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Research Note

On- and Off-Line Tuning Rules for Unconstrained SISO DMC

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The overall performance of a linear model predictive controller depends on proper adjustment of several design parameters. Most of these parameters have interdependent effects, which make their trial and error based tuning procedure very difficult. A systematic approach to overcome this problem is to reduce the number of adjustable parameters. This reduction is usually performed on the basis of sensitivity analysis, stability considerations or other objectives and constraints. The most reliable parameter, which can be independently tuned for performance improvement, is the control move suppression coefficient, λ. In this paper, some tuning rules for adjusting this parameter, on the basis of specific performance criteria, are introduced. These rules are obtained from numerical analysis of the controller performance and are, therefore, applicable regardless of the existence of an approximated first-order model. The capabilities of the rules are demonstrated using simulations of regular and adaptive Dynamic Matrix Controllers (DMCs).

INTRODUCTION

In model predictive controllers, future control moves are determined by minimizing a cost function that is based on prediction error and control move. In DMC design, future errors are predicted using a step response model for the process. Design parameters in this controller are sampling time T, prediction horizon P, control horizon M, model horizon N, control move suppression weight λ and the desired trajectory filter coefficient α. In the literature, there exist straightforward guidelines for the selection of these parameters. The guidelines are recommended to satisfy different aspects of the performance. The main considerations are stability of the closed loop system [1-3], robustness [4], control move size [5], computational load [6] and conditioning of the process matrix [5]. These guidelines may only be useful in an initial selection of control parameters and, therefore, cannot be used directly in an on-line (adaptive) design scheme. The explicit relation between the resulting control performance and the tuning parameters has been established in only a few recent papers [7-9]. Based on the reasoning found in classical works on the subject [6,10,11], all parameters other than λ have been kept fixed in these studies and were chosen to minimize the control performance sensitivity. In [7] an initial value for λ is determined such that the control move is roughly half the size of that for $\lambda = 0$. This value may, then, be adjusted manually during the operation to fine-tune the performance. In [9], the selection of λ is based on the pole restriction criteria [12]. Although the second approach, contrary to the first one, may be used in adaptive schemes of the DMC, the performance has been considered on the basis of second-order models that are not adequate for higher-order processes in general. In [8], the process is approximated as a first-order plus time delay model. Based on this approximation, a relation has been obtained between λ and the condition number of the gain matrix in DMC, which has a direct effect on the control move and, therefore, on performance. This relation can be readily used in an adaptive scheme of a DMC. However, it is only appropriate for processes that can be adequately represented by a first-order model. In the next section, a similar idea is extended to treat higher-order models. Two new approaches are also introduced, in which only the step response coefficients of the process are required.

ADJUSTABLE PARAMETERS IN DMC

In an unconstrained DMC, the control move at each control interval is determined by:

$$\Delta \mathbf{U} = (\mathbf{G}^T\mathbf{G} + \lambda\mathbf{I})^{-1}\mathbf{G}^T\mathbf{E}, \tag{1}$$

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where $\mathbf{G}$ is the $P \times M$ lower triangular Toeplitz matrix of the model, which is also called the dynamic matrix, $\mathbf{E}$ is the $P \times 1$ vector of predicted errors, $\Delta\mathbf{U}$ is the $M \times 1$ vector of future control moves and λ is the move suppression coefficient. $\mathbf{E}$ is computed using the following equations.

$$\mathbf{E} = \mathbf{Y}_d - \mathbf{Y}_p, \tag{2}$$

$$\mathbf{Y}_p = \mathbf{Y}_m + \mathbf{D}, \tag{3}$$

$$\mathbf{Y}_m = \mathbf{G}\Delta\mathbf{U} + \underline{\mathbf{G}}\ \underline{\Delta\mathbf{U}} + g_N\mathbf{U}_N, \tag{4}$$

$$\mathbf{D} = (y_p(t) - y_m(t))\begin{bmatrix}1 & 1 & 1 & \cdots & 1\end{bmatrix}, \tag{5}$$

where $\underline{\mathbf{G}}$ is the $P \times N$ Hankle matrix of the model, $\underline{\Delta\mathbf{U}}$ is the $N \times 1$ vector of past control moves, g_N is the Nth coefficient of the process step response and $\mathbf{U}_N$ is the $N \times 1$ vector of past inputs. $\mathbf{Y}_d$ is a $P \times 1$ vector with components that represent the desired output trajectory. Usually, a first-order filter is used to construct $\mathbf{Y}_d$ from the reference input, $r(t)$.

$$\mathbf{Y}_d = [y_d(t+1)y_d(t+2)\cdots y_d(t+P)]^T, \tag{6}$$

$$y_d(t+i) = \alpha y_d(t+i-1) + (1-\alpha)r(t) \tag{7}$$

$$i = 1, 2, \cdots, P-1, \quad \text{with} \quad y_d(t) = y_p(t),$$

$\mathbf{Y}_p$ has the same size as $\mathbf{Y}_d$ and contains future outputs of the process. Since these outputs are not available, they are predicted using the step response model of the process (Equation 4). The prediction is improved by adding $\mathbf{D}$ to the model output. This vector contains components that represent a mismatch between the process and its model and the measurement noise. In a simple implementation of DMC, $\mathbf{D}$ is determined by Equation 5, where $y_p(t)$ and $y_m(t)$ are process and model outputs at time t, respectively. In the case where the process has unstable poles or integrating terms, this vector is constructed in a special way [13]. Sampling time T, desired trajectory filter pole α, horizons M, P, N and the move suppression coefficient λ are the adjusting parameters in the DMC.

In order to design an auto-tuning or adaptive DMC, it is desirable to reduce the number of adjustable parameters. The remaining parameters should have easy-to-implement analytical relations with a defined performance criterion of the closed loop system. The aim of this paper is to introduce the most appropriate parameters and to obtain suitable relations for adjusting them.

Based on the sensitivity analysis [8,14] and the stability requirement [1-3], some of the parameters do not have enough variability range. A proper choice for these parameters is the following.

Sampling Time T

Variation of this parameter will affect performance, both directly and through influence on horizon parameters. Since its selection cannot be independent from the dynamics of the process, there will be little room for its variation. The following value is usually used [15]:

$$T = \frac{1}{20} \text{ to } \frac{1}{10} \tag{8}$$

of the equivalent time constant of the process, τ_e.

Horizon Parameters M, P and N

Although these parameters have wide range effects on the DMC performance, their values are mostly determined by process dynamics. For guaranteed stability, the following choices are recommended [3]:

$$P = N = (5 \times \tau_e + \text{ time delay}, d)/T, \tag{9}$$

M should be greater than the number of unstable poles and constraints

Suppression Weight λ and Desired Output Filter Pole α

By proper scaling, parameter λ can be selected independently from the process dynamics and has a wide range effect on the control performance [16]. In the following section, three different strategies are introduced to tune this parameter. Parameter α is selected in range (0,1), however, it could be used to improve performance of the controller whenever mismatch exists between the process and its model.

ANALYTICAL AND NUMERICAL EXPRESSIONS FOR λ

In [8], the following analytical relation for λ has been introduced:

$$\lambda = f k_p^2, \tag{10}$$

$$f = \frac{M}{C}(\beta - \frac{M-1}{2}), \tag{11}$$

$$\beta = P - d - \frac{3}{2}\frac{\tau_e}{T} + 2, \tag{12}$$

where k_p and d are the steady-state gain and delay of the process, respectively. An explanation for this expression is that increasing λ causes a decrease in the control move and, at the same time, decreases the condition number of the gain matrix ($\mathbf{G}^T\mathbf{G}$) in DMC. Therefore, the correlation between the decrease of the condition number C and the control move can be exploited to relate to the control performance and the condition number of the gain matrix. Although the above equations are simple, they are only applicable for processes that are approximated by a first-order, plus time delay, model. For a process that cannot be represented by a first-order model, the approach employed in [8] is not applicable. However, the existing correlation between the condition number and the control move can still be exploited by a numerical approach to the problem. This expression may be formulated using the following equations [16]:

$$\lambda = fk_p^2, \tag{13}$$

$$f = \frac{\mu_{\max}(\overline{\mathbf{G}}^T\overline{\mathbf{G}}) - C.\mu_{\min}(\overline{\mathbf{G}}^T\overline{\mathbf{G}})}{C-1}, \tag{14}$$

$$\overline{\mathbf{G}} = \frac{1}{k_p}\mathbf{G}, \tag{15}$$

where μ denotes the eigenvalue of a matrix.

In order to implement auto-tuning or adaptive DMC, it is necessary to update most of the control parameters in each control interval. However, in most cases, adjusting the parameter α alone will compensate for the performance deterioration. Otherwise a similar idea as in [9] may also be useful to reduce computational load. In a regular DMC design, since computation of the $\overline{\mathbf{G}}^T\overline{\mathbf{G}}$ eigenvalues are performed once, both of the above methods will require similar amounts of computation. However, contrary to the first approach, the second one can also be implemented for higher-order processes. The main disadvantage of both approaches is that there is not an explicit relation between control performance criteria, such as overshoot, settling time, maximum control input, etc., and the condition number of the gain matrix. The proper condition number can only be chosen using simulation results.

In the second approach presented in this paper, different performance criteria are related to the parameter λ, through curves and equations obtained by numerical analysis. The curves (equations) for second- and third-order processes are given here. For higher-order processes, a similar procedure could be applied. The following models have been used in the generation of appropriate curves.

$$G_p(s) = \frac{\omega_n^2 k_p e^{-\theta s}}{s^2 + 2\xi\omega_n s + \omega_n^2}, \tag{16}$$

$$G_p(s) = \frac{p\omega_n^2 k_p e^{-\theta s}}{(s+p)(s^2 + 2\xi\omega_n s + \omega_n^2)}. \tag{17}$$

By proper scaling of the control parameters (T, M, P, N), the behavior of the model will depend only on ξ in the second-order process and on ξ and $\phi = p/\omega_n$ in the third-order process [16]. In Figures 1 to 3 the relations

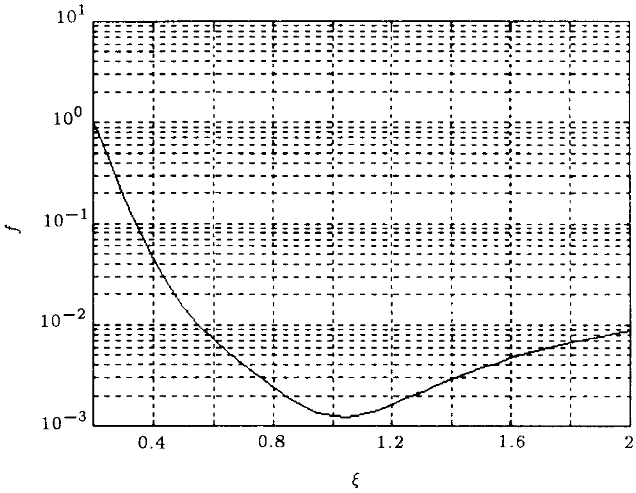

Figure 1. Curves $f(\xi)$ for the second order process and overshoot between 2% to 5% ($\lambda = fk_p^2$).

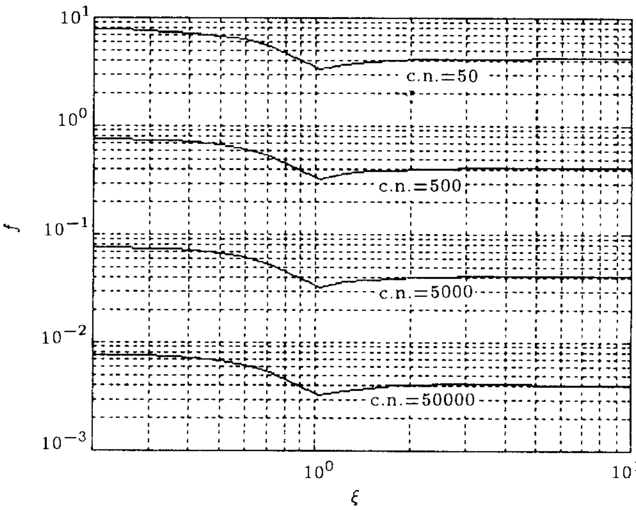

Figure 2. Curves $f(\xi)$ for the second order process and different condition number ($\lambda = fk_p^2$).

between λ and ξ are shown for different performance criteria. In all these three figures, it can easily be seen that the trends of change for f (or λ) are different for the two variation ranges of ξ. For large values of ξ, where the first-order model approximation can be applied, f has an approximately constant value, which is comparable to that obtained in [8]. For small ξ, the calculated values for f are notably different from those for large ξ. In other words, for processes that are not represented by first-order models, the value of f given in [8], is not adequate. For third-order processes, these relations are represented by two-dimensional curves, which are given in Figures 4 to 6. By implementing a proper curve fitting technique, appropriate equations may be obtained and used in auto-tuning or in adaptive design of the DMC.

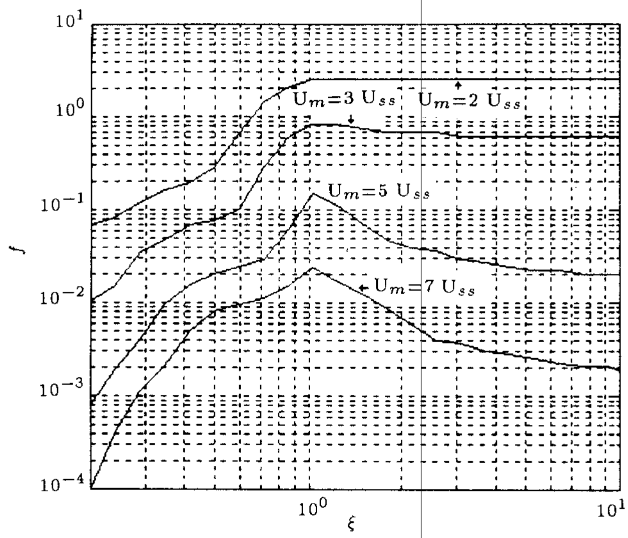

Figure 3. Curves $f(\xi)$ for the second order process and different maximum control input ($\lambda = fk_p^2$).

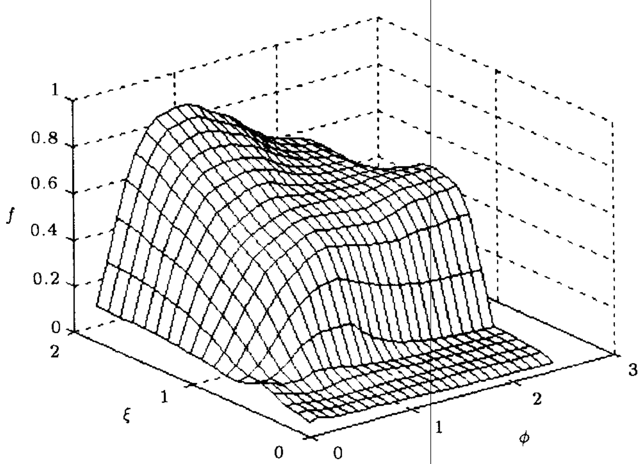

Figure 4. Curve $f(\xi, \phi)$ for the third order process and overshoot between 2% to 5% ($\lambda = fk_p^2$).

In the third approach, control parameters are adjusted in such a way that certain properties of a closed loop response are satisfied. In this paper, appropriate selections are given to obtain a desired closed loop settling time. Based on given open and closed loop settling times, the following tuning strategy can be applied.

$$T = \min(T_1, T_2)$$

where:

$$T_1 \in \left(\frac{T_{ssop}}{100}, \frac{T_{ssop}}{50}\right) \text{ and } T_2 = \frac{T_{sscl}}{15}. \tag{18}$$

This choice for sampling time satisfies requirements given in [15]. Then α is chosen, such that the desired trajectory reaches 99% of its steady-state value in the

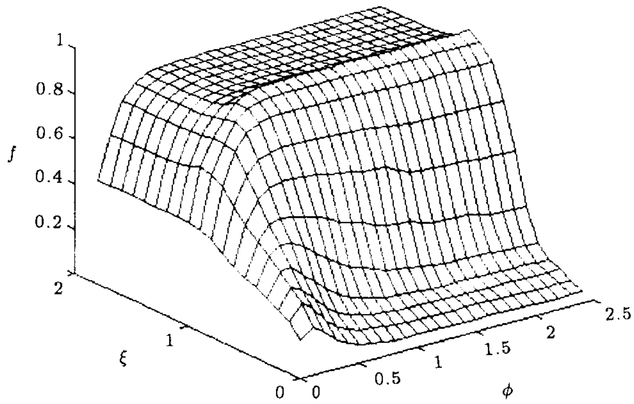

Figure 5. Curve $f(\xi, \phi)$ for the third order process and for $\mathbf{U}_{\max} = 3\mathbf{U}_{ss}(\lambda = fk_p^2)$.

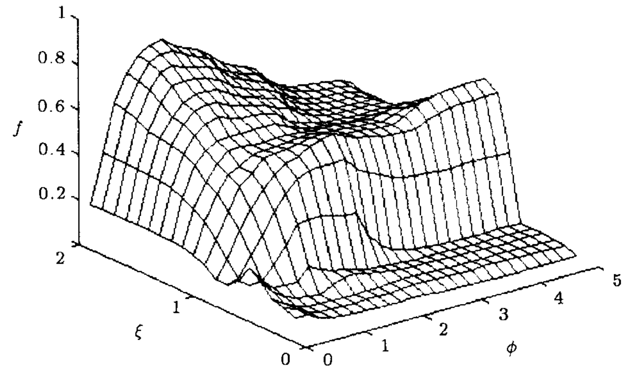

Figure 6. Curve $f(\xi, \phi)$ for the third order process and for 3% overshoot and $\mathbf{U}_{\max} \in (3 \text{ to } 5).\mathbf{U}_{ss}(\lambda = fk_p^2)$.

proper number of sample times, i.e.,

$$1 - \alpha^n = 0.99 \tag{19}$$

A proper value for n (which is equivalent to the prediction horizon) is 15 [10]. Other parameters are given as:

$$N = \frac{T_{ssop}}{T}, \quad M = 6, \tag{20}$$

$$\lambda = fk_p^2, \quad f = K\left(\frac{T_{sscl}}{T_{ssol}}\right)^2, \tag{21}$$

$$K \in (0.01, 0.05)$$

A faster closed loop response (smaller T_{sscl}), requires a higher magnitude of control move and, therefore, a small value of f. This correlation has been exploited in the derivation of the relation given in Equation 21 [16].

The difference between the second and third tuning strategies is that, in the former, the transfer function of the model is required and relations are given between properly scaled model parameters and λ. However, in the latter, these relations are

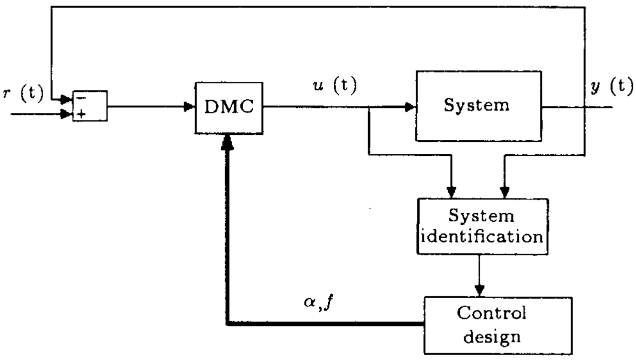

Figure 7. Block diagram of an adaptive DMC.

given between system response characteristics and λ. Therefore, existence of a transfer function model is not necessary.

ADAPTIVE DMC

The schematic diagram of an adaptive DMC is given in Figure 7. At each control interval, the step response or the transfer function model of the process is identified. Based on the identified model and using tuning rules presented in the previous sections, proper values are chosen for T, M, P and N. Then, suitable curves from given figures or their equivalent fitted equations are used to determine appropriate λ. For perfect modeling, α can be set to a constant value (usually 0.5). Otherwise, this parameter may be adjusted to improve the robustness of the control system. The following algorithm summarizes a typical implementation of an adaptive DMC:

Step 1: Select an appropriate model structure;

Step 2: Choose an appropriate performance criteria;

Step 3: Construct relevant f (or λ) curve and fit it to a proper equation;

Step 4: Estimate selected model parameters based on input/output signals;

Step 5: Assign appropriate values for control parameters T, M, P, N;

Step 6: Compute desired λ and set $\alpha = 0.5$;

Step 7: Implement DMC and compute the present control move $\Delta u(t)$;

Step 8: Apply $u(t)$ and check for control performance;

Step 9: If performance is good enough go to Step 7; if acceptable but not adequate adjust α and go to Step 7; if not acceptable go to Step 4.

ILLUSTRATIVE EXAMPLES

In the first simulation, a DMC was designed to control the following process:

$$G_p(s) = \frac{e^{-0.5s}}{s^2 + 0.8s + 1} \tag{22}$$

Tuning the controller parameters was undertaken on the basis of the condition number of the gain matrix (Equations 13 to 15). Since this process has an underdamped step response, it cannot be properly represented by a first-order plus time delay model. Therefore, the tuning strategy given in [8] will not be adequate. Results given in Figure 8 were obtained for $C = 500$ using curves in Figure 2.

In the second simulation, a time varying process with the transfer function:

$$G_p(s) = \frac{0.1(0.1s + 1)e^{-0.5s}}{s^2 + \xi(t)s + 0.25}, \tag{23}$$

was controlled by an adaptive DMC that was tuned using the second approach. The curves given in Figure 3 have been used to choose a proper value for λ. It is assumed that $\xi(t)$ has the following variations:

$$\xi(t) = 1 + 0.3\sin(0.15t). \tag{24}$$

Results of this simulation are given in Figure 9. The relation given in Equation 24 was assumed to be unknown to the controller. However, the minimum and maximum values of the variation range were used in the selection of horizon parameters.

In the third simulation, a DMC using the third approach controlled the following second-order process:

$$G_P(s) = \frac{1}{s^2 + 10s + 1}. \tag{25}$$

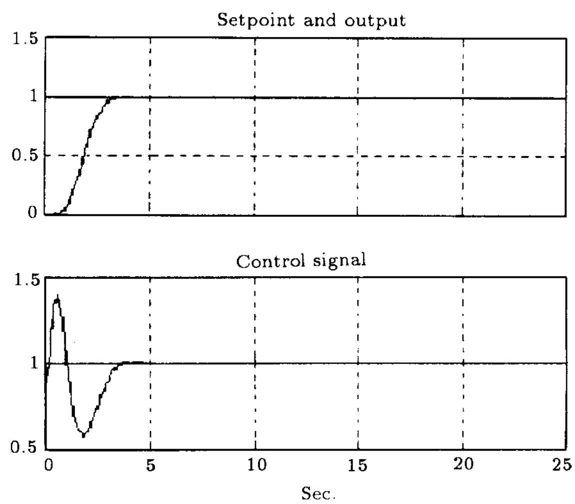

Figure 8. Step responses of a second order oscillatory process controlled by a DMC tuned for $C = 500$ ($T = 0.5^s$, $M = 6$, $P = N = 100$, $\alpha = 0.5$).

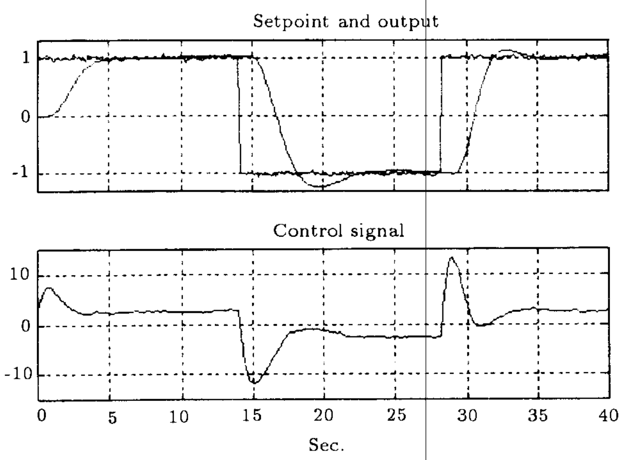

Figure 9. Simulation results for a second order time varying process controlled by an adaptive DMC tuned for $U_{\max} = 7U_{ss}$ ($T = 0.1^s$, $M = 6$, $P = N = 50$, $\alpha = 0.5$).

The desired closed loop settling time for the first 50^s of the simulation is 30^s and it is changed to 10^s after that. The following parameters were used in the simulation, which are in accordance with the proposed approach, the results of which are given in Figure 10.

$$T_{ssol} = 50^s, \quad T = 1^s, \quad M = 6, \quad N = 100,$$

$$\text{for } t < 50^s \quad T_{sscl} = 30^s, \quad P = 30,$$

$$\alpha = 0.858, \quad f = 0.01(\frac{3}{5})^2$$

$$\text{for } t > 50^s \quad T_{sscl} = 10^s, \quad P = 10,$$

$$\alpha = 0.631, \quad f = 0.01(\frac{1}{5})^2$$

In the last simulation, a second-order model was

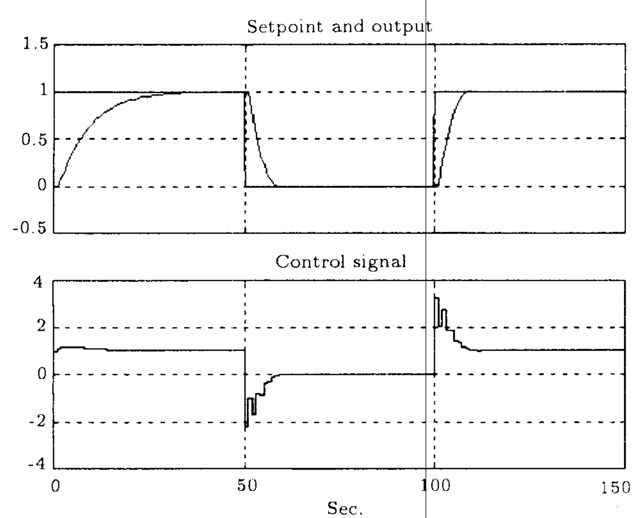

Figure 10. Simulation results for a second order process controlled by a DMC tuned for desired closed loop settling time.

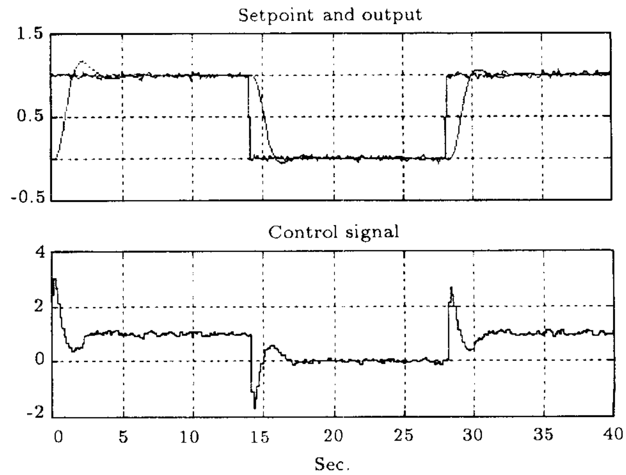

Figure 11. Simulation results for a third order process modeled by a second order model.

used to represent the following third-order process:

$$G_p(s) = \frac{1}{(0.1s + 1)(s^2 + s + 1)}. \tag{26}$$

The parameter λ was tuned based on the second tuning strategy. To fine-tune for mismatch compensation, parameter α was adjusted according to the following rule:

$$\alpha = \begin{cases} \alpha & \text{if } 0.9r \le y(t + t_1) \le 1.1r \\ \frac{\alpha+1}{2} & \text{if } y(t + t_1) > 1.1r \\ \frac{\alpha}{2} & \text{if } y(t + t_1) < 0.9r \end{cases} \tag{27}$$

where r is the reference input. It is known that increasing α will slow down the system response and decreasing α will speed up the response. This simple idea has been exploited in the proposed rule. In this simulation, the slowness of the system response is determined by output magnitude at a specific time and, therefore, will be sensitive to measurement noise. To improve the performance in practical situations, it would be better to choose other measures to evaluate the slowness. Results obtained from simulation of an adaptive DMC, using the rule given in Equation 27 and curves given in Figure 3, are shown in Figure 11. The rules in Equation 27 were applied whenever a step change appeared in the reference input.

CONCLUSIONS

In this paper, three different approaches were introduced to find an appropriate relation for adjusting the parameter λ in an auto-tuning or adaptive scheme of the DMC. This work is an extension of that found in [8], in which a first-order model approximation is used in the derivation of an analytical relation for λ. Without the approximations that were applied in [8], it is very difficult to find an analytical relation for λ even for first-order processes. In this paper, numerical analysis was proposed and, therefore, no restriction on the model order was assumed. Instead of an analytical relation, numerous graphical representations are obtained, each of which can be applied to achieve a specific property of the control performance.

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