Scientia Iranica, Vol. 13, No. 3, pp 295–302

SCIENTIA IRANICA

Improved Ant Colony Optimization Algorithm for Reservoir Operation

M.R. Jalali[1], A. Afshar* and M.A. Mariño[2]

In this paper, an improved Ant Colony Optimization (ACO) algorithm is proposed for reservoir operation. Through a collection of cooperative agents called ants, the near-optimum solution to the reservoir operation can be effectively achieved. To apply the proposed ACO algorithm, the problem is approached by considering a finite horizon with a time series of inflow, classifying the reservoir volume to several intervals and deciding for releases at each period, with respect to a predefined optimality criterion. Pheromone promotion, explorer ants and a local search are included in the standard ACO algorithm for a single reservoir, deterministic, finite-horizon problem and applied to the Dez reservoir in Iran. The results demonstrate that the proposed ACO algorithm provides improved estimates of the optimal releases of the Dez reservoir, as compared to traditional state-of-the-art Genetic Algorithms. It is anticipated that further tuning of the algorithmic parameters will further improve the computational efficiency and robustness of the proposed method.

INTRODUCTION

Ant Colony Optimization (ACO), called ant system [1,2], was inspired by studies of the behavior of ants [3]. Ant algorithms were first proposed by Dorigo [3] and Dorigo et al. [4] as a multi-agent approach to different combinatorial optimization problems, like the traveling salesman problem and the quadratic assignment problem. The ant-colony meta-heuristic framework was introduced by Dorigo and Di Caro [5], which enabled ACO to be applied to a range of combinatorial optimization problems. Dorigo et al. [6] also reported the successful application of ACO algorithms to a number of bench-mark combinatorial optimization problems. Montgomery and Randall [7,8] introduced several alternative pheromone applications. So far, very few applications of ACO algorithms to water resources problems have been reported [9,10]. Abbaspour et al. [9] employed ACO algorithms to estimate the hydraulic parameters of unsaturated soil. Maier et al. [10] used ACO algorithms to find a near global optimal solution to a water distribution system, indicating that ACO algorithms may form an attractive alternative to genetic algorithms for the optimum design of water distribution systems. In this paper, a novel way of addressing the optimum reservoir operation problem is proposed, making use of an improved ACO algorithm. To do so, the reservoir operation will be structured to fit an ACO model and the features related to ACO algorithms will be introduced. Performance of the proposed algorithm in the operation of the Dez reservoir in Iran, as well as the influence of the values of the algorithmic parameters on the performance of the ACO algorithm, will be described.

ANT COLONY BEHAVIOR

Ant colony algorithms have been founded on an observation of real ant colonies. By living in colonies, ants' social behavior is directed more to the survival of the colony as an entity rather than to that of an individual member of the colony. An interesting and significantly important behavior of ant colonies is their foraging behavior and, in particular, their ability to find the shortest route between their nest and a food source, realizing that they are almost blind. The path

1. *Mahab Ghods Consulting Engineers, Tehran, I.R. Iran.*
*. *Corresponding Author, Department of Civil Engineering, Iran University of Science and Technology, Tehran, I.R. Iran.*
2. *Department of Civil and Environmental Engineering, University of California, Davis, CA, USA.*

taken by individual ants from the nest in search for a food source, is essentially random [4]. However, when they are traveling, ants deposit on the ground a substance called pheromone, forming a pheromone trail as an indirect means of communication. By smelling the pheromone, there is a higher probability that the trail with a higher pheromone concentration will be chosen. The pheromone trail allows ants to find their way back to the food source and vice versa. The trail is used by other ants to find the location of the food source located by their nest mates. It follows that when a number of paths are available from the nest to a food source, a colony of ants may be able to exploit the pheromone trail left by the individual members of the colony to discover the shortest path from the nest to the food source and back [5]. As more ants choose a path to follow, the pheromone on the path builds up, making it more attractive to other ants seeking food and, hence, more likely to be followed.

Generally speaking, metaheuristic population-based algorithms [11] search for a global optimum by generating a population of trial solutions. Ant colony optimization, as a metaheuristic population-based algorithm, has many features which are similar to Genetic Algorithms (GAs). Table 1 compares some common and/or similar features of ACO algorithms with those of GAs, as described in detail by Maier et al. [10]. The most important difference between GAs and ACO algorithms is the way the trial solutions are generated. In ACO algorithms, trial solutions are constructed incrementally, based on the information contained in the environment and the solutions are improved by modifying the environment via a form of indirect communication called stigmergy [6]. On the other hand, in GAs, the trial solutions are in the form of strings of genetic material and new solutions are obtained through the modification of previous solutions [10]. Thus, in GAs, the memory of the system is embedded in the trial solutions, whereas, in ACO algorithms, the system memory is contained in the environment itself.

ANT COLONY OPTIMIZATION (ACO) ALGORITHMS: GENERAL ASPECTS

An interesting and very important behavior of ant colonies is their foraging behavior and, in particular, their ability to find the shortest route between their nest and a food source, realizing that they are almost blind. The path taken by individual ants from the nest to the food source is essentially random [4]. However, when they are traveling, ants deposit a substance called pheromone, forming a pheromone trail as an indirect means of communication. As more ants choose a path to follow, the pheromone on the path builds up, making it more attractive for other ants to follow.

In the ACO algorithm, artificial ants are permitted to release pheromone while developing a solution or after a solution has been fully developed, or both. As stated, the amount of pheromone deposited is made proportional to the goodness of the solution an artificial ant develops. A rapid drift of all ants toward the same part of the search space is avoided by employing the stochastic component of the choice decision policy and numerous mechanisms, such as pheromone evaporation, explorer ants and local search.

Let $\tau_{ij}(t)$ be the total pheromone deposited on path ij at time t, and $\eta_{ij}(t)$ be the heuristic value of path ij at time t, according to the measure of the objective function. Heuristic value is a measure of objective function, which along with the pheromone (τ_{ij}), will determine the transition probability from option i to j, at time period t, as follows:

$$P_{ij}(k,t) = \begin{cases} \frac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}(t)]^{\beta}}{\sum_{j=1}^{NC}[\tau_{ij}(t)]^{\alpha}[\eta_{ij}(t)]^{\beta}} & \text{if } j \in N_k(t) \\ 0 & \text{otherwise} \end{cases}, \quad (1)$$

where $P_{ij}(k,t)$ is the probability that ant k selects path ij at time period t; $N_k(t)$ is the feasible neighborhood of ant k when located at time period t and α and β are two parameters that control the relative importance of the pheromone trail and heuristic value.

Let q be a random variable uniformly distributed over [0,1] and $q_0 \in [0,1]$ be a tunable parameter. The

Table 1. Similarities of ACO and genetic algorithms

Genetic Algorithm	ACO Algorithm
Population size	Number of ants
One generation	One iteration
Trial solutions utilize the principle of survival of the fittest	It is based on foraging behavior of ant colonies
Probabilistic process is governed by crossover and mutation	Probabilistic process is defined by pheromone intensities and local heuristic information
Encouraging wider search space is achieved by mutation operator	Wider search space is guaranteed by pheromone evaporation

next option, j, that ant k chooses is [12]:

$$j = \begin{cases} \arg \max\limits_{l \in N_k(t)} \left\{ [\tau_{il}(t)]^{\alpha} \, [\eta_{il}(t)]^{\beta} \right\} & \text{if } q \leq q_0 \\ J & \text{otherwise} \end{cases}, \quad (2)$$

where J is a value of a random variable selected according to the probability distribution of $P_{ij}(k,t)$ (Equation 1). Equations 1 and 2 provide a probabilistic decision policy to be used by the ants to direct their search towards the optimal regions of the search space. The level of stochasticity in the policy and the strength of the updates in the pheromone trail determine the balance between the exploration of new points in the state-space and the exploitation of accumulated knowledge [12]. To simulate pheromone evaporation, the pheromone evaporation coefficient, (ρ), is defined, which enables greater exploration of the search space and minimizes the chance of premature convergence to sub-optimal solutions upon completion of a tour by all ants in the colony. The global trail updating is done as follows:

$$\tau_{ij}(t) \overset{\text{iteration}}{\longleftarrow} (1 \quad \rho).\tau_{ij}(t) + \rho.\Delta\tau_{ij}, \quad (3)$$

where $0 \leq \rho \leq 1$; $(1 \quad \rho)$ is evaporation (i.e., loss) rate; and the symbol $\overset{\text{iteration}}{\longleftarrow}$ is used to show the next iteration.

There are several definitions for pheromone deposition on path ij during time period t, $\Delta\tau_{ij}(t)$ [4,12]. From three well known algorithms, namely; Ant System (AS), Ant Colony System iteration-best (ACS_{ib}) and Ant Colony System global-best (ACS_{gb}), the latter was chosen in this study [13], in which:

$$\Delta\tau_{ij}(t) = \begin{cases} 1/G^{k^*_{gb}} & \text{if } (i,j) \in \text{tour done by ant } k^*_{gb} \\ 0 & \text{otherwise} \end{cases}, \quad (4)$$

where $G^{k^*_{gb}}$ is the value of the objective function for ant k^*_{gb}, which is the ant with the best performance within the past total iterations.

ACO ALGORITHMS FOR OPTIMUM RESERVOIR OPERATION

To apply ACO algorithms to a specific problem, the following steps have to be taken:

1. Problem representation as a graph or a similar structure easily covered by ants;
2. Assigning a heuristic preference to generated solutions at each time step (i.e., selected path by the ants);
3. Defining a fitness function to be optimized;
4. Selection of an ACO algorithm to be applied to the problem.

In order to apply ACO algorithms to the optimum reservoir operation problem, it is convenient to see it as a combinatorial optimization problem with the capability of being represented as a graph. The problem may be approached considering a time series of inflow, classifying the reservoir volume to several intervals and deciding for releases at each period, with respect to an optimality criterion. As depicted in Figure 1, links between initial and final storage volumes at entire periods form a graph which represents a feasible solution.

The heuristic value of this problem is defined as minimum square deviation from target demand, which is an indicator of the objective value as employed in the transition rule (Equations 1 and 2):

$$\eta_{ij}(t) = 1/([R_{ij}(t) \quad D(t)]^2 + c), \quad (5)$$

where $R_{ij}(t)$ is release at period t (provided that the initial and final storage volume are at classes i and j, respectively); $D(t)$ is demand of period t and c is a constant to avoid irregularity (dividing by zero in Equation 5). To determine $R_{ij}(t)$, the continuity equation, along with the following constraints, may be employed as:

$$R_{ij}(t) = S_i \quad S_j + I(t) \quad \text{LOSS}_{ij}(t), \quad (6a)$$

$$S_{\min} \leq S_i \leq S_{\max}, \quad (6b)$$

$$S_{\min} \leq S_j \leq S_{\max}, \quad (6c)$$

$$S_1 = S_{NT+1}, \quad (6d)$$

where S_i and S_j are the initial and final storage volumes (class i and j), respectively; $I(t)$ is the inflow to the reservoir at time period t; $\text{LOSS}_{ij}(t)$ is the loss (e.g., evaporation) at period t, provided that initial and final storage are at classes i and j, respectively; $S_{\min}$

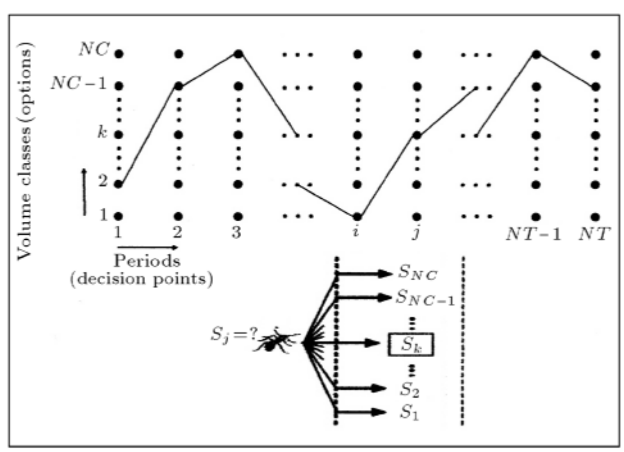

Figure 1. Decision graph of ACO algorithm for optimum reservoir operation problem.

and $S_{\max}$ are the minimum and maximum storage allowed, respectively, and NT is total number of periods. The proposed model takes "end-of-period storage" as decision variables. Therefore, the release associated with S_i and S_j may easily be determined using the continuity equation, as defined by Equation 6a. Using the transition rule (Equations 1 and 2), each ant is free to choose the class of final storage (end-of-period storage), if it is feasible through the continuity equation and storage constraints (Equations 6). In another words, selection of path ij at time period t will result in release $R_{ij}(t)$, according to the continuity equation, as defined by Equation 6a. Selection of path ij is permitted, if, and only if, end-of-period storage (i.e., S_j) satisfies Constraints 6a and 6b and the resulted release, $R_{ij}(t)$, is positive. Otherwise, option j is not feasible and may not be taken at period t by ant k.

The fitness function is a measure of the goodness of the generated solutions, according to the defined objective function. For this study, fitness function is defined as the Total Square Deviation (TSD) from the target demand:

$$\mathrm{TSD}^k = \sum_{t=1}^{\mathrm{NT}} \left[R^k(t) \quad D(t)\right]^2, \tag{7}$$

where $R^k(t)$ is release at period t recommended by ant k.

Jalali et al. [13] showed that from three different ACO algorithms, namely: the Ant System (AS), the Ant Colony System-Iteration Best (ACS_{ib}) and the Ant Colony System-Global Best (ACS_{gb}), the ACS_{gb} provides better solutions compared to two others, as applied to a reservoir management problem. In this research, 3 improved versions of ACS_{gb} are introduced and tested. A simple flow diagram of the proposed ACO algorithm for the optimum reservoir operation is depicted in Figure 2.

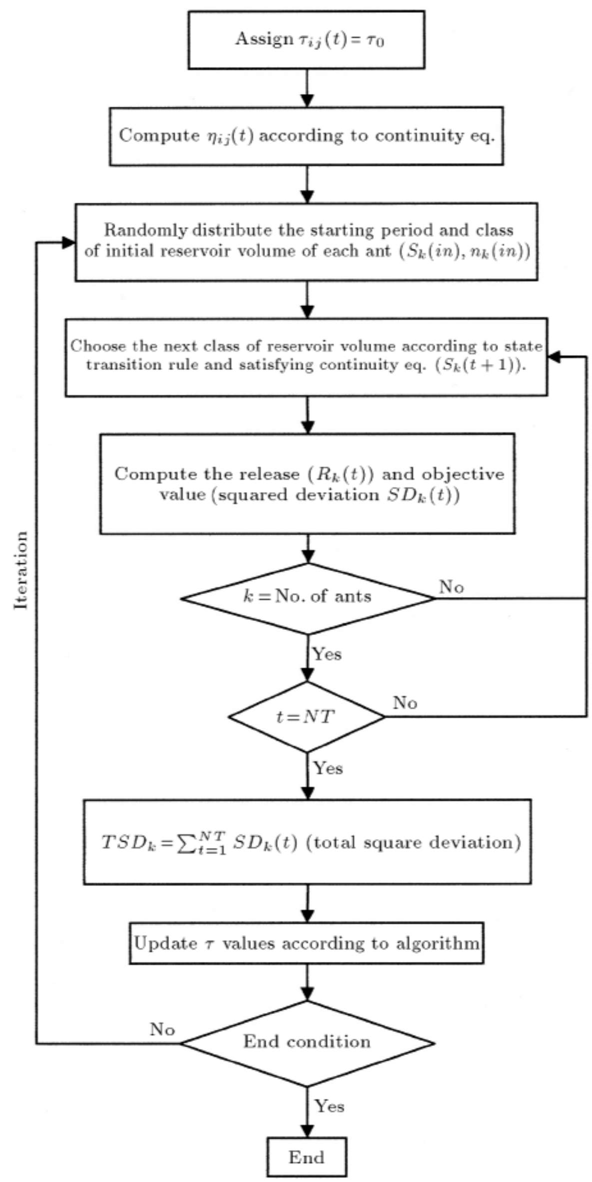

Figure 2. ACO algorithm for optimum reservoir operation.

IMPROVED ACO ALGORITHM IN RESERVOIR OPERATION

As mentioned in Equation 1, a three-dimensional (3D) pheromone, $\tau_{ij}(t)$, being defined as the total pheromone deposited on path ij at time period t, may cause a dimensionality problem in large scale and/or multi-reservoir problems. In order to reduce the extent of the dimensionality problem, one may assign the pheromone to options, rather than paths. In this case, a two-dimensional (2D) pheromone, $\tau_i(t)$, may be defined as the pheromone deposited at option i and time period t. Now, Equations 1 and 2 may be modified for the new scheme, as follows:

$$P_{ij}(t) = \begin{cases} \frac{[\tau_j(t+1)]^{\alpha}[\eta_{ij}(t)]^{\beta}}{\sum_{l=1}^{\mathrm{NC}}[\tau_l(t+1)]^{\alpha}[\eta_{il}(t)]^{\beta}} & \text{if } j \in N_k(t) \\ 0 & \text{otherwise} \end{cases}, \tag{8}$$

$$j = \begin{cases} \arg \max\limits_{l \in N_k(t)} \left\{[\tau_l(t+1)]^{\alpha}[\eta_{il}(t)]^{\beta}\right\} & \text{if } q \leq q_0 \\ J & \text{otherwise} \end{cases}. \tag{9}$$

To improve the convergence of the results, Pheromone Promotion (PP) is introduced as a new approach. Due to pheromone deposition and evaporation (Equation 3), a rapid convergence syndrome or stagnation problem may prevail, if no improvement is gained after a few iterations. Now, if a new solution with an improved objective value is identified, its pheromone must be promoted to the maximum existing pheromone concentration (i.e., available global best solution). If the existing pheromone concentration is very low, such an improved solution may not be desirable for the agents to follow. Therefore, to reduce the possibility of facing a stagnation problem, pheromone concentration, τ, for all new paths with better objective value, must be promoted.

Under some circumstances, pheromone evaporation may eventually eliminate a solution with some locally positive decision elements. To minimize this possibility, an anti-pheromone approach was proposed by Montgomery and Randall [7]. In one of the anti-pheromone approaches, Explorer Ants (EA) are assigned to explore inferior paths randomly, to find out paths with insignificant pheromone and reasonable performance. With this approach, one may regenerate solutions that have locally lost their pheromone, due to evaporation during the last iterations. Assigning some explorer ants at each iteration, they may take their paths, as follows:

$$P_{ij}(t) = \begin{cases} \frac{[\tau_{\max} \quad \tau_j(t+1)]^{\alpha}[\eta_{ij}(t)]^{\beta}}{\sum_{l=1}^{NC}[\tau_{\max} \quad \tau_l(t+1)]^{\alpha}[\eta_{il}(t)]^{\beta}} & \text{if } j \in N_k(t) \\ 0 & \text{otherwise} \end{cases}, \tag{10}$$

$$j = \begin{cases} \arg\max_{l \in N_k(t)} \left\{[\tau_{\max} \quad \tau_l(t+1)]^{\alpha}\,[\eta_{il}(t)]^{\beta}\right\} \\ \quad \text{if } q \le q_0 \\ J \quad \text{otherwise} \end{cases}, \tag{11}$$

where $\tau_{\max}$ is maximum value of τ, realized during the last iterations.

In some circumstances, A Local Search (LS) approach may cause a significant improvement in the final results, if employed after each iteration. In this approach, which is, more or less, similar to crossover in genetic algorithms, one, or a few, sections of the generated solutions after each iteration are exchanged. As an example, in a 3-opt local search algorithm [12], two elements from two different solutions are exchanged randomly, which result in two new solutions. In the present work, one of the solutions for a local search application is the present best solution and the other one selected randomly.

Finally, integration of the local search, explorer ants and pheromone promotion may facilitate the convergence of the scheme, as well as improve the final results. In fact, explorer ants may generate a string of solutions with relatively low total pheromone concentration, which is an indication of a solution with low desirability. However, within such an undesirable string, some local sections may exist that reveal low pheromone concentration, which might make some other solutions more desirable, if they were substituted in those solutions. Combinations of a local search with explorer ants provide such a possibility and minimize the chance of losing good pieces of information.

MODEL APPLICATION

To illustrate the performance of the proposed model, the Dez reservoir in southern Iran, with an effective storage volume of 2,510 MCM and average annual demand of 5,900 MCM, is selected. For illustration purposes, a period of 60 months, with an average annual inflow of 5,303 MCM, is employed. The reservoir volume is divided into 14 classes, with 200 MCM intervals. To limit the range of values of the fitness function, a normalized form of Equation 7 has been used as follows:

$$\mathrm{TSD}^k = \sum_{t=1}^{NT} \left[(R^k(t) \quad D(t))/D_{\max}\right]^2, \tag{12}$$

where $D_{\max}$ is maximum monthly demand. To make the results at different runs comparable, initial and ending storages were fixed at 1430 MCM for all runs. Therefore, the starting point for all ants was fixed at the first period with an initial storage volume of 1430 MCM. Feasible paths for the ants to follow are constrained by the continuity equation and the minimum and maximum permitted storage volume (Equations 6).

Due to the limited number of iterations (i.e., end condition criteria) and presence of the random parameter, q, in the transition rule (Equation 2), the final results for the different runs may not be the same. Therefore, the model so developed was tested with 10 different runs. In order for results to be comparable, the total number of ants (M) assigned to the problem was 100, with $\rho = 0.1$, $\alpha = 1$, $\beta = 4$ and $q_0 = 0.9$, as proposed by Jalali et al. [13]. To start with, the pheromone was uniformly distributed all over the defined paths (i.e., $\tau_0 = 1$). To normalize the value of the heuristic function, parameter c was chosen to be unity (Equation 5). The total number of iterations at each run was limited to 500.

Results of the model for different improved algorithms are presented in Table 2. Mean values of the objective function at each iteration for 10 different runs are illustrated in Figure 3. As is clear, by reducing the dimension of the pheromone from τ_{ij} to τ_i, inferior results are obtained. In this case, the TSD for the best result and its mean value for 10 runs exceeds those of τ_{ij} by 20% and 30%, respectively.

To improve model performance, Pheromone Promotion (PP), Explorer Ants (EA) and Local Search (LS) operators were included in the proposed algorithm. The mean value of TSD for 10 runs at each iteration for different versions of the improved algorithm is depicted in Figure 3 and Table 2. One may note that the pheromone promotion is a great contribution to model convergence. Individual inclusion of explorer ants and local search have improved the best result and

Table 2. Total Square Deviation (TSD) from target demand of 10 runs of several improved ACO algorithms in reservoir operation problem.

Run	3D[(a)]	2D[(b)]	2D-PP[(c)]	2D-PP-EA[(d)]	2D-PP-LS[(e)]	2D-PP-EA-LS	3D-PP-EA-LS
1	1.317	1.695	1.509	1.560	1.439	1.296	1.347
2	1.249	1.716	1.630	1.357	1.560	1.407	1.318
3	1.317	1.509	1.806	1.713	1.560	1.296	1.317
4	1.333	1.716	1.626	1.350	1.350	1.360	1.249
5	1.308	1.715	1.710	1.497	1.740	1.350	1.317
6	1.317	1.643	1.681	1.385	1.439	1.407	1.333
7	1.296	1.793	1.593	1.535	1.619	1.296	1.249
8	1.317	1.664	1.642	1.576	1.630	1.350	1.356
9	1.271	1.837	1.728	1.751	1.580	1.382	1.308
10	1.354	1.779	1.668	1.318	1.829	1.407	1.296
Mean	1.308	1.707	1.659	1.504	1.575	1.355	1.309
The best	1.249	1.509	1.509	1.318	1.350	1.296	1.249
The worst	1.354	1.837	1.806	1.751	1.829	1.407	1.356
S.D.[(f)]	0.030	0.091	0.081	0.152	0.143	0.046	0.036
C.V.[(g)]	0.023	0.053	0.049	0.101	0.091	0.034	0.028

(a) 3D: Three Dimensional Pheromone; (b) 2D: Two Dimensional Pheromone; (c) PP: Pheromone Promotion; (d) EA: Explorer Ants;(e) LS: Local Search; (f) S.D.: Standard Deviation; (g) C.V.: Coefficient of Variation.

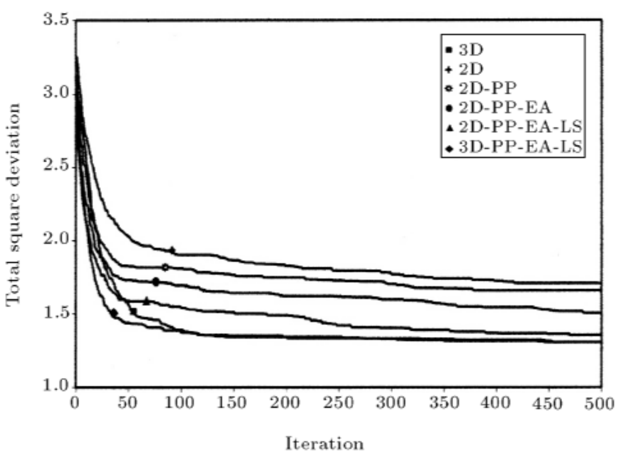

Figure 3. Convergence of several improved ACO algorithms in reservoir operation problem (averaged over 10 runs).

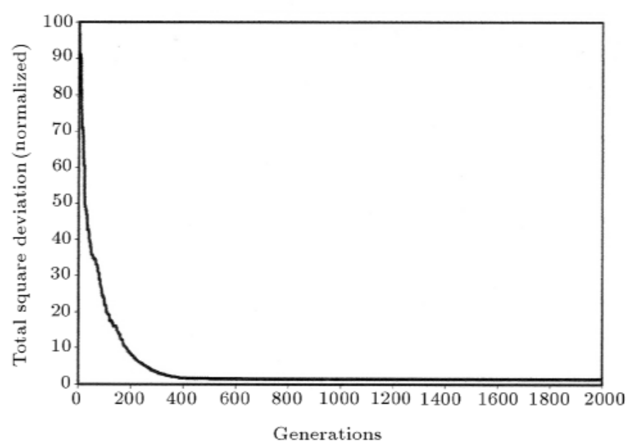

Figure 4. Evolution of the total square deviation from target demand resulted from genetic algorithm.

their mean values for 10 runs by (13%, 11%) and (10%, 5%), respectively. It is also important to note that the initial rate of convergence improved significantly, when LS and EA were employed.

Integrating LS and EA in the algorithm has significantly increased the initial and final rate of convergence, as well as resulting in highly improved results. In this case, the best result and mean value for 10 runs show 14% and 19% improvement, respectively, over only pheromone.

To compare the performance of the proposed improved ACO algorithm with that of the Genetic Algorithm (GA), the same model was solved using the Fast Messy genetic algorithm of Boulos et al. [14]. For a population size of 100 and a 2000 generation, using the best combinations of crossover and mutation probability and by employing a real value coding, a TSD of 1.38 was achieved. Results of the GA model at each generation for the problem under consideration are depicted in Figure 4. As is clear, the results of the proposed ACO algorithm, integrated with PP, EA and LS for 500 iterations and an agent size of 100, show, approximately, a 6 percent improvement over that of GA, with 2000 generations and 100 populations. Periodic reservoir release values resulted from ACO

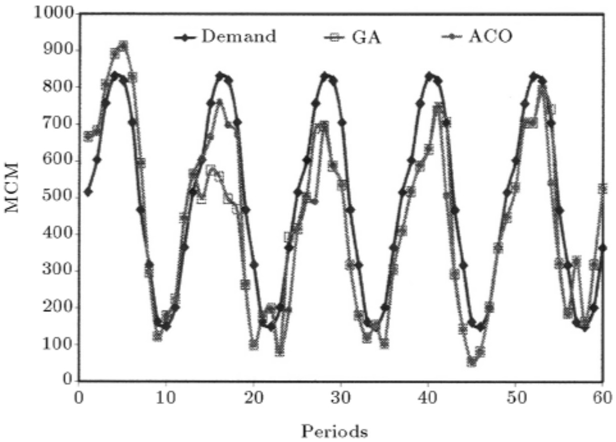

Figure 5. Reservoir release values resulted from GA and ACO models compared to demand series.

and GA compared to demand series are depicted in Figure 5.

In order to examine the effect of three-dimensional pheromone (i.e., assigning pheromone to path ij), the same problem was solved integrating PP, EA and LS operators in a 3D pheromone. Comparing the results with those of 2D-PP-EA-LS, an improvement of 3.5 percent in the best result and mean value for 10 runs was achieved, which may not be justified on a large scale or in multi-reservoir problems. Even though the best result and average value for 10 runs in simple 3D and 3D-PP-EA-LS are the same, integration of PP, EA and LS has significantly improved the initial convergence. As an example, in the first 40 iterations, the rate of convergence has increased by more than 12 percent.

CONCLUDING REMARKS

While walking from one point to another, ants deposit a substance called pheromone, forming a pheromone trail. It has been shown, experimentally [4], that this pheromone trail, once employed by a colony of ants, can give rise to the emergence of a shortest path. In general, the amount of pheromone deposited by an artificial ant is made proportional to the goodness of the solution an ant may build. To apply ACO algorithms to the reservoir operation problem, one may view it as a combinatorial optimization problem. The problem may be approached by considering a time series of inflow, classifying the reservoir volume to several intervals and deciding on the release at each period, with respect to an optimality criterion. Feasible paths for ants to follow may be constrained by the continuity equation, as well as constraints on the storage volume. Upon each tour completion, a finite number of feasible solutions will form, leaving a new value for the pheromone.

Realizing the values of the fitness function, the pheromones will be updated by a global update rule. The inclusion of Explorer Ants (EA) and a Local Search (LS), along with a new operator called Pheromone Promotion (PP), improves the performance of the classic ACO significantly. The improvement includes the final result, as well as the initial and final rate of convergence. Application of the proposed model to the Dez reservoir in Iran provided promising results. From three different pheromone updating algorithms (i.e., Ant System, Ant Colony System-iteration best and Ant Colony System-global best), the ACS_{gb} was employed in this research, which includes explorer ants, pheromone promotion and local search operators. Results of the model compare well with those of the GA and global optimum. As for any search method, the performance of the proposed model is quite sensitive to setup parameters, hence, fine tuning of the parameters is recommended.

NOMENCLATURE

ρ	pheromone persistence coefficient
$P_{ij}(t)$	transition probability from option i to option j at time period t
$\tau_{ij}(t)$	total pheromone deposited on path ij at time t
$\eta_{ij}(t)$	the heuristic value of path ij at time t
α, β	parameters that control the relative importance of the pheromone trail versus a heuristic value
q	a random variable uniformly distributed over [0,1]
q_0	a tunable parameter $\in [0,1]$
τ_0	initial value of pheromone
k^*_{gb}	the ant with the best performance within the past total iterations
$G^{k^*_{gb}}$	value of the objective function for the ant with the best performance within the past total iterations
$R_{ij}(t)$	release at period t
$D(t)$	demand of period t
c	a constant
S	storage
$I(t)$	inflow to the reservoir at time period t
$\text{LOSS}_{ij}(t)$	loss (e.g., evaporation) at period t provided that initial and final storage are at classes i and j, respectively
$S_{\min}$	minimum storage allowed
$S_{\max}$	maximum storage allowed
NT	total number of periods
TSD	total square deviation

$R^k(t)$ release at period t recommended by ant k

$D_{\max}$ maximum monthly demand

$\tau_{\max}$ maximum value of τ realized during the last iterations

REFERENCES

1. Colorni, A., Dorigo, M. and Maniezzo, V. "Distributed optimization by ant-colonies", *Proc., 1st European Conf. on Artificial Life (ECAL'91)*, Cambridge, MIT Press, Mass, USA, pp 134-142 (1991).
2. Dorigo, M. "Optimization, learning and natural algorithms", Ph.D. Thesis, Politecnico di Milano, Italy (1992).
3. Deneubourg, J.L., Pasteels, J.M. and Verhaeghe, J.C. "Probabilistic behavior in ants: A strategy of errors?", *J. Theoretical Biology*, **105**, pp 259-271 (1983).
4. Dorigo, M., Maniezzo, V. and Colorni, A. "The ant system: optimization by a colony of cooperating ants", *IEEE Trans. Syst. Man. Cybern.*, **26**, pp 29-42 (1996).
5. Dorigo, M. and Di Caro, G., *The Ant Colony Optimization Metaheuristic*, In *New Ideas in Optimization*, D. Corne, M. Dorigo, and F. Glover, Eds., McGraw-Hill, London, UK, pp 11-32 (1999).
6. Dorigo, M., Bonabeau, E. and Theraulaz, G. "Ant algorithms and stigmergy", *Future Generation Comput. Systems*, **16**, pp 851-871 (2000).
7. Montgomery, J. and Randall, M. "Anti-pheromone as a tool for better exploration of search space", *Proceedings of the Third International Workshop on Ant Algorithms, ANTS 2002*, Brussels, Belgium (2002).
8. Montgomery, J. and Randall, M. "Alternative pheromone applications for ant colony optimization", *Technical Report TR02-07*, Faculty of Information Technology, Bond University, Australia (2002).
9. Abbaspour, K.C., Schulin, R. and van Genuchten, M.T. "Estimating unsaturated soil hydraulic parameters using ant colony optimization", *Adv. Water Resour.*, **24**(8), pp 827-933 (2001).
10. Maier, H.R., Simpson, A.R., Zecchin, A.C., Foong, W.K., Phang, K.Y., Seah, H.Y. and Tan, C.L. "Ant colony optimization for design of water distribution systems", *J. Water Resour. Plng. and Mgmt.*, **129**(3), pp 200-209 (2003).
11. Blum, C. and Roli, A. "Metaheuristics in combinatorial optimization: Overview and conceptual comparison", *ACM Computing Surveys*, **35**(3), pp 268-308 (2003).
12. Dorigo, M. and Gambardella, L.M. "Ant colony system: A cooperative learning approach to the traveling salesman problem", *IEEE Transactions on Evolutionary Computation*, **1**(1), pp 53-66 (1997).
13. Jalali, M.R., Afshar, A. and Marino, M.A. "Optimum reservoir operation by ant colony optimization algorithms", *Iranian Journal of Science and Technology*, **30**(B1), pp 107-117 (2006).
14. Boulos, P.F., Wu, Z.Y., Orr, C.H. and Ro, J.J. "Least-cost design and rehabilitation of water distribution systems using genetic algorithms." *Proceedings of the AWWA IMTech Conference*, Seattle, WA, USA (Apr. 16-19, 2000).