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Missile Aerodynamic Identification Using a Two Step Estimation Procedure

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In this paper, the performance of EAM (Estimation After Modeling) and EBM (Estimation Before Modeling) techniques for aerodynamic parameter identification are compared. The results clearly show that EAM has a superior performance over EBM, but both techniques fail when there is considerable modeling uncertainty. Given this shortcoming, a two-step procedure is used that is a combination of EAM and EBM for estimating the aerodynamic parameters. Simulation results show that this new technique outperforms EAM when modeling uncertainties are considered. This technique will be applied to the problem of identifying the aerodynamic parameters of an anti-tank missile and its performance is compared with both EAM and EBM approaches.

INTRODUCTION

The aerodynamic model and its accuracy have a critical role in the guidance and control design of the aircraft, helicopter and missile. Parameters of this model are usually obtained by numerical methods or from wind tunnel test data. Another technique is based on using system identification methods and actual flight test data for estimating the aerodynamic parameters [1-3].

Note that wind tunnel tests are usually carried out under restricted conditions and simulating all possible flight conditions is quite difficult under these circumstances. Hence, using actual flight test data is the preferred approach for identifying the aerodynamic parameters. Moreover, actual flight test data can be used to validate parameters that are measured through wind tunnel experiments.

There are several techniques for identifying the aerodynamic parameters [1-19]. Basically all these techniques can be divided into two classes of Estimation Before Modeling (EBM) and Estimation After Modeling (EAM). In EAM, the aerodynamic forces and moments are modeled as functions of the state vector and some unknown parameters. Next, an estimation procedure is used to estimate these parameters. In EBM, the aerodynamic forces and moments are assumed unknown and in the first step these unknowns are estimated through an estimation algorithm. In the second step, a parameterized model is fitted to the estimated forces and moments. When modeling errors are present, EAM does not perform very well. In this paper, a strategy, called Mixed EAM-EBM (MEE) is proposed to solve the modeling error problem of EAM.

This paper is organized as follows. In the next section, EAM and EBM strategies are compared and it is shown that EAM is superior to EBM. Then, the effect of modeling error is considered and a remedy for EAM under modeling errors is presented. Also, an appropriate estimation algorithm that can be used in EAM, EBM, or MEE is presented. Moreover, a simple example is presented in which, EBM, EAM and MEE methods are described. Finally, simulation results for applying EAM, EBM, and MEE to an anti-tank missile is presented and the superior performance of mixed EAM-EBM is clearly illustrated.

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AERODYNAMIC IDENTIFICATION METHODS

Formulation

Missile equations of motion can be written in a general form as follows:

$$\dot{\underline{X}} = \underline{f}(\underline{X}, \underline{F}). \tag{1}$$

Here, $\dot{\underline{X}}$ denotes the state vector that includes the missile position, velocity, angular velocity and orientation and $\underline{F}$ denotes the forces and moments acting on the missile. Generally, $\underline{F}$ can be written as:

$$\underline{F} = \underline{F}_a + \underline{F}_T, \tag{2}$$

where $\underline{F}_a$ is the aerodynamic forces and moments and $\underline{F}_T$ is the forces and moments due to thrust vector. Aerodynamic forces and moments are a function of missile velocity, angular velocity, orientation and aerodynamic parameters $\underline{\Theta}$; hence, $\underline{F}_a$ can be denoted as:

$$\underline{F}_a = \underline{F}_a(\underline{X}, \underline{\Theta}). \tag{3}$$

There are many techniques for identifying the aerodynamic parameters $\underline{\Theta}$. An effective family of identification techniques, for this purpose is called nonlinear smoothing [6,7,9,13,17,18]. This family can be divided into two classes as follows:

1. Augment the unknown parameters $\underline{\Theta}$ to the state vector and estimate $\underline{\Theta}$ alongside with other components of the state vector. Then $\underline{\Theta}$ is assumed to be a slowly varying Markov process and the augmented equations are:

$$\dot{\underline{X}} = \underline{f}(\underline{X}, \underline{F}_a(\underline{X}, \underline{\Theta}) + \underline{F}_T),$$

$$\dot{\underline{\Theta}} = 0. \tag{4}$$

This class is usually called Estimation After Modeling (EAM).

2. Augment $\underline{F}$ to the state vector and estimate $\underline{F}$ alongside with other components of the state vector. Here, $\underline{F}$ is assumed to be a Markov process and the augmented equations are:

$$\dot{\underline{X}} = \underline{f}(\underline{X}, \underline{F}),$$

$$\dot{\underline{F}} = -a\underline{F} + \underline{\zeta}, \tag{5}$$

where $1/a$ is the time constant and $\underline{\zeta}$ is the driving process noise. After estimating F, the parameter $\underline{\Theta}$ is estimated using Equation 3 by regression methods. This class is usually called Estimation Before Modeling (EBM).

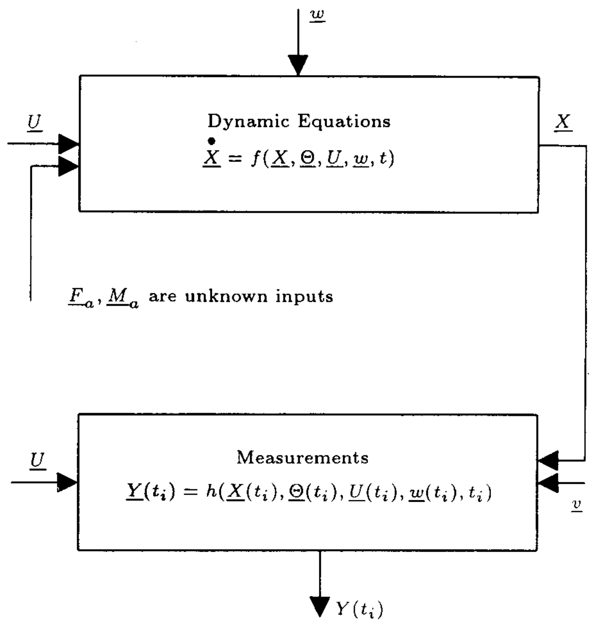

Figure 1. Estimation before modeling.

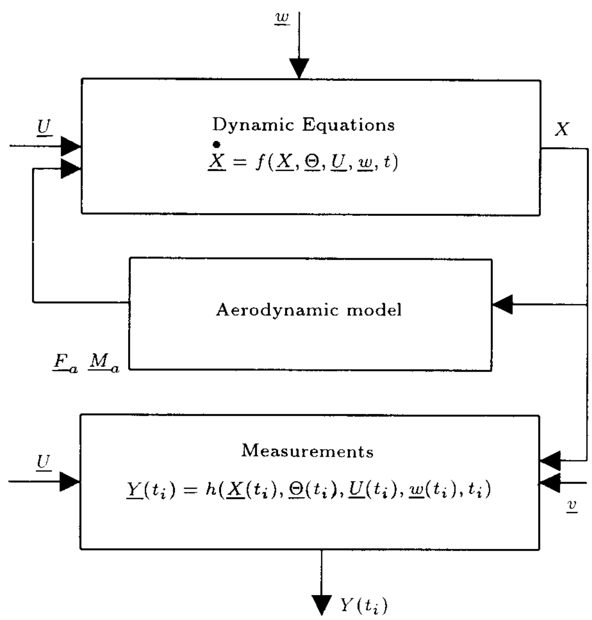

Figure 2. Estimation after modeling.

Comparison of EAM and EBM

The block diagrams of these two techniques are shown in Figures 1 and 2.

As is clear, EBM is an open loop estimation procedure. So for example, error in estimating angle of attack does not affect the error in estimating aerodynamic forces and moments. However, EAM does not have this shortcoming and simulation results also confirm the superior performance of EAM compared to EBM.

Also note that in EBM, aerodynamic forces and moments are considered as Markov processes during the estimation process. It can be shown that process noise in this method is correlated with states [16] and hence, the non-correlation assumption between states, process noise and measurement noise used in almost all identification methods is violated. (Idan [16] used another formulation to avoid this problem. In that formulation, it is assumed that angular velocity vector and linear acceleration of C.G. are measured. It can be shown that if all of these vectors cannot be measured, Idan formulation cannot be used.)

Note that the computational burden of EAM is much higher than EBM since there are more states to estimate in EAM compared to EBM.

Although both EAM and EBM require the use of nonlinear algorithms for aerodynamic parameter estimation, the model used in EAM can be more nonlinear compared to EBM and this can cause a convergence problem. Additionally, if the used aerodynamic model is changed during the identification process, the first phase of EBM that consists of estimating $\underline{F}$ remains unchanged, but this is not the case in EAM.

Identification Algorithms

In the literature, many algorithms are used to estimate aerodynamic parameters using flight test data. Some of these algorithms are as follows: Output error [3], equation error [1,3], equation decoupling [14], nonlinear smoothing identification (like extended Kalman filtering) [6,7,9,13,17,18], maximum liklihood, [1,8,10-12,15] and smoothing based identification [16,19]. The later three algorithms are used extensively.

In the nonlinear smoothing identification method, parameters are modeled as Markov processes and are augmented to state variable vector. Hence, the problem of parameter identification becomes a nonlinear smoothing or filtering in on-line applications problem. These algorithms are generally extended from the linear state estimation theory.

In Smoothing based Identification algorithm, since parameter vector $\underline{\Theta}$ is considered constant, the procedure is divided into two stages. In the first stage, states are estimated and smoothed and in the second, parameters are updated such that a cost function J is minimized. State estimation (or smoothing) is performed via backward-forward filtering. Update of the parameters in the second stage is performed with an optimization algorithm. Here, finding the gradient of the cost function J with respect to parameters is a critical task. In ML, this task is performed by computing the sensitivity matrix [10]. But in smoothing based identification algorithm, the gradient is computed efficiently using Lagrange multipliers during the smoothing procedure. When the parameters are updated, the smoothing algorithm is performed once again. This procedure is repeated until changes in parameters are very small. Based on this overview, ML estimation can be considered as a member of smoothing based identification class, if process noise is also included.

Note that all these algorithms can be used in the EAM strategy but only those which are classified as nonlinear smoothing algorithm can be used with the EBM strategy.

UNMODELING EFFECTS AND NEW METHOD

It is clear that aerodynamic parameter estimation is based on system dynamic and aerodynamic models. In the real world, these models may be inaccurate. In dynamic models, assumptions are as follows.

Product moments of inertia are assumed zero, since the missile is assumed cruciform.

The thrust curve is known.

A linear aerodynamic model or a deterministic nonlinear aerodynamic model is usually assumed. If this model is not accurate, estimated parameters deviate from true values.

(u, v, w) are linear velocity in body axis coordinate. If wind is absent, angle of attack and sideslip will be:

$$\alpha = \tan^{-1}(\frac{w}{u}), \beta = \tan^{-1}(\frac{v}{u}). \qquad (6)$$

If wind flows, angle of attack and sideslip become:

$$\alpha' = \tan^{-1}(\frac{w - w_w}{u - u_w}), \ \beta' = \tan^{-1}(\frac{v - v_w}{u - u_w}). \qquad (7)$$

Since,

$$u_w, v_w, w_w, v, w << u, \qquad (8)$$

the above equations can be approximated as:

$$\alpha' \approx \frac{w - w_w}{u - u_w} = \alpha - \frac{w_w}{u},$$

$$\beta' \approx \frac{v - v_w}{u - u_w} = \beta - \frac{v_w}{u}. \qquad (9)$$

Note that even a linear approximation to aerodynamic force C_z is modeled as:

$$C'_z = C_{za}.\alpha' + C_{zq}.q + C_{z\delta_e}.\delta_e. \qquad (10)$$

This model differs from the linear approximation given as:

$$C_z = C_{za}.\alpha + C_{zq}.q + C_{z\delta_e}.\delta_e \quad \text{assumed model,} \qquad (11)$$

and hence, an unmodeled dynamics is produced.

Every manufactured subsystem of a missile has its tolerance. For example, misalignment of motor nozzles can produce a new force and moment around C.G. of the missile. These forces and moments usually differ from one missile to another in actual flight.

Therefore, it is clear that unmodeled dynamics are always present and the estimation procedure should be capable of dealing with these modeling errors.

Mixed EBM and EAM

It was pointed out previously, that the EAM method usually results in more accurate estimations compared to EBM. However, EAM will run into trouble if there is unmodeled dynamics. This is because in this technique usually an extended kalman filter algorithm is used and the unmodeled dynamics is treated as white Gaussian noise. This assumption is usually not accurate and this is the reason for the poor performance of Kalman filtering [20-23] when modeling uncertainty is present [20-23].

Here, a new technique is suggested. Let now rewrite $\underline{F}_a$ as the sum of two components denoted by $\underline{F}_1$ and $\underline{F}_2$ as follows:

$$\underline{F}_a = \underline{F}_1(\underline{X}, \underline{\Theta}) + \underline{F}_2. \tag{12}$$

Here, $\underline{F}_1$ is a simple preliminary model for aerodynamic forces and moments and $\underline{F}_2$ denotes the rest of aerodynamic forces and moments which are not modeled in $\underline{F}_1$.

Next, $\underline{\Theta}$ and $\underline{F}_2$ are modeled as Markov process and are augmented to the state vector, for which an estimation procedure is used to estimate $\underline{\Theta}$ and $\underline{F}_2$ alongside the state vector $\underline{X}$.

In estimation phase, external forces and moments fit themselves to a preliminary modeled section and a model-free section (Markov processes). Since this procedure, unlike EBM, is not a completely open loop and uses an approximate aerodynamic model, it inherits many advantages of EAM. On the other hand, the model-free section is like EBM.

After the estimation procedure is completed, if a relation among estimated unmodeled forces, moments, state variables and inputs were found, a complementary model will be obtained and will be augmented to the preliminary aerodynamic model section. Repeating the above procedure with the improved model will result in more accurate estimated parameters, due to improved performance of EAM compared to EBM.

This procedure can be repeated until the process $\underline{F}_2$ is whitened. Note that this technique is similar to Proportional Integral (PI) [24] or Proportional Fading-Integral (PFI) Kalman filtering [25]. As is well known, in PI algorithm, an integral model is augmented. Integral section allows for the accurate estimation of disturbances caused by unknown inputs and plant perturbations. Moreover, PFI is a more robust generalization of PI algorithm.

IDENTIFICATION ALGORITHM

Extended Kalman filtering have been used as a parameter Estimator. In many papers, EKF is used to identify aerodynamic parameters using flight test data. In a comparison [7] with nonlinear recursive prediction error method and ML, it is shown that EKF is superior.

Major smoothing algorithms based on Kalman Filtering (KF) are Modified Bryson Frazier (MBF) [26] and Rauch, Tung and Stribel (RTS) [26]. Here, the Square Root (SR) algorithm is used for implementation of KF. Numerical difficulties that appear in other implementations are not seen in SRKF [27]. Square root algorithms can be divided into two different categories: Covariance SRKF and information SRKF. Since covariance of backward filtering at $t = t_N$ must be $P(t_N) = \infty$ [28], ISRKF algorithm is superior in this regard, since $\Lambda(t_N) = P^{-1}(t_N) = 0$. Also, when the initial conditions are not well-known, using the information matrix algorithm usually results in a better performance.

Frazier smoothing algorithm based on ISRKF is shown in Figure 3. The following notes should be considered:

1. The conventional estimate of R (if measurement noise is white Gaussian) using the N most recent residuals, is given by:

$$\hat{R} = \frac{1}{N-1}\sum_{i=1}^{N}(\underline{v}_i - \underline{v}_m)(\underline{v}_i - \underline{v}_m)^T, \tag{13}$$

where:

$$\underline{v}_m = \frac{1}{N}\sum_{i=1}^{N}\underline{v}_i. \tag{14}$$

Robust versions of Equation 7 (with respect to outliers and the assumption that $\underline{v}_i$ are white Gaussian noise) can be found in [29,30].

2. It can be shown that extended Kalman filtering as a parameter estimator fails (with a probability of one)

$H = \frac{\partial h}{\partial X_a}$; $F = \frac{\partial f}{\partial X_a}$; $Q = G^* G^T$
$b^-(k) = S_m^{-1}.\ \hat{\mathbf{X}}^-(k)$; $b^+(k) = S_p^{-1}.\ \hat{\mathbf{X}}^+(k)$

Time update:

$$\begin{bmatrix} N_1 \\ b^-(k+1) \end{bmatrix} = QR\left(\begin{bmatrix} 0 \\ b^+(k) \end{bmatrix}\right)$$

$$\begin{bmatrix} N_2 & N_3 \\ 0 & S_m^{-1} \end{bmatrix} = QR\left(\begin{bmatrix} R^{-1/2} & 0 \\ S_p^{-1}.F^{-1}.G & S_p^{-1}.F^{-1} \end{bmatrix}\right)$$

Measurement update:

$$\begin{bmatrix} S_p^{-1} \\ 0 \end{bmatrix} = QR\left(\begin{bmatrix} S_m^{-1} \\ G^{-1/2}.H \end{bmatrix}\right)$$

$$\begin{bmatrix} b^+(k) \\ N_4 \end{bmatrix} = QR\left(\begin{bmatrix} b^-(k) \\ R^{-1/2}.Y_m \end{bmatrix}\right)$$

ISRF:

$$\hat{\mathbf{X}}_s = (S_{fp}^{-T}\ S_{fp}^{-1} + S_{bp}^{-T}\ S_{bp}^{-1})^{-1}\ (S_{fp}^{-T}\ \hat{\mathfrak{b}}_f + S_{bp}^{-T}\ \hat{\mathfrak{b}}_b)$$

where
QR : QR decomposition
S_f and S_b are square root of forward and backward covariance matirx

Figure 3. Square root extended Kalman smoothing.

to converge to the true values of the parameters in a system whose state noise covariance is unknown.

3. Optimal estimation of Q is a difficult task. It has been shown that under special conditions, it is possible to optimally estimate Q adaptively [31]. This technique is named Residual Correlation Method (RCM). The most important restrictions in this method are:
   a) The system must be LTI.
   b) In order to find a unique solution, the number of unknown components in Q must be less than n^*m (number of states multiplied by number of inputs).

   RCM cannot be used in our case because the model is not LTI. But tuning Q parameters based on whiteness of residuals is the essential idea that is used in all Q estimation methods.

4. Note that, using simulation results, it is known that whitening the residuals increases the estimation accuracy. The relationship between Q and residuals cannot be derived analytically, but several different empirical relationships are proposed [32]. A small Q derives residuals unwhite. A large Q makes residuals white but parameter estimates degrade from their real values. Hence, small Q must be used at first, then, Q is gradually increased such that all residuals are whitened. However, changes made to individual elements of Q are based on engineering experience. For example, assume the first three elements of the state vector are the components of the velocity vector (u, v, w). Therefore, Q_{22} (process noise component in dv/dt equation) affects the a_Y accelerometer residuals and Q_{33} (process noise component in dw/dt equation) affects the a_z accelerometer residuals. These facts help in tuning Q components. Otherwise, choosing the components of Q without these physical insights is usually a very difficult task.

   The tuning of process noise covariance of these unknown forces and moments (Q_D) is critical. If covariance matrix Q_D is too large, the model-free section will be overestimated and estimated parameters deviate from true values. On the other hand, if covariance Q_D matrix is too small, the model-free section will be underestimated.

   Hence, at first, tune Q elements to obtain maximum whiteness of residuals. If residuals remain un-white, increase Q_D, until white residuals are obtained.

A SIMPLE EXAMPLE

In this section, a simple example is presented for identifying the drag coefficient of a sled. Consider the following model that describes the single degree of freedom equation of motion of a sled:

$$\dot{V} = (F_T(t) + K_d^* V^{2*} C_d)/m, \quad K_d = 1/2^* \rho^* S_x. \tag{15}$$

$F_T(t)$ is the thrust and C_d is the drag coefficient that is assumed to be constant with a value of -0.25. For simulation purpose, it is assumed that, $F_T = 350$ N, $m = 10$ kg and $K_d = 0.0039$. The velocity profile V for these values is shown in Figure 4a. Acceleration of sled is measured in discrete times ($T_s = 0.1$ sec), therefore,

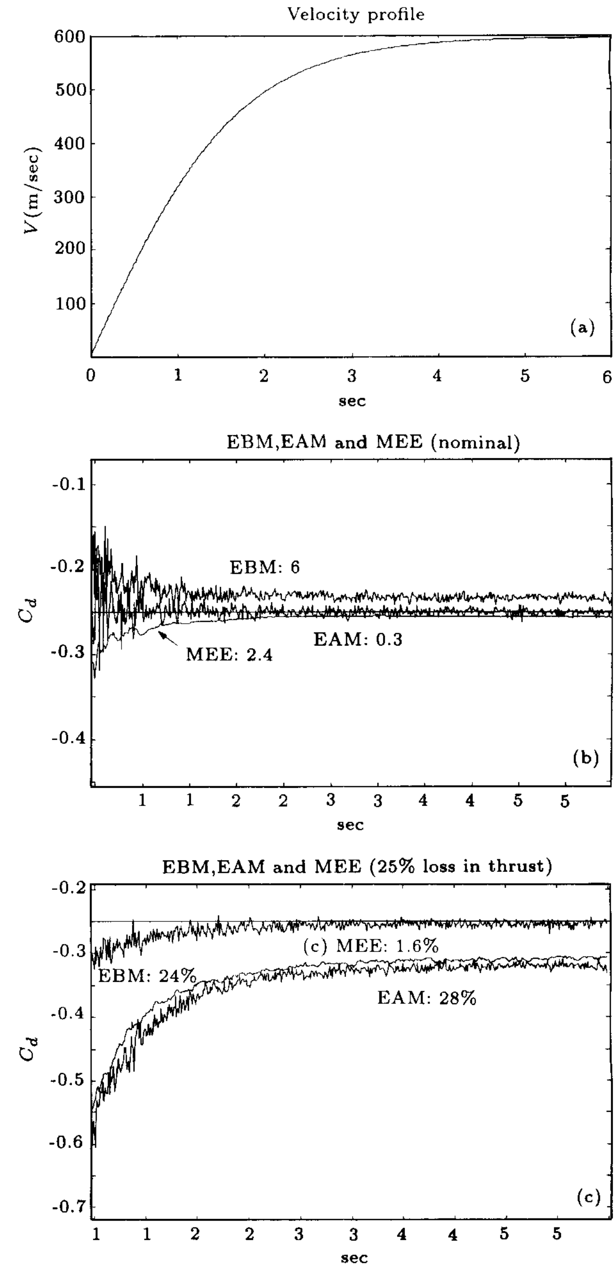

Figure 4. Comparison of EBM, EAM and MEE performance.

the measurement can be modeled as follows:

$$y_n = (F_T(t_n) + K_d^* V(t_n)^{2*} C_d)/m + v(t_n). \quad (16)$$

In simulation it is assumed that $E[v^2(t_n)] = 25\ \mathrm{m}^2/\mathrm{sec}^4$. C_d is identified using three different strategies, namely EBM, EAM and MEE and later the results are compared. Relative error between estimated C_d and true C_d is computed using the following equation:

$$\text{Relative error} = |\frac{C_m - C_t}{C_t}|, \quad (17)$$

where C_m is the mean value of $\hat{C}_d(t)$ in steady-state region and C_t is the true value.

EBM

In this method, the following model is used for estimation:

$$\overset{\bullet}{V} = F/m + w_1, \quad \overset{\bullet}{F} = w_2, \quad y_n = F(t_n)/m. \quad (18)$$

After estimation of state variables, the drag coefficient from the following relation must be extracted:

$$\hat{F}(t) = C_d(t)^* u(t) + F_T(t), \quad (19)$$

where $u(t)$ is:

$$u(t) = K_d^* \hat{V}^2. \quad (20)$$

Recursive output error method is used for identifying C_d. Results are shown in Figure 4b.

EAM

In EAM method, a preliminary aerodynamic model shown below is assumed:

$$F_a = K_d^* V^{2*} C_d. \quad (21)$$

Therefore, the following model is used for the estimation phase:

$$\overset{\bullet}{V} = (F_T(t) + K_d^* V^{2*} C_d)/m + w_1,$$

$$\overset{\bullet}{C}_d = w_2,$$

$$y_n = (F_T(t_n) + K_d^* V(t_n)^{2*} C_d(t_n))/m. \quad (22)$$

Estimation results are shown in Figure 4b. Please note that since velocity V and drag Coefficient C_d directly affect the measurement y_n, the accuracy of estimating C_d and V is increased here. In these cases, EAM performs much better compared to EBM and this is basically another interpretation of closed loop characteristics of EAM.

MEE

The above methods have good accuracy when there is no unmodeling error. For evaluating the effect of modeling errors, 25% loss in the thrust force was assumed. This unmodeling error is encountered in real world, since the real thrust force is always different from nominal thrust. Note that during the estimation phase, the nominal value for the thrust force can be used. Results of EBM and EAM methods are shown in Figure 4c. In MEE method, any unmodeled forces acting on sled is modeled as markov process, like EBM method. Therefore, the following model is used for estimation:

$$\overset{\bullet}{V} = (F_T(t) + K_d^* V^{2*} C_d + F)/m + w_1,$$

$$\overset{\bullet}{C}_d = w_2, \qquad \overset{\bullet}{F} = w_3. \quad (23)$$

Results are shown in Figure 4c. If unmodeling does not occur, F becomes zero and MEE behaves like EAM (Figure 4b). Figures 4b and c show that MEE method is more robust than EAM and EBM methods when unmodeling errors are presented. Accuracy of MEE in both cases is very good. In a complicated case, when unmodeling is due to aerodynamic model inaccuracy, a new relation between F and other state variables can be found and this new model can be added to the preliminary aerodynamic model.

Note that in practice, there are only measurements and tuning the parameters of process noise is difficult. In this example, the noise parameters based on minimizing the mean square of the residual sequence were tuned. This method works very well.

SIMULATION

ISRKF has been used for aerodynamic parameter estimation of an Anti-Tank Guided Missile. This ATGM has automatic command to Line Of Sight (LOS) guidance and is tracked with an IR tracker at the launch sight. The tracker measures Y and Z deviation of missile with respect to LOS in inertial coordinate. Control commands are sent to the missile via a connecting wire. A pulse width modulated actuator is used for applying aerodynamic forces and moments to guide the missile to LOS. A two-degree of freedom gyroscope measures roll and yaw angles. Two additional 3-axis accelerometers are mounted at fore and aft of the missile. This helps to measure a function of angular accelerations. Although pitch angle and angular rates are not measured, but our results show that pitch angle and other state variables can be estimated accurately.

An on-board high shock resistant data acquisition system is designed to record all events and outputs.

After flight, the saved data is downloaded to a computer. This method is very cost effective compared to telemetry and also there will be no interference between radio transmission and missile internal electronics. Moreover, range-independent high quality signals are recorded and there is essentially no limit on the number of channels that can be recorded.

The flight motor of the missile burns for about 2 sec and after that, the missile coasts toward the target. During the first phase, nozzle output jet flows through the wings. By assuming time-varying coefficients, the aerodynamic parameter changes can be tracked during motor burning and coasting phase. In SRKF implementation, f and h derivatives are computed by MATLAB® Symbolic Math toolbox. Sampling times of instruments are not equal. In Figure 4, EBM and EAM algorithms are compared in estimating pitch angle, v and w linear velocities. Based on other estimated variables, EAM is superior to EBM.

In the aerodynamic model a disturbance term is added in Y and Z channel with the following model:

$$C_z = C_{za}.\alpha + C_{zq}.q + C_{z\delta_e}.\delta_e + 0.1. \qquad (24)$$

This disturbance equals 20% of C_y or C_z. Figure 5 compares the results of MEE (Mixed EBM and EAM) and EAM when the mentioned disturbance is present. These figures show the superior performance of MEE as a parameter estimator. The following points are important in implementation of MEE in our application.

Since missile is cruciform, the following equalities hold:

$$C_{z\alpha} = C_{y\beta}, C_{z\delta e} = C_{y\delta r}, C_{m\alpha} = C_{n\beta},$$

$$C_{m\delta e} = C_{n\delta r}, C_{mq} = C_{nr}. \qquad (25)$$

Estimation accuracy of $C_{z\delta e}$ and $C_{m\delta e}$ are much higher than other parameters. This is because of persistent excitation of control derivatives by PWM control input.

Since angle of attack of this missile is not very high, angle of attack derivatives $C_{z\alpha}, C_{m\alpha}$ and $C^2_{d\alpha c}$ have lower estimation accuracy.

Roll angle is stabilized with phase adjustment between pulse commands. In the initial flight phase, the magnitude of input command of roll angle is high, but after a few seconds, it decreases appreciably. Therefore, the roll channel is not sufficiently excited and identification becomes inaccurate. For increasing the estimation accuracy of aerodynamic roll coefficients, an exciting scenario must be designed. It should be noted that $C_{l\delta a}$ is estimated much better than C_{lp}.

In Table 1 accuracy of estimated parameters is computed by the following equation:

$$\text{Relative error} = \frac{1}{N}\sum_{k=1}^{N}\left|\frac{C_e(k) - C_t(k)}{C_t(k)}\right|^{*} 100, \qquad (26)$$

where $C_e(k)$ is the estimation of $C_t(k)$ at kth sample. Since there is no unmodeling presented in X channel, performance of MEE is the same as EAM for estimating C_{d0} and $C_{d\alpha c2}$. But in other parameters, MEE is superior to EAM. Of course it is obvious that this superior performance is accomplished through increased computational complexity.

Test of whiteness is performed on backward filter residuals. Parzen method [31] was used for testing the whiteness of residual sequences. In this method, a band is computed based on correlation of residual sequences. If the number of times that the correlation element lies outside the specified band is less than 5% of the total, the residual sequence is declared white. In Table 2, results of whiteness test are shown for EAM and MEE methods in the presence of unmodeling errors. Note that changing the parameters of noise process in EAM could not whiten the residual sequences.

Estimation algorithm is not sensitive to initial conditions and does not diverge in any simulated condition.

An important feature of any estimation algorithm is measured by its ability in predicting a bound for the estimation error. In KF, square root of diagonal elements of covariance of error matrix is a good criterion.

Figure 6 shows the 2σ Estimated Error Bound (EEB) and real error between estimated parameters and true parameters in the simulation. In all estimated parameters, except $C_{z\alpha}, C_{m\alpha}$ and $C_{d\alpha c2}$, EEB is greater than real error.

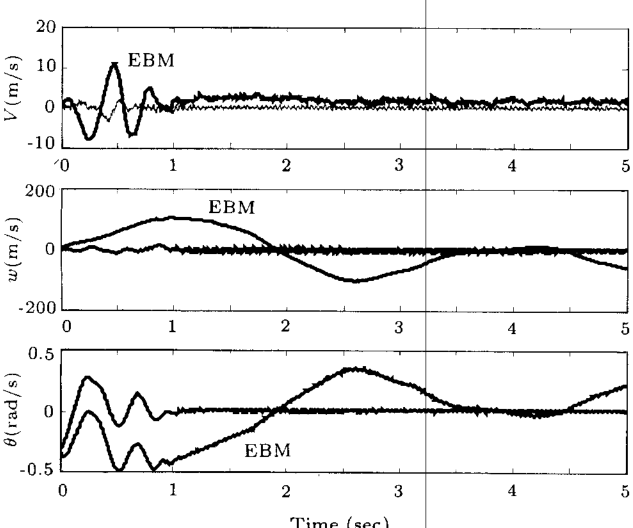

Figure 5. Estimation Error in EBM and EAM.

CONCLUSION

In a comparison of EBM and EAM methods in aerodynamic identification, it is shown that EAM methods are superior, due to their closed loop property.

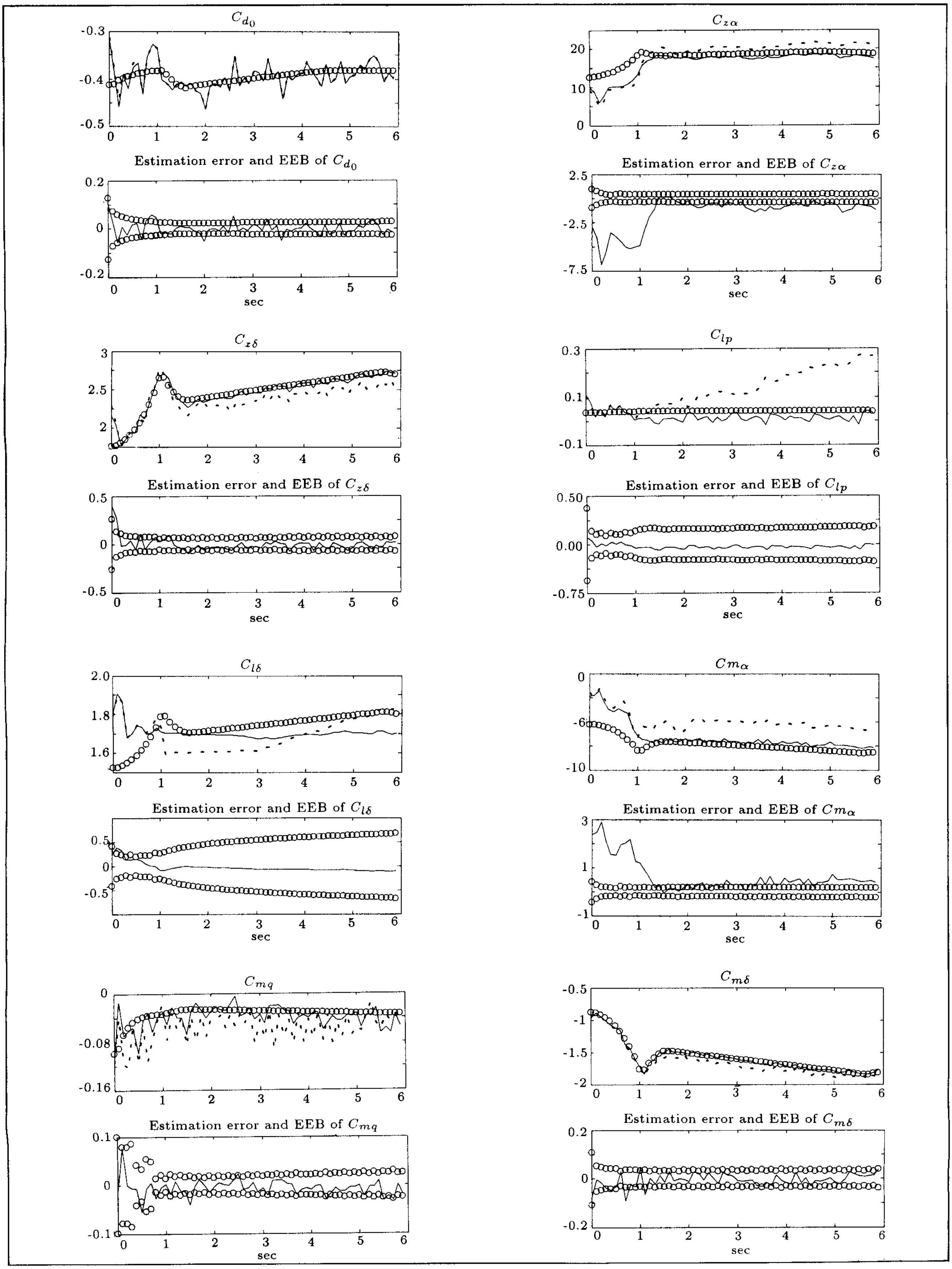

Figure 6. Comparison of MEE and EAM: True value (o), EAM (dashed), MEE (solid).

Table 1. Relative error percentage of estimated parameters in EAM and MEE methods.

Parameters	EAM	MEE
C_{d0}	6.69	6.80
$C_{d\alpha c2}$	74.89	69.22
$C_{z\alpha}$	15.26	9.41
$C_{z\delta e}$	5.63	1.97
C_{lp}	243.27	71.09
$C_{l\delta a}$	13.88	11.65
$C_{m\alpha}$	33.99	12.64
C_{mq}	74.96	34.34
$C_{m\delta e}$	3.11	0.83

Table 2. Results of whiteness test of residual sequences in EAM and MEE methods.

Measurements	EAM	MEE
A1x	1.53	1.52
A1y	72.07	2.95
A1z	57.93	2.73
roll	2.18	2.23
A2y	37.95	3.18
A2z	38.08	2.33
Yaw	8.45	2.30
Velocity	1.68	1.65
Y_i	2.98	3.00
Z_i	2.77	2.78

Here, the EKF algorithm based on EAM strategy was used for identifying time varying aerodynamic parameters. A modification of this technique was used when unmodeled dynamics was present. This modification was based on combining EAM and EBM. The superior performance of the new technique was illustrated through simulation results for an anti-tank missile.

NOMENCLATURE

(u, v, w)	components of missile velocity vector
(u_w, v_w, w_w)	components of wind velocity vector
δ_e	elevator angle
δ_a	aileron angle
δ_r	rudder angle
S_x	reference area
α	angle of attack
β	sideslip angle
$\underline{X}$	state vector
$\underline{F}$	forces and moments acting on the missile
$\underline{\Theta}$	aerodynamic parameters
$\underline{w}$	process noise vector
$\underline{v}$	measurement noise vector
$\underline{v}_i, \underline{v}(t_i)$	measurement noise vector at t_i
R	covariance matrix of measurement noise
ρ	air density
w_1, w_2, w_3	process noise
q	pitch rate.

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