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Stability and Control of Linear Systems with Multilinear Uncertainty Structure

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In this paper, linear systems with uncertain physical parameters are considered. The case where the characteristic equations of the systems are multilinearly dependent on some uncertain parameters is studied. A new procedure is proposed for the calculation of stability margins in the parameter space in general l^p-norms. The stability is defined with respect to a desired region as the location of poles in the complex plane (so-called D-stability). The procedure is simple and computationally feasible. It requires the numerical solution of a system of equations in each frequency. The Minimum Distance approach, used in several previous works for polynomials with linearly-dependent coefficients, is generalized here for the case of polynomials with multilinearly-dependent coefficients. Based on the above approach for the calculation of D-stability margins, a method is presented for the design of robust controllers. Robust placement of the closed-loop poles inside the regions of interest is addressed. The method provides a simple and computationally-tractable solution for the synthesis of robust controllers for systems with multilinear uncertainty structure.

INTRODUCTION

A common approach in the analysis and synthesis of linear systems is to deal with their characteristic equations. When the parameters of a system are uncertain, the coefficients of its characteristic equation (polynomial) are perturbed inside some ranges and that, in turn, results in the movement of the roots of the characteristic equation in the complex plane. Then, one can study the performance of the uncertain system by investigating the effects of parameter uncertainties on its characteristic equation. It is well-known that, to achieve a desired performance, the roots of the characteristic equation can be placed in an appropriate region (D) in the complex plane (so-called D-stabilization).

A fundamental type of problems encountered in robust control of uncertain systems is the calculation of maximum allowable perturbations in parameters of a D-stable system, without losing D-stability. In [1-3], general l^p-norm perturbations are considered and several algorithms are presented for the case of polynomials with linearly-dependent coefficients. However, in most applications, the coefficients of the characteristic equation of the system are dependent multilinearly or nonlinearly [4].

For polynomials with multilinear structure, the well-known mapping theorem [5] is the most powerful available tool for checking robust stability. The mapping theorem has been used in [6,7] to develop some algorithms for the calculation of stability margins. In the analysis context, in [8], it was shown that to check the stability of interval multilinear polynomials, it is sufficient to check only a set of manifolds in the parameter space. The results of [8] and [5] were used in [9] to present a numerical algorithm for the calculation of the Hurwitz-stability margin of multilinear systems. Since the mapping theorem provides the sufficient condition for stability, the method of [9] gives conservative results unless the proposed multiple decomposition of the uncertainty region is performed. The decomposition proposed in [9], on the other hand, increases the computational burden of the method. To the best of the author's knowledge, a general algorithm does not exist for the calculation of general l^p-norm margins of multilinear systems in the literature.

In this paper, a new procedure is presented to calculate bounds of l^p-perturbations in the parameters

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of systems in which the characteristic polynomials are multilinearly dependent for the preservation of D-stability. The procedure requires sweeping the contour of the region and, at each point on the contour, a system of equations must be solved. The number of equations depends upon the number of uncertain parameters of the system, which is limited in real applications. The minimum distance approach introduced in [10,11] is generalized here for the case of multilinear systems.

Another important problem in robust control is design of robust controllers. The properties of polynomials have been little used in the design of robust controllers [4]. Pole placement is one of the techniques widely used for controller design [12]. An important issue in the pole placement method is the sensitivity of the closed-loop systems to the variation of the parameters of the system. These sensitivities were taken into account in [13] by computing the variation of the poles with respect to the variation of the parameters. The robust pole placement inside the circles centered at the nominal closed-loop poles has been investigated in [14]. In this paper, the design of robust controllers for multilinear systems is posed as an optimization problem. A method is presented for the robust placement of closed-loop poles of uncertain systems inside the regions of interest. The results of this paper on D-stability margin calculation are used in the formulation and the solution of the synthesis problem. An example is also presented to demonstrate the methods. A preliminary version of this paper was presented in [15].

PRELIMINARIES

Consider the polynomial:

$$Q(\mathbf{q}, s) = a_n(\mathbf{q})s^n + \cdots + a_1(\mathbf{q})s + a_0(\mathbf{q});$$

$$a_n(\mathbf{q}) \neq 0, \tag{1}$$

where the coefficients $a_n(\mathbf{q}), \cdots, a_1(\mathbf{q}), a_0(\mathbf{q})$ are multilinear functions of the system parameters:

$$\mathbf{q} = \begin{bmatrix} q_1 & q_2 & \cdots & q_m \end{bmatrix}, \ \mathbf{q} \in \mathbf{R^m}.$$

The results of the well-known Zero Exclusion Theorem is used in this paper for the calculation of D-stability margins.

Theorem 1 [4] (Zero Exclusion)

Suppose a family of invariant-degree polynomials (Equation 1) with $\mathbf{q} \in \mathbf{Q}$, where $\mathbf{Q}$ is an uncertainty set which is pathwise connected. Let D be an open subset of the complex plane and C_D be its contour. Furthermore, assume that the family of polynomials has at least one D-stable member. Then, the family of polynomials is D-stable if, and only if:

$$0 \notin Q(\mathbf{q}, u); \ \ \forall \mathbf{q} \in \mathbf{Q}, \ \ \forall u \in C_D.$$

To check the necessary and sufficient conditions of Theorem 1 for D-stability, the contour of the region must be swept. The point u on the contour C_D can be expressed as a function of a sweeping parameter ζ, i.e., $u = u(\zeta)$; $\zeta \in \Omega$. For instance, the contour of the unit circle can be represented by $u = e^{j\zeta}$; $\zeta \in [0, 2\pi)$. The substitution of $u = u(\zeta)$ in Equation 1 results in:

$$Q\big(\mathbf{q}, u(\zeta)\big) = Q_R(\mathbf{q}, \zeta) + jQ_I(\mathbf{q}, \zeta), \tag{2}$$

where:

$$Q_R(\mathbf{q}, \zeta) = Re\Big[Q\big(\mathbf{q}, u(\zeta)\big)\Big], \tag{3}$$

and:

$$Q_I(\mathbf{q}, \zeta) = \mathrm{Im}\Big[Q\big(\mathbf{q}, u(\zeta)\big)\Big]. \tag{4}$$

Define the nominal parameter vector:

$$\overline{\mathbf{q}} = \begin{bmatrix} \overline{q}_1 & \overline{q}_2 & \cdots & \overline{q}_m \end{bmatrix}, \tag{5}$$

at which $Q(\overline{\mathbf{q}}, s)$ is D-stable. The calculation of the minimal distance of $\overline{\mathbf{q}}$ to the instability region in the system parameter space will be addressed in the next section. The weighted l^p-distance (norm) of two arbitrary points $\mathbf{q}, \mathbf{q}' \in \mathbf{R^m}$, is defined by:

$$\delta_p(\mathbf{q}, \mathbf{q}') = \left[\sum_{k=0}^{m} \Big(|q_k - q_k'|/w_k\Big)^p\right]^{1/p}, \tag{6}$$

where $w_k > 0$, $k = 1, \cdots, m$ are weights and $1 < p < \infty$ is a constant. If $\widetilde{\mathbf{q}}$ represents a perturbed value of the parameter vector, $\delta_p(\overline{\mathbf{q}}, \widetilde{\mathbf{q}})$ gives the l^p-norm magnitude of perturbation from $\overline{\mathbf{q}}$ to $\widetilde{\mathbf{q}}$.

D-STABILITY MARGINS

In the wake of the Minimum Distance approach presented in [11] for the calculation of D-stability margins for polynomials with independent and linearly-dependent coefficients, a relevant approach is taken in this paper for the multilinear case.

From Theorem 1, it is conferred that the contour of the D-stability region must be swept. At an arbitrary value of the sweeping parameter, ζ^*, which corresponds to the point u^* on the contour, the minimum distance of the nominal parameter vector $\overline{\mathbf{q}}$ to the set of parameter vectors $\mathbf{q}^*$, at which D-stability conditions of Theorem 1 fail, can be found by solving the following optimization problem:

$$\rho_p(\zeta^*) \doteq \min_{\mathbf{q}^*} \delta_p(\overline{\mathbf{q}}, \mathbf{q}^*), \tag{7a}$$

subject to:

$$Q_R(\mathbf{q}^*, \zeta^*) = 0, \tag{7b}$$

$$Q_I(\mathbf{q}^*, \zeta^*) = 0. \tag{7c}$$

Several relevant optimization methods have been used in [16,17] for the case of polynomials with linearly-dependent coefficients. Here, the more general case of polynomials with multilinear coefficients is considered. At each ζ^*, the perturbations $\delta_p(\overline{\mathbf{q}}, \widetilde{\mathbf{q}})$ must be kept smaller than the minimum distance $\rho_p(\zeta^*)$, to ensure D-stability of the perturbed polynomials. Then, the D-stability margin of the nominal polynomial $Q(\overline{\mathbf{q}}, s)$ is obtained, sweeping ζ^* in its entire range, i.e., $\min_{\zeta^*} \rho_p(\zeta^*), \zeta^* \in \Omega$.

A necessary condition of Theorem 1 for D-stability, is the invariance of the degree of the perturbed polynomials. This means that the perturbation of the parameters must not result in the nullification of $a_n(\mathbf{q})$. To satisfy this condition, the perturbations $\delta_p(\overline{\mathbf{q}}, \widetilde{\mathbf{q}})$ must be less than the optimal value:

$$\eta_p \doteq \min_{\hat{\mathbf{q}}} \delta_p(\overline{\mathbf{q}}, \hat{\mathbf{q}}), \tag{8a}$$

constrained by:

$$a_n(\hat{\mathbf{q}}) = 0. \tag{8b}$$

The following theorem is next stated.

If the perturbations in the parameters of the system are smaller than both minima of Equations 7 and 8, considering that is D-stable, all conditions of Theorem 1 are satisfied and the family of perturbed polynomials are D-stable. The above discussion is concluded by the following theorem.

Theorem 2

The family of polynomials:

$$P = \left\{Q(\mathbf{q}, s), \mathbf{q} \in B_p(\overline{\mathbf{q}}, \xi_p)\right\},$$

where:

$$B_p(\overline{\mathbf{q}}, \xi_p) = \left\{\widetilde{\mathbf{q}} : \delta_p(\widetilde{\mathbf{q}}, \overline{\mathbf{q}}) < \xi_p\right\}, \tag{9}$$

is the l^p-hypersolid of parameter uncertainty, D-stable if, and only if, $Q(\overline{\mathbf{q}}, s)$ is D-stable and $\xi_p < \gamma_p$, where:

$$\gamma_p = \min\left\{\eta_p, \min_{\zeta^*} \rho_p(\zeta^*) : \zeta^* \in \Omega\right\}. \tag{10}$$

Proof

If the conditions of the theorem are satisfied, the requirements of Theorem 1 for D-stability are satisfied since:

1. There exists at least one stable member $Q(\overline{\mathbf{q}}, s)$ in the family of polynomials;
2. There is not any change in the degree of perturbed polynomials if the perturbation $\delta_p(\widetilde{\mathbf{q}}, \overline{\mathbf{q}})$ is less than η_p;
3. The perturbed polynomials $Q(\widetilde{\mathbf{q}}, s)$ do not vanish as the contour of the region is swept. If $\xi_p < \gamma_p$, where γ_p is the minimum distance of $\overline{\mathbf{q}}$ to the set of $\mathbf{q}^*$ at which zero exclusion fails (Equations 7b and 7c), the distance of any point $\widetilde{\mathbf{q}} \in B_p(\overline{\mathbf{q}}, \xi)$ to the nominal $\overline{\mathbf{q}}$, is less than D-stability margin γ_p.

Therefore, the family of polynomials satisfies the necessary and sufficient conditions of D-stability. ■

In the rest of this section, the optimizations in Equations 7 and 8 are performed. Optimization (Equations 7), with two constraints, will be discussed. Similar steps can be taken to perform the optimization in Equations 8.

The method of Lagrange multipliers is used to perform the optimization (Equations 7). The functional:

$$F(\mathbf{q}^*) = \left[\delta_p(\overline{\mathbf{q}}, \mathbf{q}^*)\right]^p + \lambda Q_R(\mathbf{q}^*, \zeta^*) + \mu Q_I(\mathbf{q}^*, \zeta^*),$$

is defined. At the critical points, one has:

$$\frac{\partial F}{\partial q_k^*} = -\operatorname{sgn}(\overline{q}_k - q_k^*)\frac{p}{w_k^p}|\overline{q}_k - q_k^*|^{p-1} + \lambda\frac{\partial Q_R(q_k^*, \zeta^*)}{\partial q_k^*} + \mu\frac{\partial Q_I(q_k^*, \zeta^*)}{\partial q_k^*} = 0,$$

$$k = 1, \cdots, m.$$

Then, with some manipulation of the equations, it is obtained that:

$$\Delta_k \doteq \left|\overline{q}_k - q_k^*\right| = \left[\frac{w_k^p}{p}\left|\lambda\frac{\partial Q_R(q_k^*, \zeta^*)}{\partial q_k^*} + \mu\frac{\partial Q_I(q_k^*, \zeta^*)}{\partial q_k^*}\right|\right]^{1/(p-1)},$$

$$S_k \doteq \operatorname{sgn}(\overline{q}_k - q_k^*) = \operatorname{sgn}\left(\lambda\frac{\partial Q_R(q_k^*, \zeta^*)}{\partial q_k^*} + \mu\frac{\partial Q_I(q_k^*, \zeta^*)}{\partial q_k^*}\right).$$

At ζ^*, corresponding to a point on the contour, the following system of equations must be solved to obtain the closest distance to the instability border:

$$q_k^* = \overline{q}_k - S_k\Delta_k \quad k = 1, \cdots, m,$$

$$Q_R(q^*, \zeta^*) = 0,$$

$$Q_I(q^*, \zeta^*) = 0, \tag{11}$$

where $(m+2)$ unknowns are q_k^*; $k = 1, \cdots, m$ and the Lagrange multipliers λ and μ. The critical points q_k^*; $k = 1, \cdots, m$ obtained from the system of Equations 11 can be substituted in Definition 6 to obtain the minimum distance of the nominal parameter vector $\overline{q}_k$ to the instability border:

$$\rho_p(\zeta^*) = \min_{\mathbf{q}^*} \delta_p(\overline{\mathbf{q}}, \mathbf{q}^*).$$

SYNTHESIS OF ROBUST CONTROLLERS

Consider a standard feedback control system (Figure 1). The plant transfer function is defined by:

$$G(\mathbf{q}, s) = \frac{N_G(\mathbf{q}, s)}{D_G(\mathbf{q}, s)}, \tag{12}$$

where $N_G(\mathbf{q}, s)$ and $D_G(\mathbf{q}, s)$ are polynomials multilinear in $\mathbf{q}$. The parameters of the plant are uncertain and are perturbed inside the l^p-hypersolid (Equation 9), where $\overline{\mathbf{q}}$ is the nominal value of the parameter vector. In most applications, the bounds of perturbations of the parameters of the system can be estimated a priori and the perturbation hypersolid is predefined for the design problem. For $p = \infty$, the hypersolid of Equation 9 is a box and for $p = 2$, the hypersolid is a hypersphere.

The controller is represented by the ratio of two fixed polynomials, i.e.,

$$C(\mathbf{x}, s) = \frac{N_C(\mathbf{x}, s)}{D_C(\mathbf{x}, s)}, \tag{13}$$

where:

$$N_C(\mathbf{x}, s) = \sum_{k=0}^{l} c_k s^k,$$

$$D_C(\mathbf{x}, s) = \sum_{k=0}^{l} d_k s^k,$$

and the controller vector is defined by:

$$\mathbf{x} \doteq \left[c_l \cdots c_1 \; c_0 \quad d_l \cdots d_1 \; d_0\right]. \tag{14}$$

The characteristic polynomial of the closed-loop control system is given by:

$$Q_{CL}(\mathbf{q}, \mathbf{x}, s) = N_G(\mathbf{q}, s) N_C(\mathbf{x}, s) + D_G(\mathbf{q}, s) D_C(\mathbf{x}, s).$$

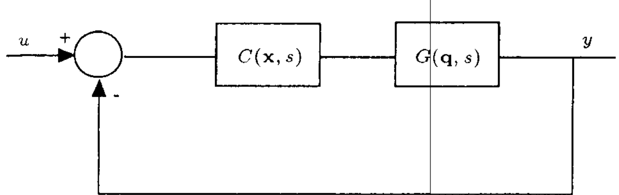

Figure 1. Standard feedback control configuration.

To comply with the definition of the uncertain characteristic polynomial given in Equation 1, noting that the controller $\mathbf{x}$ is fixed, one may write:

$$Q_{CL}(\mathbf{q}, \mathbf{x}, s) = Q(\mathbf{q}, s) = a_n(\mathbf{q}) s^n + \cdots + a_1(\mathbf{q}) s + a_0(\mathbf{q}),$$

where the coefficients are multilinear in the plant parameters $\mathbf{q}$. Then, the results of the previous section can be used for this feedback control system.

For the robust synthesis of the system, the Robust Pole Assignment (RPA) can be defined as [18]. Given the uncertain plant model of Equation 12 and a region D in the complex plane, find a controller $C(\mathbf{x}, s)$ such that all closed-loop poles lie in D for every $\mathbf{q} \in B_p(\overline{\mathbf{q}}, \xi_p)$.

It is well known that various performances for a system can be achieved by the placement of the roots of its characteristic polynomial in appropriate regions. The results of the previous section on D-stability of polynomials with linear uncertainty structure, can be used directly for the solution of the RPA problem for the feedback configuration described in this section.

The main aim in RPA is to ensure that the roots of the family of characteristic polynomials lie inside D. However, a set of controllers, rather than a unique controller, usually satisfies this requirement. Then, to select a controller among all D-stabilizing controllers, an additional objective can be introduced [13]. In this paper, the controller design is posed as the following optimization problem:

$$\min_{\mathbf{x}} f(\mathbf{x}, D), \tag{15a}$$

subject to:

$$Q_{CL}(\overline{\mathbf{q}}, \mathbf{x}, s) \text{ is } D\text{-stable}, \tag{15b}$$

$$\xi_p \leq \gamma_p, \tag{15c}$$

where γ_p is obtained from Equation 10.

The function $f(\mathbf{x}, D)$ can be any convex function whose optimal value is desired. Several examples for this function are presented in [13,18]. One of the common objectives for Equation 15a is $f(\mathbf{x}, D) = f(\mathbf{x}) = \delta_p(\mathbf{x}, \overline{\mathbf{x}})$, where $\overline{\mathbf{x}}$ is the nominal controller vector. The objective is the distance of the designed controller to the nominal controller. This distance is to be minimized.

A necessary condition of Theorem 2 for D-stability is the existence of at least one D-stable member in the family of polynomials. Constraint 15b is added to ensure that the closed-loop system is D-stable at the nominal values of the parameters $\overline{\mathbf{q}}$.

When Inequality 15c is satisfied, it is ensured that the roots of the characteristic polynomials corresponding to all plants with $\mathbf{q} \in B_p(\overline{\mathbf{q}}, \xi_p)$, lie inside D. As mentioned in the proof of Theorem 2, this constraint is imposed to guarantee that the size of the uncertainty hypersolid (Equation 9) is smaller than D-stability margin γ_p.

EXAMPLE

The following feedback control system introduced in [9] is examined in this paper. The plant transfer function is given by:

$$G(\mathbf{q}, s) = \frac{s^2+s+1}{s^3+q_2s^2+4s+q_1}\,\frac{6.6s^3+13.5s^2+15.5s+20.4}{s^3+q_3s^2+3.5s+2.4},$$

with the nominal parameter vector:

$$\overline{\mathbf{q}} = \begin{bmatrix}2 & -3 & 3.5\end{bmatrix},$$

and the weights $w_i = 1; i = 1, 2, 3$.

Calculation of Stability Margin

Assuming the fixed nominal controller:

$$C(\overline{\mathbf{x}}, s) = C(s) = \frac{s+2}{s+1}; \quad \overline{\mathbf{x}} = \begin{bmatrix}1 & 2 & 1 & 1\end{bmatrix}, \qquad (16)$$

it is desired to calculate the Hurwitz-stability margin of the nominal parameters of the plant. The contour of the region is the imaginary axis and a point on the contour can be described by the sweeping function $u(\zeta) = j\zeta;\ \zeta \in [0, \infty)$. At each ζ^*, the system of Equation 11 is formed and solved to compute the minimum distance $\rho_p(\zeta^*)$. In Figure 2, the minimum distances are plotted for the nominal parameter vector in a selected range for $p = \infty$. The minimum of $\rho_\infty(\zeta^*)$ is obtained as 0.640 at $\zeta^* = 1.92$. For this system, considering that the coefficient of the highest order coefficient of the characteristic equation is unperturbed ($a_n = 1$), $\eta_p = \infty$ is obtained. Therefore, the Hurwitz-stability margin is calculated from Theorem 2 as:

$$\gamma_\infty = \min\{\infty, 0.640\} = 0.640.$$

Investing more computational effort, a close value was obtained in [9] by dividing the uncertainty domain into smaller regions and applying the results of the mapping theorem.

In contrast to most of the current methods, which consider the interval perturbations ($p = \infty$), the method described in this paper can be used for any norm. For special cases of $p = 1, 2$, the stability margins $\gamma_1 = 2.77$ and $\gamma_2 = 0.94$ are obtained.

Design of a Robust Controller

For the sample plant, it is assumed that the parameters are uncertain in the ranges between:

$$\mathbf{q}_{\min} = \begin{bmatrix}1 & -4 & 2.5\end{bmatrix},$$

and :

$$\mathbf{q}_{\max} = \begin{bmatrix}3 & -2 & 4.5\end{bmatrix}.$$

Therefore, the uncertainty set is the box $B_\infty(\overline{\mathbf{q}}, \xi_\infty)$ with the size $\xi_\infty = 1$. The nominal controller (Equation 16) does not stabilize the uncertain plant. The locations of the roots of the characteristic equation, using this controller, is shown in Figure 3 for five evenly-spaced points in the ranges of the uncertain parameters (between $\mathbf{q}_{\min}$ and $\mathbf{q}_{\max}$).

For this system, Optimization Problem 15 is defined as follows. Objective 15a is given by $f(\mathbf{x}, D) = f(\mathbf{x}) = \delta_2(\mathbf{x}, \overline{\mathbf{x}})$. The controller parameters c_1 and d_1 are kept unchanged by assigning small numbers as their

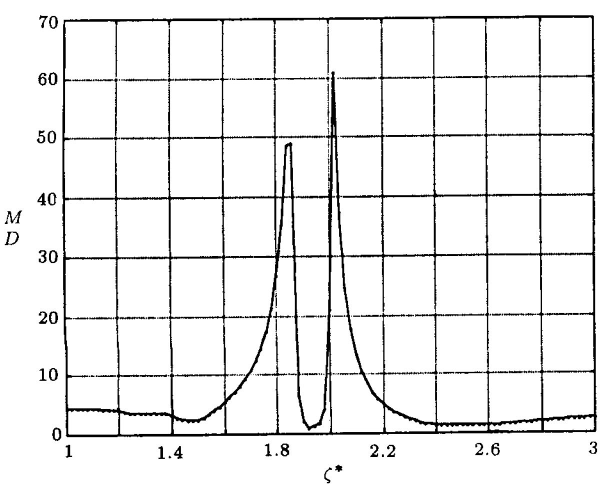

Figure 2. Minimum distances for the example.

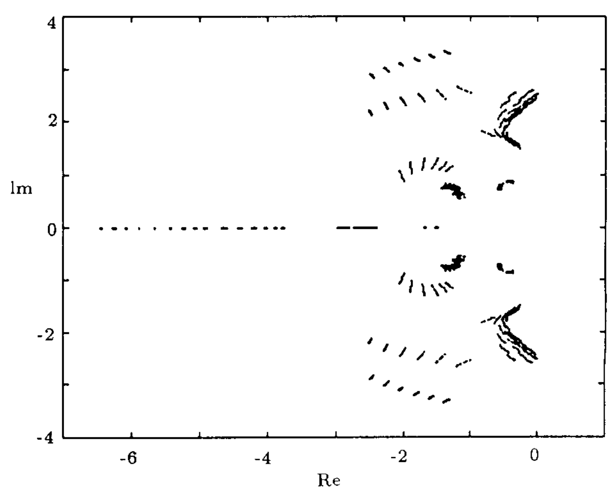

Figure 3. Closed-loop poles with the nominal controller.

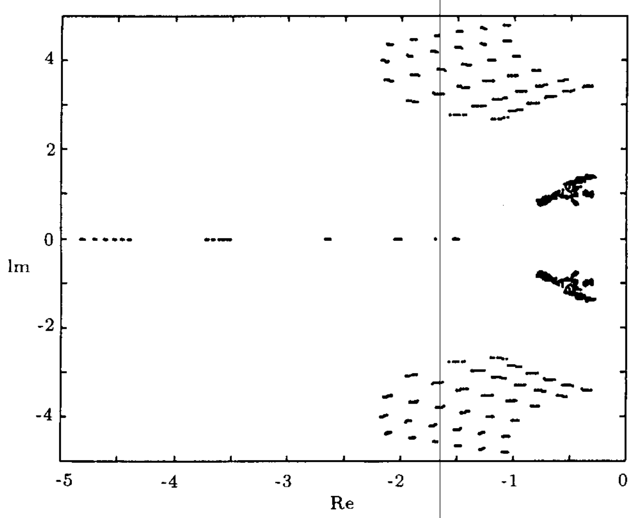

Figure 4. Closed-loop poles with the optimal controller.

weights. The weights in $\delta_2(\mathbf{x}, \overline{\mathbf{x}})$ are then defined as $[10^{-9} \quad 1 \quad 10^{-9} \quad 1]$. The region D in Equation 15b is assumed to be the left-hand plane. The solution of the above optimization problem leads to the optimal controller:

$$\mathbf{x}_{\mathrm{Opt}} = [1 \quad 2.878 \quad 1 \quad 0.102] \, .$$

The closed-loop poles of the control system with the controller $\mathbf{x}_{\mathrm{opt}}$ and the uncertain parameters ($\mathbf{q}$ evenly spaced in ten intervals) are plotted in Figure 4. It can be seen that the controller places all the poles inside the left-hand plane and D-stabilizes the plant.

DISCUSSION AND CONCLUSION

In this paper, a new method has been presented for the calculation of D-stability margins in the parameter space for polynomials with multilinearly-dependent coefficients. Compared with some existing methods, such as that of [6,9], the method presented here is more straightforward and computationally-efficient. It requires sweeping of the contour and, at each point on the contour, it requires the solution of the system of Equation 11. The number of equations depends on the number of uncertain parameters of the system. In real physical applications, this number is limited. Then, the computational burden of the method is contained.

In contrast to previous works [10,11], the margins calculated by Theorem 2 are not conservative, since D-stability conditions are both necessary and sufficient. This comes from the fact that in the optimization (Equation 7), both real and imaginary parts of the characteristic polynomial are constrained simultaneously, while in previous works, the constrains were considered separately to accommodate for analytical optimization. However, the methods of [10,11] require much less computations for the simpler case of linear uncertainty structure. The simplicity and the computational feasibility of the proposed method provides the grounds for its use in iterative design algorithms. In this paper, a new method has also been presented for the design of robust controllers for multilinear systems. The robust placement of closed-loop poles in the regions of interest has been addressed. The method can be used to achieve optimal controllers that D-stabilize an uncertain plant with a multilinear uncertainty structure. Considering that few methods are available for robust control of systems with multilinear uncertainty structures, this work provides feasible tools to deal with such systems.

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