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Reliability Analysis of Dynamic Structures

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In this paper, the concepts to compute reliabilities for stationary and ergodic conditions in the presence of time-invariant, non-ergodic parameters will first be reviewed. Focus will be on numerical techniques like FORM and numerical integration. The effect of correlation between the environmental processes within a sea state is studied, as opposed to between sea state correlation which proves be unimportant. The system reliability of jack-up structures is discussed using a combination of dynamic simulation of a stick model and the static analysis of a detailed jack-up model. First, the dynamic time domain simulation, the quasi-static time-domain simulation and the design wave analysis are applied for different sea-states using a stick structural model. In the second phase, the concepts of Dynamic Amplification Factors (DAF) and Calibration Factors (CF) are introduced for extreme responses of a detailed structural model of the jack-up structure. To utilize the analogical quantities for stick and detailed structural models, the design wave responses of the detailed structural model are linked with the extreme responses and the probabilistic models of dynamic effects using a stick structural model.

INTRODUCTION

To ensure a rational design of deep water platforms, the dynamic response of slender offshore structures to environmental loading has received significant attention in the recent years. In this paper, various aspects including linear dynamics, nonlinear hydrodynamic force, free surface effects, P-δ and P-Δ effects (second order effects), complex structural details for a jacking system and the soil-structure interaction are encountered. In all these features, the main aim is concentrated on implied safety levels of jack-up structures in harsh environmental conditions. Different consequences of failure may also be a reason to differentiate the safety levels for jack-up platforms rather than for the fixed type units.

Studies of the structural behavior of jack-up platforms have been made by Kjeoy et al. [1], Liu [2] and Karunakaran [3]. It is recognized that the jack-up concept is sensitive to dynamic response effects. However, the jack-up structures are presently designed for deeper waters and harsher environments, the dynamic effects changing significantly for different response quantities. The dynamic amplification factors of SDOF obtained for a regular sea-state will fail in the prediction of extreme responses. To solve the problem, Karunakaran [3] used a Monte Carlo simulation technique for the long term reliability analysis of a jack-up structure against foundation failure and overturning moment.

Regarding structural non-linearity, the state of stresses in the jack-up components may, in some cases, result in plastic deformations. For frame structures, the common system reliability is based on two basic assumptions. Firstly, the dynamic effects are neglected in evaluating the response and, secondly, the loads are assumed to be proportional and attain their maximum values simultaneously. In the practical designs, the post failure behavior of a real offshore platform and the behavior of real structures near collapse can be very complex and expensive to assess with the dynamic simulation of the detailed jack-up models.

For the reliability analysis of fixed offshore platforms, a number of methods have been developed to obtain the system reliability based on the most dominant mechanisms (see [4-7]). De et al. [6] and Dalane et al. [8] developed approximate methods in

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the case of jacket type platforms for the effect known as "fixed load pattern" in literature. For dynamically sensitive structures such as jack-up platforms in deep water, there is not yet a developed method for the contribution of dynamic loads in system reliability analysis (the state-of-the-art by Nikolaidis et al. [5] and Moan [9]). This paper is aimed at presenting some ideas for the reliability analysis of dynamically sensitive platforms.

STOCHASTIC DYNAMIC ANALYSIS

The stochastic dynamic analysis of a stick model with Monte Carlo simulation requires a very large sample size, depending on the acceptable statistical error. Importance sampling techniques, based on the random selection of sea-state variables, have been used for the reliability analysis of a stick jack-up structural model (see [3]). Such a procedure of Monte Carlo simulation is mostly suitable for static problems and not for stochastic dynamic analysis. The combined sampling scheme and dynamic simulation is amenable and time consuming and is, therefore, recommended for reliability analysis of a static system. In this paper, to simplify the task of stochastic dynamic analysis, the uncertainties of system parameters are assumed less important than the uncertainty of environmental variables. The reliability analysis is thus carried out by the conditional limit state functions on the structural parameters and the uncertainties of the system variables are treated in a further step by the assumption of a one by one relation of excitation and response quantities of the static structures.

A realistic reliability analysis of the jack-up structure must account for the variation of load with time and the uncertainty of the structural resistance. In the domain of stochastic mechanics, two basic types of random variables can be distinguished namely, the time invariant random variables, $Z(t)$ = constant in time and the time-dependent random variables, $X(t)$. Usually, some of the time-dependent random variables may be kept constant for a short term realization while, for the long term simulation, these variables are also time variant (see Figure 1). According to Bjerager et al. [10], the time dependent stochastic process may be classified by two sets of environmental processes, a fast process $X(t)$ and a slow process $Y(t)$.

The slow part of the process represents the intensity of the process, such as the significant wave height (H_s) and spectral peak period (T_P). The fast part of the process represents the deviations from the mean value within the slow process, such as sea surface elevation at a specific location. The fast process is defined conditionally on the value of the slow process: $X(t) = X(t, Y(t))$, e.g., the wave motion process,

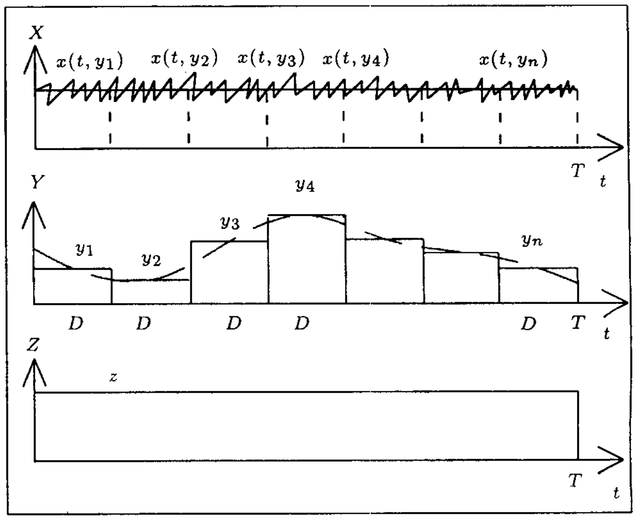

Figure 1. Identification of fast, slow, constant process in time domain analysis.

which is considered for a given value of the significant wave height.

Further, the slow process is assumed stationary for the average period (the sea state duration), i.e. $Y(t)$ is approximated by a sequence of random vectors $y_1, y_2, y_3, \cdots, y_n$ as shown in Figure 1. The stochastic process $Y(t)$ is given by N pulses with rectangular pulse duration $D = T/N$. The problem of combination of N sea states is approximated with the failure events of conditionally independent short term responses.

STRUCTURAL MODELS FOR RELIABILITY ANALYSIS

A jack-up platform system can be split into four subsystems, namely the hull, the legs, the leg-hull interface (jacking system) and the leg-foundation interface. The reason behind this classification is as follows. The jack-up structures have been built using a steel frame hull structure supported by three or four legs. The way of connecting the deck to the legs by the jacking system has great influence on the stress distribution near the deck-leg connection, due to the interaction of leg inclination and the directional moment distribution. An accurate model for the connection of the hull structure to the jacking system can only be obtained choosing the deck structural model in different elevations.

For the purpose of this study, two models of jack-up structures are used:

1. A simplified (stick) 3-D finite element model of the jack-up structure exposed to the design wave loads in the static analysis method and to the irregular sea-states in the stochastic nonlinear dynamic anal-

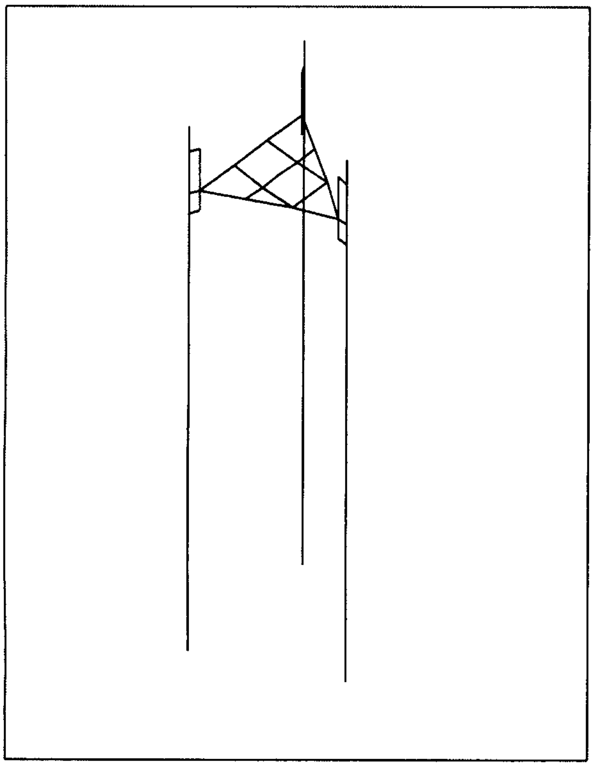

Figure 2. The 3-D stick model of the jack-up structure.

ysis methods. The parametric study for a 3-D stick structural model has been presented for the three-leg jack-up structure located in the Hutton area of the North Sea [11]. The finite element representation of the structural model, with the element numbers, is presented in Figure 2. The overall leg truss work is idealized as a string of beam elements with equivalent stiffness properties. The computer model adopted here is a three-dimensional space frame with 59 nodes and 71 elements. The prototype is a three-leg jack-up rig that is triangular in plane and rests on spudcans at the base of each leg.

The leg-hull connection may be discretized by linear springs located at the position of lower and upper guide levels. The realistic model of the leg-hull interface is a complicated three-dimensional structure with clearances in leg-guides, backlash in the jacking system and several kinds of interactions. For jack-ups equipped with a fixation system, in lieu of backlash and guide tolerance, the jacking system behavior is roughly linear. The fixation system provides a clamping mechanism in which the model is represented by a totally rigid connection.

2. A detailed 3-D finite element model of the jack-up structure with the limited number of simulations in the single design wave method. Using shell elements for the platform, interface elements for the jacking system and 3-D beam elements for the jack-up legs results in a model that accurately represents most aspects of a jack-up structure [12,13]. The detailed structural models are characterized by the type of jacking system and the bracing mode. The original structure is the K-braced three-legged jack-up structure with tubular chords connected to the platform through the fixed type jacking system with a fixation system. Each chord member has one rack fitted in the split tubular section. For the fixed type jacking units with a fixation system, the initial failure occurs for the chord members below the lower guide level. To investigate the effect of the bracing mode, the structural analysis is carried out with an X-braced configuration. Further, the structural model is altered in the second design with a floating type model. As a typical jack-up design, the floating type jacking system is considered without the fixation mechanism. The finite element models of the stick and the detailed models are shown in Figures 2 and 3.

The finite element model for the prototype model is constructed from typical spring, beam and plate elements (see Figure 3). The model contains 1800 nodes connected by 1128 plate elements and 1990 beam or spring elements. The tubular chords of each leg (totally 216 elements) are connected with three types of diagonal, horizontal and inner bracings modeled by 845 tubular elements. Besides structural and reliability analysis of all detailed leg parts, the effect of the

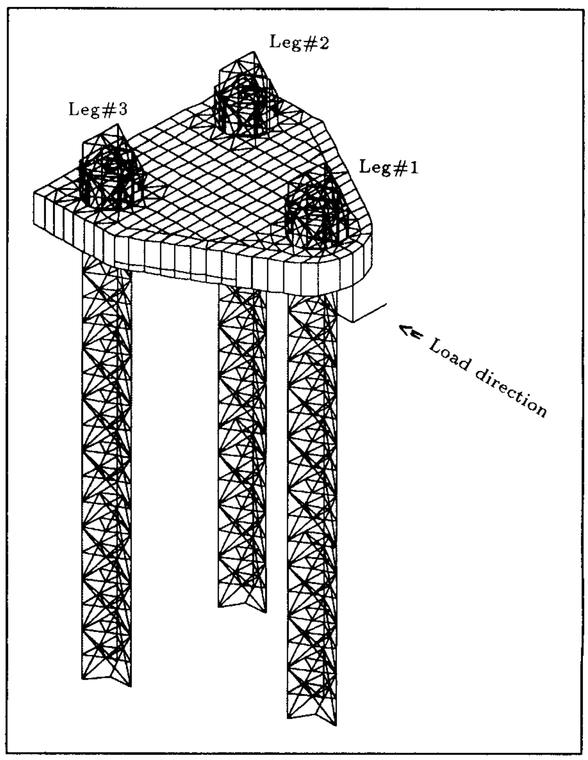

Figure 3. The 3-D detailed model with detailed leg, hull and jacking system.

jacking mechanism and the redundancy of bracing components can be better identified with the overall detailed models.

RELIABILITY ANALYSIS FOR A SINGLE ELEMENT

The dynamic simulation of the detailed structural model is too complex, therefore, in this study, only the static design wave responses are analyzed for this model. A reliability analysis of the detailed model is, however, formulated with combined dynamic simulation of the stick model. Using the stick structural model, dynamic and quasi-static responses are found for estimation of dynamic amplification factors and calibration factors of different short term sea-states.

Short-Term Reliability

The statistical characteristics of responses are estimated conditionally to the sea-states and the Dynamic Amplification Factors (DAFs) and Calibration Factors (CFs) are defined based on the conditional sea-states for the same response quantities. Mathematically, the random DAF and CF are written as:

$$\hat{\text{DAF}}|H_s = \frac{\max(\hat{\sigma}_d|H_s)}{\max(\hat{\sigma}_{qs}|H_s)}, \tag{1}$$

$$\hat{\text{CF}}|H_s = \frac{\max(\hat{\sigma}_{qs}|H_s)}{\hat{\sigma}_{dw}(\hat{H}|H_s)}, \tag{2}$$

in which $\hat{\text{DAF}}|H_s, \hat{\text{CF}}|H_s$ are random; DAF and CF are conditioned on the sea-state with a significant wave height H_s. The max $(\hat{\sigma}_d|H_s)$ and max $(\hat{\sigma}_{qs}|H_s)$ represent the maximum stress component in a certain short term period (3 hours) for dynamic and quasi-static analyses responses and $\hat{\sigma}_{dw}$ is the design wave stress response based on the maximum wave height of the same short term sea-state. For the dynamic and quasi-static analysis, the Gumbel time variant distribution can be used and the response quantity of the design wave analysis can be given for the same stick structural model.

The term quasi-dynamic is used for the random response of the detailed model because the dynamic effects of the stick model are assumed to represent the sensitivity of dynamic effects for the more detailed structural model. The closed form relation between design wave responses of the detailed structural model is combined with the random dynamic amplification factor and calibration factor of the stick structural model at the same structural position. For an individual short term sea state, the conditional failure probability can be computed from the following equation:

$$\begin{aligned} P(F_{s|y,z}) &= P\{M \le 0|H_s\} \\ &= P\{g(\sigma_y, \hat{\sigma}_A, \hat{\sigma}_B) \le 0|H_s\}, \end{aligned} \tag{3}$$

in which:

$$\begin{aligned} \hat{\sigma}_A &= \hat{\text{DAF}}_A \times \hat{\text{CF}}_A \times \hat{\sigma}_A(\hat{H}|H_s), \\ \hat{\sigma}_B &= \hat{\text{DAF}}_B \times \hat{\text{CF}}_B \times \hat{\sigma}_B(\hat{H}|H_s), \end{aligned} \tag{4}$$

where $\hat{\text{DAF}}_A, \hat{\text{CF}}_A$ and $\hat{\sigma}_A$ are random DAF_A, CF_A and σ_A (the dynamic amplification factor, the calibration factor and the design wave response of axial stress, respectively) conditioned on the sea-state with a significant wave height H_s. Similarly, $\hat{\text{DAF}}_B, \hat{\text{CF}}_B$ and $\hat{\sigma}_B$ are random DAF_B, CF_B and σ_B (the dynamic amplification factor, the calibration factor and the design wave response of bending stress, respectively) conditioned on the sea-state with a significant wave height H_s. The design of tubular leg elements is performed according to the Norske Veritas Classification Notes (DNV-OS 1982, see [14]):

$$\begin{aligned} I.R. = &\frac{\hat{\sigma}_A}{\sigma_y}\left[1 + \frac{1}{\left(1 - \frac{\hat{\sigma}_A}{\sigma_{xkE}}\right)}\left(\frac{\sigma_y}{\sigma_{xcE}} - 1\right)\left(1 - \frac{\sigma_{xcE}}{\sigma_{xkE}}\right)\right] \\ &+ \sqrt{\left(\frac{\hat{\sigma}_{B1}}{\sigma_y}\right)^2 + \left(\frac{\hat{\sigma}_{B2}}{\sigma_y}\right)^2} \le 1, \end{aligned} \tag{5}$$

where $I.R.$ indicates the interaction equation, $\hat{\sigma}_A$ is the axial stress, $\hat{\sigma}_{B_1}$ and $\hat{\sigma}_{B_2}$ are bending stresses associated with two normal axes of a tubular section. Stresses σ_y, σ_{xcE} and σ_{xkE} are the yield stress, the characteristic compressive stress and the Euler buckling stress, respectively. The limit state function for buckling of tubular elements is expressed as:

$$M = g(\sigma_y, \hat{\sigma}_A, \hat{\sigma}_B) = -\log(I.R.). \tag{6}$$

The probabilistic models of dynamic effects are introduced into the static responses of more detailed structural models. With respect to the structural characteristics, the physical and geometrical nonlinearities are involved in the response analysis of jack-up structures. A long term response analysis, which includes the contribution from all possible sea-states, has been employed in the present analysis. Implying the probabilistic distribution of dynamic effects, the system reliability analysis of the detailed structural model has been performed with stress analysis of a complex model in the static state.

Long-Term Reliability

If the conditional failure probability for an individual sea-state with significant wave height H_s is given by Equation 3, the probability of failure for an arbitrary sea-state can be found by the following integration:

$$P(F_{s|z}) = \int_Y P\{M \leq 0 | Z = z, H_s\} f(H_s) dH_s$$
$$= \int_Y P\{g(\sigma_y, \hat{\sigma}_A, \hat{\sigma}_B\} \leq 0 | Z = z, H_s\} f(H_s) dH_s, \quad (7)$$

in which $F_{s|z}$ is the conditional short term probability of failure for a given set of system variables $Z = z, \hat{\sigma}_A$ and $\hat{\sigma}_B$ are random axial and bending stresses given by Equation 4. The long term reliability is now defined in terms of the failure event of an arbitrary sea-state using the multiplication law of probability,

$$P(F_{L|s}) = 1 - \{1 - P(F_{s|z})\}^N, \quad (8)$$

where N is the number of sea-states in the long term period. For a large number of sea- states, the long term reliability can be formulated by using the independency of failure events for arbitrary sea-states. The long term probability of failure can be rewritten by substitution of Equation 7 in Equation 9 using the Poisson type distribution for the failure events of the arbitrary sea-states.

$$P(F_{L|s}) \approx 1 - \exp\{-N.P(F_{s|z})\}. \quad (9)$$

By integration of Equation 8 or 9 for non-ergodic random variables, the reliability of single elements of structure are evaluated. Mathematically, the failure probability can be integrated as follows:

$$P(F_L) = \int_z P(F_{L|z}) f_z(z) dz. \quad (10)$$

For the random design wave responses of $\hat{\sigma}_{A,dw}$ and $\hat{\sigma}_{B,dw}$ (axial and bending stress in legs), using the Morison equation for hydrodynamic load calculation, the closed form solution between static hydrodynamic load and the axial or bending stress in jack-up legs are used in the probabilistic analysis. In case of dynamic simulations, an approximate methodology, based on the sensitivity of dynamic response with the static design wave responses, has been introduced in [15].

SOLUTION TECHNIQUES FOR SYSTEM RELIABILITY

For a jack-up structure, six groups of elements are distinguished, the chords ($i = 1$), the braces ($i = 2$), the jacking pinions ($i = 3$), the shock pads ($i = 4$), the fixation system ($i = 5$) and the hull plate elements ($i = 6$). Z_1 is the vector of random variables common to all individual failure events, $Z_2^{(i)}$ is the set of random variables common to the components of subgroup i (such as chord elements), and $Z_3^{(ij)}$ is the set of random variables belonging to the component j of subgroup i and no others. Graphically, these random variables are displayed in Figure 4 where $m = 6$ is the number of groups composed of the same elements and n is the number of elements per group.

Note that there is no restriction in the choice to which each random variable belongs and the only important point is that a set of random variables belonging to the component j of subgroup i are mutually independent to allow a straightforward solution. In cases where the random variables, Z_3, are correlated with Z_1 and Z_2, the conditional distribution function of Z_3, with respect to the others, has to be found and linear interpolation may be used for values of z_1 and z_2 between the chosen set.

The general idea behind this methodology is that the probability of failure of the system of the jack-up structure is obtained conditional to the failure probability of the correlated variables.

Suppose that for given random variables of $Z_1 = z_1, Z_2^{(i)} = z_2$, the limit state function is defined conditional to the set of correlated variables. The conditional probability of failure is then calculated for the mutually independent random variables. The conditional probability of failure $P_F | Z_1 = z_1, Z_2^{(i)} = z_2$ is obtained in the FORM routine, since according to the results of the reliability analysis of single elements, the FORM approach is quite satisfactory for the component analysis. If the components are integrated in the system, the conditional probability of failure may also be defined for the system. Mathematically, the

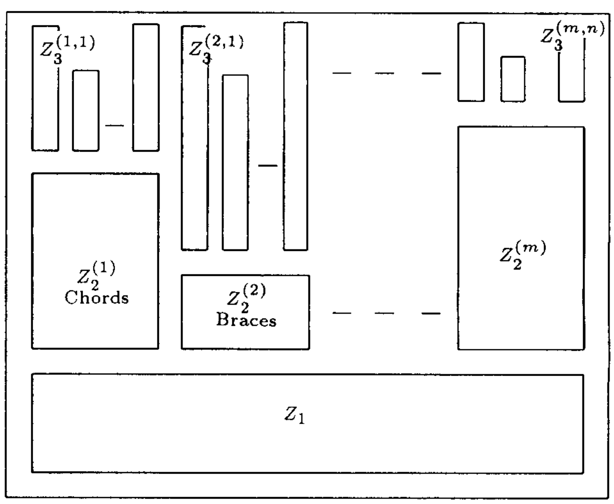

Figure 4. Random variables for system reliability; m = no. of groups, n = no. of components for the group.

conditional probability of failure can be written as:

$$P_{f,sys}(z_1, z_2, Z_3) = P(f_{sys|z_1,z_2})$$

$$= 1 - \prod_{i=1}^{i=m} \prod_{j=1}^{j=n} \left[1 - P\left[g_{ij}(z_1, z_2^{(i)}, Z_3^{(ij)}) \leq 0\right]\right], \qquad (11)$$

for series systems, and:

$$P_{f,sys}(z_1, z_2, Z_3) = P(f_{sys|z_1,z_2})$$

$$= \prod_{i=1}^{i=m} \prod_{j=1}^{j=n} P\left[g_{ij}(z_1, z_2^{(i)}, Z_3^{(ij)}) \leq 0\right], \qquad (12)$$

for parallel systems. In this formulation, i refers to the group number with a maximum number of $m = 6$ and n refers to the number of elements in each group. Capital letters refer to stochastic variables, while small letters reflect the deterministic variables. The limit state function for the whole system is also calculated by the conditional failure probability as proposed by Wen and Chen [16] and Bjerager et al. [10]. The conditional system probability of failure can be determined from the following limit state function:

$$g_{sys}(Z_1, Z_2, Z_{sys}) = Z_{sys} - \Phi^{-1}\left(P_{f,sys}(z_1, z_2, Z_3)\right)$$

$$= Z_{sys} - \Phi^{-1}\left(P(f_{sys|Z_1=z_1, Z_2=z_2})\right). \qquad (13)$$

Substituting Equations 11 and 12 into Equation 13, the limit state functions of series and parallel systems may be obtained. Such a formulation has been used for the system probability analysis of Tension Leg Platforms in a recent thesis [17]. An alternative to the limit state function defined by Equation 13, is to perform a series of FORM analysis for component analysis of $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$ with deterministic z_1 and z_2 and, then, to evaluate the probability of failure from the numerical integration.

$$P(M \leq 0) = P(F_{sys})$$

$$= \int_{z_1=-\infty}^{\infty} \int_{z_2=-\infty}^{\infty} \left[P(f_{sys|z_1,z_2})\right] f_{z_1}(z_1).f_{z_2}(z_2).dz_1.dz_2, \qquad (14)$$

where the conditional probability of failure can be calculated by reasonable accuracy using a FORM routine. Substitution of Equations 11 and 12 into Equation 14, the probability of failure of a series system is represented by:

$$P(F_{sys}) = \int_{z_1=-\infty}^{\infty} \int_{z_2=-\infty}^{\infty} \left[1 - \prod_{i=1}^{i=m} \prod_{j=1}^{j=n} \left[1 - P[g_{ij}(z_1, z_2^{(i)}, Z_3^{(ij)}) \leq 0]\right] f_{z1}(z_1).f_{z2}(z_2^{(i)})\right].dz_1.dz_2^{(i)}, \qquad (15)$$

and, similarly, for the parallel system.

$$P(F_{sys}) = \int_{z_1=-\infty}^{\infty} \int_{z_2=-\infty}^{\infty} \prod_{i=1}^{i=m} \prod_{j=1}^{j=n} P\left[g_{ij}(z_1, z_2^{(i)}, Z_3^{(ij)}) \leq 0\right] f_{z1}(z_1).f_{z2}(z_2^{(i)}).dz_1.dz_2^{(i)}. \qquad (16)$$

System Reliability Analysis, Level 1

The failure events of the undamaged structure are used in order to estimate the system reliability in Level 1. An upper bound approximation of the system probability of failure in Level 1 is defined by the series system of n components failures which are conditional to the system parameters Z_1 and Z_2:

$$P(F_{L,sys,1|z_1,z_2}) = \sum_{i=1}^{n} P(F^i_{L|z_1,z_2})$$

$$= \sum_{i=1}^{n} \{1 - \{1 - P(F^i_{S|z_1,z_2})\}^N\}$$

$$\approx \sum_{i=1}^{n} \{1 - \exp\{-N.P(F^i_{S|z_1,z_2})\}\}. \qquad (17)$$

in which $P(F_{L,sys,1|z_1,z_2})$ is the system probability of failure at Level 1, conditional to $(Z_1 = z_1, Z_2 = z_2)$. The conditional long term probability of failure for element $(i), P(F_{L|z_1,z_2^i})$ is calculated in terms of the probability of failure for an arbitrary sea-state $P(F_{s|z_1,z_2^i})$, as described in the previous section. Note that the integration of failure probability for sea-state variables (H_s) has been carried out in the calculation of failure events for an arbitrary sea-state. In order to treat the correlation effects between different safety margins in terms of Z_1 and Z_2 variables, a solution technique is developed based on the conditional failure events using uncorrelated random variables. The system probability of failure is then integrated for all of the random variables, including Z_2 time independent parameters common in a group of elements (chords or braces, etc.) and random variables, Z_1, common for all elements. The conditions $Z_1 = z_1$ and $Z_2 = z_2$ are integrated by application of the conditional failure probabilities in a numerical integration technique and the long term system probability of failure at Level 1 becomes:

$$P(F_{L,sys,1})=\int_{z_1}\int_{z_2}P(F_{L,sys,1|z_1,z_2})f_{z1}(z_1)f_{z2}(z_2)dz_2dz_1$$

$$=\int_{z_1}\int_{z_2}\sum_{i=1}^{n}P(F^i_{L|z_1,z_2})f_{z1}(z_1)f_{z2}dz_2(z_2)dz_1$$

$$=\int_{z_1}\int_{z_2}\sum_{i=1}^{n}\left\{1-\{1-P(F^i_{s|z_1,z_2})\}^N\right\}$$

$$f_{z1}(z_1)f_{z2}(z_2)dz_2dz_1,$$

$$P(F_{L,sys,1})\approx\int_{z_1}\int_{z_2}\sum_{i=1}^{n}\left\{1-\exp\{-N.P(F^i_{S\backslash z_1,z_2})\}\right\}$$

$$f_{z1}(z_1)f_{z2}(z_2)dz_2dz_1. \tag{18}$$

While the structural state is changed by the failure of the first element, the conditional limit state function of other elements must include the random strength parameter of the damaged element. For Level 2, the limit state function of element (i) in an arbitrary sea-state (Equation 7) is formulated conditionally on the random strength of the damaged element:

$$P\left(F^i_{S|z_1,z_2}/F^{d_1}_{S|z_1,z_2}\right)$$

$$=\int_Y P\{M_i\leq 0|M_{d_1}<0;H_s\}f(H_s)dH_s$$

$$=\int_Y P\{g(\sigma^i_y,\sigma^{d_1}_y,\hat{\sigma}^i_{A|d_1},\hat{\sigma}^i_{B|d_1})$$

$$\leq 0|M_{d_1}<0;H_s\}f(H_s)dH_s. \tag{19}$$

In Equation 19, the design wave responses are conditional to significant wave height; $Y\approx(H_s)$:

$$\hat{\sigma}^i_{A|d_1}=\mathrm{D\hat{A}F}^i_{A|d_1}\times\mathrm{\hat{C}F}^i_{A|d_1}\times\hat{\sigma}^i_{A|d_1}(\hat{H}|H_s),$$

$$\hat{\sigma}^i_{B|d_1}=\mathrm{D\hat{A}F}^i_{B|d_1}\times\hat{C}F^i_{B|d_1}\times\hat{\sigma}^i_{B|d_1}(\hat{H}|H_s), \tag{20}$$

where d_1 is the most important failure element at Level 1 and $\mathrm{D\hat{A}F}^i_{A|d_1}$, $\mathrm{\hat{C}F}^i_{A|d_1}$, and $\hat{\sigma}^i_{A|d_1}$ are random $\mathrm{DAF}^i_{A|d_1}$, $\mathrm{CF}^i_{A|d_1}$ and $\sigma^i_{A|d_1}$ of the dynamic amplification factor, the calibration factor and the design wave response of axial stress with damaged condition of element 1, which are all conditional to the sea-state with a significant wave height H_s. Similarly, $\mathrm{D\hat{A}F}^i_{B|d_1}$, $\mathrm{CF}^i_{B|d_1}$ and $\hat{\sigma}^i_{B|d_1}$ are random $DAF^i_{B|d_1}$, $\mathrm{CF}^i_{B|d_1}$ and $\sigma^i_{B|d_1}$ (the dynamic amplification factor, the calibration factor and the design wave response of bending stress with a damaged condition of element 1) conditional to the sea-state with a significant wave height H_s. The failure events are conditional to the failure of the damaged element with the limit state function ($M_{d_1}<0$).

System Reliability, Higher Levels

For the system reliability at Level 2, the structural state will be changed from the initial undamaged state to the damaged state. In conventional system reliability, the vector of the wave load is modeled as a constant static load. Similar to the nonlinear pushover analysis of fixed platforms, a load multiplication factor, λ, is usually increased until the failure of the structure. For dynamic problems, the load multiplication factor, λ, has been taken into account for nodal forces at all time intervals.

The extension of system reliability to Level (m) is also treated in a similar manner with the application of dynamic amplifications of the specified structural state. For structural state (m), the short term failure probability of element (i) in an arbitrary sea-state is conditional to the damaged state of most likely to fail elements ($i=d_1,d_2\cdots,d_{m-1}$). The failure probability for an arbitrary sea-state is written as:

$$P(F^i_{S|z_1,z_2}|F^{d_1}_{S|z_1,z_2}\bigcap F^{d_2|d_1}_{S|z_1,z_2}\bigcap F^{d_3|d_1,d_2}_{S|z_1,z_2}\bigcap\cdots)$$

$$=\int_Y P\{M_i\leq 0|M_{d_1}<0\bigcap M_{d_2|d_1}<0\bigcap\cdots;H_s\}f(H_s)dH_s$$

$$=\int_Y P\{g(\sigma^i_y,\sigma^{d_1}_y,\sigma^{d_2}_y,\cdots,\hat{\sigma}^i_{A|d_1,d_2,\ldots},\hat{\sigma}^i_{B|d_1,d_2,\ldots})$$

$$\leq 0|M_{d_1}<0\bigcap M_{d_2|d_1}<\sigma\bigcap\cdots;H_s\}f(H_s)dH_s,$$

$$i\neq d_1,d_2,\cdots,d_{m-1}. \tag{21}$$

In this formula, the design wave responses are conditional to the significant wave height H_s and the damaged conditions of elements $(1,2,...,m-1)$:

$$\hat{\sigma}^i_{A|d_1,d_2,\cdots}=\mathrm{D\hat{A}F}^i_{A|d_1,d_2,\cdots}\times\mathrm{\hat{C}F}_{A|d_1,d_2,\cdots}$$

$$\times\hat{\sigma}^i_{A|d_1,d_2,\cdots}(\hat{H}|H_s),$$

$$\hat{\sigma}^i_{B|d_1,d_2,\cdots}=\mathrm{D\hat{A}F}^i_{B|d_1,d_2,\cdots}\times\mathrm{\hat{C}F}^i_{B|d_1,d_2,\cdots}$$

$$\times\hat{\sigma}^i_{B|d_1,d_2,\cdots}(\hat{H}|H_s). \tag{22}$$

In terms of the conditional failure events of the arbitrary sea-states given z_1 and z_2, an upper bound for the system probability of failure is written as:

$$P(F_{L,sys,m|z_1,z_2}) = \sum_{i=1}^{n} \left\{ \begin{array}{l} [1-\{1-P(F^{i^*}_{S|z_1,z_2})\}^N].[1-\{1-P(F^{(d_{m-1})^*}_{S|z_1,z_2}\}^N] \\ \cdots [1-\{1-P(F^{d_1}_{S|z_1,z_2}\}^N] \end{array} \right\}$$

$$i \neq d_1, d_2, \cdots, d_{m-1}. \tag{23}$$

where:

$$P(F^{i^*}_{S|z_1,z_2}) = P(F^{i}_{S|z_1,z_2}|F^{d_1}_{S|z_1,z_2} \bigcap F^{d_2|d_1}_{S|z_1,z_2} \bigcap F^{d_3|d_1,d_2}_{S|z_1,z_2} \bigcap \cdots \bigcap F^{d_{m-1}|d_1,d_2,\cdots}_{S|z_1,z_2});$$

$$P(F^{(d_{m-1})^*}_{S|z_1,z_2}) = P(F^{d_{m-1}|d_1,d_2,\cdots}_{S|z_1,z_2}|F^{d_1}_{S|z_1,z_2} \bigcap F^{d_2|d_1}_{S|z_1,z_2} \bigcap F^{d_3|d_1,d_2}_{S|z_1,z_2} \bigcap \cdots),$$

$$i \neq d_1, d_2, \cdots, d_{m-1}, \tag{24}$$

$P(F_{s|z_1,z_2^i})$ is the probability of failure for undamaged elements in an arbitrary sea-state for element ($i = m, \cdots, n$) and $P(F_{S|z_1,z_2}; d_{m-1}|d_1, d_2, \cdots, d_{m-2})$ indicates the failure event of element (m) for an arbitrary sea-state when the damaged elements $(1, 2, \cdots, m-2)$ have already failed. By using a type of Poisson distribution for the failure events of an arbitrary sea-state:

$$P(F_{L,sys,m|z_1,z_D2}) = \sum_{i=1}^{n} \left\{ \begin{array}{l} [1-\exp\{-N.P(F^{i^*}_{S|z_1,z_2})\}].[1- \\ \exp\{-N.P(F^{(d_{m-1})^*}_{S|z_1,z_2})\}] \cdots [1-\exp\{-N.P(F^{d_1}_{S|z_1,z_2})\}] \end{array} \right\},$$

$$i \neq d_1, d_2, \cdots, d_{m-1}. \tag{25}$$

The unconditional probability of failure is obtained by integration for the space variables Z_1 and Z_2. In the detailed model, the design wave analysis is based on structural analysis using the most extreme wave height in 50 years and the load pattern is assumed to be constant for other design wave heights. The effect of the wave pattern on system reliability can be included by using the fragility analysis method, which uses conditional design wave responses to extreme wave height. In this article, fragility analysis has not been included in calculating system reliability.

Using independent mechanical and reliability modules, application of the deterministic strength characteristics of $\sigma_y^{d_1}, \sigma_y^{d_2}, \cdots$ is illustrated for the system reliability analysis. By integration of Equation 23 or 25 for non-ergodic random variables, the system reliability of the structure is evaluated at different levels of structural safety. Upper and lower bounds on the failure probabilities of the detailed structural model are calculated, based on application of the mean value of strength parameters and application of minimum failure events in the parallel systems.

EXAMPLE OF SYSTEM RELIABILITY ANALYSIS OF A JACK-UP

Static analysis methods are usually carried out for failure of detailed leg models and the effect of the detailed models of the hull and jacking system is often neglected using dynamic simulations. To study the behavior of jack-up structures with realistic structural models, a detailed structural model has been developed with a 3-D model for the leg-hull connection. That system made it possible to accurately measure the number of different structural behaviors for fixed and floating jacking systems with or without a fixation system. Among other models of jacking systems, this unique numerical model showed that the mechanical behavior of a jacking system can be accurately studied using the proposed model. However, in this respect, one should take into account other uncertain characteristics and some experimental tests are needed to calibrate the accuracy of equivalent strength characteristics.

In the evaluation of system reliability, the random dynamic amplification factors DAF_R are used from the dynamic simulations of the stick models for both elastic behavior and initial failure state (1). For structural states (2) and higher, the dynamic amplification factors have been derived with the results of failure state (1) from the stick model and the ratio of daf_λ and $daf_{(1)}$ for the SDOF model is as follows:

$$\mathrm{DAF}_\lambda = \frac{daf_\lambda}{daf_{(1)}}.\mathrm{DAF}_{(1)}. \tag{26}$$

In this formula, DAF_λ indicates the dynamic amplification of the stick model for the load factor, $\mathrm{DAF}_{(1)}$ and $daf_{(1)}$ are the dynamic amplification factors of the stick model and SDOF in the structural state (1) and daf_λ is the dynamic amplification factor of SDOF with implying load factor λ (state (2) and higher). For the aim of simplification, the ratios of $daf_\lambda/daf_{(1)}$ are assumed to be deterministic. Since dynamic amplification is obtained as the ratio of the extreme dynamic response to the extreme quasi-static response, using this assumption, the number of random variables for each response quantity is decreased to two stochastic variables defining $\mathrm{DAF}_{(1)}$instead of six random variables. The CFs are determined by a similar approach and are assumed as deterministic variables in system reliability analysis.

For the structural system, a better estimate of reliability for the whole system may be identified using the reliability of a series system with number of failure elements. To include the post failure behavior of structural elements, reliability is measured for a series system where each element in the series system is modeled by parallel systems with 2 failure elements. The structural analysis is performed for two states, the original structure and the modified structural state with a sequence of structural failure in the most likely element ($\lambda = \lambda_{MLM} = 2.58$). Each pair of failure event contain the failure elements of the most likely to fail element (from first analysis) and the failure event of n-1 elements with the modified state of structure. The reliability at Level 2 is then measured by the union of events in the series system with n-1 elements.

In structural analysis of the detailed jack-up, some states of failure occur for two elements simultaneously. Thus, from a deterministic search algorithm, the failure events for these pairs are equally distributed and fully correlated. Here, the failure events of both elements are assumed to have potential collapse and the one most likely to fail is substituted in the failure tree analysis.

System reliability of the structure at Level 3 is defined on the basis of a series system where the elements are parallel systems, each with three failure elements. By increasing the load multiplication factor ($\lambda = 2.68 > \lambda_{MLM}$), the second failure element (or elements) are determined, in which the failure occurs for two structural elements identified with two previous fail sequences. While both most likely to fail elements at Level 1 and Level 2 are in a state of failure, the failure probabilities are calculated for the remaining $n-2$ elements. Each triple of failure elements at Level 3, contains: a) The failure event of the most likely to fail element (from first analysis), b) The failure event of the most likely to fail element (by modified state of structure at Level 1) and c) The failure event of $n-2$ elements with the second sequence of structural failure.

The same procedure is applied for system reliability at higher levels. Three cases are considered, Case K, jack-up with a K-bracing and fixed jacking system, Case XA, jack-up with a floating jacking system mounted on relatively weak springs and Case XB, jack-up with a floating system and 10 times the stiffness of the jacking pinions in Case XA. The state of structure has been identified for load factors ($\lambda = 2.78, 2.88, 3.00, 3.30, 3.60$) by which system reliability may be carried out for Levels 4 to 8. The results of system reliability for all three cases within different levels of reliability analysis, are shown in Tables 1 and 2, respectively.

Failure events in the parallel system are calculated with the assumption of deterministic strength parameters. For example, in the limit state function $g(2/1) < 0$, the strength characteristics of element 1 are given at the design point of the limit state function

Table 1. System reliability for X-braced type, floating jacking mechanism without fixation system, Cases XA and $XB, \sigma = 8\%\mu$.

System, Level	Probability of Failure	Upper Bound
$XA, 0$	5.99E-5	-
$XA, 1$	6.74E-4	~1.291E-3
$XA, 2$	1.666E-4	~6.665E-4
$XB, 0$	5.998E-5	-
$XB, 1$	2.110E-4	~4.9.16E-4
$XB, 2$	1.125E-5	~2.661E-4
$XB, 3$	7.539E-6	~1.473E-4

Table 2. System reliability for K-braced type, fixed jacking mechanism with fixation system, Case $K, \sigma = 8\%\mu$.

System, Level	Probability of Failure	Upper Bound
$K, 0$	3.255E-5	-
$K, 1$	1.312E-4	~3.830E-4
$K, 2$	1.004E-4	~8.870E-4
$K, 3$	1.432E-5	~2.450E-5
$K, 4$	1.023E-5	~1.938E-5
$K, 5$	7.871E-6	~1.945E-5
$K, 6$	6.889E-6	~1.918E-5
$K, 7$	6.576E-6	~1.886E-5

$g(1) < 0$. With this condition, the correlation of failure modes is neglected for the ductile structural behavior and the results correspond to the lower bound of the system reliability for a parallel system. In order to obtain the upper bound, the minimums of the failure probabilities in the parallel system are used in the calculation of the system reliability of the jack-up structures. The uncertainties of strength variables are assumed as $\sigma = 8\%\mu$ and $\sigma = 5.65\%\mu$ and the results of reliability analysis are given in Tables 3 and 4.

The simple upper bound of series system has also been evaluated for the uncorrelated system events in the parallel systems given in Tables 2 to 5. As can be seen, the system reliability in Levels 0 and 1 are correlated to the failure probabilities obtained from simple bounds. For Levels 2 and higher, the simple upper bounds of the system reliability can be narrowed with the application of the minimum failure events in the parallel systems as given in Tables 3 and 4.

Finally, the complexity measures and redundancy factors have been determined for jack-ups with and without fixation systems. The quantitative ratio of any first failure probability, P_{AFF}, to the most-likely to fail

Table 3. System reliability for X-braced type, floating jacking mechanism without fixation system, the minimum failure events in the parallel systems, Cases XA and XB.

System, Level	**Probability of Failure $\sigma = 8\%\mu$**	**Probability of Failure $\sigma = 5.65\mu$**
$XA,0$	5.998E-5	5.861E-5
$XA,1$	6.748E-4	6.471E-4
$XA,2$	2.545E-4	2.447E-4
$XB,0$	5.998E-5	5.861E-5
$XB,1$	2.110E-4	1.987E-4
$XB,2$	7.667E-5	7.458E-5
$XB,3$	5.170E-5	5.027E-5

Table 4. System reliability for K-braced type, fixed jacking mechanism with fixation system, the minimum failure events in the parallel systems, Case K.

System, Level	**Probability of Failure $\sigma = 8\%\mu$**	**Probability of Failure $\sigma = 5.65\%\mu$**
$K,0$	3.255E-5	3.186E-5
$K,1$	1.312E-4	1.036E-4
$K,2$	2.227E-4	2.122E-4
$K,3$	7.412E-5	7.509E-5
$K,4$	6.024E-5	5.534E-5
$K,5$	5.214E-5	4.656E-5
$K,6$	3.538E-5	3.178E-5
$K,7$	3.327E-5	2.980E-5

member, P_{MLM}, is defined as the Complexity Measure (CM). The system redundancy factors are also defined as the ratio of the probability of any first failure to the probability of system failure:

$$CM = \frac{P_{AFF}}{P_{MLM}}; \quad RF = \frac{P_{AFF}}{P_{sys}}. \tag{27}$$

CONCLUSIONS

The application of deterministic variables for strength parameters has been illustrated for the system reliability of a jack-up system. The simple upper bound of series system has been evaluated for the uncorrelated system events in the parallel systems and it has been found that the system reliability in levels zero and one are bounded with the failure probabilities obtained from simple bounds. For Levels 2 and higher, the simple upper bounds of the system reliability can be narrowed with the application of the minimum failure events in the parallel systems.

For a floating jacking system without a fixation system (Cases XA and XB), the independency of failure events increase with a decrease in the stiffness of the jacking pinions. This is the reason for the high complexity measure in Case XA compared to Cases K and XB (compare the CM values, $CM = 11.25$ for Case XA and $CM = 3.5 \sim 4$ for Cases K and XB).

For jack-ups with a floating jacking system mounted on relatively weak springs, the redundancy will be lowered ($RF = 2.165 \sim 4$) contrary to the complexity measures as mentioned above. However, for jack-ups with a more rigid jacking system, the redundancy factor is found in the range of $RF = 4 \sim 28$, depending on the system reliability method used. This shows that the application of system reliability at higher levels is important, since this type of jack-ups behave in a ductile manner.

The system failure probability may differ considerably from the probability of failure of most likely to fail element P_{MLM}. For a jack-up with a floating jacking system without a fixation system, the system failure probability may be larger than the probability P_{MLM}, when the initial failure occurs in the jacking system (Case XA). When the sequences of failure are found in the jacking system, the system failure probability can be as high as 2.5×10^{-4} per year. For sequences of failure in jack-up legs, the failure probabilities are reduced to below 3.5×10^{-5} per year.

For jack-ups with a floating type jacking system mounted on weak springs (Case XA without the fixation mechanism), a brittle collapse behavior has been observed leading to the higher probability of failure (for example 1.7×10^{-4} per year). For this type of structure, the consequences of failure may be different from the initial failure, depending on the actual stiffness of the jacking system. On the other hand, for jack-ups with more rigid jacking systems, the inclusion of higher order levels will result in the reduction of the system probability of failure compared to the probability of the most likely to fail element P_{MLM}. For the parametric study of a stiff floating jacking system without a fixation system (Case XB), the failure probabilities in the element level and system levels were 6×10^{-5} and 7.54×10^{-6}, while for the jacking system with a fixation system (Case K), the failure probabilities have been found in the range of 3.3×10^{-5} to 6.6×10^{-6} per year.

NOMENCLATURE

CF_a, CF_b	calibration factors for axial stress and bending stress
$\mathrm{DAF}_a, \mathrm{DAF}_b$	dynamic amplification factors, axial stress and bending stress

$daf_\lambda, daf_{(1)}$ dynamic amplification factor for the load multiplication factor λ, dynamic amplification factor for structural state (1), SDOF model

$\mathrm{DAF}_\lambda, \mathrm{DAF}_{(1)}$ dynamic amplification factor for the load multiplication factor λ, dynamic amplification factor for structural state (1), stick model

$I.R.$ interaction equation for tubular elements

Level (m) system reliability based on a series system of parallel sub-systems with (m) failure modes should not be confused with level of reliability method

$M = g()$ safety margin using interaction equation

$P(f_{sys|z_1,z_2})$ conditional system probability of failure for given system variables z_1 and z_2

$P(F_{sys})$ system probability of failure

$P(F_{S|y,z})$ conditional short term probability of failure for a given random process y and system variables z

$P(F_{S|z})$ conditional probability of failure for an arbitrary sea-state given system variables z

$P(F_{L|z})$ conditional long term probability of failure for a given system variables z

$P(F^i_{S|z})$ conditional probability of failure of element (i) for an arbitrary sea-state for a given system variables z

$P(F^i_{L|z})$ conditional long term probability of failure of element (i) for a given system variables z

$P(F^{d1}_{S|z})$ conditional probability of failure of damaged element (1) for an arbitrary sea-state using a given system variables z

$P(F^{d1}_{L|z})$ conditional long term probability of failure of damaged element (1) for a given system variables z

$P(F_{L,sys,m|z})$ conditional system long term probability of failure for a given system variables z

SDOF single degree of freedom

Z_{sys} an auxiliary standard normal variable

λ load multiplication factor

λ_{MLM} load multiplication factor for failure of most-likely to fail member

σ_y yield stress

σ^{d1}_y deterministic yield stress of damaged element 1

σ_{xcE} the characteristic compressive stress, given in terms of the Euler buckling stress and the yield stress of welded section

σ_{xkE} the Euler buckling stress

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