

Theoretical Analysis of Spectral Hole-Burning in All-Optical Gain-Stabilized Multi-Channel Fiber Amplifiers

A.R. Bahrampour* and M. Mahjoei¹

In this paper, spectral hole burning effects and gain dynamics of all-optical gain-clamped multi-channel fiber amplifiers are modeled. The Cabezas and Treat simple model is used to write the propagation and rate equations of an inhomogeneous laser medium. The governing equations are an uncountable system of partial differential equations. After some mathematical manipulations, averaging over the fiber amplifier length and introducing an approximation method, the system of infinite partial differential equations are converted to a finite system of ordinary differential equations. The model is applied for hole-burning effects and transient response analysis of the surviving channels and relaxation-oscillations of the compensating (laser) signal of a WDM Erbium Doped Fiber Amplifier (EDFA). The results are in qualitative agreement with the published experimental results.

INTRODUCTION

Trivalent rare-earth ions doped optical fibers find a major field of application as traveling-wave fiber amplifiers for optical fiber communications as an alternative to semiconductor laser amplifiers [1,2]. The Er³⁺-Doped Fiber Amplifier (EDFA) with a flattened gain is a key device for Wavelength Division Multiplexing (WDM) transmission systems and has been used in WDM transmission experiments at over 1Tb/s [3,4]. The main problem facing WDM optical fiber networks with fiber amplifier cascades is gain dynamics. These amplifiers are generally operated near saturation and since the total output power of a saturated fiber amplifier is nearly constant, the output power of each channel will depend on the number of channels present. When the number of channels changes as a result of network reconfiguration, it will induce transients to gain in other surviving channels through transient cross saturation in the amplifier. One of the important schemes which is demonstrated to control the unwanted

power excursions of surviving channels in rare earth ions doped fiber amplifiers is the gain clamping by an all-optical feedback loop [5,6]. The main goal of the scheme is that the maximum value of the power excursions of the surviving channels should be less than a fraction of a dB for any possible change in channel loading. In an all-optical feedback loop with homogeneously broadening active medium, for all wavelengths in the gain bandwidth, gain clamping is obtained. However, in an inhomogeneously broadened system it is achieved only for wavelengths in the hole burning bandwidth. Hence, gain clamping bandwidth (i.e., the spectral hole burning behavior) is strongly dependent on the broadening type of the active medium. Inhomogeneous broadening effects, such as spectral hole burning, are observed in some fiber amplifiers [7-9]. The homogeneous model is used by previous investigators [10-12]. In this paper, the Cabezas and Treat simple inhomogeneous model [13] is used to describe the spectral hole burning and relaxation oscillation in Erbium Doped Fiber Amplifiers (EDFA) ignoring the population of the lower laser level in the analysis. For this reason the differences between our theoretical calculation and published experimental results are noticeable. Finally a new method for the numerical solution of the inhomogeneous medium

* Corresponding Author, Department of Physics, Valiasr University, Rafsanjan, I.R. Iran.

1. Department of Physics, Shahid Bahonar University, Kerman, I.R. Iran.

balance equations is introduced. The results of the numerical solution are in good qualitative agreement with the experimental results, presented in [10].

GOVERNING EQUATIONS

Gain saturation in amplifiers with hole burning and cross relaxation (c.r.) has been studied by Cabezas and Treat [13] whose analysis is presented. By integrating the light intensity spectral density distribution of the beam $I(\nu, r, \varphi, z)$ on the radial and azimuthal coordinates, the beam total power spectral density $p(\nu, z)$ at position z in the fiber amplifier is given:

$$p(\nu, z) = \int_0^{2\pi} \int_0^\infty I(\nu, r, \varphi, z) r dr d\varphi.$$

The normalized optical density spectral is defined as:

$$i(\nu, r, \varphi) = I(\nu, r, \varphi, z)/p(\nu, z).$$

It is assumed that $I(\nu, r, \varphi, z)$ is separable such that for each optical mode, $i(\nu, r, \varphi)$ is independent of z [11]. Let $n(\nu, r, \varphi, z)$ be the number of inverted ions per unit volume per unit frequency interval in the fiber amplifier which is called spectral inversion. The rate of change of the inverted population spectrum is given by:

$$\begin{aligned} \frac{\partial n(\nu, r, \varphi, z)}{\partial t} = & -n(\nu, r, \varphi, z) \int_0^\infty \sigma^h(\nu, \nu') i(\nu', r, \varphi) \\ & \frac{p(\nu', z)}{h\nu'} d\nu' - \frac{n(\nu, r, \varphi, z)}{\tau} + w[N_0 g(\nu) \\ & - n(\nu, r, \varphi, z)] + F[g(\nu) \int_0^\infty n(\nu', r, \varphi, z) d\nu' \\ & - n(\nu, r, \varphi, z)], \end{aligned} \quad (1)$$

where $\sigma^h(\nu, \nu')$ is the stimulated emission cross-section of the atomic line centered at ν , ν' is the stimulating frequency and τ is the lifetime of the upper laser level. The first term on the right hand side is the loss due to stimulated emission and the second is the loss due to spontaneous emission. In the third term, N_0 is the total density of particle available for pumping, and $g(\nu)$ is the probability distribution of transition wavelength fluctuations caused by inhomogeneous broadening centered at ν_a and normalized to unity is the fraction able to be pumped giving $N_0 g(\nu) - n(\nu, r, \varphi, z)$ as the actual number pumped. The pump rate w is proportional to the pump power and is of the same dimension as the Einstein parameter $A = \frac{1}{\tau}$, like F , in the last term, which gives the cross relaxation rate. The total number of inverted atoms per unit volume is given by:

$$N(r, \varphi, z) = \int_0^\infty n(\nu, r, \varphi, z) d\nu. \quad (2)$$

Assuming that the erbium ion distribution is radially symmetric and decreases monotonically from $r = 0$, the equivalent radius of the doped region is [14]:

$$b_{eff} = \left(\frac{1}{2} \int_0^\infty \frac{N_0(r, \varphi, z)}{N_0(0, 0, 0)} r dr d\varphi \right)^{\frac{1}{2}},$$

and the average density is:

$$\bar{n}(\nu, z) = \frac{\int_0^{2\pi} \int_0^\infty n(\nu, r, \varphi, z) r dr d\varphi}{\pi b_{eff}^2}.$$

The overlap integral of the $n(\nu)$ population is:

$$\Gamma(\nu, \nu', z) = \frac{\int_0^{2\pi} \int_0^\infty i(\nu', r, \varphi) n(\nu, r, \varphi, z) r dr d\varphi}{\bar{n}(\nu, z)}.$$

If the erbium ions are well confined to the center of the optical modes, with a Gaussian approximation to the optical mode, the overlap integral ($\Gamma(\nu, \nu')$) is nearly constant and independent of the frequency ν and ν' [14].

Now integrate both sides of the rate equation over the fiber core, divide by the effective area and from the definition of $\Gamma(\nu, \nu')$ and $\bar{n}(\nu, z)$:

$$\begin{aligned} \frac{\partial \bar{n}(\nu, z)}{\partial t} = & -\bar{n}(\nu, z) \Gamma \int_0^\infty \frac{\sigma^h(\nu, \nu') p_k(\nu', z)}{h\nu' \pi b_{eff}^2} d\nu' - \frac{\bar{n}(\nu, z)}{\tau} \\ & + w[\bar{N}_0 g(\nu) - \bar{n}(\nu, z)] + F[g(\nu) \int_0^\infty \bar{n}(\nu', z) d\nu' - \bar{n}(\nu, z)] \\ & \forall \nu \in [0, \infty]. \end{aligned} \quad (3)$$

The bandwidth of the atomic line is smaller than the inhomogeneous line width of the medium and is assumed to be the minimum frequency interval for which cross relaxation processes are infinitely fast.

Light in the amplifier can be considered to be propagating as a number of laser beams of narrow frequency bandwidth centered at the optical wavelengths $\lambda_k = \frac{c}{\nu_k}$ ($k = 1, 2, \dots, m$). Then, it can be assumed that the optical power spectrum $p(\nu, z)$ of the light in the amplifier can be written as follows:

$$p(\nu, z) = \sum_{k=1}^m p_k(z) \delta(\nu - \nu_k). \quad (4)$$

For this power spectrum the rate equation is rewritten as follows:

$$\begin{aligned} \frac{\partial \bar{n}(\nu, z)}{\partial t} = & -\bar{n}(\nu, z) \Gamma \sum_{k=1}^m \frac{\sigma^h(\nu, \nu_k) p_k(z)}{h\nu_k S} - \frac{\bar{n}(\nu, z)}{\tau} \\ & + w[\bar{N}_0 g(\nu) - \bar{n}(\nu, z)] \\ & + F[g(\nu) \int_0^\infty \bar{n}(\nu', z) d\nu' - \bar{n}(\nu, z)], \end{aligned} \quad (5)$$

where $S = \pi b_{eff}^2$ is the effective area of the fiber core. Both sides of the above equation are integrated over the frequency interval $[0, \infty]$ and the differential equation of the total population inversion (N) is obtained as:

$$\begin{aligned} \frac{\partial \bar{N}(z)}{\partial t} = & -\Gamma \sum_{k=1}^m \frac{p_k(z)}{h\nu_k S} \int_0^\infty \bar{n}(\nu, z) \sigma^h(\nu, \nu_k) d\nu \\ & - \frac{\bar{N}(z)}{\tau} + w[\bar{N}_0 - \bar{N}(z)]. \end{aligned} \quad (6)$$

In order to obtain the set of integrals on the right hand side of the Equation 3, the moment functions $q_{i_1, i_2, \dots, i_j}(z)$ are defined as follows:

$$\begin{aligned} q_{i_1, i_2, \dots, i_j}(z) = & \int_0^\infty \sigma^h(\nu, \nu_{i_1}) \sigma^h(\nu, \nu_{i_2}) \dots \sigma^h(\nu, \nu_{i_j}) \bar{n}(\nu, z) d\nu, \\ i_\xi = & 1, 2, \dots, m \\ \xi = & 1, 2, \dots, j \\ j = & 1, 2, \dots \end{aligned}$$

where j is called the order of moment functions. The rate Equation 3 is multiplied by $\sigma(\nu, \nu_{i_1}) \sigma(\nu, \nu_{i_2}) \dots \sigma(\nu, \nu_{i_j})$ and is integrated over the frequency variable (ν), then the differential equations of these variables are obtained as:

$$\begin{aligned} \frac{\partial q_{i_1, i_2, \dots, i_j}(z)}{\partial t} = & -\Gamma \sum_{k=1}^m \frac{p_k(z)}{h\nu_k S} q_{i_1, i_2, \dots, i_j, k}(z) - \frac{q_{i_1, i_2, \dots, i_j}(z)}{\tau} \\ & + w[\bar{N}_0 \alpha_{i_1, i_2, \dots, i_j} - q_{i_1, i_2, \dots, i_j}(z)] \\ & + F[\bar{N} \alpha_{i_1, i_2, \dots, i_j} - q_{i_1, i_2, \dots, i_j}(z)], \\ i_\xi = & 1, 2, \dots, m \\ \xi = & 1, 2, \dots, j \\ j = & 1, 2, \dots \end{aligned} \quad (7)$$

where the parameters $\alpha_{i_1, i_2, \dots, i_m}$ are given by the following relation:

$$\begin{aligned} \alpha_{i_1, i_2, \dots, i_j} = & \int_0^\infty g(\nu) \sigma^h(\nu, \nu_{i_1}) \sigma^h(\nu, \nu_{i_2}) \dots \sigma^h(\nu, \nu_{i_j}) d\nu \\ i_j = & 1, 2, \dots, m \\ j = & 1, 2, \dots \end{aligned}$$

The uncountable system of the rate equations is converted to a countable system of partial differential

Equations 6 and 7. The remaining equations describe the propagation of the beams through the fiber, i.e.:

$$\begin{aligned} \frac{u_k}{v_k} \frac{\partial p_k}{\partial t} + \frac{\partial p_k}{\partial z} = & u_k \int_0^\infty d\nu \int_0^{2\pi} d\varphi \int_0^\infty r dr \\ & \times i(\nu_k, r, \varphi) n(\nu, r, \varphi, z) \sigma^h(\nu, \nu_k) \\ & \times (p_k(z) + \mu h \nu_k \Delta \nu_k) - u_k l_k p_k, \\ k = & 1, 2, \dots, m+2, \end{aligned} \quad (8)$$

where each beam is travelling either in the forward ($u_k = 1$) or backward ($u_k = -1$) direction. Here $\mu h \nu_k \Delta \nu_k$ is the contribution of spontaneous emission from the local n population and it amplifies through the amplifier. The number of modes μ is normally 2, as in the case of the optical fiber supporting only the two polarization states of the lowest order optical mode [15], and l_k is the intrinsic fiber loss. After some mathematical manipulations and using the overlap integral the propagation equations are rewritten as follows:

$$\begin{aligned} \frac{u_k}{v_k} \frac{\partial p_k}{\partial t} + \frac{\partial p_k}{\partial z} = & u_k p_k \Gamma \int_0^\infty \sigma^h(\nu, \nu_k) \bar{n}(\nu, z) d\nu \\ & + u_k \mu h \nu_k \Delta \nu_k \int_0^\infty \bar{n}(\nu, z) \sigma^h(\nu, \nu_k) d\nu - u_k l_k p_k, \\ k = & 1, 2, \dots, m. \end{aligned} \quad (9)$$

For an open loop system, the first term on the left hand side of Equation 8 is negligible in comparison with $\frac{\partial p_k}{\partial z}$, while for transient times of a closed loop system Figure 1, it is of the order of $\frac{\frac{n_l(t)}{n_{l,th}} - 1}{\tau_l}$ where $\frac{1}{\tau_l}$ is the cavity decay rate at oscillating wavelength, λ_l , $n_{l,th}$ is the threshold population inversion for lasing at λ_l and $\frac{n_l(t)}{n_{l,th}}$ is the instantaneous ratio of laser gain to cavity loss [16] and is not ignorable. Hence, in transient response analysis of closed loop network, the propagation equations must be considered in the form of Equation 9. Coupling terms on the right hand side of Equation 5 can be arranged into three groups: lasers, amplified signals and pumps. The amplification

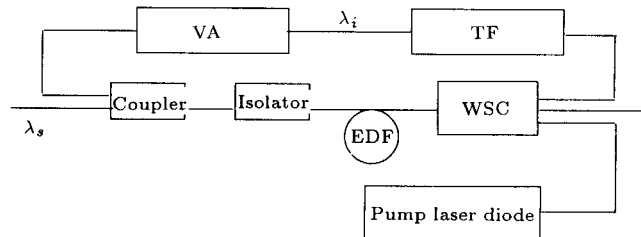


Figure 1. Scheme of an all-optically gain stabilized fiber amplifier, VA (variable attenuator), TF (tunable filter), WSC (wavelength selective coupler) [18].

process of the second group is represented by $p_k(z, t) = \exp(G_k(z, t))f_k(t)$ where $G_k(z, t)$ is the gain function till the position z and time t and $f_k(t)$ is input power at wavelength λ_k . By neglecting the Amplification of Spontaneous Emission (ASE) term in Equation 9 this equation is rewritten for the amplifying signals in terms of gain G_k as follows:

$$\frac{u_k}{v_k} \frac{\partial G_k}{\partial t} + \frac{\partial G_k}{\partial z} = u_k \Gamma q_k - u_k l_k - \frac{u_k}{v_k} \frac{f_k}{f_k},$$

$$k = 1, 2, \dots, m_1 \quad (10)$$

where $\frac{df_k}{dt}$ is denoted by \dot{f}_k and m_1 is the number of amplifying signals. The propagation equation for the lasers power p_l is as follows ($u_l = 1$):

$$\frac{1}{v_l} \frac{\partial p_l}{\partial t} + \frac{\partial p_l}{\partial z} = \Gamma p_l q_l - l_l p_l + \mu h \nu_l \Delta \nu_l q_l,$$

$$l = m_1 + 1, m_1 + 2, \dots, m_2, \quad (11)$$

where $m_2 - m_1$ is the number of oscillating wavelengths. The propagation equation for the pump powers is as follows [11]:

$$u_p \frac{\partial p_p}{\partial z} = -(\alpha_p + l_p) p_p,$$

$$p = m_2 + 1, m_2 + 2, \dots, m,$$

where $(m - m_2)$ is the number of pump lasers and $\alpha_p = \sigma_a(\lambda_p) \Gamma N_0$ is the absorption coefficient at pump wavelength.

Now, another simplification can be applied. This simplification is based on the averaging of the governing equations over the length of the active medium (L). Integrating Equations 7, 10 and 11 over the length of the active medium and using the $\langle \rangle$ notation for the averaged functions ($\langle f \rangle = \frac{1}{L} \int_0^L f(z, t) dz$) the following is obtained [17]:

$$\frac{d\bar{G}_k}{dt} = -\frac{v_k}{\eta_k L} \bar{G}_k + \frac{v_k \Gamma \langle q_k \rangle}{\eta_k L} - \frac{v_k l_k}{\eta_k L} - \frac{1}{\eta_k L} \frac{\dot{q}_k}{q_k},$$

$$k = 1, 2, \dots, m_1, \quad (12)$$

where $\bar{G}_k = G_k(L, t)$ for a beam entering at $z = 0$. While for beams entering at $z = L$, $\bar{G}_k = G_k(0, t)$ and $\eta_k \equiv \frac{\langle G_k \rangle}{\bar{G}_k} \simeq 0.75$ [18]:

$$\frac{d\langle p_l \rangle}{dt} = \Gamma v_l \langle p_l \rangle \langle q_l \rangle - v_k (l_l + \gamma_l) \langle p_l \rangle,$$

$$l = m_1 + 1, m_1 + 2, \dots, m_2, \quad (13)$$

where $\gamma_l \simeq (1 - f_l)/L$ and f_l is the feedback rate at oscillation wavelength [18]. Furthermore:

$$\frac{d\langle q_{i_1, i_2, \dots, i_j} \rangle}{dt} = -\Gamma \sum_k \frac{\langle p_k \rangle}{h \nu_k S} \langle q_{i_1, i_2, \dots, i_j, k} \rangle$$

$$- \frac{\langle q_{i_1, i_2, \dots, i_j} \rangle}{\tau} + \dot{w} \left[\bar{N}_0 \alpha_{i_1, i_2, \dots, i_j} - \langle q_{i_1, i_2, \dots, i_j} \rangle \right]$$

$$+ F[\alpha_{i_1, i_2, \dots, i_j} \langle \bar{N} \rangle - \langle q_{i_1, i_2, \dots, i_j} \rangle],$$

$$i_\xi = 1, 2, \dots, j$$

$$\xi = 1, 2, \dots, m_2$$

$$j = 1, 2, \dots \quad (14)$$

and:

$$u_p (p_p(L, t) - p_p(0, t)) = -L(\alpha_p + l_p) \langle p_p \rangle,$$

$$P = m_2 + 1, m_2 + 2, \dots, m. \quad (15)$$

The systems of ordinary differential Equations 12 to 14 and 6 are called averaged balance equations.

NUMERICAL SOLUTION OF THE BALANCE EQUATIONS

In an erbium doped fiber amplifier the ${}^4I_{11/2} - {}^4I_{15/2}$ transition corresponds to the 980 nm pump band and ${}^4I_{13/2} - {}^4I_{15/2}$ transition corresponds to 1520 – 1570 nm signal band and the resonant pumping in the 1460 – 1500 nm band. Other pump bands and the potential for more complex phenomena such as pump Excited State Absorption (ESA) are associated with other energy levels of Er^{3+} . Negligible ESA occurs for 980 nm, or 1480 nm pump amplifiers [19]. In this work the measurement of only absorption and emission spectra does not establish the relative contributions of homogeneous and inhomogeneous broadening to the observed line width. This is important to know because it may significantly affect the pumping and saturation behavior of the amplifier. Generally, the observed cross-section spectra $\sigma_{a,e}(\nu)$ are the convolution of homogeneous cross-section $\sigma_{a,e}^h(\nu)$ with the probability distribution $g(\nu)$ of the transition wavelength fluctuations caused by inhomogeneous broadening [20,21], i.e.:

$$\sigma_{a,e}(\nu) = \int_{-\infty}^{+\infty} \sigma_{a,e}^h(\nu') g(\nu - \nu') d\nu', \quad (16)$$

or:

$$\sigma_{a,e}(\nu) = \sigma_{a,e}^h(\nu) * g(\nu),$$

where $*$ denotes the convolution integral. The probability distribution $g(\nu)$ of the transition wavelengths is assumed to be Gaussian centered at ν_a [20] i.e.:

$$g(\nu) = (k/\Delta\nu) \text{Exp}\{-4Ln2[(\nu - \nu_a)/\Delta\nu]^2\},$$

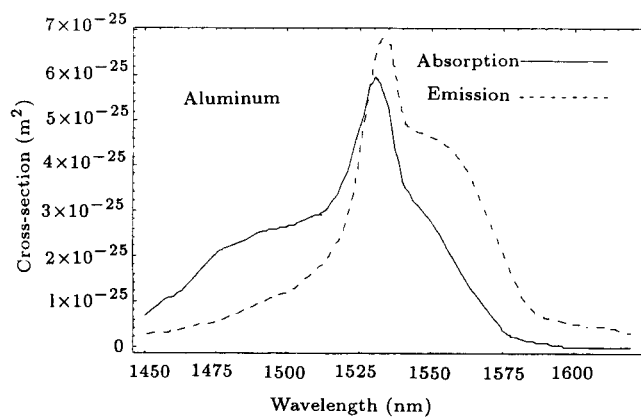
$$k = (4\pi Ln2)^{1/2} / \pi,$$

where $\Delta\nu$ is inhomogeneous broadening bandwidth. By applying the Fourier transform method, the solution of Equation 16 is obtained as:

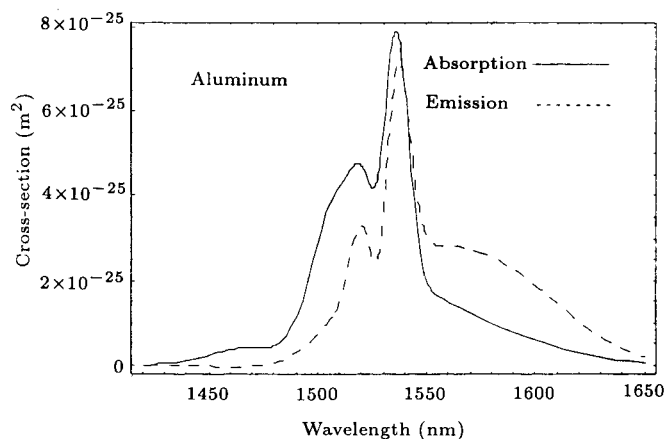
$$\sigma_{a,e}^h(\nu) = \mathcal{F}^{-1} \left\{ \frac{\mathcal{F}[\sigma_{a,e}(\nu)]}{\mathcal{F}[g(\nu)]} \right\}, \quad (17)$$

where \mathcal{F} and \mathcal{F}^{-1} are used to denote the Fourier and inverse Fourier transform which are calculated by the FFT algorithm. Measurements of aluminosilicate and fluoride glasses have shown the inhomogeneous line width to be comparably less than the homogeneous line broadening. Germanium silicate glasses, appear to have greater inhomogeneous line width (~ 8 nm) than the room temperature homogeneous line width (3–4 nm) [14]. Two examples of the room-temperature absorption and stimulated emission spectra obtained from erbium-doped silica glass fibers are shown in Figure 2 [11]. The fiber of Figure 2a had only germanium as an index-raising co-dopant while the fiber of Figure 2b had aluminum added to improve the

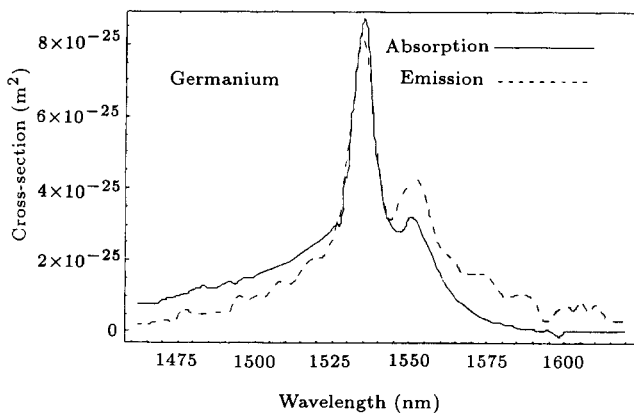
solubility of the Er^{3+} in glass. Adding Al also broadens the amplifier gain spectra [22]. In this work for typical absorption and emission spectra, which are shown in Figure 2, the homogeneous cross-sections ($\sigma_{a,e}^h$) are obtained and results which are shown in Figure 3 are used for the solution of the Cabezas and Treat balance equations. By neglecting the population of lower laser level, the present work can be applied to 1480 nm pumped erbium doped fiber amplifier. In the preceding section it has been shown that the governing equations on an all-optical gain clamped fiber amplifier are an uncountable system of partial differential equations. Then, after some mathematical manipulations and averaging over the fiber length, this system of partial differential equation is converted to a countable system of ordinary differential equations. Since the moment functions are rapidly decreasing functions of the order of moment functions (j), the truncation method with respect to the order of moment functions is a suitable method for the numerical solution of the governing system of ordinary differential equations. The number



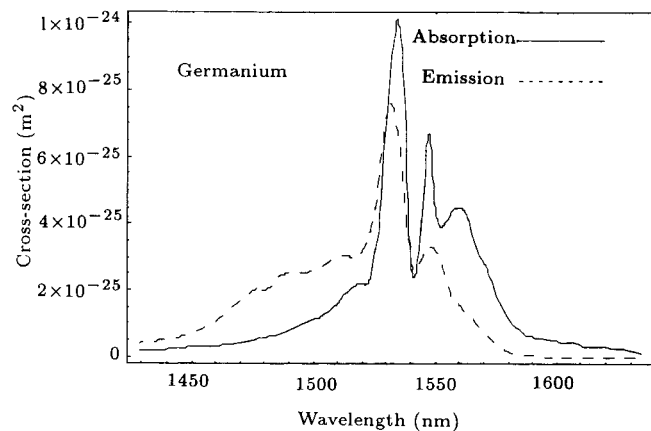
a) Typical emission and absorption cross-section spectra in an erbium-doped fiber co-doped with aluminum



a) Erbium doped fiber co-doped aluminum



b) Typical emission and absorption cross-section spectra in an erbium-doped fiber co-doped with germanium [11]



b) Erbium doped fiber co-doped germanium

Figure 2. A typical experimental emission and absorption in an Erbium doped fiber co-doped with aluminum or co-doped with germanium.

Figure 3. Theoretical homogeneous emission and absorption cross-section $\sigma_{e,a}^h$ obtained by the deconvolution method.

of ordinary differential equations (\mathcal{N}) of the truncated system is given by:

$$\mathcal{N} = m_2 + \sum_{i=0}^j s(i, m_2), \quad (18)$$

where $s(i, m)$ is the number of independent components of a symmetric tensor of order i in an m -dimensional space (see Table 1). The number of ordinary differential equations of the truncated system (\mathcal{N}) is a strongly increasing function of the order of moment function (j). Table 2 shows the variation of the number of equations (\mathcal{N}) with respect to the number of amplifying signals, laser and the order of moment functions. The truncated system of equations are rapidly convergent and low order moment functions give good approximation for the solution of the governing system of equations. Error is defined by the norm topology induced by the inner product on the L^2 Hilbert space.

$$\epsilon = \int_0^\infty |n_{j+1}(\nu) - n_j(\nu)|^2 d\nu. \quad (19)$$

The experimental cross-sections for (Al, Ge) $\sigma_e(\lambda)$ calculated from the fiber fluorescence and absorption spectrum [21] are shown in Figure 2 and it is assumed a Gaussian distribution with $1/e$ width $\Delta\nu_i = \Delta\lambda_i/\lambda^2$ and $\Delta\lambda_i = 11.5$ nm is the inhomogeneous line width

Table 1. Number of independent components of a symmetric tensor of order i ($i=1, 2$) in an m -dimensional space ($s(i, m)$) as a function of (m).

i	$s(i, m)$
1	m
2	$m + m(m-1)/2!$
3	$m + m(m-1) + m(m-1)(m-2)/3!$
4	$m + 3/2m(m-1) + m(m-1)(m-2)/2! + m(m-1)(m-2)(m-3)/4!$
5	$m + 2m(m-1) + m(m-1)(m-2) + m(m-1)(m-2)(m-3)/3! + m(m-1)(m-2)(m-3)(m-4)/5!$

Table 2. Number of governing equations (\mathcal{N}) versus the order moment functions (J) and number of propagating wavelengths.

m	$s(1, m)$	$s(2, m)$	$\mathcal{N} = 1 + m + s(1, m) + s(2, m)$
1	1	1	4
2	2	3	8
3	3	6	13
4	4	10	19
5	5	15	26
6	6	21	34
7	7	28	43
8	8	36	53
9	9	45	64
10	10	55	76

for $g(\nu)$ inhomogeneous broadening distribution [21]. Then by the homomorphic deconvolution method [23] the homogeneous cross-section spectra $\sigma_e^h(\lambda)$ are calculated and demonstrated in Figure 3. This shows the contribution of several transitions between the $^4I_{13/2}$ and $^4I_{15/2}$ Stark manifolds. The truncated governing system of differential equations is solved by the fourth order Runge-Kutta method for different values of j and results for an optical inverter are shown in Figure 4, which are in qualitative agreement with Fatehi's experimental results [24]. The homogeneous approximation is inaccurate at high input or oscillating power. Also if the model is applied to an eight-input all-optically controlled WDM amplifier system, spec-

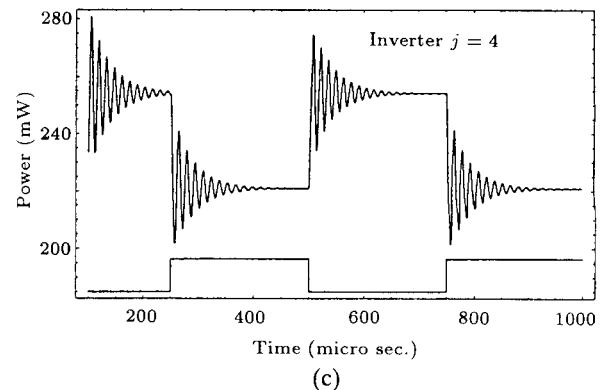
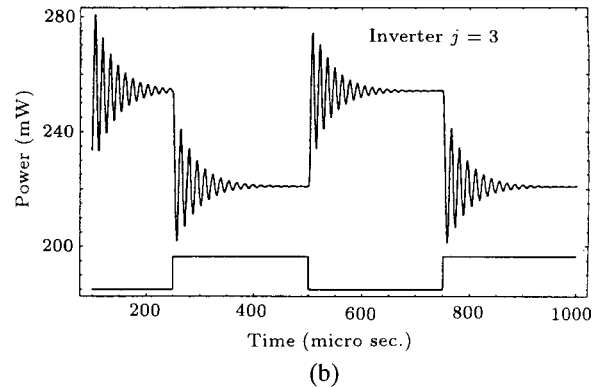
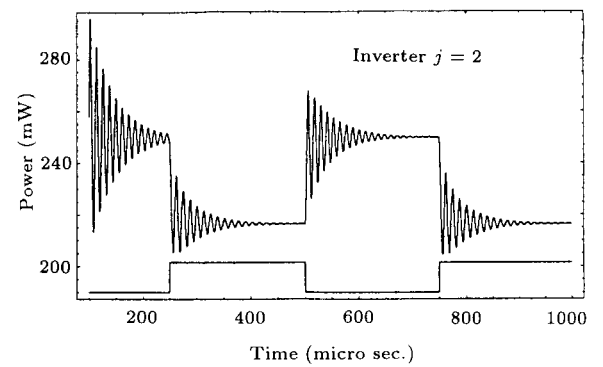


Figure 4. Theoretical step response of an optical fiber inverter.

tral hole burning and relaxation oscillation in surviving channels are obtained as experimentally observed by Luo et al. [10]. Figure 5 shows transient response of the surviving signal output power for three different laser wavelengths. When the 1557.8 nm signal is modulated on and off, corresponding to the worst case scenario addition and loss of seven of the eight WDM channels as shown in Figures 4 and 5, numerical results for different values of j are indistinguishable and $j = 3$ gives an accurate result.

The relaxation oscillation which appears in Figure 5 can be described by the homogeneous models [12].

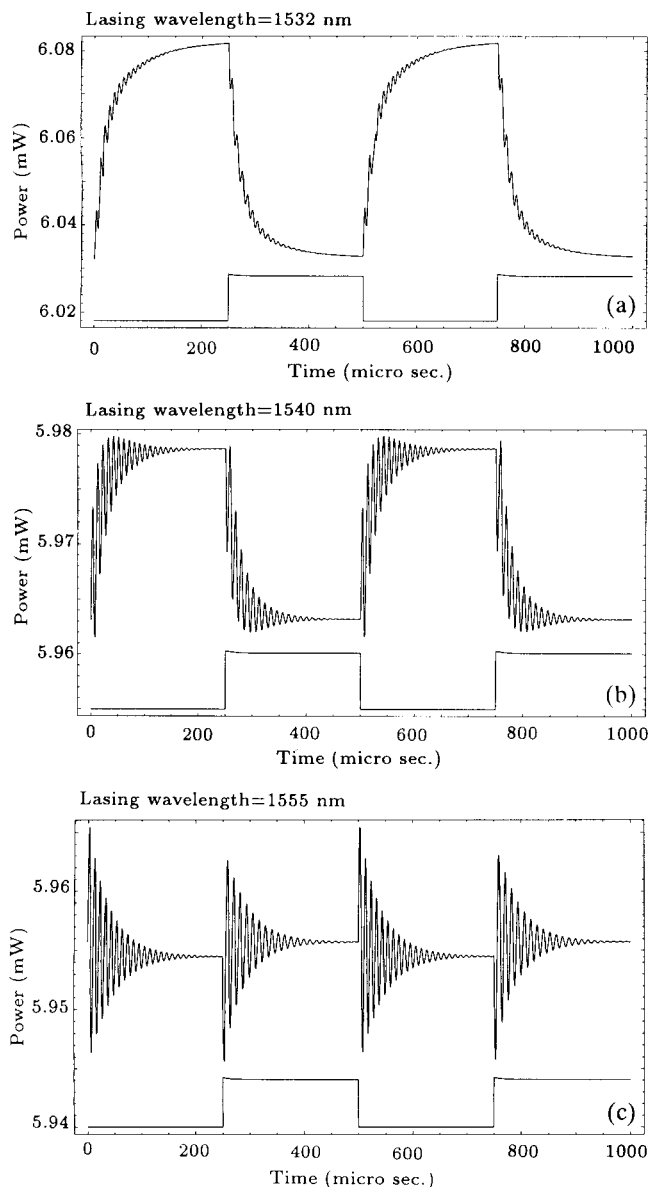


Figure 5. Theoretical transient response of surviving channel output power to dropping/adding seven of eight channels (a) The lasing wavelength is 1532 nm, (b) The same as (a) except that the lasing wavelength is 1540 nm, and (c) The same as (a) except that the lasing wavelength is 1555 nm.

At $\lambda_l = 1532$ nm (Figure 5a), the laser AGC suppresses transients in the surviving channel, but not completely. The steady state value of the surviving signal with and without 1557.8 nm signal present, differs by as much as 0.04 mWatt. This failure of the laser AGC arises from spectral hole burning, i.e., inhomogeneity of the erbium gain medium, which cannot be described by the homogeneous model. The transition between these two gain levels requires an order from 100 to 200 μ sec reflecting the slow gain dynamics of the erbium gain medium.

CONCLUSION

In this paper, a theoretical model has been presented for analysis of inhomogeneous all-optical gain-stabilized optical fiber amplifier. The calculation algorithm presented is an ultra fast numerical method. The number of governing differential equations depends on the number of the input signals and is much less than the number of equations in the conventional method for the solution of inhomogeneous broadening lasers. This simple equation provides a qualitative description of the relaxation oscillation and spectral hole-burning effects in Luo et al. [10] experimental observations in a WDM fiber amplifier system.

ACKNOWLEDGMENT

The authors are grateful to Dr. M.T. Fatehi (Lucent Tech., U.S.A.) and Mr. R. Farahi (Kerman University, Iran) for their fruitful discussions. The financial support provided by International Center for Science & High Technology & Environmental Science (ICST, Kerman, Iran) is also gratefully acknowledged.

REFERENCES

1. Koester, C.J. and Snitzer, E. "Amplification in a fiber laser", *Appl. Opt.*, **3**, p 1182 (1964).
2. Stone, J. and Burrus, C.A. "Neodymium doped silica laser in end-pumped fiber geometry", *Appl. Phys. Lett.*, **23**, p 388 (1973).
3. Onaka, H., Miyata, H., Ishikawa, G., Otsuka, K., Ooi, H., Kai, Y., Kinoshita, S., Seino, M., Nishimoto, H. and Chikama, T. "1.1 Tb/s WDM transmission over 150 Km, 1.3 μ m zero-dispersion single-mode fiber", in *Proc. OFC'96*, Post deadline Paper, **PD19** (1996).
4. Yano, Y., Ono, T., Fukuchi, K., Ito, T., Yamazaki, H., Yamaguchi, M. and Emura, K. "2.6 Terabit/s WDM transmission experiment using optical duo binary coding", in *Proc. ECOC'96*, Post deadline papers, Thb 3,1, (1996).
5. Ziringibl, M. "Gain control in erbium-doped fiber amplifiers by an all-optical feedback loop", *Electron. Lett.*, **27**, pp 560-561 (1991).

6. Landousies, B., Georges, T., Delevaque, E., Lebref, R. and Monerie, M. "Low power transient in multi channel equalized and stabilized gain amplifier using passive gain control", *Electron. Lett.*, **32**, pp 1912-1913 (1996).
7. Luo, G., Zyskind, J.L., Sun, Y., Srivastava, A.K., Sulhoff, J.W. and Ali, M.A. "Relaxation-oscillations and spectral hole burning in laser automatic gain control of EDFA", in *Proc. OFC '97*, Dallas, Tx. WF4.
8. Joindet, I. and Durpe, F. "Spectral hole burning insilica-based and in fluoride-based optical fiber amplifiers", *Electron. Lett.*, **33**, pp 1239 (July 1997).
9. Desurvire, E., *IEEE Photonics Technol. Lett.*, **1**(10), p 293 (1989).
10. Luo, G., Syskind, J.L., Nagel, J.A. and Ali, M.A. "Experimental and theoretical analysis of relaxation-oscillation and spectral hole burning effects in all-optical gain clamped EDFA's for WDM networks", *J. Lightwave Tech.*, **16**, pp 527-533 (1998).
11. Jarabo, S. and Alvarez, J.M. "Experimental cross-section of erbium-doped silica fibers pumped at 1480 nm", *Applied Optics*, **37**(12), p 20 (April 1998).
12. Bahrapour, A.R., Khosravi, M. and Farrahi R.M. "Analysis of gain dynamics in EDFA and in optical fiber inverter using a simplified model", *J. of Scientia Iranica*, Sharif University of Technology, **7**(1), pp 32-40 (2000).
13. Cabezas, A.Y. and Treat, R.P. "Effect of spectral hole-burning and cross relaxation on the gain saturation of laser amplifiers", *J. of Applied Physics*, **37**(9) (1966).
14. Giles, C.R., Desurvire, E. "Modeling erbium-doped fiber amplifiers", *J. Lightwave Tech.*, **9**, pp 271-283 (1991).
15. Desurvire, E. "Analysis of erbium-doped fiber amplifiers pumped in ${}^4I_{15/2} - {}^4I_{13/2}$ band", *IEEE, Phot. Tech. Lett.*, **1**(10) (Oct. 1989).
16. Sigman, A.E., *Lasers*, University Science Books, Mill Valley, CA.
17. Tarasov, L.V., *Laser Physics*, Mir, Moscow (1983).
18. Khosravi, M. "A theoretical analysis of relaxation-oscillation behavior of an all-optical fiber inverter", M.S. Thesis, University of Shahid Bahonar, Kerman, I.R. Iran (1999).
19. Bjarklev, A., *Optical Fiber Amplifiers*, Artech. House Inc (1993).
20. Yariv, A., *Quantum Electronics*, John Wiley and Sons, New York (1989).
21. Desurvire, E., Sulhoff, J.W., Ziskind, J.L. and Simpson, J.R. "Spectral dependence of gain saturation and effects of inhomogeneous broadening in erbium-doped alumino silicate fiber amplifiers", *Tech. Dig. of Optical Amplifiers and Their Applications OAA '90*, Monterey, **6-8**, Post deadline Paper PD9 (Aug. 1990).
22. Desurvire, E., Giles, C.R. and Simpson, J.R., *IEEE Photonics Technol. Lett.*, **2**(4), p 246 (1990).
23. Oppenheim, A.V. and Schafer, R.W., *Discrete Time Signal Processing*, Prentice-Hall International, Inc. (1989).
24. Fatehi, M.T. and Giles, C.R. "Erbium doped amplifiers with wavelength-selective optical feedback", *IEEE Phot. Tech. Lett.*, **8**, pp 1012-1014 (1996).