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A New Method for Load Steps Calculation During Power System Restoration

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An important subject in load pick up during power system restoration is the selection of suitable load steps, especially in the initial phase of restoration. Operators are often concerned with the size of the load that can be safely picked up and the effectiveness of the generation reserve. Load pick up in small increments tends to prolong the restoration period. With large increment, there is always the risk of falling into a frequency decline and to a recurrence of the system outage. In this paper, a novel method for dynamic calculation of load steps is presented. The proposed method considers the dynamics of system components, as well as their limitations and uses a suitable form for frequency behavior. It could be used to assist the operator in the on-line selection of suitable load steps and/or to compute the minimal frequency for a given load step. Simulation tests, carried out on a 39-bus New England test system, verify the effectiveness of the proposed method.

INTRODUCTION

The problem of restoring power systems after a complete or partial blackout is as old as the power industry itself. In recent years, due to economic competition and deregulation, power systems are operated closer and closer to their limits. At the same time, power systems have increased in size and complexity. Both factors increase the risk of major power outages. After a blackout, power needs to be restored as quickly and reliably as possible and, consequently, detailed restoration plans are necessary [1-4].

Power system restoration following a complete or a partial system collapse, is the process of restoring power plants, re-energising the transmission network, restoring customer loads and accomplishing this process as rapidly as possible without causing any further failure or equipment damage [1,2]. Power system operations require definite strategies to handle service interruption, where the goal is to minimize the impact on consumers. The strategy developed with this proposal includes [2,4,5]:

- Sectionalisation of the power system into islands,
- Restoration of each island,
- Synchronisation of islands.

The idea behind the proposed strategy is that simultaneous restoration will result in speedy restoration.

In a typical restoration procedure in each island, the stages of the restoration process are summarized as follows [4-6]: In the first stage, the system status is assessed, initial cranking sources are identified and critical loads are located. In the second stage, restoration paths are identified and the system is energised. These islands are then interconnected to provide a more stable system. In the final stage, the bulk of unserved loads is restored.

This paper concentrates on the initial stages of restoration, where operators are often searching for the size of the load, which can be safely picked up, and the effectiveness of the generation reserve [2,7,8]. Load pick up in small increments prolongs the restoration duration. Should an attempt be made to pick up a larger load block than can be accommodated by the response capability of the generators on line, the block of load already restored, which is subject to under-frequency load shedding, will be lost. If the frequency swing is severe, one or more generators may trip, precipitating another shutdown [2,7,9]. Hence, it is desirable to determine the magnitude of the load blocks

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which can be restored as a guideline for operators. In previous works [7,8,10], the dynamics of loads and the limitations of the prime mover are not considered, thus the evaluation of frequency behavior is not complete and exact, especially for calculation of the minimum frequency point during load pick up. Operation of under frequency load shedding relays and the generators protective relays depend on the minimum frequency point. In this paper, a new method for calculation of load steps is presented, which determines the maximum load pick up within the allowable system frequency dip for each bus and each stage of load restoration. This method considers the dynamics of the frequency response of the prime mover and load. Also, a suitable form is used for the behavior of frequency during load reconnection, which transforms the differential equations of the system into algebraic equations to find the minimum frequency. The proposed method is suitable for the on-line guiding of the operator in selecting load steps.

To evaluate long-term stability and to analyze frequency behavior, especially during restoration of customer loads, a simulation program is needed [1,2,11]. In order to make the Long-Term Dynamics (LTD) simulation, a dynamic load flow is used with suitable models for power plants, loads and relays.

The organisation of the paper is as follows. The next section describes power system modeling, i.e., prime mover, network and load modeling for LTD simulation. Then, the Dynamic load flow is discussed and the frequency response of the power system after load pick up and the proposed calculation algorithms are presented. Finally, the simulation results, based on the 39 bus New England test system are provided.

POWER SYSTEM MODELING FOR LTD SIMULATION

System frequency can exhibit large excursions due to load pick up. Generators usually have protective relays or other protections to trip them off-line if the frequency exceeds 110 percent or drops below 95 percent [5]. While both frequency limits are important, it is the lower one that causes greater concern [5]. Frequency decline is due to inadequate system response, following loss of generation or the picking up of a larger load than can be absorbed by the generators online. When a load is restored or a generator trips, the magnitude of the frequency excursion is determined by the response reserve of the power system. This reserve largely comes from 3 sources: Generation reserve of prime movers, load shedding and demand relief [12]. Prime movers of generating units will respond to a fall in frequency by increasing their power output, within a 5 to 10 second interval after the contingency. Customer loads already restored, that are subject to under-frequency load shedding, will be lost if the frequency swing is severe [5]. The frequency sensitivity of the system demand can provide substantial amounts of reserve immediately following a contingency, due to the nature and characteristics of the load. Thus, to analyze the frequency behavior during load restoration, it is essential to model prime movers, protective relays and the load. Then, the frequency changes will be calculated using long-term analysis.

LTD analytical studies address system performance under large-scale upsets which, invoke the actions of slow processes, a protective system and controls not present in conventional transient stability programs [13]. A typical range of a long-term time period can exceed several tens of minutes. The LTD simulation program should include, in addition to the models used in conventional transient stability simulations, adequate representation of the prime mover and energy supply systems. It should also include appropriate models for the wide range of protection and control systems that are invoked when the system is in the emergency and extreme states. The choice of mathematical models for power plant prime movers, generation control and protection systems, load representation, under frequency control and other types of automation, are discussed in this section.

Prime Mover Modeling

Hydro electric units are simulated using a second order model that includes the water hammer effect, as well as transient and steady-state droops and gate rate limit [14,15]. Combustion turbine units are represented by governor, governor rate limit, turbine and power limit models. Dynamic models for steam turbine units, including turbine boiler and the associated control systems, are used for simulation [14].

Network Modeling

In long term stability studies, synchronous generators are modeled using the electromotive force magnitude, E, behind the transient impedance, X'_d. It is, however, important to consider their reactive capability limits [11]. The IEEE standard models are used for excitation systems, which consider saturation, under and over excitation [16]. Basic protective schemes are utilized such as: Under voltage load-shedding relay, under voltage generator protection, turbine under frequency protection, under frequency load-shedding relay and Volt per Hertz (V/H) protection.

Load Modeling for Restored Loads

The load powers are functions of bus voltage and frequency. In the LTD simulation program, loads are modeled using a static model, consisting of ZIP (constant impedance, current and power load model) terms plus two voltage-frequency dependent terms [17]:

- Active component:

$$\begin{aligned} P =&P_0[K_{Pz}.(V/V_0)^2 + K_{Pi}.(V/V_0) + K_{Pc} \\ &+ K_{P1}.(V/V_0)^{n_{Pv1}}.(1 + n_{Pf1}.\Delta f) \\ &+ k_{P2}.(V/V_0)^{n_{Pv2}}.(1 + n_{Pf2}.\Delta f)]. \end{aligned} \tag{1}$$

- Reactive component:

$$\begin{aligned} Q =&Q_0[K_{qz}.(V/V_0)^2 + K_{Qi}.(V/V_0) + K_{qc} \\ &+ K_{q1}.(V/V_0)^{n_{qv1}}.(1 + n_{qf1}.\Delta f) \\ &+ k_{q2}.(V/V_0)^{n_{qv2}}.(1 + n_{qf2}.\Delta f)], \end{aligned} \tag{2}$$

where P_0 and Q_0 are the initial active and reactive load powers from the base case power flow.

This model provides the flexibility to model various types of load. For instance, the frequency dependent terms could be used for static representation of two different types of motors. Alternatively, the frequency dependent terms could be used to represent a motor and, also, fluorescent lighting.

Cold Load Pick up Modeling

Representation of reconnected loads plays a significant role in the safe evolution of the restoration plan, with particular reference to island frequency behavior. In fact, there is a close relationship between the duration of the outage and the power demand profile. In [18], experimental data are used to model equivalent load demands after cold load pick up with linear (active power) and exponential (reactive power) fitting. Load real and reactive powers are, therefore, as:

$$P(t)=\begin{cases} 0, & t<t_0 \\ P_{init}-\frac{P_{init}-P_{fin}}{T_d}.(t-t_0), & t_0\leq t<t_0+T_d \\ P_{fin}, & t>t_0+T_d \end{cases} \tag{3}$$

$$Q(t) = \begin{cases} 0, & t < t_0 \\ Q_{init} - (Q_{init} - Q_{fin}).(1 - e^{-\frac{t-t_0}{\tau}}), & t \geq t_0 \end{cases} \tag{4}$$

where t_0 is the instant of insertion of the cold load, T_d is the time interval of the real power linear decay, τ is the time constant of the reactive exponential decay, P_{init}, Q_{init} are the peak value at cold load pick up and P_{fin}, Q_{fin} are the final steady-state values of the cold load. The active power component has a decreasing linear form until T_d seconds after load pick up, where T_d is dependent on the duration of load outage.

DYNAMIC LOAD FLOW

Dynamic load flow, based on the uniform frequency power system model, is a special type of power flow. The main differences between the dynamic and static load flow are:

1. The active power of slack generator is determined in dynamic load flow;
2. Transient impedance, Z, is considered as a series element of an electrical network for all generators;
3. In dynamic load flow, an additional variable is defined, which is named acceleration power and shows a mismatch between generation and consumption.

Acceleration power determines the system frequency, according to the swing equation.

For the LTD simulation, in each time step, the electromotive force magnitude, E, behind the transient impedance, X'_d, as well as the mechanical power, P_M are given for generation buses. P_M and E are determined from the differential equation describing the transient processes in prime movers and generators, which take into account variable limits and possible control actions.

LOAD STEP CALCULATION

The minimum frequency, after load pick up, determines the activation of under frequency load shedding relays and generators protective relays. Thus, it is an important calculation of minimal frequency, as a dynamic constraint during load pick up. In this section, the approach for finding minimum frequency after load reconnection is presented. Then, a load step calculation algorithm is denoted.

After load pick up, power consumption increases and system frequency decreases. A swing equation for the power system, based on a uniform frequency, determines the frequency variation [19]:

$$P_{acc} = 2H_{sys}.\omega.\frac{d\omega}{dt}, \tag{5}$$

where:

$$P_{acc} = \sum_{i=1}^{ng} P_{g,i} - \sum_{i=1}^{nl} P_{l,i} - P_{loss}, \tag{6}$$

$$P_{g,i} = P_{g0,i} + P_{gov,i}, \tag{7}$$

$$P_{l,i} = P_{l0,i} + \Delta P_{l,i}. \tag{8}$$

H_{sys} is the total inertia constant of the power system, $P_{g0,i}$ the active power generation of ith unit, before load pick up, $P_{gov,i}$ the increment of active power generation of the ith unit after load pick up, due to its governor response, $P_{l0,i}$ the load power of bus no. i before load pick up, $\Delta P_{l,i}$ the load power that is picked up at ith bus and P_{loss} is the active power loss of the electrical network.

Equation 5 shows the frequency response of the system after load pick up. Experience shows that system frequency changes after switching a load has a quadratic form, in terms of time, until the minimum point (Figure 1):

$$\omega - \omega_0 = a.t^2 + b.t, \quad 0 \leq t \leq t_{\min}, \tag{9}$$

where ω_0 is the initial frequency of the power system, $\omega_{\min}$ is the minimum frequency of the system after load pick up, $t_{\min}$ is the time to reach the minimum frequency.

With this assumption, the minimum frequency point can be calculated with no need to solve the differential equations of the system and, hence, speed of calculation increases significantly. In the minimum frequency point, it is shown that:

$$\omega(t = t_{\min}) = \omega_{\min}, \quad \left.\frac{d\omega}{dt}\right|_{(t=t_{\min})} = 0. \tag{10}$$

According to Equations 9 and 10, it can be written:

$$t_{\min} = \frac{2(\omega_{\min} - \omega_0)}{b}, \quad a = -\frac{b^2}{4(\omega_{\min} - \omega_0)}. \tag{11}$$

From Equation 9, at initial time, it is concluded that:

$$\left.\frac{d\omega}{dt}\right|_{t=0^+} = b. \tag{12}$$

By substituting the above equation in Equations 5 to 8, b is calculated as follows:

$$b = \frac{1}{2\omega_0 H_{sys}}.\Big(\sum_{i=1}^{ng} P_{g0,i} - \sum_{i=1}^{nl} P_{l0,i}(\omega_0) - \Delta P_{l0}(\omega_0) - P_{loss}(\omega_0)\Big). \tag{13}$$

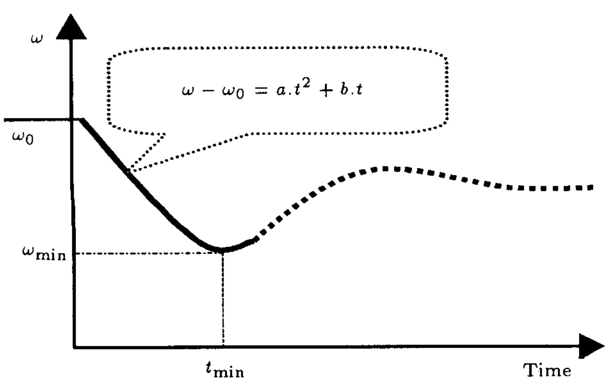

Figure 1. Quadratic form of frequency change.

Load pick up value at the minimum frequency point can be expressed as:

$$\Delta P_{l,i}(\omega_{\min}) = \sum_{i=1}^{ng} P_{g0,i} + \sum_{i=1}^{ng} P_{gov,i} - \sum_{i=1}^{nl} P_{l0,i} - P_{loss}. \tag{14}$$

In the Appendix, using Equations 9 and 11, P_{gov} is expressed in terms of b and $\omega_{\min}$ at the minimum frequency point. Equation 14 results from Equation 5, at the minimum frequency point, only for load pick up in the ith bus. Assuming steady-state, ω, before load pick up and using Equations 6 and 13, yields:

$$P_{loss}(\omega_0) = \sum_{i=1}^{ng} P_{g0,i} - \sum_{i=1}^{nl} P_{l0,i}(\omega_0), \tag{15}$$

$$b = \frac{1}{2\omega_0 H_{sys}}.[-\Delta P_{l0}(\omega_0)]. \tag{16}$$

In Equation 14, P_{loss} is needed for $\omega_{\min}$, however with a good approximation, it can be calculated by ω_0 in Equation 15, because the effect of frequency changes in the power loss of the electrical network can be neglected. Thus, Equation 14 can be written as follows:

$$\Delta P_{l,i}(\omega_{\min}) = \sum_{i=1}^{ng} P_{gov,i} - \sum_{i=1}^{nl} P_{l0,i}(\omega_{\min}) + \sum_{i=1}^{nl} P_{l0,i}(\omega_0). \tag{17}$$

There are two expressions for the value of the picked up load at the minimum point, that depend on the relation between T_d and $t_{\min}$. Using Equation 3, the change in the system load, after load pick up, can be expressed as:

$$\Delta P_{l,i}(\omega_{\min}) = \begin{cases} P_{fin}.\left(k_p + \frac{2(k_p-1).(\omega_0-\omega_{\min})}{b.T_d}\right), & t_{\min} < T_d \quad \text{(18a)} \\ P_{fin}, & t_{\min} > T_d \quad \text{(18b)} \end{cases}$$

where k_p is the ratio of initial to final active power of the load.

There are two problems during load pick up: (a) Calculation of the maximum load pick up for a given frequency decline, (b) Computation of the minimum frequency for picking up an estimated low voltage AC network load. For finding a solution to these problems, Equation 17 is used in an iterative algorithm. This equation contains 4 terms: The value of cold load at minimum frequency (Equation 18a or 18b), governor system response for all generators (Appendix) and the values of restored loads at minimum and initial frequency (Equation 1).

Figure 2 shows a flow chart of the algorithm for case (a), which has two sections. The initial assumption is that $t_{\min}$ is smaller than T_d. Then, Equation 18a is used for the value of cold load in Equation 17. After finding the load step and calculating $t_{\min}$, if $t_{\min}$ is greater than T_d, then Equation 18b is used for the value of cold load in Equation 17.

The initial guess value for P_{fin} is calculated as follows:

$$P_{fin}^{(0)} = -\frac{\omega_{\min}}{\sum_{i=1}^{ng} R_i}, \tag{19}$$

where R_i is the permanent droop coefficient of the ith unit.

A similar iterative algorithm is used for calculation of minimum frequency for a given load step.

SIMULATION RESULTS AND DISCUSSION

In this section, the new method is used for the calculation of load step and minimum frequency for the 39-bus New England system (Figure 3) and the results are compared with LTD simulation results. The system includes 39 buses, 10 generators, 35 lines and 12 transformers and is divided into 2 subsystems (islands) for simultaneous restoration. The first island includes generators in buses 30, 33, 34, 35, 36, 37, 38 and the second island includes generators in buses 31, 32, 39. Lines between buses 1 and 39, 3 and 4, 15 and 16, are out of service and divide the system into 2 islands.

Minimum Frequency Calculation

In this case, the load step is selected equal to one per unit and minimum frequency is calculated by the proposed method in each island.

Table 1 shows the results of calculations and simulation, where buses are selected according to the restoration scenario for this system. The third and fourth columns of this table, respectively, show minimum frequency calculation and LTD simulation results show minimum frequency for the load step equal to one (p.u) at each load bus.

The Error (%) in Table 1 is calculated as follows:

$$\text{Error}(\%) = \frac{\omega_{\min}^{*} - \omega_{\min}^{**}}{\omega_{\min}^{**}} \times 100, \tag{20}$$

where $\omega_{\min}^{*}$ and $\omega_{\min}^{**}$ are minimum frequency that have been calculated by the proposed method and LTD simulation, respectively.

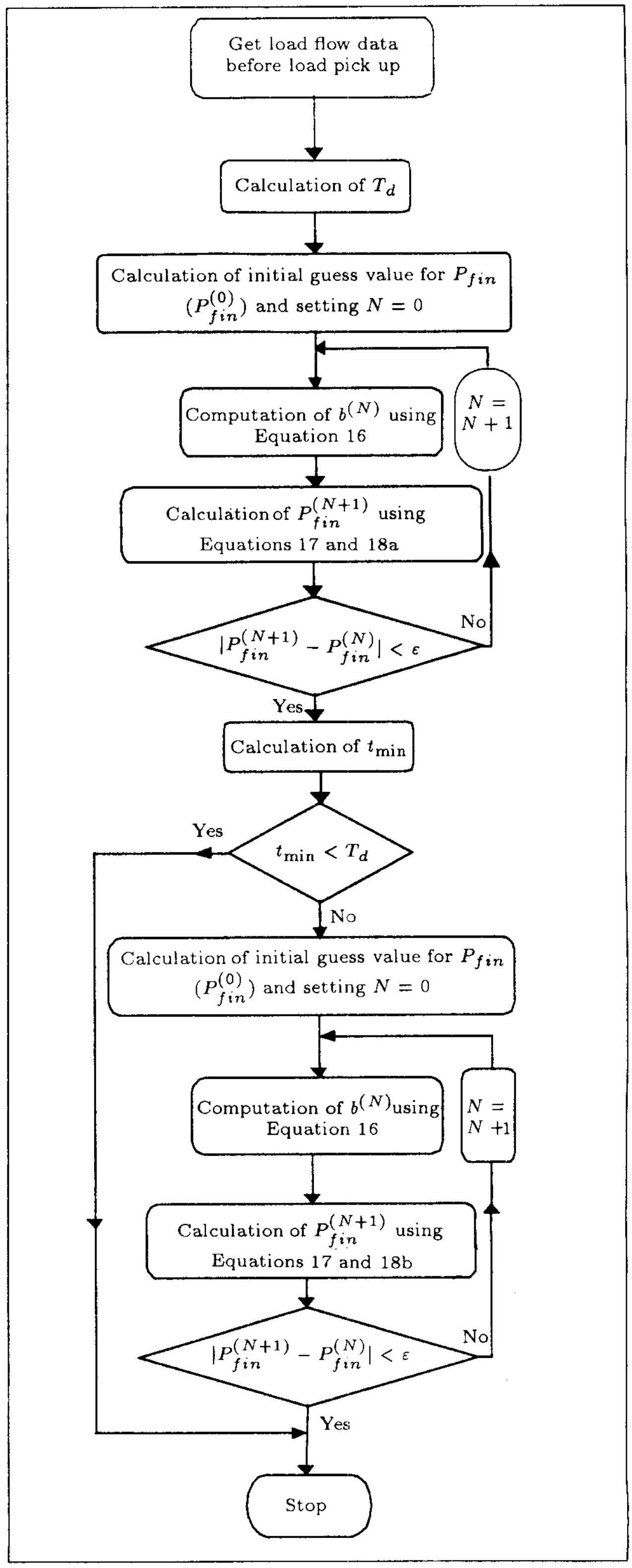

Figure 2. Flow chart of load step calculation.

Load Step Calculation

In practice, for calculation of load steps, the minimum allowable frequency is selected according to the characteristics of the power system (inertia constant of

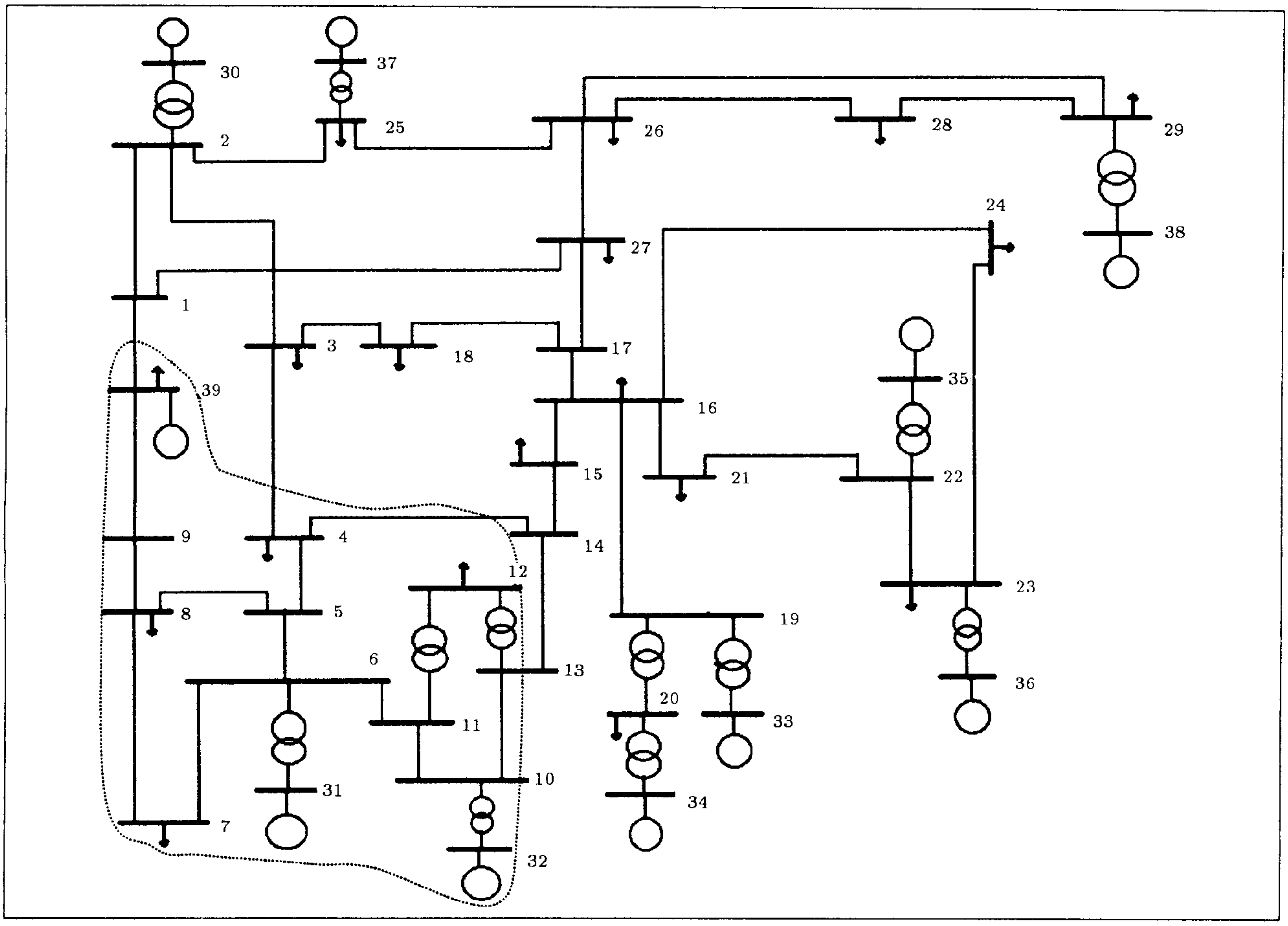

Figure 3. One line diagram of 39-bus New England test system.

generators, settings of under frequency load shedding relays and generators protective relays,...). Minimum frequency is selected equal to 59.64 Hz (or -0.6%) for the 39-bus test system and then the load step is calculated for a number of load buses during system restoration. Table 2 shows the results of load step calculation by the proposed method and the results of minimum frequency from LTD simulation for these load steps. Figure 4 shows the frequency response of the system after load pick up in load bus no. 39 at $t = 13$ (sec). This figure confirms the assumption of quadratic form for frequency change until the minimum frequency point.

The results of these cases and other test cases show the effectiveness and accuracy of the proposed method.

In practice, in the control center, the SCADA/EMS provides data of the current network topology and in the restoration scenario the load which must be picked up is determined. Then, the calculation algo-

Table 1. Minimum frequency calculation results.

Bus No.	No. of Island	$\omega^{*}_{\min}$ (Hz) (The Proposed Method)	$\omega^{**}_{\min}$ (Hz) (LTD Simulation)	Error (%)
3	1	59.78	59.77	4.3
8	2	59.51	59.45	10.9
15	2	59.50	59.45	9.1
18	1	59.78	59.76	8.3
27	1	59.77	59.76	4.2
39	2	59.55	59.50	10.0

* Minimum frequency calculation for load step = 1 p.u

** LTD simulation results for $\omega_{\min}$ after load pick up with load step = 1 p.u

Table 2. Load step calculation results.

Bus No.	No. of Island	Load Step* (p.u)	$\omega^{**}_{\min}$ (Hz)	Error (%)
3	1	1.5842	59.652	3.3
7	2	0.6530	59.688	13.3
8	2	0.6532	59.688	13.3
15	2	0.6547	59.676	10
18	1	1.5302	59.664	6.6
27	1	1.5037	59.652	3.3
28	1	1.5317	59.664	6.6
39	2	0.7266	59.676	10.0

* Load step calculation for $\omega_{\min} = 59.64$ Hz.

** LTD simulation results for $\omega_{\min}$ after load pick up with the calculated load step.

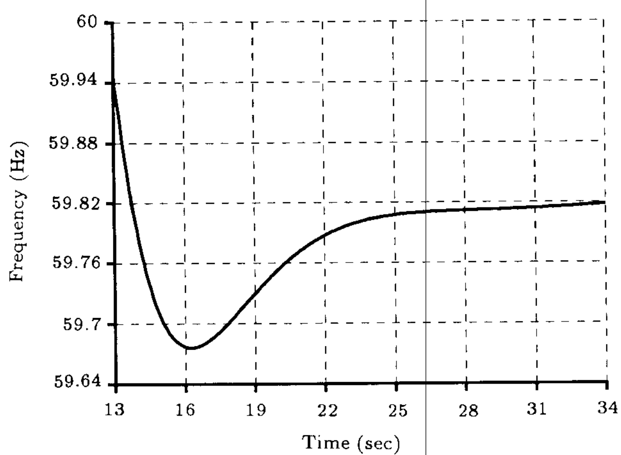

Figure 4. Frequency response after load pick up in load bus no. 39.

rithm computes the load step, or minimum frequency, dependent on the load condition. The algorithm obtains the dynamic data of units and loads from the data base. The minimum frequency for the load step calculation can be determined by the algorithm (as a constant value) or by operators.

CONCLUSION

In this paper, modeling of loads, power plants and relays are explained and then dynamic load flow is used for the simulation of long-term dynamics during system restoration. Furthermore, a new method for dynamic calculation of load steps is presented. This method is suitable for the on-line guiding of the operator in selecting suitable load steps and in calculating the minimum frequency after reconnection of a given load, since it requires lesser computation time than full simulation. Simulation tests on the 39-bus New England system verify the effectiveness of the proposed method.

Also, this method can be used for dynamic security assessment during a forced outage of a generating unit by predicting the operation of an under-frequency load shedding system.

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APPENDIX

For load step calculation, a general model of a turbine and governor system, as shown in Figure A1, is used.

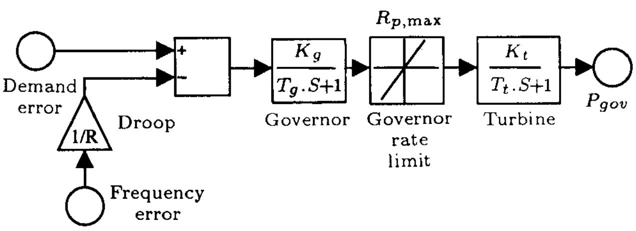

Figure A1. Governor and turbine model for load step calculation.

Using this block diagram, the differential equation of a governor system can be written as:

$$P_{gov} + T'.\frac{dP_{gov}}{dt} + T''.\frac{d^2P_{gov}}{dt^2} = K(\omega - \omega_0), \tag{A1}$$

where:

$$T' = T_t + T_g, \quad T'' = T_t.T_g, \quad K = -\frac{K_g.K_t}{R}. \tag{A2}$$

Substituting Equation 11 in Equation A1 at the frequency minimum point, yields:

$$\begin{aligned} P_{gov}(t = t_{\min}) &= \frac{1}{(\omega_{\min} - \omega_0)}[K_1.e^{\frac{2S_1}{b}(\omega_{\min}-\omega_0)} \\ &+ K_2.e^{\frac{2S_2}{b}(\omega_{\min}-\omega_0)}] + K_3.e^{\frac{2S_1}{b}(\omega_{\min}-\omega_0)} \\ &+ K_4.e^{\frac{2S_2}{b}(\omega_{\min}-\omega_0)} + K(\omega_{\min} - \omega_0) \\ &+ \frac{K.b^2}{2(\omega_{\min} - \omega_0)}(T'' - T'^2), \end{aligned} \tag{A3}$$

where:

$$\begin{aligned} K_1 &= Kb^2.\frac{(T'' - T'^2).S_2 - T'}{2(S_1 - S_2)}, \\ K_2 &= -Kb^2.\frac{(T'' - T'^2).S_1 - T'}{2(S_1 - S_2)}, \\ K_3 &= -Kb^2.\frac{T'^2.S_2 + 1}{S_1 - S_2}, \\ K_4 &= -Kb.\frac{T'.S_1 + 1}{S_1 - S_2}, \end{aligned} \tag{A4}$$

$$\begin{aligned} S_1 &= \frac{\sqrt{T'^2 - 4T''} - T'}{2T''}, \\ S_2 &= -\frac{\sqrt{T'^2 - 4T''} + T'}{2T''}. \end{aligned} \tag{A5}$$

If governor reaches its rate limit, then:

$$\begin{aligned} P_{gov}(t = t_{\min}) &= R_{p\max}.t_{\min} \\ &= R_{p\max}.\frac{2(\omega_{\min} - \omega_0)}{b}. \end{aligned} \tag{A6}$$