} \\ &\quad + (w_{1,y}N_{yy0})_{,y} \\ &\quad + (w_{1,x}N_{xy0})_{,y} = 0, \end{aligned}$$

$$\begin{aligned} &M_{xx1,x} + M_{xy1,y} - Q_{xz1} + \frac{4}{h^2}R_{xz1} \\ &\quad - \frac{4}{3h^2}(P_{xx1,x} + P_{xy1,y}) = 0, \end{aligned}$$

$$\begin{aligned} &M_{xy1,x} + M_{yy1,y} - Q_{yz1} + \frac{4}{h^2}R_{yz1} \\ &\quad - \frac{4}{3h^2}(P_{xy1,x} + P_{yy1,y}) = 0. \end{aligned} \tag{16}$$

Denoting the prebuckling modes by N_{ij0}, M_{ij0}, P_{ij0}, Q_{ij0} and R_{ij0} and the buckling modes by N_{ij1}, M_{ij1}, P_{ij1}, Q_{ij1} and R_{ij1}, it follows that [7]:

$$\begin{aligned} \begin{pmatrix} N_{xx0} \\ N_{yy0} \\ N_{xy0} \end{pmatrix} &= \begin{pmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xx0} \\ \epsilon^0_{yy0} \\ \epsilon^0_{xy0} \end{pmatrix} \\ &+ \begin{pmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{pmatrix} \begin{pmatrix} k^0_{xx0} \\ k^0_{yy0} \\ k^0_{xy0} \end{pmatrix} \\ &+ \begin{pmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{pmatrix} \begin{pmatrix} k^2_{xx0} \\ k^2_{yy0} \\ k^2_{xy0} \end{pmatrix} - \begin{pmatrix} N^T_{xx0} \\ N^T_{yy0} \\ N^T_{xy0} \end{pmatrix}, \end{aligned}$$

$$\begin{aligned} \begin{pmatrix} M_{xx0} \\ M_{yy0} \\ M_{xy0} \end{pmatrix} &= \begin{pmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xx0} \\ \epsilon^0_{yy0} \\ \epsilon^0_{xy0} \end{pmatrix} \\ &+ \begin{pmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{pmatrix} \begin{pmatrix} k^0_{xx0} \\ k^0_{yy0} \\ k^0_{xy0} \end{pmatrix} \\ &+ \begin{pmatrix} F_{11} & F_{12} & F_{16} \\ F_{12} & F_{22} & F_{26} \\ F_{16} & F_{26} & F_{66} \end{pmatrix} \begin{pmatrix} k^2_{xx0} \\ k^2_{yy0} \\ k^2_{xy0} \end{pmatrix} - \begin{pmatrix} M^T_{x0} \\ M^T_{y0} \\ M^T_{xy0} \end{pmatrix}, \end{aligned}$$

$$\begin{pmatrix} P_{xx0} \\ P_{yy0} \\ P_{xy0} \end{pmatrix} = \begin{pmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xx0} \\ \epsilon^0_{yy0} \\ \epsilon^0_{xy0} \end{pmatrix} + \begin{pmatrix} F_{11} & F_{12} & F_{16} \\ F_{12} & F_{22} & F_{26} \\ F_{16} & F_{26} & F_{66} \end{pmatrix} \begin{pmatrix} k^0_{xx0} \\ k^0_{yy0} \\ k^0_{xy0} \end{pmatrix} + \begin{pmatrix} H_{11} & H_{12} & H_{16} \\ H_{12} & H_{22} & H_{26} \\ H_{16} & H_{26} & H_{66} \end{pmatrix} \begin{pmatrix} k^2_{xx0} \\ k^2_{yy0} \\ k^2_{xy0} \end{pmatrix} - \begin{pmatrix} P^T_{x0} \\ P^T_{y0} \\ P^T_{xy0} \end{pmatrix},$$

$$\begin{pmatrix} Q_{xz0} \\ Q_{yz0} \end{pmatrix} = \begin{pmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xz0} \\ \epsilon^0_{yz0} \end{pmatrix} + \begin{pmatrix} D_{44} & D_{45} \\ D_{45} & D_{55} \end{pmatrix} \begin{pmatrix} k^1_{xz0} \\ k^1_{yz0} \end{pmatrix} - \begin{pmatrix} Q^T_{x0} \\ Q^T_{y0} \end{pmatrix},$$

$$\begin{pmatrix} R_{xz0} \\ R_{yz0} \end{pmatrix} = \begin{pmatrix} D_{44} & D_{45} \\ D_{45} & D_{55} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xz0} \\ \epsilon^0_{yz0} \end{pmatrix} + \begin{pmatrix} F_{44} & F_{45} \\ F_{45} & F_{55} \end{pmatrix} \begin{pmatrix} k^1_{xz0} \\ k^1_{yz0} \end{pmatrix} - \begin{pmatrix} Q^T_{x0} \\ Q^T_{y0} \end{pmatrix},$$

$$\begin{pmatrix} N_{xx1} \\ N_{yy1} \\ N_{xy1} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xx1} \\ \epsilon^0_{yy1} \\ \epsilon^0_{xy1} \end{pmatrix} + \begin{pmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{pmatrix} \begin{pmatrix} k^0_{xx1} \\ k^0_{yy1} \\ k^0_{xy1} \end{pmatrix} + \begin{pmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{pmatrix} \begin{pmatrix} k^2_{xx1} \\ k^2_{yy1} \\ k^2_{xy1} \end{pmatrix},$$

$$\begin{pmatrix} M_{xx1} \\ M_{yy1} \\ M_{xy1} \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xx1} \\ \epsilon^0_{yy1} \\ \epsilon^0_{xy1} \end{pmatrix} + \begin{pmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{pmatrix} \begin{pmatrix} k^0_{xx1} \\ k^0_{yy1} \\ k^0_{xy1} \end{pmatrix} + \begin{pmatrix} F_{11} & F_{12} & F_{16} \\ F_{12} & F_{22} & F_{26} \\ F_{16} & F_{26} & F_{66} \end{pmatrix} \begin{pmatrix} k^2_{xx1} \\ k^2_{yy1} \\ k^2_{xy1} \end{pmatrix},$$

$$\begin{pmatrix} P_{xx1} \\ P_{yy1} \\ P_{xy1} \end{pmatrix} = \begin{pmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xx1} \\ \epsilon^0_{yy1} \\ \epsilon^0_{xy1} \end{pmatrix} + \begin{pmatrix} F_{11} & F_{12} & F_{16} \\ F_{12} & F_{22} & F_{26} \\ F_{16} & F_{26} & F_{66} \end{pmatrix} \begin{pmatrix} k^0_{xx1} \\ k^0_{yy1} \\ k^0_{xy1} \end{pmatrix} + \begin{pmatrix} H_{11} & H_{12} & H_{16} \\ H_{12} & H_{22} & H_{26} \\ H_{16} & H_{26} & H_{66} \end{pmatrix} \begin{pmatrix} k^2_{xx1} \\ k^2_{yy1} \\ k^2_{xy1} \end{pmatrix},$$

$$\begin{pmatrix} Q_{xz1} \\ Q_{yz1} \end{pmatrix} = \begin{pmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xz1} \\ \epsilon^0_{yz1} \end{pmatrix} + \begin{pmatrix} D_{44} & D_{45} \\ D_{45} & D_{55} \end{pmatrix} \begin{pmatrix} k^1_{xz1} \\ k^1_{yz1} \end{pmatrix},$$

$$\begin{pmatrix} R_{xz1} \\ R_{yz1} \end{pmatrix} = \begin{pmatrix} D_{44} & D_{45} \\ D_{45} & D_{55} \end{pmatrix} \begin{pmatrix} \epsilon^0_{xz1} \\ \epsilon^0_{yz1} \end{pmatrix} + \begin{pmatrix} F_{44} & F_{45} \\ F_{45} & F_{55} \end{pmatrix} \begin{pmatrix} k^1_{xz1} \\ k^1_{yz1} \end{pmatrix}. \tag{17}$$

The linear strain-displacement relations for the deviation components reduce to:

$$\epsilon^0_{xx1} = u_{1,x}$$

$$k^0_{xx1} = \phi_{1,x} \qquad k^2_{xx1} = \frac{-4}{3h^2}(\phi_{1,x} + w_{1,xx})$$

$$\epsilon^0_{yy1} = v_{1,x}$$

$$k^0_{yy1} = \psi_{1,x} \qquad k^2_{yy1} = \frac{-4}{3h^2}(\psi_{1,x} + w_{1,yy})$$

$$\epsilon^0_{xy1} = u_{1,y} + v_{1,x}$$

$$k^0_{xx1} = \phi_{1,y} + \psi_{1,x} \qquad k^2_{xy1} = \frac{-4}{3h^2}(\phi_{1,y} + \psi_{1,x} + 2w_{1,xy}).$$

STABILITY EQUATIONS IN TERMS OF DISPLACEMENTS

For symmetric (about the midplane) cross-ply plates, the following plate stiffness coefficients are identically zero [8]:

$$B_{ij} = E_{ij} = 0$$

$$A_{16} = A_{26} = D_{16} = D_{26} = F_{16} = F_{26} = H_{16} = H_{26} = 0$$

$$A_{45} = D_{45} = F_{45} = 0. \tag{18}$$

Stability equations in terms of the displacement components are obtained by substitution of Equation 17 into 16 with the aid of Equations 18 and 19:

$$A_{11}u_{1,xx} + A_{12}v_{1,xy} + A_{66}(u_{1,yy} + v_{1,xy}) = 0,$$

$$A_{66}(u_{1,xy} + v_{1,xx}) + A_{12}u_{1,xy} + A_{22}v_{1,yy} = 0,$$

$$A_{44}(\phi_{1,x}+w_{1,xx})+D_{44}(\frac{-4}{h^2})(\phi_{1,x}+w_{1,xx})$$
$$+A_{55}(\psi_{1,y}+w_{1,yy})+D_{55}(\frac{-4}{h^2})(\psi_{1,y}+w_{1,yy})$$
$$+(\frac{-4}{h^2})\{D_{44}(\phi_{1,x}+w_{1,xx})+F_{44}(\frac{-4}{h^2})(\phi_{1,x}+w_{0,xx})$$
$$+D_{55}(\psi_{1,y}+w_{1,yy})+F_{55}(\frac{-4}{h^2})(\psi_{1,y}+w_{1,yy})\}$$
$$(\frac{4}{h^2})\{F_{11}\phi_{1,xxx}+F_{12}\psi_{1,xxy}$$
$$+H_{11}(\frac{-4}{3h^2})(\phi_{1,xxx}+w_{1,xxxx})$$
$$+H_{12}(\frac{-4}{3h^2})(\psi_{1,xxy}+w_{1,xxyy})+2F_{66}(\phi_{1,xyy}+$$
$$\psi_{1,xxy})+2H_{66}(\frac{-4}{3h^2})(\phi_{1,xyy}+$$
$$\psi_{1,xxy}+2w_{1,xxyy})$$
$$+F_{12}\phi_{1,xyy}+F_{22}\psi_{1,yyy}+$$
$$H_{12}(\frac{-4}{3h^2})(\phi_{1,xyy}+w_{1,xxyy}),$$
$$+H_{22}(\frac{-4}{3h^2})(\psi_{1,yyy}+w_{1,yyyy}+\frac{h^2}{4}w_{2,yyyy})\}$$
$$+(N_{xx0}w_{1,x}+N_{xy0}w_{1,y})_{,x}+(N_{yy0}w_{1,y}$$
$$+N_{xy0}w_{1,x})_{,y}=0,$$

$$D_{11}\phi_{1,xx}+D_{12}\psi_{1,xy}+F_{11}(\frac{-4}{3h^2})(\phi_{1,xx}+w_{1,xxx}$$
$$+\frac{h^2}{4}w_{2,xxx})+F_{12}(\frac{-4}{3h^2})$$
$$(\psi_{1,xy}+w_{1,xyy}+\frac{h^2}{4}w_{2,xyy})$$
$$+D_{66}(\phi_{1,yy}+\psi_{1,xy})+F_{66}(\frac{-4}{3h^2})(\phi_{1,yy}+\psi_{1,xy}$$
$$+2w_{1,xyy})-A_{44}(\phi_1+w_{1,x})-D_{44}(\frac{-4}{h^2})(\phi_1+w_{1,x})$$
$$+\frac{4}{h^2}\{D_{44}(\phi_1+w_{1,x})$$
$$+F_{44}(\frac{-4}{h^2})(\phi_1+w_{1,x})\}+\frac{-4}{3h^2}\{F_{11}\phi_{1,xx}+F_{12}\psi_{1,xy}$$
$$+H_{11}(\frac{-4}{3h^2})(\phi_{1,xx}+w_{1,xxx}+\frac{h^2}{4}w_{2,xxx})+H_{12}$$
$$(\frac{-4}{3h^2})(\psi_{1,xy}+w_{1,xyy})+F_{66}(\phi_{1,yy}+\psi_{1,xy})+H_{66}$$
$$(\frac{-4}{3h^2})(\phi_{1,yy}+\psi_{1,xy}+2w_{1,xyy}+\frac{h^2}{2}w_{2,xyy})\}=0,$$

$$D_{66}(\phi_{1,xy}+\psi_{1,xx})+F_{66}(\frac{-4}{3h^2})(\phi_{1,xy}+\psi_{1,xx}$$
$$+2w_{1,xxy}+\frac{h^2}{2}w_{2,xxy})+D_{12}\phi_{1,xy}+D_{22}\psi_{1,yy}$$
$$+F_{12}(\frac{-4}{3h^2})(\phi_{1,xy}+w_{1,xxy}+\frac{h^2}{4}w_{2,xxy})$$
$$+F_{22}(\frac{-4}{3h^2})(\psi_{1,yy}+w_{1,yyy})-A_{55}(\psi_1+w_{1,y})$$
$$-D_{55}(\frac{-4}{h^2})(\psi_1+w_{1,y})+\frac{4}{h^2}\{D_{55}(\psi_1+w_{1,y})$$
$$+F_{55}(\frac{-4}{h^2})(\psi_1+w_{1,y})\}-\frac{4}{3h^2}\{F_{66}(\phi_{1,xy}+\psi_{1,xx})$$
$$+H_{66}(\frac{-4}{3h^2})(\phi_{1,xy}+\psi_{1,xx}+2w_{1,xxy})+F_{12}\phi_{1,xy}$$
$$+F_{22}\psi_{1,yy}+H_{12}(\frac{-4}{3h^2})(\phi_{1,xy}+w_{1,xxy})$$
$$+H_{22}(\frac{-4}{3h^2})(\psi_{1,yy}+w_{1,yyy})\}=0. \qquad (19)$$

These equations are related to the thermal forces through the prebuckling terms, such as N_{xx0}, through Equation 17.

In the next section, three cases of thermal buckling are discussed and the critical temperatures are calculated. The edges are assumed to be simply supported and also displacements in x and y directions are prevented, therefore, the boundary conditions are [8]:

$$w_1(x,0)=w_1(x,b)=w_1(0,y)=w_1(a,y)=0,$$
$$P_{yy1}(x,0)=P_{yy1}(x,b)=P_{xx1}(0,y)=P_{xx1}(a,y)=0,$$
$$M_{yy1}(x,0)=M_{yy1}(x,b)=M_{xx1}(0,y)=M_{xx1}(a,y)=0$$
$$u_1(x,0)=u_1(x,b)=v_1(0,y)=v_1(a,y)=0,$$
$$\phi_1(x,0)=\phi_1(x,b)=\psi_1(0,y)=\psi_1(a,y)=0, \qquad (20)$$

where a and b are the plate dimensions. The displacement components for a rectangular orthotropic plate satisfying the simply supported edge conditions are considered as [8]:

$$w_1=w_{1mn}\sin\alpha_m x\sin\beta_n y,$$
$$u_1=u_{1mn}\cos\alpha_m x\sin\beta_n y,$$
$$v_1=v_{1mn}\sin\alpha_m x\cos\beta_n y,$$
$$\phi_1=\phi_{1mn}\cos\alpha_m x\sin\beta_n y,$$
$$\psi_1=\psi_{1mn}\sin\alpha_m x\cos\beta_n y, \qquad (21)$$

where $\alpha_m=\frac{m\pi}{a}$ and $\beta_n=\frac{n\pi}{b}$. Substitution of these assumed solutions into the five stability Equation 20 and setting the determinant of the matrix of the coefficients equal to zero leads to the critical temperature.

CRITICAL UNIFORM TEMPERATURE RISE

Consider a rectangular orthotropic plate of dimensions a, b and thickness h. The initial uniform temperature of the plate is assumed to be T_i. The temperature is uniformly raised to a final value of T_f such that the plate buckles. To find the critical $\Delta T = T_f - T_i$, the prebuckling stresses are [7]:

$$N_{ij0} = -N_{ij0}^T = -\int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{kl}\acute{\alpha}_{ij}\Delta T dz = -A_{ij}\acute{\alpha}_{ij}\Delta T., \tag{22}$$

or:

$$\begin{aligned} N_{xx0} &= -N_{xx0}^T = -(A_{11}\alpha_{xx} + A_{12}\alpha_{yy})\Delta T, \\ N_{yy0} &= -N_{yy0}^T = -(A_{12}\alpha_{xx} + A_{22}\alpha_{yy})\Delta T, \\ N_{xy0} &= -N_{xy0}^T = -A_{66}\alpha_{xy}\Delta T = 0. \end{aligned} \tag{23}$$

Substituting Equations 22 and 24 into Equation 20 yields a system of five homogeneous equations for w_{1mn}, u_{1mn}, v_{1mn}, ϕ_{1mn} and ψ_{1mn}, i.e.,

$$[K_{ij}]\begin{pmatrix} u_{1mn} \\ v_{1mn} \\ w_{1mn} \\ \phi_{1mn} \\ \psi_{1mn} \end{pmatrix} = 0, \tag{24}$$

in which K_{ij} is a symmetric matrix with the following components [6]:

$$\begin{aligned} K_{11} &= \alpha_m^2 A_{11} + \beta_n^2 A_{66}, \\ K_{12} &= \alpha_m\beta_n(A_{12} + A_{66}), \\ K_{13} &= 0, \\ K_{14} &= 0, \\ K_{15} &= 0, \\ K_{22} &= \beta_n^2 A_{22} + \alpha_m^2 A_{66}, \\ K_{23} &= 0, \\ K_{24} &= 0, \\ K_{25} &= 0, \\ K_{33} &= \alpha_m^2 A_{44} + \beta_n^2 A_{55} - \frac{8}{h^2}(\alpha_m^2 D_{44} + \beta_n^2 D_{55}) \\ &\quad + (\frac{4}{h^2})^2(\alpha_m^2 F_{44} + \beta_n^2 F_{55}) + (\frac{4}{3h^2})^2 \\ &\quad [\alpha_m^2 H_{11} + 2\alpha_m^2\beta_n^2(H_{12} + 2H_{66}) + \beta_n^4 H_{22}] \\ &\quad - \{(A_{11}\alpha_{xx} + A_{12}\alpha_{yy})\alpha_m^2 \\ &\quad + (A_{12}\alpha_{xx} + A_{22}\alpha_{yy})\beta_n^2\}\Delta T, \\ K_{34} &= \alpha_m A_{44} - \frac{8}{h^2}\alpha_m D_{44} - \frac{4}{3h^2} \\ &\quad [\alpha_m^3 F_{11} + \alpha_m\beta_n^2(F_{12} + 2F_{66})] + (\frac{4}{h^2})^2\alpha_m F_{44} \\ &\quad + (\frac{4}{3h^2})^2[\alpha_m^3 H_{11} + \alpha_m\beta_n^2(H_{12} + 2H_{66})] \\ K_{35} &= \beta_n A_{55} - \frac{8}{h^2}\beta_n D_{55} - \frac{4}{3h^2} \\ &\quad [\beta_n^3 F_{22} + \alpha_m^2\beta_n(F_{12} + 2F_{66})] + (\frac{4}{h^2})^2\beta_n F_{55} \\ &\quad + (\frac{4}{3h^2})^2[\beta_n^3 H_{22} + \alpha_m^2\beta_n(H_{12} + 2H_{66})] \\ &\quad - \{(D_{11}\alpha_{xx} + D_{12}\alpha_{yy})\alpha_m^2 \\ &\quad + (D_{12}\alpha_{xx} + D_{22}\alpha_{yy})\beta_n^2\}\Delta T \\ K_{44} &= A_{44} + \alpha_m^2 D_{11} + \beta_n^2 D_{66} - \frac{8}{h^2}D_{44} \\ &\quad + (\frac{4}{h^2})^2 F_{44} - \frac{8}{3h^2}(\alpha_m^2 F_{11} + \beta_n^2 F_{66}) \\ &\quad + (\frac{4}{3h^2})^2(\alpha_m^2 H_{11} + \beta_n^2 H_{66}) \\ K_{45} &= \alpha_m\beta_n(D_{12} + D_{66}) - \frac{8}{3h^2}\alpha_m\beta_n(F_{12} + F_{66}) \\ &\quad + (\frac{4}{3h^2})^2\alpha_m\beta_n(H_{12} + H_{66}) \\ K_{55} &= A_{55} + \alpha_m^2 D_{66} + \beta_n^2 D_{22} - \frac{8}{h^2}D_{55} \\ &\quad + (\frac{4}{h^2})^2 F_{55} - \frac{8}{3h^2}(\alpha_m^2 F_{66} + \beta_n^2 F_{22}) \\ &\quad + (\frac{4}{3h^2})^2(\alpha_m^2 H_{66} + \beta_n^2 H_{22}). \end{aligned} \tag{25}$$

The critical value of ΔT is found by setting $|K_{ij}| = 0$ and is:

$$\Delta T = \frac{\delta - \gamma}{(A_{11}\alpha_{xx} + A_{12}\alpha_{yy})\alpha_m^2 + (A_{12}\alpha_{xx} + A_{22}\alpha_{yy})\beta_n^2}, \tag{26}$$

in which δ and γ are:

$$\delta = \frac{K_{12}(K_{12}K_{33} - K_{13}K_{23}) - K_{13}(K_{12}K_{23} - K_{22}K_{13})}{K_{22}K_{33} - K_{23}^2},$$

$$\gamma = \alpha_m^2 A_{44} + \beta_n^2 A_{55} - \frac{8}{h^2}(\alpha_m^2 D_{44} + \beta_n^2 D_{55}) + (\frac{4}{h^2})^2(\alpha_m^2 F_{44} + \beta_n^2 F_{55}) + (\frac{4}{3h^2})^2 [\alpha_m^2 H_{11} + 2\alpha_m^2\beta_n^2(H_{12} + 2H_{66}) + \beta_n^4 H_{22}]. \quad (27)$$

Equation 26 is used to determine the critical temperature of symmetric orthotropic thin plates.

CRITICAL GRADIENT THROUGH THE THICKNESS TEMPERATURE

Assume linear temperature variation across the plate thickness as:

$$T(z) = \Delta T \frac{(z + \frac{h}{2})}{h}, \quad (28)$$

where z is measured from the middle plane of the plate. For simply supported edges, the prebuckling forces in the symmetric orthotropic plate are [7]:

$$N_{xx0} = -N_{xx0}^T = -(A_{11}\alpha_{xx} + A_{12}\alpha_{yy})\frac{\Delta T}{2},$$

$$N_{yy0} = -N_{yy0}^T = -(A_{12}\alpha_{xx} + A_{22}\alpha_{yy})\frac{\Delta T}{2},$$

$$N_{xy0} = -N_{xy0}^T = 0. \quad (29)$$

Using a similar method, the critical temperature difference across the thickness is obtained by setting the determinant of the coefficient equal to zero, which yields:

$$\Delta T = \frac{\delta - \gamma}{(A_{11}\alpha_{xx} + A_{12}\alpha_{yy})\frac{\alpha_m^2}{2} + (A_{12}\alpha_{xx} + A_{22}\alpha_{yy})\frac{\beta_n^2}{2}}. \quad (30)$$

Equation 30 is used to determine the critical temperature of symmetric orthotropic thin plates.

CRITICAL TEMPERATURE WITH LINEAR VARIATION IN THE X-DIRECTION

Consider a rectangular symmetric orthotropic plate of dimensions a and b under temperature difference across the x-direction and with simply supported edges, where the motion of the edges in x and y directions are prevented. Assume a linear temperature variation along the x-direction:

$$T(x) = \Delta T \frac{x}{a}, \quad (31)$$

where $\Delta T = T(a) - T(0)$. The prebuckling forces are [7]:

$$N_{xx0} = -N_{xx0}^T = -(A_{11}\alpha_{xx} + A_{12}\alpha_{yy})\Delta T \frac{x}{a},$$

$$N_{yy0} = -N_{yy0}^T = -(A_{12}\alpha_{xx} + A_{22}\alpha_{yy})\Delta T \frac{x}{a},$$

$$N_{xy0} = -N_{xy0}^T = 0,$$

$$R_{xy0} = -R_{xy0}^T = 0. \quad (32)$$

To calculate the critical temperature ΔT, the displacement equations of stability (Equation 19) are used and the Galerkin method with the help of Equation 22 is employed to determine the critical buckling temperature. The left-hand side of the five Equations 22 are designated by $R1, R2, R3, R4$ and $R5$, respectively. Considering Equation 22, the Galerkin method leads to the following equations:

$$\int_0^a \int_0^b R1 \cos\alpha_m x \sin\beta_n y dx dy = 0,$$

$$\int_0^a \int_0^b R2 \sin\alpha_m x \cos\beta_n y dx dy = 0,$$

$$\int_0^a \int_0^b R3 \sin\alpha_m x \sin\beta_n y dx dy = 0,$$

$$\int_0^a \int_0^b R4 \cos\alpha_m x \sin\beta_n y dx dy = 0,$$

$$\int_0^a \int_0^b R5 \sin\alpha_m x \cos\beta_n y dx dy = 0. \quad (33)$$

Equations 33 result into five homogeneous equations for the constant coefficients $u_{1mn}, v_{1mn}, w_{1mn}, \phi_{1mn}$ and ψ_{1mn}. Using a similar method, the critical temperature difference in the x-direction is:

$$\Delta T = \frac{\delta - \gamma}{(A_{11}\alpha_{xx} + A_{12}\alpha_{yy})\frac{\alpha_m^2}{2} + (A_{12}\alpha_{xx} + A_{22}\alpha_{yy})\frac{\beta_n^2}{2}}. \quad (34)$$

Equation 34 is used to obtain the thermal buckling load of symmetric orthotropic thin plates.

RESULTS AND CONCLUSION

In this paper, results regarding the thermal buckling temperature of a plate under three cases of loadings are provided in a closed form by Equations 26, 30 and 34. The buckling modes m and n appear in the definition of the parameters α_m and β_n. The buckling load is the minimum ΔT for all values of m and n. For composite

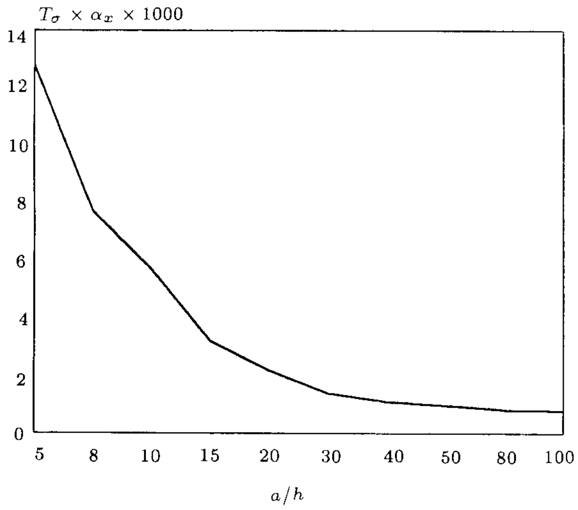

Figure 1. Buckling thermal load versus a/h.

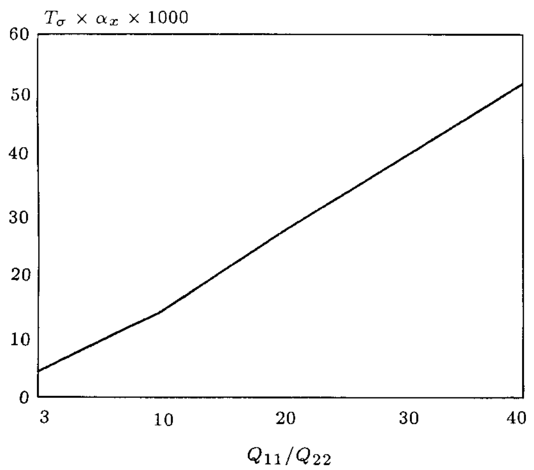

Figure 2. Buckling thermal load versus Q_{11}/Q_{22}.

materials, the lowest buckling load occurs at higher buckling modes [10].

Figure 1 shows the critical buckling temperature, T_{cr}, for the uniform temperature rise of an orthotropic plate (material specification is T300/N5208), with ply angles [0/90/90/0] versus a/h ratio. The curve illustrates that with increasing a/h ratio, the critical buckling temperature decreases. This result is qualitatively identical with the result provided by Chang and Leu [6], where an orthotropic plate with [+45/-45] ply angle arrangment has been studied. The case has been checked for other types of thermal loading and it is found that with increasing a/h ratio, the critical buckling temperature decreases.

Figure 2 demonstrates critical buckling temperature for the uniform temperature rise of an orthotropic plate (material specification is T300/N5208), with ply angles [0/90/90/0] and $a/h = 10$, versus $\frac{Q_{11}'}{Q_{22}}$ ratio. As is observed from this figure, by increasing $\frac{Q_{11}'}{Q_{22}}$ ratio, the critical buckling temperature increases, which is qualitatively identical with the result given by Chang and Leu [6]. Other types of thermal loading have also been investigated and it is found that with increasing a/h ratio, the critical buckling temperature increases.

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