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Optimization of Reliability Determined Hydro-Thermal Power Systems

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In this paper, the optimization of long term operation and determination of the reliability level of a hydro-thermal power system are integrated into a unified model. Inflow to reservoirs is modeled as a random variable. Furthermore, the demand for energy is also assumed to be a random variable with normal distribution. In order to minimize the total cost, the reliability level of the system is determined, rather than considering it as a priori input data. Since the resulting model is a large-scale stochastic nonlinear programming, it is necessary to develop a special method to solve it. This method, which provides an optimal solution within three stages, consists of a decomposition technique, Lagrangian relaxation and nonlinear and dynamic programming methods. To test the method, it has been implemented in Khuzestan power system and the results are compared with the existing operation procedures.

INTRODUCTION

Due to the importance of long-term optimization of power system operations, many researchers have devoted intensive efforts to this area of study for more than four decades. However, the resulted mathematical models are usually developed on the basis of assumptions that are sometimes far from realistic. This is mainly because the system is quite complicated to be analyzed mathematically. In recent years, the trend is toward improving the assumptions and making them more realistic, as well as modeling the total generation of the power system rather than just one part of it, for example, thermal power plants, separately.

Although, in short-term operation, demand for energy can be assumed to be deterministic, in long-term operation uncertainties are so high that this assumption is not realistic any more. In previous research regarding hydro-thermal power systems, demand for energy is usually assumed to be deterministic. Although stochastic demands are considered in some papers, when it comes to the calculation of energy shortage, the supply is compared with the expected value of demand, or its expected value, as a "load duration curve". Practically, deterministic demands are being dealt with. On the other hand, in a hydro-thermal power system, usually inflows are treated as stochastic parameters for long-term studies.

In this research, a hydro-thermal power system is investigated which has stochastic demand with variable variance. The objective is to optimize long-term operation and determine the reliability level of this system simultaneously. In previous research, optimization of power system operation and determination of an optimal level of system reliability are considered separately, while both are incorporated in one model in this study. Therefore, what makes this research distinguishable from previous studies are the following two major points:

a) Both inflow to reservoirs and demand for energy are assumed to be stochastic;

b) Reliability is incorporated in the model and its level is not given in advance as an input data, but is determined such that total cost is minimized.

Since the model turns out to be a very large nonlinear stochastic programming, the classical methods are not capable of solving it. Therefore, a new method is specially developed for this model, in which a decomposition technique, Lagrangian relaxation, dynamic programming and nonlinear algorithms are applied. To test the method, it has been implemented in Khuzestan power system, which has the largest hydro-thermal system in Iran. The results are compared with existing operation procedures.

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Literature Survey

As mentioned before, many researchers have worked either on long-term optimization of hydro-thermal power system operation or on reliability evaluation. Therefore, only some studies similar to the research undertaken here are mentioned, although neither the model nor the method described here are the same.

To review the literature in the area of reservoir management and operations, one can see [1]. Among the recent works regarding the development of mathematical models, assumptions made by Chao-an Li et al. [2], Sherkat et al. [3] and Ardekaniaan [4] are close to the ones described in this paper. However, these models are developed on the basis of deterministic energy demand, while in [5,6], the demand uncertainty is treated as load duration curve. Also, in [7] a Stochastic Dual Dynamic Programming (SDDP) is introduced, in which the expected cost-to-go-function is approximated by piecewise linear functions. Although, in their model, inflows are treated as stochastic parameters, the demand is assumed to be deterministic. Jacobs et al. [8] present an in depth analysis of stochastic hydro scheduling for multi-reservoir systems by using Benders decomposition technique. They improved the traditional Benders decomposition, but adopted linear approximations to production functions. In addition, they didn't consider demand uncertainty in their proposed method for scenario generation.

While reliability is explicitly considered as a constraint in some models, the objective function does not include interruption costs explicitly, e.g. [9]. Xiao Ying et al. [10] introduce a new concept of comprehensive satisfaction degree by using multi-objective fuzzy dynamic programming, with different weights of economy and reliability. Nahman and Bulatovic [11] minimize the sum of operation and interruption costs and, also, the criterion for committed reserve capacity aimed at minimizing total cost. However, reliability level is not a decision variable in their model.

Due to the complexity of analytical methods, many researchers analyze the hydro-thermal power systems by applying simulation techniques. Ubeda and Allan [12] applied stochastic sequential simulation to assess hydro-thermal system reliability without considering optimization in their proposed algorithm. There are also some optimization models for short-term operation that consider system reliability. Wang et al. [13] consider a multi-area power system for which the desired reliability of each area, measured by LOLP, is included in the model as a constraint. In their model the target value for LOLP (ε) is an input parameter.

This paper is organized in the following way. The statement of the problem and its assumptions as well as the mathematical model are presented. Next, an approach to determine the system reliability is discussed, along with the presentation of the main theorem, regarding integration of long-term operation and reliability in a unified model. Then, the idea behind the method, as well as the details of the algorithm, are presented. Finally, the proposed method is tested for a real case, i.e., Khuzestan power system and the results are described.

THE PROBLEM

Consider a power system which consists of M thermal and N hydro power plants. The inflow of water is a random variable and demand for energy is assumed to be stochastic. Each hydro power plant has a multi-purpose reservoir which performs other functions such as water supply and flood control. The objective is to determine the turbine discharges of hydro power plants, as well as the energy output of thermal power plants, in order to minimize the total expected cost of the system during a horizon of T periods. In this study, each period is taken as a month.

The Objective Function

The total expected cost of the system (the objective function) is comprised of three parts as follows.

Cost of Energy Generation by Thermal Power Plants

The generation cost of a thermal power plant is approximated by a quadratic function with respect to its energy output x and is modeled as follows:

$$GC(x) = ax^2 + bx + c,$$

where a, b and c are constant parameters. Therefore, the total thermal generation cost of M thermal power plants in the system, over T periods under consideration, is given as:

$$\text{Total thermal generation cost} = \sum_{m=1}^{M} \sum_{t=1}^{T} GC_m(x_{mt}),$$

where $GC_m(\cdot)$ is the generation cost function of thermal power plant m and x_{mt} is its energy generation in period t in MWH. No cost is associated with energy generation of hydro power plants in the objective function, since it is expense free (except the fixed cost which is independent of the output level).

Terminal Cost of Reservoirs

The excessive usage of a reservoir water at a certain period results in less hydro energy generation capability during the next period. In this case part of the demand in the next period must be satisfied by more

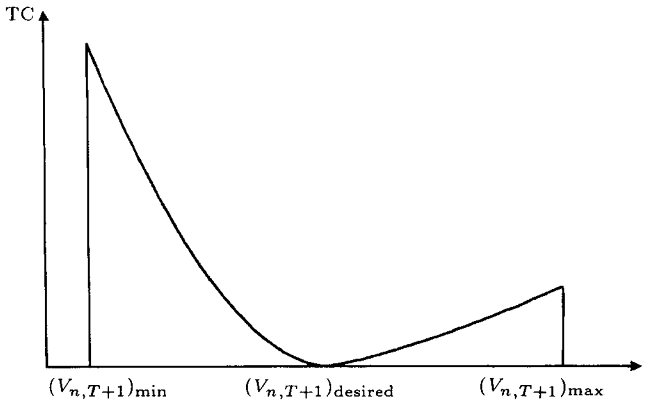

Figure 1. Typical terminal cost function of a reservoir.

expensive thermal generation. Thus, the optimization model determines the optimal values for turbine discharges, such that the total expected cost is minimized. However, during the last period, in order to preserve the water content of each reservoir at some desired level, a cost function called "terminal cost" is defined. The objective of this cost function is to penalize the deviations from the desired volume and is mainly used to prevent excessive usage of water during the last period. The general structure of this cost function is shown in Figure 1, in which $(V_{n,T+1})_{\min}$, $(V_{n,T+1})_{\text{desired}}$ and $(V_{n,T+1})_{\max}$ are minimum, desired and maximum allowable values for the volume of water in reservoir n in the beginning of period $T+1$, respectively. The penalty cost for values below V_{desired} is determined by estimating the marginal cost of energy generation in period $T+1$. It is assumed that the volumes above V_{desired} may not be acceptable for management, due to some operational reasons and, therefore, a penalty cost is associated with them, although it can also be set equal to zero. Consequently, if $TC_n(v)$ denotes the terminal cost of reservoir n as a function of its volume v, the total terminal cost is given by summing over all N reservoirs in the system, i.e.,

$$\text{Total terminal cost} = \sum_{n=1}^{N} TC_n(v_{n,T+1}),$$

where $v_{n,T+1}$ is the storage volume of reservoir n in the beginning of period $T+1$.

Expected Cost of Energy Interruption

In order to penalize the damage caused by energy shortage, the expected cost of energy interruption (EIC) is also considered in the objective function. If γ_t denotes the probability that the total generation is greater than or equal to demand in period t, then the Expected Energy Not Supplied ($EENS$) in period t is calculated as follows:

$$EENS_t = (EENS_t|\text{Demand} > \text{total generation})$$
$$\times \text{Probability}\{\text{Demand} > \text{total generation}\},$$

or:

$$EENS_t = (1-\gamma_t)\int_{ES_t>0} \left[D_t - \left(\sum_{m=1}^{M} x_{mt} + \sum_{n=1}^{n} u_{nt}\right)\right] dF_D(d_t)$$
$$= (1-\gamma_t) \int_{ES_t>0} (ES_t) dF_D(d_t), \quad (1)$$

where D_t is the random variable representing energy demanded in period t, u_{nt} is the energy generation of hydro power plant n in period t, ES_t is the energy shortage in period t and is defined as $ES_t = D_t - \left(\sum_{m=1}^{M} x_{mt} + \sum_{n=1}^{N} u_{nt}\right)$ and $F_D(d_t)$ is the distribution function of demand in period t.

To express the damages caused by energy shortage in terms of costs, it is necessary to estimate the Interrupted Energy Assessment Rate ($IEAR$). As Billinton and Li show [14], $IEAR$ is an important factor in generating system reliability assessment and is quite stable for a given system, i.e., it generally does not vary significantly when the system demand level and other factors change. Consequently, for different demand levels, EIC can be obtained by multiplication of $IEAR_t$ by $EENS_t$ and summing over all periods. Therefore, the third component of the objective function is given by,

Total expected cost of energy interruption =

$$EIC = \sum_{t=1}^{T} IEAR_t \times EENS_t. \quad (2)$$

Constraints

As mentioned before, the reservoirs are multi-purpose and, thus, other requirements such as flood control and supply of water for irrigation and domestic consumption must be satisfied. A lower and an upper bound on the turbine discharge are considered in each period to ensure the supply of water and satisfaction of the maximum allowable release from each of the turbines. Concerning flood control, it should be noted that determining maximum and minimum allowable values of reservoir volume in each period is usually performed, based on a given risk level for flood control and the results are provided as a set of values which are known as "reservoir rule curves". Therefore, lower and upper bounds on the volume of reservoirs in each period are determined, based on the values of their rule curves. There are also bounds on the maximum and minimum allowable generation for thermal and hydro power plants. Furthermore, there should be a balance between water inflow and outflow of a reservoir, which is represented by water balance constraint.

In most of the conventional studies regarding hydro-thermal power systems optimization, the demand for energy is treated as a deterministic quantity, directly or indirectly. Therefore, a deterministic energy balance equation relates demand and supply. Since in this model, demand for energy is assumed to be stochastic and normally distributed, it is satisfied by the following chance constraint:

$$Pr\Big(\sum_{m=1}^{M} x_{mt} + \sum_{n=1}^{N} u_{nt} \geq D_t\Big) \geq \gamma_t,$$

$$t = 1, \cdots, T.$$

In other words, the demand in period t is satisfied only with a probability of γ_t. The above chance constraints are equivalent to the following deterministic inequalities:

$$\sum_{m=1}^{M} x_{mt} + \sum_{n=1}^{N} u_{nt} \geq F_{D_t}^{-1}(\gamma_t),$$

$$t = 1, \cdots, T, \tag{3}$$

where, $F_{D_t}^{-1}(\gamma_t)$ is the inverse distribution function of demand in period t.

Hydro Energy Generation and Water Inflow Model

The energy output of a hydro power plant is modeled as follows:

$$u_{nt}(y) = \beta_n . \bar{h}_t . y_{nt},$$

where, β_n is the efficiency of the plant $n, \bar{h}_t$ is the average head in period t and y_{nt} is the turbine discharge volume of hydro power plant n in period t in Million Cubic Meters (MCM). It is assumed that the water inflow to a reservoir in period t is independent of inflow to others, but this depends on its inflow in period $t-1$. More specifically, water inflow to a reservoir follows a Markov Chain pattern and is modeled as follows [2,3]:

$$Z_{nt} = \rho_{n,t-1} \times Z_{n,t-1} + (1 - \rho_{n,t-1}^2)^{0.5} \times \varepsilon_{nt},$$

where,

$$Z_{nt} = \frac{Ln(\omega_{nt}) - \bar{\omega}_{nt}}{\Omega_{n,t}},$$

and ω_{nt}, which represents actual inflow to reservoir, is a lognormal random variable with mean $\bar{\omega}_{nt}$ and parameter $\Omega_{n,t}$. Furthermore, ε_{nt} is a random variable generated from normal distribution $N(0,1)$ to model the noise in inflow and ρ is the correlation coefficient between periods t and $t-1$.

The Mathematical Model

Referring to derivations provided in the previous sections, the mathematical model of a long-term hydro-thermal coordination problem, as considered in this paper, is given as follows:

Minimize

$$E\Big[\sum_{m=1}^{M}\sum_{t=1}^{T} GC_m(x_{mt}) + \sum_{n=1}^{N} TC_n(v_{n,T+1})\Big] + \sum_{t=1}^{T}(IEAR_t)(1-\gamma_t)\int_{ES_t>0}\Big[D_t - \Big(\sum_{m=1}^{M} x_{mt} + \sum_{n=1}^{N} u_{nt}\Big)\Big] dF_D(d_t), \tag{4}$$

subject to:

$$v_{n,t+1} = v_{nt} + Q_{nt} - y_{nt} - s_{nt}$$

$$n = 1, \cdots, N; \quad t = 1, \cdots, T, \tag{5}$$

$$\underline{x_{mt}} \leq x_{mt} \leq \overline{x_{mt}}$$

$$m = 1, \cdots, M; \quad t = 1, \cdots, T, \tag{6}$$

$$\underline{y_{nt}} \leq y_{nt} \leq \overline{y_{nt}}$$

$$n = 1, \cdots, N; \quad t = 1, \cdots, T, \tag{7}$$

$$\underline{u_{nt}} \leq u_{nt} \leq \overline{u_{nt}}$$

$$n = 1, \cdots, N; \quad t = 1, \cdots, T, \tag{8}$$

$$\underline{v_{nt}} \leq v_{nt} \leq \overline{v_{nt}}$$

$$n = 1, \cdots, N; \quad t = 1, \cdots, T, \tag{9}$$

$$Pr\Big(\sum_{m=1}^{M} x_{mt} + \sum_{n=1}^{N} u_{nt} \geq D_t\Big) \geq \gamma_t$$

$$t = 1, \cdots, T, \tag{10}$$

where $E[.]$ represents the expected value with respect to reservoir inflows. It is also assumed that the expected amount of spillage in each period is determined, such that the objective function of the problem is minimized and no attempt is made to minimize the spillage in each period separately. This is due to the fact that if an operation policy spills more water, its expected hydro generation will reduce; therefore, it must use more thermal generation to satisfy demand which, in effect, will increase the expected total cost of the system. In addition, it is assumed that all reservoir releases, except spillage, are through turbines.

OPTIMAL LEVEL OF RELIABILITY

It is a common practice to select a reliability level for each period, e.g., γ_t, as an input parameter of the system. However, in order to minimize the total expected cost of the system, this important factor is determined. In fact, in this paper, the reliability level γ_t, is considered as a decision variable. It is clear that system operation cost is an increasing function of γ_t, while expected cost of energy interruption (EIC) is a decreasing function of it. Thus, the total cost assumes its minimum value at a single specific level, say γ_t^*, which, clearly, is not at either bound of $\gamma_t = 0$ or $\gamma_t = 1$.

Theorem 1

Let demand be normally distributed with mean $\bar{D}_t$ and standard deviation of σ_t. If, for a given feasible solution, the following set of equations hold, then the solution is optimal,

$$(IEAR_t) \times \frac{\partial EENS_t}{\partial \gamma_t} + \mu_t^* \times \sqrt{2\pi}\sigma_t \mathrm{Exp}\left(\frac{(\lambda_t - \overline{D}_t)^2}{2\sigma_t^2}\right) = 0, \quad t = 1, \cdots, T, \tag{11}$$

where, $\lambda_t = F_{D_t}^{-1}(\gamma_t)$ and μ_t^* is the optimal Lagrange multiplier for period t.

Summary of Proof

Using NOMENCLATURE section, the total cost of the objective function (Equation 4) is $OC(\gamma) + EIC(\gamma)$, where $EIC(\gamma)$ is defined by Equation 2 and,

$$OC(\gamma) = \text{Minimize } E\left[\sum_{m=1}^{M}\sum_{t=1}^{T} GC_m(x_{mt}) + \sum_{n=1}^{N} TC_n(v_{n,T+1})\right], \tag{12}$$

subject to Constraints 3 and 5 to 9.

By taking derivatives from both terms of the objective function, with respect to γ_t and considering the fact that for optimal solution, γ_t is neither zero nor one, it is implied that, for optimal solution,

$$\frac{\partial OC(\gamma)}{\partial \gamma_t} + (IEAR_t) \times \frac{\partial EENS_t}{\partial \gamma_t} = 0, \quad t = 1, \cdots, T, \quad 0 < \gamma_t < 1, \tag{13}$$

using chain rule, the following is obtained:

$$\frac{\partial OC(\gamma)}{\partial \gamma_t} = \frac{\partial OC(\gamma)}{\partial F_D^{-1}(\gamma_t)} \times \frac{\partial F_D^{-1}(\gamma_t)}{\partial \gamma_t},$$

and applying the concept of shadow prices, it is known that:

$$\frac{\partial OC(\gamma)}{\partial F_D^{-1}(\gamma_t)} = \mu_t^*, \quad t = 1, \cdots, T,$$

also for normal distribution,

$$\frac{\partial F_D^{-1}(\gamma_t)}{\partial \gamma_t} = \sqrt{2\pi}\sigma_t \mathrm{Exp}\left(\frac{(\lambda_t - \bar{D}_t)^2}{2\sigma_t^2}\right),$$

and, by substitution, the proof is complete. ■

THE ALGORITHM

The model is a large nonlinear stochastic programming problem. Therefore, obtaining an optimal solution is quite complicated. An algorithm, on the basis of Lagrangian relaxation and decomposition technique, is proposed to solve the problem.

Framework

The problem is solved hierarchically in three stages, moving forward from first to third stage and then backward to the first stage.

Stage 1

For a given level of reliability γ_t, The Master program is obtained by Equation 12 as its objective function, subject to Constraints 3 and 5 to 9.

Stage 2

By applying Lagrangian relaxation for Constraint 3, the corresponding dual problem is,

$$\max_{\mu} \min_{x,y} E\left[\sum_{m=1}^{M}\sum_{t=1}^{T} GC_m(x_{mt}) \sum_{n=1}^{N} TC_n(v_{n,T+1})\right] + \sum_{t=1}^{T} \mu_t \left[F_{D_t}^{-1}(\gamma_t) - \left(\sum_{m=1}^{M} x_{mt} + \sum_{n=1}^{N} u_{nt}\right)\right],$$

subject to Constraints 5 to 9. The expectation is taken with respect to reservoir inflows.

Stage 3

The problem of Stage 2 is separable, in terms of hydro and thermal power plant subproblems. The thermal subproblem is, for $m = 1, \cdots, M$:

$$\text{Minimize} \quad \sum_{t=1}^{T}\left[GC_m(x_{mt}) - \mu_t \times x_{mt}\right]$$

subject to:

$$\underline{x_{mt}} \leq x_{mt} \leq \overline{x_{mt}}, \quad t = 1, \cdots, T,$$

and the hydro subproblem is, for $n = 1, \cdots, N$:

$$\text{Minimize } E\left[TC_n(v_{n,T+1}) - \sum_{t=1}^{T} \mu_t \times u_{nt}(v_{nt}, y_{nt})\right]$$

subject to Constraints 5 and 7 to 9.

Since thermal power plant generations in each period are independent of their generations in the other periods, thermal subproblem can also be decomposed in time. Therefore, the solution of thermal subproblem for period t, x^*_{mt}, is found by equating the derivative of $[GC_m(x_{mt}) - \mu_t \times x_{mt}]$ to zero and comparing the result, $\tilde{x}_{mt}$, with the lower and upper bounds. Then the optimal solution is:

$$x^*_{mt} = \min\{\overline{x_{mt}}, \max\{\underline{x_{mt}}, \tilde{x}_{mt}\}\}.$$

Dynamic programming is applied to solve the hydro subproblem starting from terminal cost function as illustrated in Figure 1 and moving backward to period $t = 1$. The functional optimization is described by the following relation:

$$B_{nt}(v_{nt}, Q_{n,t-1}) =$$

$$\underset{Q_{nt}|Q_{n,t-1}}{\mathrm{E}} \left(\min_{y_{nt}} \left[B_{n,t+1}(v_{n,t+1}, Q_{nt}) - \mu_t \times u_{nt}\right]\right),$$

$$E_{Q_{nt}}$$

where $B_{nt}(v_{nt}, Q_{n,t-1})$ is the expected cost-to-go function for reservoir n in period t, knowing the inflows in period $t-1$. In order to calculate the expected hydro generation in each period, the method suggested in [2,3] is employed. In this method, a number of historical inflow sequences are used and the above functional optimization is carried out for each of them. Then, the expected hydro generation for each power plant is calculated as the average of hydro generation for the same power plant over all inflow sequences.

After obtaining hydro and thermal subproblem optimal solutions, they are coordinated by updating Lagrange multipliers. This can be done by applying any one of the existing methods, such as subgradient or variable metric method. The subgradient method is used here, in which multipliers are updated according to the following formula:

$$\mu_t^{k+1} = \mu_t^k + \alpha^k \times \Delta_t^k, \quad t = 1, \cdots, T,$$

where,

$$\Delta_t^k = F_{D_t}^{-1}(\gamma_t) - \left(\sum_{m=1}^{M} x_{mt} + \sum_{n=1}^{N} u_{nt}\right), \quad t = 1, \cdots, T,$$

$$\alpha^k = \frac{1}{\alpha_0 + \delta.k},$$

and μ_t^k is the Lagrange multiplier for period t in iteration k. α^k is the scalar correction step in iteration k and α_0 and δ are system dependent parameters.

After updating Lagrange multipliers, Stage 2 is reconsidered and the updated problem is solved. As long as the convergence is not reached, moving between the second and third stages continues. Then, the optimality is tested according to Theorem 1. If optimality conditions are not satisfied, the procedure continues by moving between Stages 1 and 2 again. The method is summarized as follows and shown in the flow diagram of Figure 2:

1) Select an initial level of reliability of supply and set up OC (γ_t) defined as Master program;
2) Select initial values for Lagrange multipliers, which are close to the thermal system marginal cost in each period;
3) Solve hydro and thermal subproblems with given reliability level and the corresponding Lagrange multipliers;
4) If the convergence is not reached, modify Lagrange multipliers by applying subgradient method and go to Step 3. The convergence criterion is:

   $$|\Delta_t^k - \Delta_t^{k-1}| \leq \varepsilon_1, \quad t = 1, \cdots, T,$$

   where ε_1 is an arbitrary tolerance;
5) With optimal values obtained from Step 3, check for optimality condition of Theorem 1. This is done by finding the roots of Equation 11, which are $\lambda_t^*, t = 1, \cdots, T$, while other variables are given their corresponding values in iteration k. If in iteration k and for $t = 1, \cdots, T, |\gamma_t^k - \gamma_t^{k-1}| \leq \varepsilon$, stop. Otherwise go back to Step 3 with the new values for reliability levels, which are $\gamma_t^{k+1} = F_D(\lambda_t^*), t = 1, \cdots, T$.

It should be noted that $\overline{\gamma_t}$ and $\underline{\gamma_t}$ are being considered as the upper and lower bounds for γ_t, $t = 1, \cdots, T$, up to the current iteration, respectively. If the new estimate for the optimal reliability level in period t does not lie within $[\underline{\gamma_t}, \overline{\gamma_t}]$, then its value is set equal to $(\underline{\gamma_t} + \overline{\gamma_t})/2$. In this way, the algorithm monotonically improves either the lower or upper bound of the optimum reliability level for period t in each iteration.

Algorithm Convergence

To prove that the algorithm is convergent, first it is shown that γ_t, obtained as roots of Equation 11, fluctuates from one side of γ_t^* to the other. On the other hand, the procedure to adjust γ_t when it is not within $[\underline{\gamma_t}, \overline{\gamma_t}]$, forces the absolute deviation of γ_t from γ_t^* to be reduced constantly.

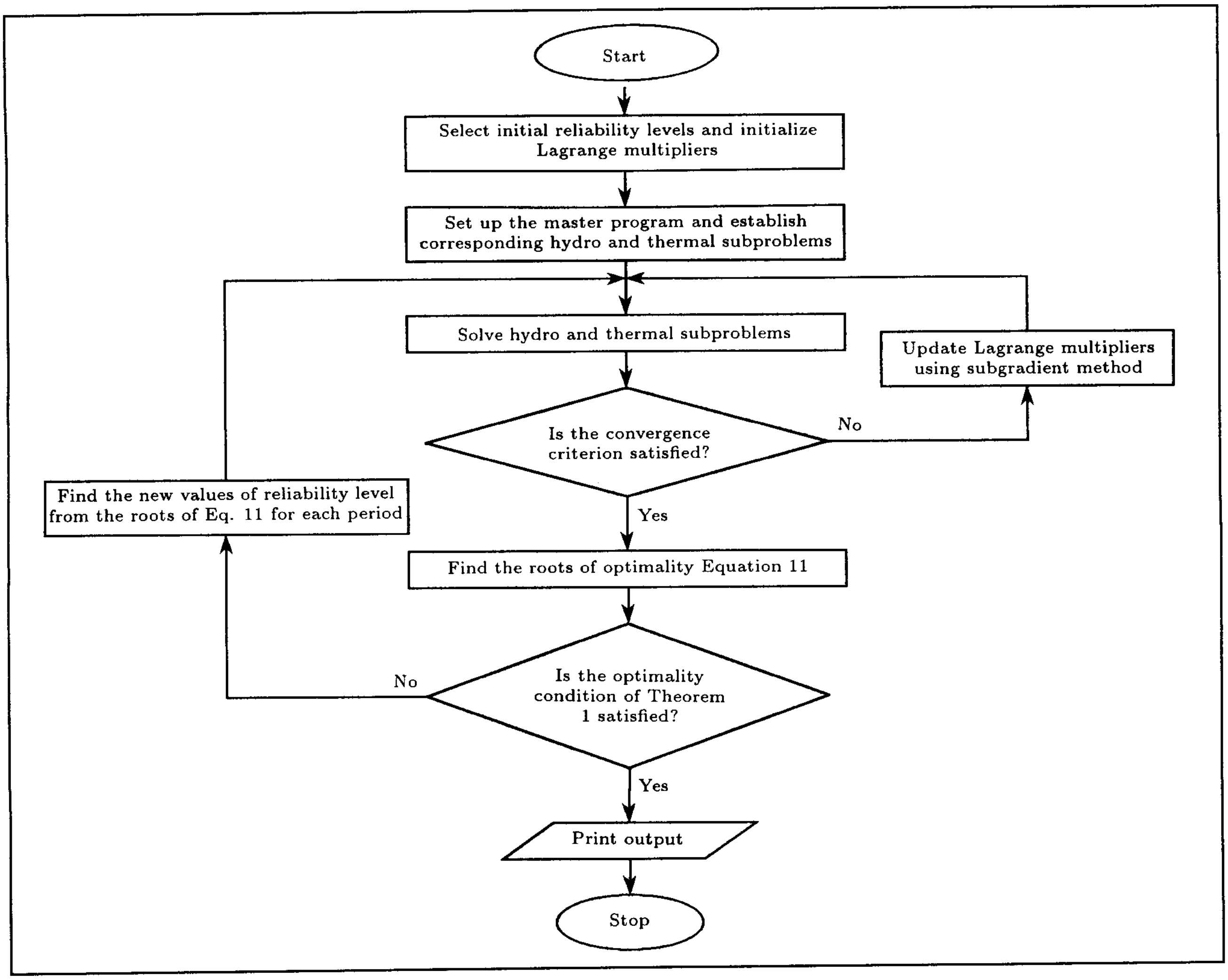

Figure 2. Flow diagram of the proposed method.

Theorem 2

Suppose in iteration k and for some t, $1 \leq t \leq T$, $\gamma_t^k > \gamma_t^*$. Then, the next estimate of γ_t^*, i.e. γ_t^{k+1}, will be less than γ_t^*. (The converse is also true.)

Summary of Proof

Let the corresponding values of μ_t^k, x_{mt}^k and u_{nt}^k as well as λ_t^k, the roots of Equation 11, be obtained from the algorithm. Then:

i) Considering the fact that both operating cost and $F_{D_t}^{-1}(\gamma_t)$ are monotonically increasing functions of γ_t, it is implied that $\mu_t^k > \mu_t^*$ and $x_{mt}^k + u_{nt}^k > x_{mt}^* + u_{nt}^*$.

ii) Since $x_{mt}^k + u_{nt}^k$ is an increasing function of γ_t^k, it is implied from Equation 1 that $\frac{\partial EENS_t}{\partial \gamma_t^k} > \frac{\partial EENS_t}{\partial \gamma_t^*}$. Therefore, the first term of Equation 11 is greater than the same term with γ_t^*.

iii) Thus, the second term of Equation 11 must be less than the corresponding term with optimal solution. On the other hand, since $\lambda_t = F_{D_t}^{-1}(\gamma_t)$ is a monotonically increasing function of γ_t for each period, it follows that $\gamma_t^{k+1} \leq \gamma_t^*$. (Proof of the converse is similar.) ■

Preprocessing of Input Data

The method is implemented after the required data is gathered and preprocessed. Although preparation of such data needs a great deal of work, fortunately power companies have almost all the necessary information from historical data collected in their data bases. Thus, in this section there is a summary of how to prepare and process the input data required for the method.

1. By applying Maximum Likelihood Estimation (MLE) method, estimate $\bar{D}_t$ and σ_t (mean and standard deviation of energy demanded) for $t = 1, \cdots, T$, from historical data;
2. Estimate parameters of the distribution of water inflow for each period, as well as the correlation

coefficient for successive periods from historical data;

3. Obtain cost functions for thermal energy generation of each thermal power plant from operation data by applying MLE method;
4. Given seasonal limitations, fuel constraints, rule curves of reservoirs, availability and maintenance schedules for generating units, establish the corresponding lower and upper bound constraints;
5. Determine the modified parameters for demand probability distribution by considering the information regarding energy interchange contracts with neighboring regions;
6. Calculate $IEAR_t$ by using the customer damage function obtained from regional surveys. This can be done, for example, by Monte Carlo simulation. As mentioned before, $IEAR_t$ does not vary significantly when the system demand level and other factors change.

CASE STUDY

Several examples are tested in order to examine the robustness of the proposed algorithm. Among them is an interconnected power system located in Khuzestan, a southern province of Iran. There exist 12 thermal units and 2 hydro power plants (including 12 hydro units) in this power system, with a total capacity of 3778 MW, of which 1520 MW is produced by hydro power plants. This province has a maximum load of 2887 MW with a load factor of 82% at the peak period. The energy demanded at the peak period has a mean value of 1636 GWh. It is assumed that the variance of energy demand distribution is increasing with respect to time and, in the first month, forecast error has a standard deviation equal to 2% of the mean energy demand, which linearly increases to 5% at the last period.

The reservoirs of existing hydro power plants have a total storage volume of 6240 MCM, while there is a vital demand of 150 CM/S of discharge water for irrigation and domestic consumption that must be satisfied. It should be emphasized that the distribution of turbine discharges among different water users in different periods of the year is usually performed on downstream of the reservoirs. Therefore, only the sum of the needs of different water users are considered at each period and incorporated as a lower bound on the discharges of the turbines. On the other hand, because of the extreme heat during four months of the year, the power supply is critically needed; thus great damage cost is envisaged for these months, which is found via regional surveys.

Table 1 gives the average of energy demanded and the corresponding $IEAR$ for various months. In the fifth column, the minus sign indicates import of energy.

Table 1. Data from Khuzestan power system.

Month	IEAR ($/KWh)	Average Load (MW)	Demand (GWh)	Export (GWh)
1	1	1506.9	738.6	595
2	1.6	1726.8	1086.9	600
3	1.8	1928.2	1419.2	480
4	2	2004.6	1561.3	400
5	2	2062.8	1636.1	350
6	1.4	1982.4	1574.8	110
7	1.05	1542.9	1245.1	-10
8	1.05	1662.4	818.2	-89
9	1.1	1695.8	799.5	-20
10	1.2	1728.8	918.2	18
11	1.3	1754.4	925.9	185
12	1.1	1684.6	870.9	508

Results

The results of implementing the proposed method for Khuzestan power system is shown in Table 2, where γ_t^* and μ_t^* are optimal reliability level and optimal Lagrange multiplier in period t, respectively. The expected total cost obtained by the method is $191294, which includes cost of energy interruption. The actual operation cost of Khuzestan system to meet the demand of Table 1 has been $417210 (based on 1$ = 8000 Iranian Rials) for the same year as optimized by the model, which does not include interruption costs. It should be emphasized that the actual operation cost cannot be compared directly with the results of the proposed model. This is due to the fact that in

Table 2. The optimal solution for Khuzestan power system.

Month	γ_t^*	μ_t^*	OC_t ($)	EIC_t ($)	Total $Cost_t$ ($)
1	0.882	8.453	3228	171	3399
2	0.886	12.665	8834	329	9164
3	0.869	18.669	18242	610	18852
4	0.866	21.741	24450	811	25261
5	0.864	22.334	25756	932	26688
6	0.842	20.788	22422	943	23365
7	0.829	17.620	16338	686	17024
8	0.826	18.201	17380	819	18199
9	0.834	17.343	15855	830	16685
10	0.847	16.201	13940	813	14753
11	0.865	14.004	10621	727	11348
12	0.872	10.220	6038	518	6556

the actual operation of a power system, some short-term considerations, such as frequency control of the whole power system by hydro power plants and online transmission constraints, must be taken into account. These short-term considerations are not modeled in long-term studies because of the degree of the details required for their incorporation [5]. However, they greatly affect hydro generation of the system and will increase actual operation costs.

Nevertheless, the actual operation cost can act as an upper bound for the value obtained by the model. If the results obtained by the model show a potential improvement in system operations, then they should still be tested using a special measure called "value of stochastic solutions", which will be explained at the end of this section.

In order to observe the difference between stochastic and deterministic treatment of demand, the proposed method was also implemented for the above system for the following cases:

a) Considering the energy demand in each period as a deterministic quantity, which is equal to its expected value at the same period; in fact, $\gamma_t^* = 0.5$.
b) Considering an extra supply equal to 10% of average demand as reserve for each period.

The results for the above cases are summarized in Tables 3 and 4. Note that Tables 2 to 4 are developed based on the assumption that the actual water year 1999-2000 (1378 in the Iranian calendar) has been a dry year. Therefore, a value for the initial inflow to each reservoir is specified, which is sufficiently lower than the average inflow of the corresponding period during the last 30 years. The distribution function of inflow to each reservoir is then determined, based on the assumption that inflow follows a Markov Chain

Table 3. The optimal solution for Case (a) deterministic demand.

Month	μ_t^*	OC_t ($)	EIC_t ($)	Total $Cost_t$ ($)
1	8.308	2904	5032	7936
2	12.200	8258	10464	18722
3	18.198	17374	14165	31539
4	21.175	23235	17850	41085
5	21.609	24158	19899	44057
6	20.073	20962	14416	35378
7	17.126	15481	8725	24206
8	17.605	16312	10028	26340
9	16.642	14664	11386	26050
10	15.277	12485	13408	25893
11	13.007	9275	15621	24896
12	9.272	5114	12741	17855

Table 4. The optimal solution for Case (b), considering extra supply.

Month	γ_t^*	μ_t^*	OC_t ($)	EIC_t ($)	Total $Cost_t$ ($)
1	0.9999	8.873	4198	1E-7	4198
2	0.9999	13.916	10498	1E-5	10498
3	0.9998	19.998	20811	3E-4	20811
4	0.9996	23.122	27546	4E-3	27546
5	0.9990	23.640	28758	3E-2	28758
6	0.9979	21.978	24967	0.1	24967
7	0.9962	18.609	18131	0.2	18131
8	0.9938	19.208	19263	0.6	19264
9	0.9907	18.333	17620	1.7	17622
10	0.9869	17.148	15517	4.0	15521
11	0.9824	14.885	11893	8.8	11902
12	0.9772	10.938	6797	12.3	6809

pattern, as mentioned in the previous section and, by application of correlation coefficients calculated, using the historical inflow data of the last 30 years.

Total expected costs for Cases (a) and (b) are equal to $323957 and $206027 respectively. This means that there is an increase of 69% in total cost as compared with that obtained using the proposed method, if the demand is considered as a deterministic quantity, i.e., Case (a). Similarly, if Case (b) is adopted, then the cost is 8% higher than that of the proposed method. Also it can be seen that if uncertainty in forecasting demand is increased, further savings will be obtained by the proposed method.

To investigate the convergence behavior of the algorithm, 108 different runs were performed on the Khuzestan power system, using different data. It was found that the initial values of Lagrange multipliers have a great impact on the number of iterations of the algorithm in Stage 2. In most cases, the number of iterations in Stage 2 was between 5 to 20. The number of iterations, moving between Stages 1 and 2, was always less than 6. Total CPU time with a Pentium-Pro 200 PC varied from 1h:20m:16s to 2h:43m:39s, with a mean of 2h:11m:8s.

The procedure suggested by Kall and Wallace [15] was also applied to obtain a measure of the value of stochastic solutions. To do this, first the problem was solved with random inflow and demand for each period replaced by their expected values. This gives a solution for each period, while only the solution for the first period is adopted. Then the solution was implemented for the first period and the total expected cost was calculated for this period, using a number of randomly generated reservoir inflows and demand for the first period. In the next stage, with the initial conditions obtained from each random variable realization, the deterministic model from period two onwards was again solved and the solution was implemented for the second period. This procedure was repeated until the last period was reached, where the terminal cost for each realization of the inflow sequence for the last period could be calculated. Calculation of interruption cost for each period was carried out by computing the difference between the generation level of that period, as obtained by implementing the deterministic solution and the value of randomly generated demand for the same period. The result of calculation was multiplied by interrupted energy assessment rate ($IEAR$) for the corresponding period. The sum of the operating and interruption costs for each period and the terminal cost for the last period will give the value of the objective function for each realization of random variables.

Due to the fact that inflow data was available for the last 30 years, they were used, together with randomly generated demand, using a normal random number generator that has the same mean as used by the deterministic model and the same variance as used by the stochastic model. The mean value of the objective function for 30 samples was \$302259.2, while the value of the objective function for the stochastic model was \$191294. Comparing the results with those obtained using the stochastic model, it can be concluded that using a deterministic model when demand and reservoir inflows are uncertain, is not a proper choice.

CONCLUSION

In this research, a new model was developed for long-term operation of hydro-thermal power systems. In the proposed model, not only the optimal amounts of turbine discharge of hydro power plants and the energy output of thermal power plants were obtained, but also the system reliability was determined simultaneously, in order to minimize the total cost. To solve the resulting problem, a method was also developed, applying decomposition technique, Lagrangian relaxation and nonlinear and dynamic programming. The problem was solved within three stages. To see the efficiency of the proposed method, it was also tested for a real case.

An extension of this research is to consider the forced outage rate of thermal and hydro units in the model and to enhance the savings from system operation optimization. It is also possible to extend the model to the cascaded reservoirs in multi-area power systems, including transmission constraints. As mentioned previously, hydro energy generation is calculated using constant efficiency and average head during each period. However, efficiency of a hydro power plant depends on its head and the head itself depends on the reservoir volume, which is a function of turbine discharge. Therefore, considering variable efficiency and head relationships for hydro energy generation is another research extension of this work.

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NOMENCLATURE

T	number of operation periods
M	number of thermal power plants
N	number of hydro power plants
x_{mt}	energy output of thermal power plant m in period t
y_{nt}	water discharged from turbines of hydro power plant n in period t
u_{nt}	energy output of hydro power plant n in period t
v_{nt}	water content of reservoir n in the beginning of period t
$TC_n(v)$	terminal cost of reservoir n as a function of its water content v
Q_{nt}	inflow of water to reservoir n in period t
$\underline{x_{mt}}, \overline{x_{mt}}$	minimum and maximum allowable values for x_{mt}
$\underline{y_{nt}}, \overline{y_{nt}}$	minimum and maximum allowable values for y_{nt}
$\underline{u_{nt}}, \overline{u_{nt}}$	minimum and maximum allowable values for u_{nt}
$\underline{v_{nt}}, \overline{v_{nt}}$	minimum and maximum allowable values for v_{nt}
s_{nt}	water spilled from reservoir n in period t
D_t	energy demand in period t
$\bar{D}_t, \sigma_t$	mean and standard deviation of energy demand in period t
$F_{D_t}(.)$	distribution function of demand D_t
$IEAR_t$	interrupted energy assessment rate in period t
γ_t	the level of reliability for period t

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