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Performance Evaluations and Comparisons of Several LDPC Coded MC-FH-CDMA Systems

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In this paper, the application of regular Low-Density Parity-Check (LDPC) codes in Multi-Carrier Frequency-Hopping (MC-FH) CDMA systems is studied. To this end, different well-known constructions of regular LDPC codes are considered and the performance of LDPC coded MC-FH-CDMA systems, based on these constructions, are evaluated and compared in a frequency-selective slowly Rayleigh fading channel. These results are compared with those previously reported for super-orthogonal convolutionally coded MC-FH-CDMA systems. The simulation results indicate that the LDPC coded MC-FH-CDMA system significantly outperforms the uncoded and super-orthogonal convolutionally coded schemes. To alleviate the restrictions imposed by well known LDPC construction methods when applied to the coded MC-FH-CDMA system considered, a new semi-random construction is proposed and its performance is evaluated in the coded scheme. The simulation results indicate that this new construction substantially outperforms other well-known construction methods in the application considered.

INTRODUCTION

Recently, a multicarrier frequency-hopping CDMA (MC-FH-CDMA) system, using the concept of frequency diversity on a phase-coherent Frequency-Hopping Spread Spectrum (FHSS) system [1], was proposed [2] as a candidate for future high-rate multimedia wireless multiple access communication systems [3,4]. In this system, the total given bandwidth is equally divided into N_s subbands, each containing N_h different orthogonal carrier frequencies. At each bit interval, i, for each user, k, N_s carriers are chosen from N_s distinct subbands. These N_s carriers are modulated with the ith data bit of the user, using binary phase shift-keying (BPSK) modulation. These modulated carriers are then added together and transmitted through the channel. For the next signaling interval, each of these N_s carriers independently hops in its subband and another frequency in the subband is chosen. The frequency-hopping pattern, which is determined by the dedicated signature sequence of the user, is assumed to be known in the receiver. After dehopping at the receiver side, N_s subband correlators separate the signal transmitted in different subbands. These N_s detected signals are used to make a decision on the transmitted data bit. To this end, the correlators' weighted outputs are simply combined and the result is compared to threshold zero to make a decision on the transmitted data bit. Note that the modulating and demodulating of the N_s carriers simultaneously can be implemented easily by using the Inverse Fast Fourier Transform (IFFT) and the Fast Fourier Transform (FFT), respectively.

In the MC-FH-CDMA system described above, the diversity is obtained via both multicarrier transmission and frequency-hopping. The conventional Fast Frequency-Hopping (FFH) systems, which transmit one carrier at a time and change it in a fraction of a bit duration, make coherent demodulation relatively difficult. However, the MC-FH-CDMA system allows slower carrier frequency hopping and imposes each carrier to hop solely in a fraction of the total given bandwidth. This system is thoroughly studied and analyzed in [5]. Thus, with slow frequency-hopping, with a period at least equal to the bit duration, a

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coherent reception will be feasible in a slowly fading channel [2]. (Note that in some applications, FFH systems are more plausible and desirable, such as military applications. With fast hopping, it is much more difficult for the transmission to be intercepted by an undesired user.) To visualize the difference, the traditional slow and fast frequency hopping schemes are depicted in Figure 1.

Even though coherent detection gives a somewhat superior performance compared to the non-coherent case, in a case of fast fading, a non-coherent reception is inevitable. Single user performance of MC-FH and FFH systems using non-coherent detection has been evaluated in [6], in which it has been shown that the non-coherent MC-FH system outperforms the FFH system when the channel delay spread is severe, while the FFH system is superior to the MC-FH system for a fast fading channel. The multiuser performance of a MC-FH system, with coherent and non-coherent detections in additive white Gaussian noise (AWGN) and a frequency-selective slowly Rayleigh fading channel, is evaluated in [7]. The results in [7] have shown that the coherent detection substantially outperforms the non-coherent detection. In addition, it has been shown that [8] the multiuser performance of MC-FH and FFH systems is almost identical, when utilizing non-coherent detection. In this paper, it is assumed that the coherent reception is feasible and the focus is on coherent detection.

To exploit the given bandwidth more efficiently, in [5], the authors have proposed to use a practical low rate error correcting code in the MC-FH-CDMA system, which does not require any additional bandwidth to that needed in an uncoded MC-FH-CDMA system. The idea is as follows: Instead of sending the N_s carriers at each bit interval with an identical phase ($0°$ or $180°$), which is determined by the corresponding input data bit, these carriers can

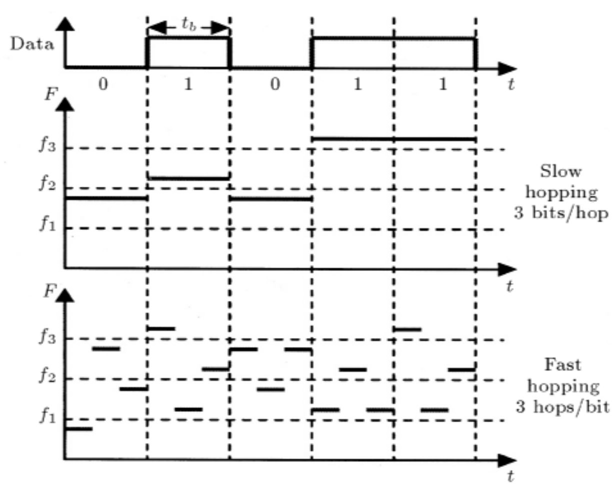

Figure 1. Slow and fast frequency hopping schemes.

be sent with different phases, the values of which are determined with output symbols of an encoder. In fact, with the above insight, the uncoded scheme is considered as a coded scheme with a repetition block code of rate $1/N_s$. Since a repetition code is not a good code, it is expected that applying a more powerful code with the same rate, $1/N_s$, substantially improves the system performance without requiring any bandwidth expansion in addition to that needed by the uncoded scheme. The error correcting code considered in [5] is a super orthogonal convolutional code [9], as its path generating function is available for performance evaluation. The authors have shown [5] that the super orthogonal convolutionally coded MC-FH-CDMA system substantially outperforms the uncoded scheme.

On the other hand, LDPC codes, originally introduced by Gallager [10] and rediscovered by MacKay and Neal [11], have received considerable attention recently and their applications in various communication systems, including OFDMA, Mobile Satellite and CDMA systems, have been considered. The interest in these codes is due to their near Shannon limit performance and their simple descriptions, implementations and decoding algorithms.

In this paper, the application of a LDPC code in a MC-FH-CDMA system is considered and the performance of the LDPC coded system is evaluated, using the coded scheme introduced in [5] and described above, in a Rayleigh fading channel. Different constructions of regular LDPC codes, are examined, namely Gallager [10], MacKay [11] and semi-random [12] constructions. It will be shown that for the application considered, these constructions have some practical limitations. To alleviate those limitations, a modified semi-random construction is proposed. The simulation results indicate that the LDPC coded systems in the cases considered perform better than the super orthogonal coded systems. Furthermore, the results show that the proposed modified semi-random construction displays a better performance than the other well-known regular construction methods for the application considered, despite its very low encoding complexity.

The outline of the paper is as follows. In the following section, the LDPC coded MC-FH-CDMA system and its conventional single user receiver structure are described. Then, a brief description of LDPC codes and their different constructions are presented. After that, the performance of a LDPC coded MC-FH-CDMA system in Rayleigh fading channels is evaluated and some simulation results are provided. Then, the proposed modified semi-random construction of LDPC codes is described and its performance in the MC-FH-CDMA system is evaluated. Finally, the paper is concluded.

SYSTEM DESCRIPTION

In this section, first, a brief description of the MC-FH-CDMA system is provided, as introduced in [1,5]. In this system, every transmitter sends N_s carriers for each data bit using BPSK modulation. The carriers are spaced apart in sequential subbands. The total given frequency bandwidth is equally partitioned into N_s subbands, where each subband contains N_h different frequency carriers spaced apart by f_d. f_d is chosen such that every pair of carriers is orthogonal, i.e., $f_d = 1/T_s$, where T_s denotes a bit time duration. Thus, the total frequency carriers available for multiple-access communications are $N_s N_h$. In this system, the equivalent baseband signal transmitted by user k can be written as follows:

$$s^{(k)}(t) = \sum_i \sum_{l=0}^{N_s \quad 1} \sqrt{2P_w} d_{l,i}^{(k)} e^{j2\pi(f_l + c_{l,i}^{(k)} f_d)(t \quad iT_s)} P_{T_s}(t \quad iT_s), \tag{1}$$

where index l indicates the subband number, P_w is the transmit power of each carrier and $\{d_{l,i}^{(k)}\}$ is the transmitted binary sequence of user k. This sequence modulates the dedicated carriers. $\{c_{l,i}^{(k)}\}$ is the pseudorandom sequence of user k, which determines the carrier frequency selected from each subband, l, during the ith bit interval. The elements of this sequence are independent and identically distributed (i.i.d) random variables, which take on integer values in the interval $[0, N_h \quad 1]$. $P_\lambda(t)$ is a rectangular pulse over the interval $[0, \lambda]$ with amplitude equal to 1. f_l is the first carrier frequency in subband l and is equal to $lN_h f_d$, $l = 0, 1, 2, \cdots, N_s \quad 1$.

In an uncoded scheme, sequence $d_{l,i}^{(k)}$ is N_s repetitions of the user information data sequence, i.e., $d_{l,i}^{(k)} = D_i^{(k)}$, for $l = 0, 1, \cdots, N_s \quad 1$, where $D^{(k)}$ is the ith information bit of user k. Thus, the uncoded scheme can be considered as a coded scheme with a simple repetition block code of rate $1/N_s$. Since the repetition code is not a good code, a more powerful code can be used to improve system performance. Use of a LDPC code is suggested with rate $1/N_s$. In the coded scheme considered, $d_{l,i}^{(k)}$ in Equation 1 is the lth code bit of user k at information bit interval i. Thus, in this coded scheme, the phase of each carrier is determined by the corresponding coded symbol. Figure 2 presents a block diagram of the transmitter structure. In this figure, the LDPC encoder is implemented in the same way as any linear block code, by knowing its generator or parity-check matrix.

The channel is considered as a frequency-selective slowly Rayleigh fading channel with an AWGN. However, each carrier is assumed to experience a frequency-non selective fading channel, i.e., $f_d << (\Delta f)_c$, where $(\Delta f)_c$ is the coherence bandwidth of the channel. Since, with very high probability, the distance among adjacent carriers is large as they are located in different subbands, note that the average frequency distance between adjacent carriers is $N_h f_d$.) it can well be assumed that the dedicated carriers experience independent flat fading. In addition, the Doppler shift is assumed to be small enough to have a slow fading. The above assumptions are reasonable for most practical applications. Propagation measurements in wireless environments, including indoor, open rural, suburban and urban areas, show that the delay spread is typically distributed over the range of [0.1 μs, 3 μs] [13]. So, for instance, at a bit rate equal to 1Mbps for indoor

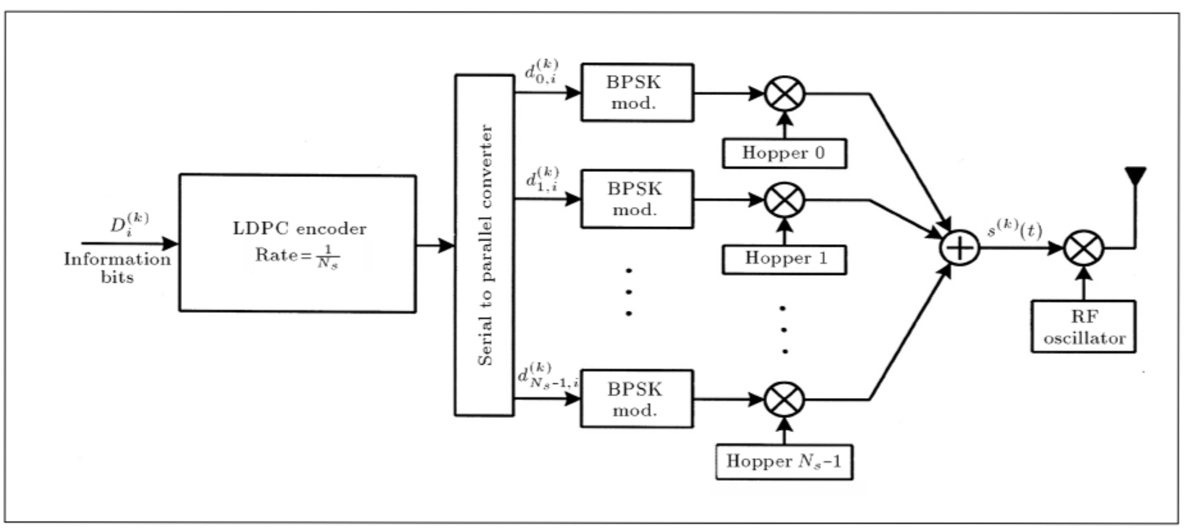

Figure 2. Transmitter block diagram of user k.

environments, which has the delay spread of 0.1 μs, the coherence bandwidth is larger than the symbol rate. As a result, each carrier experiences a flat fading. Furthermore, for N_h greater than 20, the average frequency distance between adjacent carriers (i.e., $N_h f_d$) is greater than the coherence bandwidth and, as a result, each carrier experiences independent fading. Note that for future very high rate wireless communication networks, to make the carriers experience independent fading, a serial-to-parallel converter (like in OFDMA and MC-CDMA systems) might be required before MC-FH-CDMA modulation, in order to reduce the symbol rate in each parallel branch to much less than the coherence bandwidth of the channel.

Due to the random construction and very sparse property of the parity-check matrix of a LDPC code, the decoder of a LDPC code has a built-in "interleaver" [14]. So, one can well assume that the successive symbols transmitted by the same carrier in a subband also experience independent fading. Let $g_{l,i}^{(k)} = \alpha_{l,i}^{(k)} e^{j\theta_{l,i}^{(k)}}$ be the channel complex gain observed by the lth carrier of user k at bit interval i. Then, $g_{l,i}^{(k)}$s for different values of k, l and i are independent. Furthermore, since the channel is assumed to be slowly-fading, i.e., the complex channel gain corresponding to each carrier does not change significantly during several bit intervals, the channel gain can be well estimated at the receiver. However, in the current work, it is assumed that perfect estimations of the channel parameters are available and the effects of parameter estimation errors are not considered in the performance evaluations. Also, the effects of the synchronization errors are not considered. For some recent work on channel estimations and synchronization problems in a MC-FH CDMA system and their error effects on the system performance, please see [4,15].

As the channel is assumed to have Rayleigh distribution, $\alpha_{l,i}^{(k)}$'s are independent from $\theta_{l,i}^{(k)}$'s, with $\overline{\alpha_{l,i}^{(k)^2}} \triangleq \Omega$. Also, $\theta_{l,i}^{(k)}$'s will have a uniform distribution over the interval $[0, 2\pi]$. The received signal, due to user k, is equal to:

$$s_{\text{rec}}^{(k)}(t) = \sum_{i} \sum_{l=0}^{N_s\;\;1} g_{l,i}^{(k)} \sqrt{2P_w} d_{l,i}^{(k)} e^{j2\pi(f_l + c_{l,i}^{(k)} f_d)(t\;\;iT_s)} P_{T_s}(t\;\;iT_s), \tag{2}$$

For simplicity of presentation, a synchronous system with perfect power control is assumed. Then, the total received signal can be written as:

$$r(t) = \sum_{k=1}^{N_u} s_{\text{rec}}^{(k)}(t) + \nu(t), \tag{3}$$

where N_u is the number of active users and $\nu(t)$ is AWGN with a two sided power spectral density of $N_0/2$. By substituting Equation 2 in Equation 3, one has:

$$r(t) = \sum_{k=1}^{N_s} \sum_{i} \sum_{l=0}^{N_s\;\;1} \sqrt{2P_w} g_{l,i}^{(k)} d_{l,i}^{(k)} e^{j2\pi(f_l + c_{l,i}^{(k)} f_d)(t\;\;iT_s)} P_{T_s}(t\;\;iT_s) + \nu(t). \tag{4}$$

In the following, the receiver structures for uncoded and coded schemes are described. Without any loss of generality, it is assumed that the desired user is user 1.

Uncoded Scheme

For this system, a single user Maximal Ratio Combining (MRC) receiver is considered. Let the desired user be user 1. The well-known MRC receiver makes a decision at each bit interval, i, according to the following rule:

$$\underbrace{\text{Re}\left\{ \sum_{m=0}^{N_s\;\;1} g_{m,i}^{(1)*} \overbrace{\overbrace{\frac{1}{T_s} \int_{iT_s}^{(i+1)T_s} r(t) e^{\;\;j2\pi(f_m + c_{m,i}^{(1)} f_d)t} dt}^{\text{Subband correlator output}}}^{Z_{m,i}} \right\}}_{\text{Decision variable} \triangleq Z_i} \overset{1}{\underset{0}{\gtrless}} 0. \tag{5}$$

Coded Scheme

The block diagram of the receiver for user 1 is shown in Figure 3. In this receiver, at each information bit interval, i, the N_s received coded symbol signals carried by the N_s carriers of that interval are first demodulated and, then, are given to the dehopper followed by a subband correlator. The subband correlator outputs, as defined in Relation 5, are multiplied by the conjugate of the channel gains $g_{m,i}^{1)}$'s. The real part of these results, i.e., $\text{Re}(Z_{m,i})$'s in Relation 5, are given to the parallel to serial converter and then passed to the LDPC decoder. In fact, $\text{Re}(Z_{m,i})$ denotes the received sample (the decoder soft input) corresponding to the mth transmitted code bit at the ith information bit interval.

LDPC CODES

LDPC codes, originally invented and investigated by Gallager [10] in 1962, are linear block error-correcting codes based on very sparse parity check matrices. $A(N, d_v, d_c)$-regular LDPC code is a linear binary code determined by the condition that each code bit participates in exactly d_v parity-check equations and that each such check equation consists of exactly d_c code bits. In other words, the corresponding parity-check matrix, H, has d_v ones in each column and

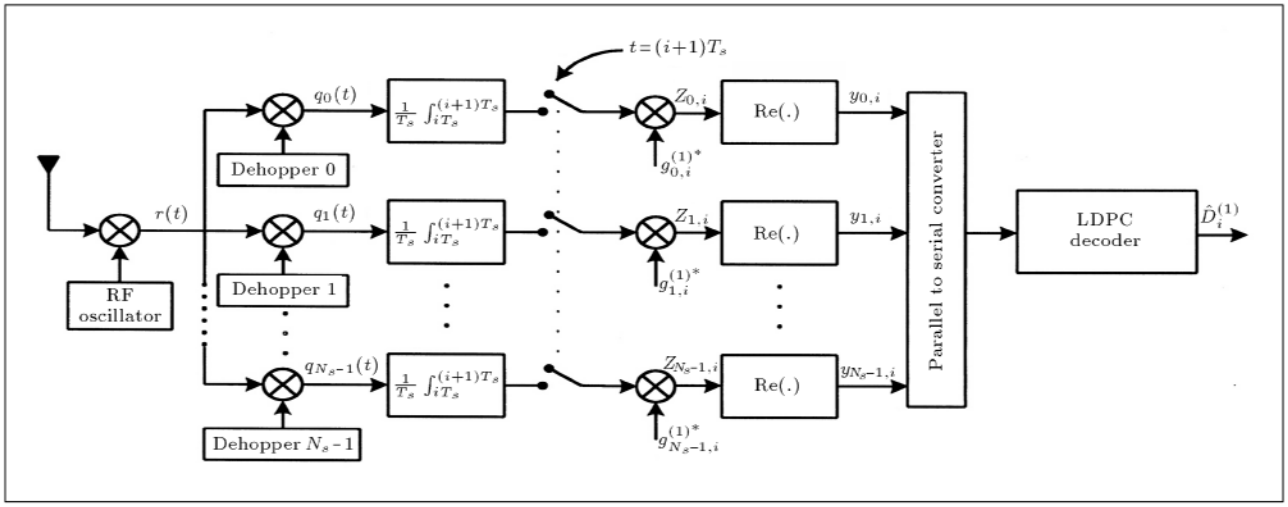

Figure 3. Receiver block diagram (coded scheme) of user 1.

d_c ones in each row. Thus, the number of parity bits and information bits are, respectively, equal to $M = N.d_v/d_c$ and $K = M - N$, where N is the length of the code. In other words, as both $N.d_v$ and $M.d_c$ represent the number of edges in the corresponding bipartite graph of the parity check matrix, they are equal. Therefore, the rate, R, of the code is given by $R = 1 - (M/N) = 1 - (d_v/d_c)$.

LDPC Encoding

In the following, the three well-known construction methods of regular LDPC codes are briefly described, namely, Gallager, MacKay and semi-random constructions, which are applied to the coded MC-FH-CDMA systems in the next sections.

Gallager Construction

In this construction, the parity-check matrix is divided horizontally into d_v equal size submatrices, each containing a single '1' in each column. The first submatrix is constructed as follows: the ith row contains 1's in columns $(i-1)d_c + 1$ to id_c. The subsequent submatrices are merely random column permutations of the first one.

MacKay Construction

MacKay has presented [11] several construction methods for regular LDPC codes. The best one is used, for which the parity-check matrix, H, is generated randomly, with column Hamming weight d_v, row Hamming weight as uniform as possible, and overlap between any two columns is no greater than one.

Semi-Random Construction

The major problem with regular LDPC codes is their very high encoding complexity. Since the parity-check matrix of the code is sparse, the associated generator matrix will be dense and, as a result, the encoding process requires a high number of computations. To overcome this problem, the concept of semi-random LDPC codes has been introduced [12]. In semi-random construction, to produce a (N,K) code, the following parity check matrix, $H_{MN} = [h_{ij}]$, is used:

$$H_{M\times N} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & \alpha^1_{M'\times K} \\ 1 & 1 & 0 & \cdots & 0 & \vdots \\ 0 & 1 & 1 & \cdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 1 & 1 & \alpha^{d_\nu}_{M'\times K} \\ \multicolumn{5}{c}{\underbrace{\hspace{10em}}_{M}} & \end{bmatrix},$$

where $M = N - K$ and $M' = M/d_v$. Each component matrix, $\alpha^i_{M'\times K}$, has a random construction with column weight equal to one and row weight equal to d'_c, where $d'_c = Kd_\nu/M$. As can be observed, in this construction, the parity check matrix, H, consists of two parts, only one part of which is generated randomly. For the systematic codeword, i.e., $V = (p_1, \cdots, p_M, u_1, \cdots, u_k)$, parity bits are computed as:

$$\begin{cases} p_1 = \sum_{j=1}^{K} u_j h_{1,M+j} & \pmod 2 \\ p_m = p_{m-1} + \sum_{j=1}^{K} u_j h_{m,M+j} & \pmod 2 \\ \quad 2 \le m \le M \end{cases} \tag{6}$$

where h_{ij} is the element in the ith row and jth column of the parity-check matrix and $u_1, \cdots, u_k$ are information bits. Efficient encoding is achieved by directly computing the parity-check bits from Equation 6 without any requirement to compute the generator matrix.

One of the advantages of the semi-random construction is its Unequal Error Protection (UEP). In [16], Davey has shown that in an irregular LDPC code, in which symbols do not all participate in the same number of parity-checks, those symbols participating in the highest number of parity-check equations receive the most information during the decoding process. As a result, the value of those symbols is determined more quickly and more accurately than the "less privileged" symbols. This property is called as Unequal Error Protection (UEP). In the semi-random construction, information bits have a degree of $d_v (d_v > 2)$ and parity bits have a degree of 2, except the parity bit, p_M, which is of degree one. Therefore, the information bits participate in the highest number of parity-check equations and will be decoded more quickly with higher reliability. Because of this property, as seen later, the semi-random code gives a better performance in comparison with the other two regular LDPC codes when applied to MC-FH-CDMA systems, despite its lower encoding complexity.

LDPC Decoding

Iterative soft decoding of LDPC codes can be implemented using the Belief Propagation (BP) algorithm [11]. In this algorithm, information is exchanged between neighboring nodes in the graph by passing messages along the edges. At the initial stage, this algorithm requires the knowledge of the *a priori probability* of the transmitted code bit. Let $P(d_n = z)$ be the *a priori probability* of code bit d_n. The value of this probability is fully related to the modulation used and the channel characteristics. In the following, in the Binary Phase Shift Keying (BPSK) modulation, it is assumed that 0 is mapped to -1 and 1 is mapped to 1. Let y_n be the nth received sample at the LDPC decoder input, i.e., the real part of $Z_{m,i}$, as defined in Relation 5, corresponding to code bit d_n. Then, the Log Likelihood Ratio (LLR) of y_n is computed as follows:

$$\mathrm{LLR} = \log \frac{p(y_n/d_n = 0)}{p(y_n/d_n = 1)}. \tag{7}$$

Then, from Equation 7, the *a priori probability* of the corresponding code bit, d_n, is easily obtained as:

$$P(d_n = z) = 1/(1 + \exp((2z \quad 1)^{*}\mathrm{LLR})). \tag{8}$$

In the following section, for performance evaluation, the *a priori probability* of the code bit is first computed. Then, by using these values at the initial stage of the BP algorithm, the performance of the coded system is evaluated by simulation.

PERFORMANCE EVALUATION (RAYLEIGH FADING CHANNEL)

A Priori Probability Evaluation for the LDPC Decoder

As mentioned before, a priori probabilities of the code bits are required by the LDPC decoder implemented by the BP algorithm. To this end, one must first compute the distribution of the interference term in the received samples at the LDPC decoder input. Through this section and subsequent sections, similar to [5], the discrete moment generating function (mgf) approach is used for computing the distribution. Without any loss of generality, the received signal at the information bit interval is considered to be zero. So, the subindex zero is dropped from $Z_{m,0}$, defined in Relation 5. From Equation 4, the received signal at bit interval zero is equal to:

$$r(t) = \sum_{k=1}^{N_u} \sum_{l=0}^{N_s \quad 1} g_l^{(k)} d_l^{(k)} \sqrt{2P_w} e^{j2\pi(f_l + c_l^{(k)} f_d)t} + \nu(t), \tag{9}$$

where, for the sake of simplicity, $g_{l,0}^{(k)}$, $d_{l,0}^{(k)}$ and $c_{l,0}^{(k)}$ have been replaced with $g_l^{(k)}$, $d_l^{(k)}$ and $c_l^{(k)}$, respectively. For the desired user (user 1), from Equations 4 and 5, the subband correlator output, i.e., Z_m, is simply obtained as follows:

$$\begin{aligned} Z_m &= \frac{g_m^{(1)^*}}{T_s} \int_0^{T_s} r(t) e^{\quad j2\pi(f_m + c_m^{(1)} f_d)t} dt \\ &= \frac{g_m^{(1)^*}}{T_s} \int_0^{T_s} \left\{ \sum_{k=1}^{N_u} \sum_{l=0}^{N_s \quad 1} g_l^{(k)} d_l^{(k)} \sqrt{2P_w} \right. \\ &\quad \left. e^{j2\pi(f_l \quad f_m + (c_l^{(k)} \quad c_m^{(1)}) f_d)t} + \nu(t) e^{\quad j2\pi(f_m + c_m^{(1)} f_d)t} \right\} dt. \end{aligned} \tag{10}$$

As the carriers are orthogonal, the integral in Equation 10 is nonzero only for $l = m$. Consequently, one has:

$$\begin{aligned} Z_m &= |g_m^{(1)}|^2 d_m^{(1)} \sqrt{2P_w} \\ &\quad + \sum_{k=2}^{N_u} \frac{g_m^{(k)} g_m^{(1)^*} d_m^{(k)} \sqrt{2P_w}}{T_s} \int_0^{T_s} e^{j2\pi(c_m^{(k)} \quad c_m^{(1)}) f_d t} dt + \nu_m, \end{aligned} \tag{11}$$

where ν_m is the complex noise component. In Equation 11, the first term is due to the desired user signal and the second and third terms are the multiple-access interference and white Gaussian noise components, respectively. The real part of the subband correlator's output, i.e., $\mathrm{Re}(Z_m)$, is simply obtained as:

$$y_m = \mathrm{Re}(Z_m) = d_m^{(1)}(\alpha_m^{(1)})^2\sqrt{2P_w}$$
$$+\underbrace{\overbrace{\sum_{k=2}^{N_u}\frac{\alpha_m^{(1)}\alpha_m^{(k)}d_m^{(k)}\sqrt{2P_w}\cos(\theta_m^{(k)}\quad\theta_m^{(1)})}{T_s}\int_0^{T_s}e^{j2\pi(c_m^{(k)}\quad c_m^{(1)})f_d t}dt}^{I_m^{(k)}}}_{I_m}$$
$$+\,\eta_m, \tag{12}$$

where $g_m^{(k)} = \alpha_m^{(k)}e^{j\theta_m^{(k)}}$, $I_m^{(k)}$ is the interference caused by interfering user k at mth subband correlator's output, and I_m is the total interference due to all interfering users at the mth subband correlator output of user 1. In Equation 12, η_m, the noise component, is a Gaussian random variable with zero mean and variance σ_η^2 equal to $N_0(\alpha_m^{(1)})^2/T_s$ (or, equivalently, equal to $N_sP_w(\alpha_m^{(1)})^2/\gamma_b$), where γ_b is the received signal to noise ratio per bit.

It is necessary to obtain the mgf of I_m conditioned on $g_m^{(1)}$. To this end, the mgf of the interference caused by each interfering user k must be determined. Then, as the interfering user components are independent, the mgf of the total interference is computed by multiplying the mgfs of different interfering users' components. Under the assumption of full power control, the mgfs of the interference, due to different users, are identical. Therefore, it is sufficient to determine the mgf of the interference caused by only one user.

Since random variable $\theta_m^{(k)}\quad\theta_m^{(1)}$ has uniform distribution on interval $[\quad\theta_m^{(1)}, 2\pi\quad\theta_m^{(1)}]$ and $\alpha_m^{(k)}$ has a Rayleigh distribution with $\overline{\alpha_m^{(k)2}} = \Omega = 1$ and, also, $\alpha_m^{(k)}$ and $\theta_m^{(k)}\quad\theta_m^{(1)}$ are independent, it can be concluded that $\alpha_m^{(k)}\cos(\theta_m^{(k)}\quad\theta_m^{(1)})$ has a Gaussian distribution with zero mean and variance equal to 1/2 [17]. As a result, it can easily be shown that random variable $\alpha_m^{(1)}d_m^{(k)}\sqrt{2P_w}\alpha_m^{(k)}\cos(\theta_m^{(k)}\quad\theta_m^{(1)})$ has also a Gaussian distribution with zero mean and variance equal to $P_w(\alpha_m^{(1)})^2$. On the other hand, the integral in Equation 12 is equal to T_s for $c_m^{(k)} = c_m^{(1)}$ and zero for other values of $c_m^{(k)}$. Considering the distribution of $\{c_m^{(k)}\}$'s, the probability that $c_m^{(k)}$ is equal to $c_m^{(1)}$ is $\alpha = 1/N_h$ and the probability that $c_m^{(k)}$ is not equal to $c_m^{(1)}$ is $\beta = 1\quad\alpha = 1\quad 1/N_h$. Consequently, the moment generating function of $I_m^{(k)}$, conditioned on $\alpha_m^{(1)} = \mathrm{Re}(g_m^{(1)})$, is computed as:

$$\Phi_{I_m^{(k)}|\alpha_m}(s) = \beta + \alpha\exp\left(\frac{s^2P_w\alpha_m^2}{2}\right), \tag{13}$$

where $\alpha_m^{(1)}$ has been replaced with α_m. Then, from Equations 12 and 13, the mgf of I_m is simply obtained as:

$$\Phi_{I_m|\alpha_m}(s) = \prod_{k=2}^{N_u}\Phi_{I_m^{(k)}|\alpha_m}(s)$$
$$= \left(\beta + \alpha\exp\left(\frac{s^2P_w\alpha_m^2}{2}\right)\right)^{N_u\quad 1}. \tag{14}$$

Now, from Equation 12, one can easily compute the conditional mgfs of the soft received samples, i.e., y_m, conditioned on α_m and the corresponding transmitted code symbol, i.e., $d_m^{(1)}$, as follows:

$$\Phi_{y_m|\alpha_m,d_m^{(1)}}(s) = \Phi_{y_m|\alpha_m,\pm1}(s) = \exp(\pm s\alpha_m^2\sqrt{2P_w})$$
$$.\left(\beta + \alpha\exp\left(\frac{s^2P_w\alpha_m^2}{2}\right)\right)^{N_u\quad 1}.\exp\left(\frac{s^2P_w}{2}.\frac{N_s}{\gamma_b}.\alpha_m^2\right)$$
$$= \sum_{i=0}^{N_u\quad 1}\binom{N_u\quad 1}{i}\beta^{N_u\quad 1\quad i}\alpha^i\exp\Bigg(\pm s\sqrt{2P_w}$$
$$+ (iP_w + P_w.\frac{N_s}{\gamma_b})\frac{s^2}{2}\Bigg)(\alpha_m^2), \tag{15}$$

where the second equality simply follows from polynomial expansion of the second multiplication term. Without loss of generality, $\sqrt{2P_w} = 1$ is set. Since α_m has a Rayleigh distribution, α_m^2 have a chi-square distribution of order 2. Taking the expectation on α_m's and using some algebraic manipulations lead to:

$$\Phi_{y_m|\pm1}(s) = \sum_{i=0}^{N_u\quad 1}\frac{\binom{N_u\quad 1}{i}\beta^{N_u\quad 1\quad i}\alpha^i}{1\mp s\quad\left(i+\frac{N_s}{\gamma_b}\right)\frac{s^2}{4}}$$
$$= \sum_{i=0}^{N_u\quad 1}\frac{\binom{N_u\quad 1}{i}\beta^{N_u\quad 1\quad i}\alpha^i}{\sqrt{1+i+\frac{N_s}{\gamma_b}}}$$
$$\left(\frac{1}{s\quad p_{i,1}} + \frac{1}{p_{i,2}\quad s}\right), \tag{16}$$

where $p_{i,1}$ and $p_{i,2}$ are given, as follows:

$$p_{i,1} = \frac{2}{\pm1\quad\sqrt{1+i+\frac{N_s}{\gamma_b}}},$$
$$p_{i,2} = \frac{2}{\pm1+\sqrt{1+i+\frac{N_s}{\gamma_b}}}. \tag{17}$$

$p_{i,1}$s are all negative and $p_{i,2}$s are all positive. Also, the Region Of Convergence (ROC) of an mgf always includes the axis, $s = j\omega$. Therefore, from Equation 16, the conditioned probability density function (pdf) of soft received samples, i.e., y_ms, can easily be obtained using an inverse Laplace transform, as follows:

$$f_{y_m/\pm 1} = \sum_{i=0}^{N_u - 1} \frac{\binom{N_u - 1}{i} \beta^{N_u - 1 - i} \alpha^i}{\sqrt{1 + i + \frac{N_s}{\gamma_b}}}$$

$$.\exp\left(\frac{2}{i + \frac{N_s}{\gamma_b}} \left(\pm y_m - |y_m| . \sqrt{1 + i + \frac{N_s}{\gamma_b}} \right) \right). \tag{18}$$

Now, one can easily use Equations 7, 8 and 18 to compute the *a priori probability* of a transmitted code bit, which is required by the LDPC decoder.

Computation of a Priori Probabilities Under the Gaussian Assumption

If one assumes that the distribution of interference at the output of the subband correlator is Gaussian, the mean and variance of this random variable should be determined, in order to compute the a priori probabilities. In terms of mgf, the mean and variance of an arbitrary random variable, X, can be expressed as:

$$\overline{X} = E\{X\} = \frac{d\Phi_X(s)}{ds}|_{S=0},$$

and:

$$\sigma_X^2 = E\{X^2\} - (E\{X\})^2$$

$$= \frac{d^2\Phi_X(s)}{ds^2}|_{S=0} - \left(\frac{d\Phi_X(s)}{ds}|_{S=0} \right)^2.$$

Using Equation 16, it can be shown that:

$$\frac{d\Phi_{I_m|\alpha_m}(s)}{ds}|_{S=0} = 0,$$

and:

$$\frac{d^2\Phi_{I_m|\alpha_m}(s)}{ds^2}|_{S=0} = \alpha P_W (N_u - 1)\alpha_m^2.$$

So, at the weighted output of each subband correlator, m, one has a Gaussian signal, with the following mean and variance:

$$\begin{cases} \overline{y_m} = \pm \alpha_m^2 \sqrt{2P_w} \\ \sigma_{y_m}^2 = \left(\alpha(N_u - 1) + \frac{N_s}{\gamma_b} \right) P_w \alpha_m^2. \end{cases} \tag{19}$$

Therefore:

$$\Phi_{I_m|\alpha_m, d_m^{(1)}}(s) = \exp\left\{ \left(\pm s + \frac{s^2}{4} \left(\alpha(N_u - 1) + \frac{N_s}{\gamma_b} \right) \right) \alpha_m^2 \right\},$$

where $\sqrt{2P_w} = 1$. After taking expectation on α_ms and doing some manipulations, the conditional pdf of y_m can be obtained, as follows:

$$f_{y_m/\pm 1} = \frac{1}{\sqrt{1 + \frac{N_u - 1}{N_h} + \frac{N_s}{\gamma_b}}}$$

$$.\exp\left(\frac{2}{\frac{N_u - 1}{N_h} + \frac{N_s}{\gamma_b}} \left(\pm y_m - |y_m| . \sqrt{1 + \frac{N_u - 1}{N_h} + \frac{N_s}{\gamma_b}} \right) \right). \tag{20}$$

This formula simplifies the computational complexity of the *a priori probability* formula in Equation 18. In the next subsection, the performance of LDPC codes in a MC-FH-CDMA system is evaluated, based on both exact and Gaussian approximated evaluations of the a priori probabilities.

Simulation Results

In this section, some simulation results are presented to evaluate the performance of LDPC coded MC-FH-CDMA systems. MacKay, Gallager and semi-random constructions of the LDPC codes are considered. The belief propagation algorithm is used for decoding of the LDPC codes, in which Equations 7, 8 and 18 are first used to compute the *a priori probability* of the transmitted code bit. Under Gaussian assumption, the *a priori probability* is calculated, using Equation 20 instead of Equation 18. Then, the BP algorithm uses this *a priori probability* for its initial stage, to decode the transmitted code bits.

A processing gain of $N_h N_s = 320$ and signal to noise ratio per bit (γ_b) of 12 dB are assumed. Figures 4a and 4b present the plots of the Bit Error Rate (BER) versus the number of users for uncoded and coded schemes in a Rayleigh fading channel at a number of subbands, i.e., N_s, equal to 2 and 4, respectively. The Rayleigh parameter, Ω, is set to one and the maximum number of decoding iterations (i_m) for LDPC codes is set to 1000. To evaluate the bit error probability, 100000 blocks are transmitted. In these figures, the performance of LDPC coded schemes, with a data block length of 500 and $d_v = 3$, is compared with the lower bound of BER of the super-orthogonal convolutional coded schemes, as reported in [5]. (Note that the performance evaluation of the super-orthogonal coded schemes in [3] is based on the analytical results in which only the lower and upper bound of BER can be computed using the path generating function of the super-orthogonal code.) Even though the super-orthogonal codes show a good performance at BER=10^{-3}, which is suitable for voices at lower BER, required for high-quality services such as data, LDPC codes perform substantially better. For instance, at BER=10^{-5}, the number of users supported

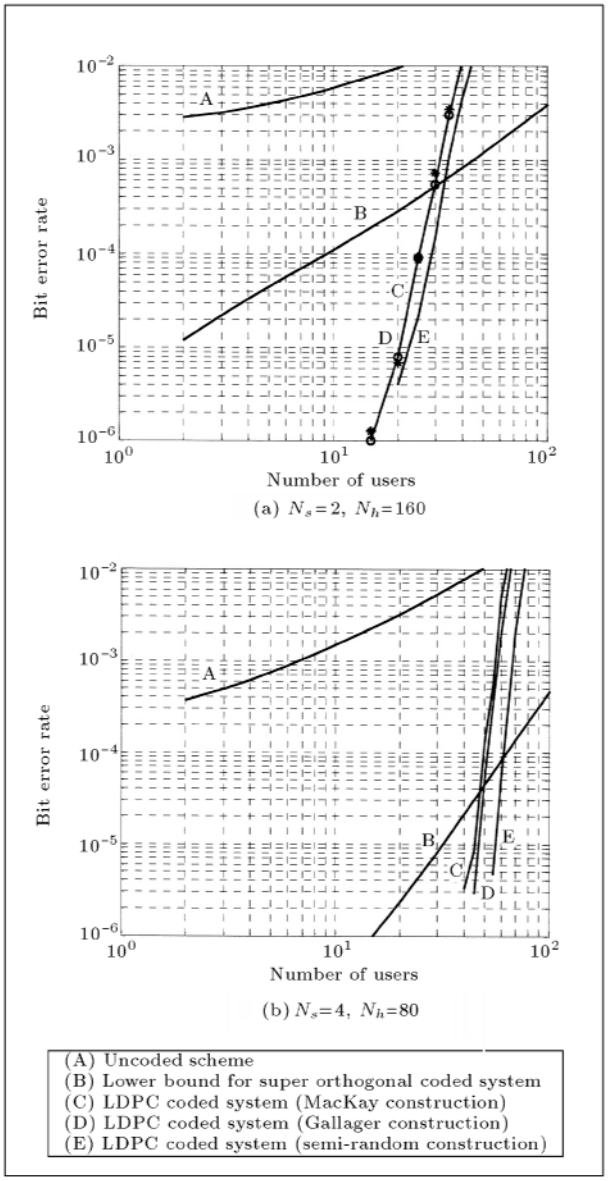

Figure 4. Bit error rate versus the number of users in MC-FH-CDMA system. γ_b is set to 12dB. The average number of decoding iterations is 214.407 for (a) and 339.518 for (b).

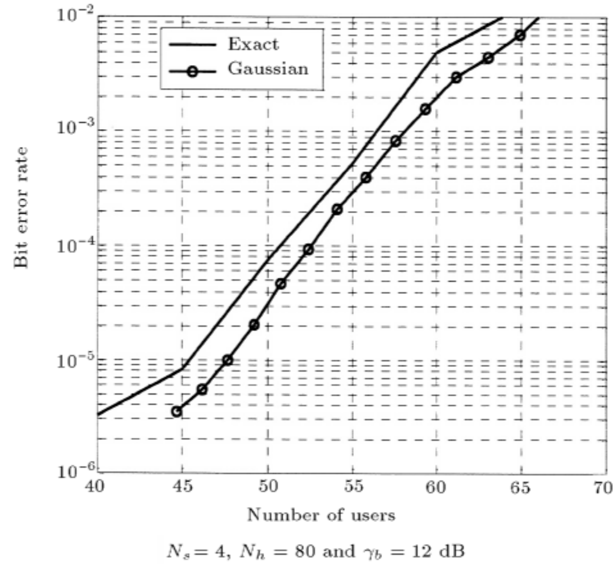

Figure 5. Bit error rate versus the number of users in LDPC coded MC-FH-CDMA system (MacKay construction).

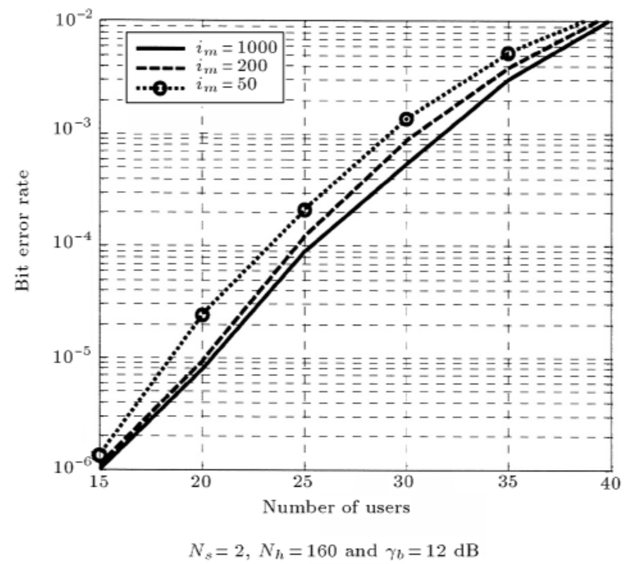

Figure 6. BER versus the number of users in LDPC coded MC-FH-CDMA system (MacKay construction) parameterized by the maximum number of iterations (i_m). The average numbers of decoding iterations for $i_m = 1000$, 200 and 50 are 214.407, 25.673, and 18.965, respectively.

by the semi-random code is about 25 (for $N_s = 2$) and 60 (for $N_s = 4$), whereas, by the super-orthogonal codes, it is, at most, 2 (for $N_s = 2$) and 30 (for $N_s = 4$). From these figures, it can also be realized that the semi-random code, despite its encoding simplicity, performs better than the more complex Gallager and MacKay codes.

The performance of the Gaussian distribution assumption for multiuser interference at the output of the subband correlator is illustrated in Figure 5, where the bit error rate versus the number of users for $N_s = 4$ under MacKay construction is evaluated. As can be realized, the Gaussian assumption, while simplifying the computation of *a priori probabilities*, performs well in the LDPC coded MC-FH-CDMA system.

To consider the effect of a maximum number of decoding iterations in the performance of a LDPC decoder, the BER versus the number of users parameterized by the maximum number of iterations is shown in Figure 6. It can be realized that the performance of a LDPC decoder with $i_m = 200$ is close to the performance of the LDPC decoder with $i_m = 1000$.

MODIFIED SEMI-RANDOM CONSTRUCTION

The well-known constructions of regular LDPC codes, briefly described in the previous section, impose some restrictions. That is that M (the number of parity bits) must be divisible by d_v. The semi-random construction also requires the divisibility of $K \times d_v$ by M. Furthermore, in the application considered (MC-FH-CDMA), the code rate must be equal to $1/N_s$, in order not to have any bandwidth expansion, due to the coding applied. Thus,

$$\frac{K \times d_v}{M} = \frac{R \times d_v}{1 \quad R} = \frac{d_v}{N_s \quad 1}, \tag{21}$$

where the first equality follows from definition of the code rate, $R = \frac{K}{N} = \frac{K}{K+M}$, and the second equality follows from the requirement of the coded MC-FH-CDMA, i.e., $R = \frac{1}{N_s}$. As a result, this requirement imposes the limitation of the divisibility of d_v by $N_s \quad 1$.

To overcome these restrictions for the application considered, a new construction will be presented, based on the semi-random code, which will be called "modified semi-random" construction. In this method, an appropriate value is chosen for d_v (without any restriction) and d'_v is computed as follows:

$$d'_v = \left\lceil \frac{K \times d_v}{M} \right\rceil. \tag{22}$$

The deterministic part of the parity check matrix, H, is similar to that of the semi-random construction and the random part is generated by matrix $H'_{K \times M}$ with weight d'_v per column, using the MacKay construction. So, the matrix, H, for this construction is given as follows:

$$H_{M \times N} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & | & \\ 1 & 1 & 0 & \cdots & 0 & | & \\ 0 & 1 & 1 & \cdots & \vdots & | & H'^{T}_{M \times K} \\ \vdots & \vdots & \ddots & \ddots & 0 & | & \\ 0 & \cdots & 0 & 1 & 1 & | & \end{bmatrix}.$$

Note that, in this construction, the value of d_v does not depend on the value of N_s. As a result, for the application considered, its value can be selected small enough for good performance [10], even for high values of N_s.

The performance of the LDPC coded MC-FH-CDMA system has been evaluated using the proposed construction by simulation. Figures 7a and 7b present the plots of BER versus the number of users for different constructions of LDPC codes. As can be realized, the simple modified semi-random construction outperforms the other constructions, especially at high values of N_s. For instance, at $N_s = 4$ and BER equal to $10^{\ 5}$, the number of users supported by MacKay, Gallager, semi-random and modified semi-random constructions are about 46, 47, 57 and 65, respectively.

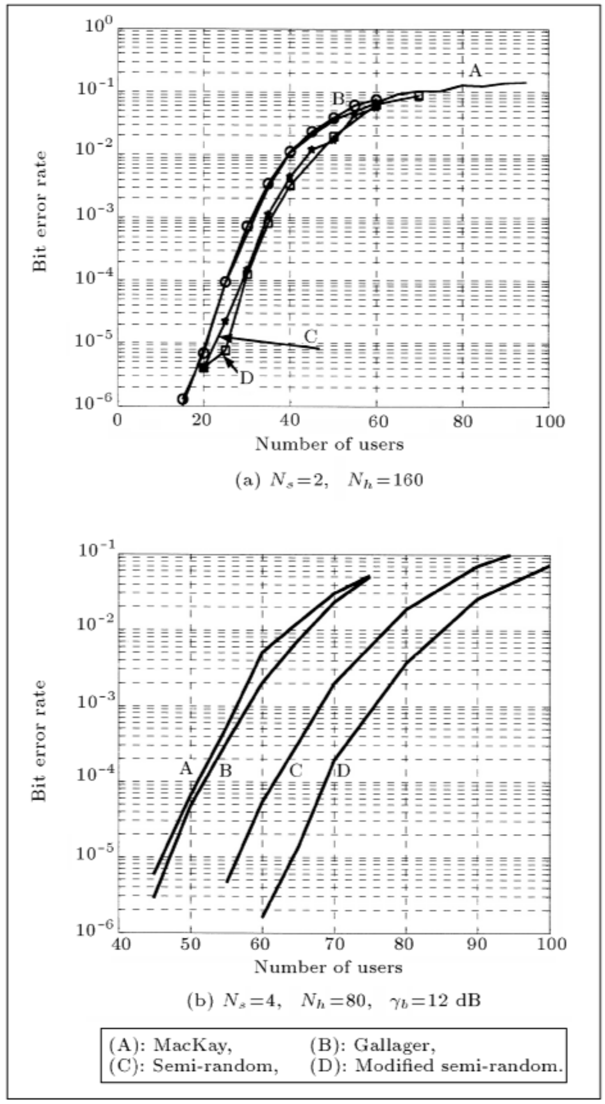

Figure 7. Performance of different construction methods of regular LDPC codes for MC-FH-CDMA system.

CONCLUSION

In this paper, a LDPC coded multi-carrier frequency-hopping CDMA scheme was first considered, which does not require any additional bandwidth to that needed by an uncoded spread spectrum MC-FH-CDMA system. Then, the performance of the coded system was evaluated, using different constructions of LDPC codes in a Rayleigh fading channel. The simulation results have indicated that the coded schemes substantially outperform the uncoded scheme. Furthermore, it has been realized that the LDPC coded scheme performs substantially better than the super-orthogonal convolutionally coded scheme (previously reported in [5]) at low BERs. It has also been observed that the semi-random LDPC codes perform very well, despite their simple encoding structures. Then, to overcome the restrictions imposed by the well-known constructions of LDPC codes for the application considered, a modified semi-random construction has been proposed. Despite its encoding simplicity, which is the same as that of the semi-random construction, the proposed modified construction outperforms previous construction methods.

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