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# Analytical Model for the Ultimate Bearing Capacity of Foundations from Cone Resistance

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By application of Cone Penetration Test (CPT) data for shallow foundation (footing) design, the problems of providing representative undisturbed samples and, rather, $\varphi \quad N$ coefficient relations will be eliminated. An analytical model, based on a general shear failure mechanism of the logarithm spiral type, has been developed for calculating, directly, the bearing capacity of footings, $q_{ult}$ from cone resistance, $q_c$. The transform of the failure mechanism from a shallow to a deep foundation and the scale effect have been considered in the proposed method. Six current CPT direct methods for determining the bearing capacity of footings have been investigated. The proposed method and others were compared to the measured capacity, ranging from 1.7 to 15 kg/cm$^2$, of 28 footings compiled in a database with a range of diameter from 0.3 to 3 m located in different soils. The graphical and cumulative probability approaches for the validation of the methods indicates optimistic results for the bearing capacity estimation of the proposed method, which is simple and routine.

## INTRODUCTION

One of the main steps for the safe and economic design of foundations is determination of ultimate bearing capacity. The maximum load that can be applied to subgrade soil from the foundation without occurrence of shear or punching failure, keeping settlement to a limited range and avoiding serviceability damage to super structures.

Currently, four approaches are being used to determine the bearing capacity of shallow and deep foundations; static analysis, in-situ testing methods, full-scale loading tests and using presumed values recommended by codes and handbooks. Among these approaches, a theoretical solution; i.e. static analysis, is more common and employed first, where other approaches are realized as being supplementary to static analysis.

In-situ soil testing has shown an increase in recent years in geotechnical engineering practice. This is due to the rapid development of field testing instruments, improved understanding of soil behavior and the subsequent realization of the limitations and inadequacies of some conventional laboratory testing. The Standard Penetration Test, SPT, is still the most commonly used in-situ test. However, many problems and limitations exist for SPT, with respect to performance, interpretation and repeatability. These are due to the uncertainty of the energy delivered by various SPT hammers to the anvil system, test procedures and operator-equipment effects.

In contrast to SPT, the Cone Penetration Test, CPT, is simple, fast, relatively economical, and it supplies continuous records with depth. The results are interpretable on both an empirical and analytical basis and a variety of sensors can be incorporated by use of a cone penetrometer. Evaluating the bearing capacity of foundations from CPT data is one of the earliest applications of this sounding and includes two main approaches; direct and indirect methods. Direct CPT methods apply the measured values of cone bearing for bearing resistance with some modifications regarding scale effects i.e., the influence of foundation width to cone diameter ratio. Indirect CPT methods employ friction angle and undrained shear strength values estimated from CPT data based on bearing capacity and/or cavity expansion theories.

The analogy of the cone penetrometer and pile has caused a concentration of research work in geotechnical practice on the application of CPT data for deep foundations. However, in practice, in civil engineering projects, the majority of foundations are spread and shallow. Hence, in this paper, for shallow foundations (footings), different CPT direct methods for determin-

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ing bearing capacity are investigated. By utilizing a database, including the full scale loading test results of footings and CPT soundings performed close to footing locations, the capability of the predictive methods has been compared and validated.

## CPT INDIRECT METHODS FOR BEARING CAPACITY OF FOOTINGS

Terzaghi [1] presented an applicable formula, usually referred to as the "3N bearing capacity formula". For a general shear failure mechanism beneath strip footings, the Terzaghi bearing capacity equation is as follows:

$$q_{ult} = CN_c + \overline{q}N_q + 0.5\gamma BN_\gamma, \tag{1}$$

where:

- $q_{ult}$ ultimate unit resistance or bearing capacity of footing,
- $C$ cohesion parameter,
- $\overline{q}$ surcharge around foundation, equal to $\gamma D_f$,
- $\gamma$ average effective unit weight of the soil below and around the foundation,
- $B$ foundation width,
- $D_f$ embedment depth of foundation,
- $N_c, N_q, N_\gamma$ non-dimensional bearing capacity factors as exponential functions of $\varphi$,
- $\varphi$ soil internal friction angle.

Terzaghi also recommended a revised formula for the different shape of foundations and, also, in case of occurring punching failure. Later, Meyerhof [2], Hansen [3] and Vesić [4], applied modifications for shape, depth, inclination, ground and base factors to the original Terzaghi bearing capacity formula, which was conservative.

In common geotechnical engineering practice, the following equation is used [5]:

$$q_{ult} = CN_cs_ci_cd_c + \overline{q}N_qs_qi_qd_q + 0.5\gamma BN_\gamma s_\gamma i_\gamma d_\gamma, \tag{2}$$

where:

- $s_c, s_q, s_\gamma$ non-dimensional shape factors,
- $i_c, i_d, i_\gamma$ non-dimensional inclination factors,
- $d_c, d_q, d_\gamma$ non-dimensional depth factors.

The ultimate bearing capacity is divided into a safety factor usually equal to 3 and the result is called the "safe bearing capacity", which is used for foundation design. However, to satisfy settlement criteria, it might be employed even greater values of safety factors and the outcome would be called "allowable bearing capacity".

Perhaps it should be emphasized that the forenamed equations pertain to the loading of footing under long-term service conditions, that is, fully drained conditions. Rapid loading of foundations in normally consolidated clays and silts will create pore pressures that reduce soil strength. Practice has established rules for the analysis of such "undrained" conditions, normally assessing the stability in a slip circle analysis with an input of undrained shear strength, as follows:

$$q_{ult} = S_uN_C + \overline{q}, \tag{3}$$

where:

- $S_u$ undrained shear strength,
- $N_C$ cohesion coefficient ranges from 5 to 6 for shallow foundations.

Several correlations have been suggested for determining $\varphi$-angle or $S_u$ from CPT data. In turn, the bearing capacity can be calculated from Equation 2 or 3. Other than empirical and semi-empirical correlations between cone resistance and soil friction angle, two theories; i.e. bearing capacity or cavity expansion, are used.

Numerous methods have been developed for evaluating $\varphi$-angle from $q_c$. Schmertmann [6] proposed a relationship between the $\varphi$-angle and relative density ($D_r$), for different grain size characteristics. This approach requires the approximate nature to which $D_r$ can be estimated from CPT data. Experience has shown that $D_r$ $q_c$ correlations are sensitive to soil compressibility and in-situ horizontal stresses. Robertson and Campanella [7] compared measured cone resistance, $q_c$, to measured friction angle values, which were obtained from drained triaxial compression tests performed at confining stresses equal to in-situ horizontal stress $\sigma'_{h0}$ in the calibration chamber. A simple set of relationships is shown in Figure 1, where $q_c$ increases linearly with effective overburden stress for a constant $\varphi$-angle. The results were obtained for uncemented moderately incompressible silica sands. For highly compressible sands, the chart shown in Figure 1 would tend to predict a low friction angle.

Bearing capacity solutions are basically based on plane strain conditions, linear strength envelopes and incompressible materials. Two main available methods were developed by Janbu and Senneset [8] and Durgunoglu and Mitchell [9]. The solution by Janbu and Senneset depends on the shape of the failure zone, but the Durgunoglu and Mitchell method accounts for the effect of in-situ horizontal stress and cone roughness, as shown in Figure 2. The cavity expansion method, developed by Vesic [10], accounts for soil compressibility and volume change characteristics. Research by Michell and Keaveny [11] has shown that the cavity expansion method appeared to model

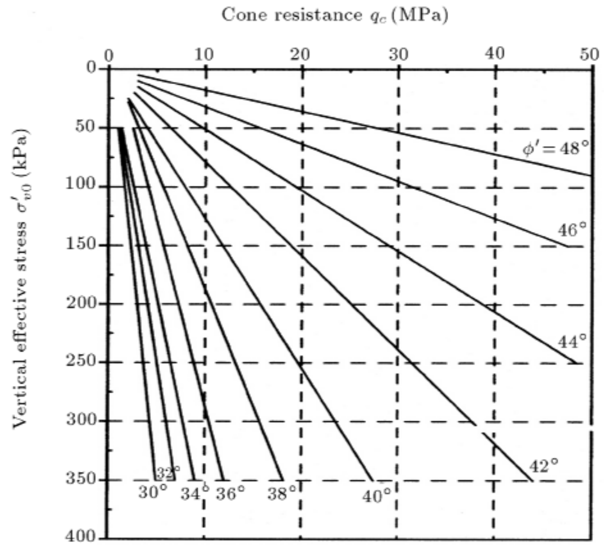



**Figure 1.** Correlation between peak friction angle $\phi$ and $q_c$ for uncemented quartz sands [7].

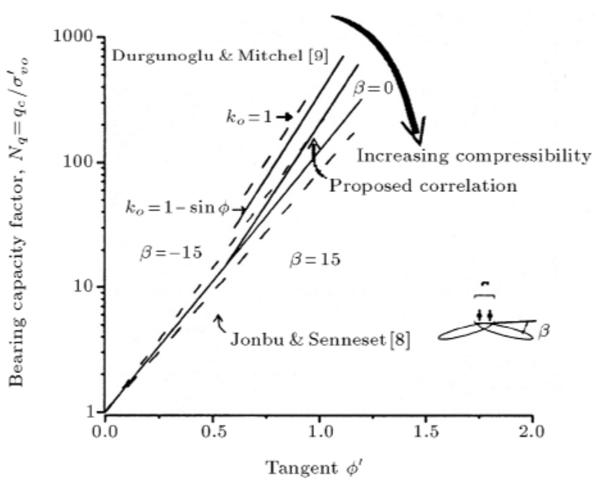



**Figure 2.** Relationship between bearing capacity factor and friction angle from large calibration chamber tests [8,9].

the measured response extremely well and could predict the response in compressible sands. However, the cavity expansion approach requires a knowledge of soil stiffness and volumetric strains in the plastic region, which are both difficult to estimate or derive.

Also, for interpretation of the undrained shear strength of clays from CPT or CPTu results, a large amount of work has been reported, based on theoretical solutions or empirical correlations. The relationship between cone resistance, $q_c$, and $S_u$ from theory is, as follows:

$$q_c = N_{Co} S_u + \sigma_0, \tag{4}$$

where, $N_{Co}$ is cone factor and $\sigma_0$ is the in-situ total stress. Depending on which theory is used, $\sigma_0$ may be replaced by $\sigma_{v_0}$, $\sigma_{h_0}$ or $\sigma_{\text{mean}}$.

Since cone penetration performance is a complex phenomenon, the theoretical solutions make several simplifying assumptions regarding soil behavior, boundary conditions and failure mechanisms. The output of theoretical solutions requires to be verified from actual field or laboratory test data. Therefore, empirical correlations are preferred. The $S_u$ from empirical approaches can be estimated from $q_c$, $f_s$ or $u$, as follows [12]:

$$S_u = \frac{q_c \quad \sigma_{v0}}{N_k}, \tag{5}$$

$$S_u = \frac{\Delta_u}{N_u}, \tag{6}$$

$$S_u = N_s f_s, \tag{7}$$

where:

| | |
|---|---|
| $N_k$ | is cone factor ranging from 10-15 for N.C. clays and 15-20 for O.C. clays, |
| $\Delta u = u \quad u_o$ | $u$ is mobilized penetration pore water pressure and $u_o$ is hydrostatic pressure, |
| $N_u$ | a factor ranging from 2-20 depending on soil parameters, |
| $f_s$ | CPT sleeve friction, |
| $N_s$ | a coefficient ranging from 0.8-1. |

According to Lunne et al. [13], the use of the piezocone to correct the cone resistance for pore pressure effects and employing the reference values of undrained shear strength have helped in improving the quality of correlations in general. Moreover, use of site-specific empirical correlations still seems to be the well defined procedure for the interpretation of $S_u$ from CPT or CPTu data.

Above all, soil parameters estimated from the CPT data include errors due to disregarding horizontal stress, assumption of the plane strain failure mechanism for the axisymmetric nature of deep cone penetration, soil compressibility, rate of penetration and strain softening.

## DIRECT CPT METHODS FOR BEARING CAPACITY OF FOOTINGS

Direct CPT methods, which are initiated theoretically or empirically, relate the ultimate bearing capacity of soils, $q_{ult}$, to the cone point resistance, $q_c$, with some modification factors. Among different methods, the following are commonly used by geotechnical engineers.

Schmertmann [6] proposed bearing capacity factors based on Terzaghi's basic formula for non-cohesive soils from CPT data, as follows:

$$q_{ult} = \overline{q} N_q + 0.5 \gamma B N_\gamma, \tag{8}$$

$$N_q = N_\gamma = 1.25 \overline{q}_c, \tag{9}$$

$$\overline{q_c} = \sqrt{q_{c1} \times q_{c2}}, \tag{10}$$

where:

$q_{c1}$ arithmetic average of $q_c$ values in an interval between the footing base and 0.5 $B$ beneath the footing base,

$q_{c2}$ arithmetic average of $q_c$ values in an interval between 0.5 $B$ to 1.5 $B$ beneath the footing base.

Meyerhof [14] suggested a direct method for estimating $q_{ult}$ from cone resistance in sands and clays, as follows:

$$q_{ult} = \overline{q_c} \left( \frac{B}{12.2} \right) \left( 1 + \frac{D_f}{B} \right), \tag{11}$$

where, $\overline{q}_c$ is the arithmetic average of $q_c$ values in a zone, including the footing base and $1.5B$ beneath the footing. A safety factor of at least 3 is recommended by Meyerhof to obtain the allowable bearing pressure.

Owkati [15] proposed separated equations for the ultimate bearing capacity of sands, as follows:

$$q_{ult} = 28 \quad 0.0052\,(300 \quad \overline{q_c})^{1.5}, \quad \text{for strip footings}, \tag{12}$$

$$q_{ult} = 48 \quad 0.009\,(300 \quad \overline{q_c})^{1.5}, \quad \text{for square footings}, \tag{13}$$

where, $\overline{q}_c$ is the arithmetic average of $q_c$ values in an interval between the footing base and $1.5B$ beneath, in terms of kg/cm$^2$.

CFEM [5], the Canadian Foundation Engineering Manual, recommended an equation for the evaluation of allowable bearing capacity using:

$$q_a = 0.10\, \overline{q}_c. \tag{14}$$

Also, based on CFEM, the safety factor of 3 has been suggested. Hence, the ultimate bearing capacity is:

$$q_{ult} = 0.30\, \overline{q}_c. \tag{15}$$

Eslaamizaad and Robertson [16], based on the Meyerhof method (Equation 11), by using some case histories and CPT soundings close to foundation locations in cohesionless soils, proposed a relationship between $q_{ult}$ and $q_c$, as follow:

$$q_{ult} = k \overline{q}_c, \tag{16}$$

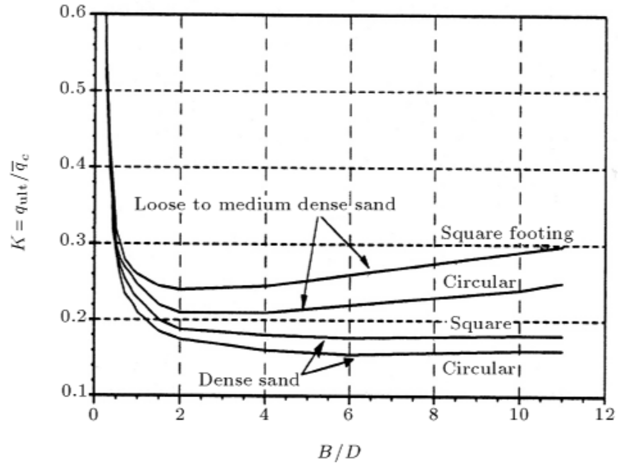



**Figure 3.** Correlation between ratio of bearing capacity to average cone resistance with width factor [16].

where, $k$ is a correlation factor and is a function of $B/D_f$, the shape of the footing and sand density, as shown in Figure 3.

Tand et al. [17] employed a few full scale load tests with CPT data and suggested the ultimate bearing capacity of shallow footings on lightly cemented medium dense sand by the following equation:

$$q_{ult} = R_k q_c + \sigma_{v0}, \tag{17}$$

where, $R_k$ values range from 0.14 to 0.2, depending on the footing shape and depth, and $\sigma_{v_0}$ is the initial vertical stress at the footing base.

## NEW DIRECT CPT METHOD

Attempts to predict pile toe capacity from CPT data were generally successful, whether the unit toe resistance was taken directly from the $q_c$ value at pile toe level or taken as the average of $q_c$ values for some distance above and below the pile toe. However, there is still some skepticism regarding application of small scale cone penetrometers to large scale deep foundations, which is referenced as the scale effect. Considerable insight into this problem was provided by DeBeer [18] in a thorough study of the effect of pile or penetrometer size on the ultimate toe bearing capacity. By considering penetrometers of different size, DeBeer showed that all penetrometers should ultimately attain the same point of resistance and the depth at which this constant value occurs must be proportional to the penetrometer size.

The explanation offered by DeBeer for this phenomenon was that the bearing capacity failure which occurs beneath the cone tip gradually changes from that of a shallow foundation to that of a deep one, as shown in Figure 4 [19].

During the transition between the shallow and deep type base failure, the point resistance increases

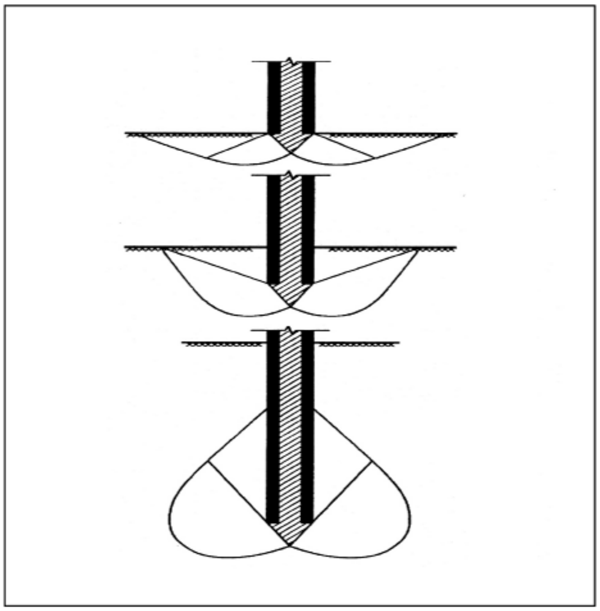


**Figure 4.** Shear failure transformation from shallow to deep mechanism [19].

linearly with depth, but, after the deep foundation condition is reached, point resistance only increases slightly with increased penetration. The depth at which this condition is met is termed the critical depth ($D_c$). The correct value of $D_c$ is more difficult to determine. Meyerhof [14], DeBeer [18] and Eslami and Fellenius [20] have shown that the $D_c$ is a function of the foundation size and soil density. Thus, it is more convenient to express this depth in terms of the ratio of the foundation size to the critical depth, i.e., $(D/B)_c$. Both experimental and theoretical studies have denoted that $(D/B)_c$ for sands is a function of the sand density and varies from about 5 for loose sands to about 15 for dense sands.

Research by DeBeer [18], Meyerhof [14] and Eslami and Fellenius [20] indicated that a type of logarithmic spiral failure zone and attributed rupture surface to reach the penetrometer body needs a penetration depth of almost 10 times that of the penetrometer diameter to fully mobilize the ultimate unit toe resistance. In other words, a penetration depth of at least 10 footing width or cone diameter is required to transform a shallow to a deep mechanism of a rupture surface. In the former, the rupture surface extends to the ground level but, for the latter, the rupture surface reaches the penetrometer or footing body at a deep soil level.

A direct CPT method by Eslami and Gholami [21] has been developed, based on an analytical model for determining the ultimate bearing capacity, $q_{ult}$, of the shallow foundations from the CPT cone resistance, $q_c$. Regarding the basic bearing capacity formula, the length ($L$) and the confining and depth of the rupture surface, i.e. embedment depth ($A$), play a major role in mobilized foundation bearing capacity. These two key factors can be regarded in a rational manner for connection of the shallow rupture surface to the deep, entirely mobilized, failure zone. Therefore, the $q_{ult}$ to $q_c$ in the direct approach can be correlated by a general rule, as follows:

$$q_{ult} = \overline{\alpha} \times \overline{q}_{c,g} = (\alpha_1 + \alpha_2)/2 \times \overline{q}_{c,g}, \tag{18}$$

$$\overline{\alpha} = (\alpha_1 + \alpha_2)/2, \tag{19}$$

$$\alpha_1 = \frac{L_\theta}{L_o}, \tag{20}$$

$$\alpha_2 = \frac{A_\theta}{A_o}, \tag{21}$$

where:

| | |
|---|---|
| $\overline{q}_{c,g}$ | geometric average of $q_c$ values from footing base to $2B$ beneath footing depth, |
| $\alpha_1$ | modification transforming length ratio, |
| $\alpha_2$ | modification transforming area embedment (depth) ratio, |
| $\overline{\alpha}$ | average of length and area modification factor, |
| $L_o, L_\theta, A_o, A_\theta$ | have been shown in Figures 5 and 6. |

In the following, the details of determining the forenamed parameters and ratios will be discussed.

In practice, natural soil deposits produce a cone point resistance profile with high and low values. The cone resistance variations reflect the alteration of soil strength and stiffness. Therefore, when determining foundation bearing capacity, which is a function of the subsoil conditions in a zone above and below the foundation base, an average must be determined that is representative of the influence zone. The extent of

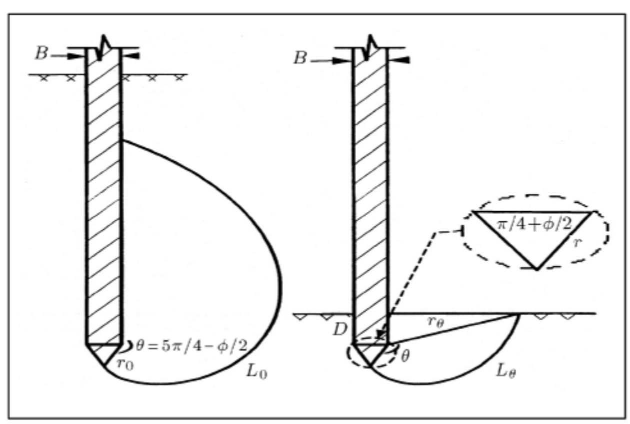



**Figure 5.** Comparison of rupture surface length for shallow and deep conditions.

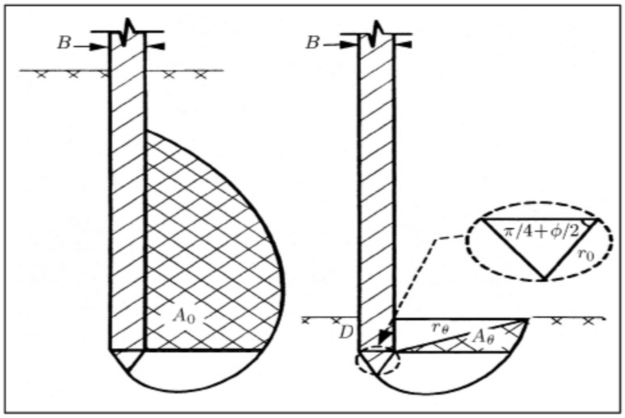



**Figure 6.** Comparison of embedment depth (surcharge) for shallow to deep conditions.

the influence zone, regarding scale effect, should be a function of the foundation width, $B$. Based on the modified bearing capacity equations by Meyerhof [14], Hansen [3] and Vesic [4], the extent of the rupture surface beneath the foundation base is about $B/2$ tan $(45 + \varphi/2)$, which, approximately, recommends $1.5B$ to $2B$ in geotechnical practice. Consequently, the CPT $q_c$-values should be averaged along the influence zone extending from $2B$ below the foundation base to ground level.

According to arguments by Eslami and Fellenius [20], the filtering and smoothing of peaks and troughs for $q_c$-values can be achieved by calculating the geometrical average, $q_{c.g}$ (geomean), within the influence zone. By the geomean of the $q_c$ data, a filtered representative value is obtained which is unaffected by bias and, therefore, repeatable. The geomean of $q_c$ values must be calculated in a zone in the vicinity of the foundation base.

According to Figure 5 and with an assumption of log spiral general shear failure, the radius of the rupture surface, $r$, by realizing geometric and trigonometric relationships, can be calculated as:

$$r = r_0 e^{\theta \mathrm{tg}\varphi}, \tag{22}$$

$$r = \frac{B}{2}\mathrm{tg}(\pi/4 + \varphi/2) \times \frac{1}{\cos(\pi/4 - \varphi/2)} e^{\theta \mathrm{tg}\varphi}, \tag{23}$$

where, $r$ is the radius of the logarithmic spiral; $r_0$ is the radius of the log-spiral for $\theta = 0$, $\theta$ is the angle between a radius and $r_0$, as shown in Figure 5, and $\varphi$ is the angle between radius and normal at any point on the spiral assumed equal to the friction angle of the soil, which is pointed out by Meyerhof [14].

Figure 5 presents the pattern of rupture surfaces regarded in the proposed method. The depth of embedment is derived as:

$$D = y = r_0 e^{\theta \mathrm{tg}\varphi} \times \sin\left(\theta - \frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{\varphi}{2}\right)\right). \tag{24}$$

By integrating the curve length and using substitution factor "$m$" as follows:

$$m = \frac{B}{2}\mathrm{tg}(\pi/4 + \varphi/2) \times \frac{1}{\cos(\pi/4 - \varphi/2)}, \tag{25}$$

the relative depth and the ratio of foundation depth to foundation width can be obtained by the following equation:

$$\frac{D_f}{B} = \frac{\mathrm{tg}(\pi/4 + \varphi/2) e^{\theta \mathrm{tg}\varphi}}{2\cos(\pi/4 - \varphi/2)} \times \sin(\theta - 3\pi/4 + \varphi/2). \tag{26}$$

Therefore, the length of the rupture surface becomes:

$$L_\theta = \frac{m\sqrt{1 + \mathrm{tg}^2\varphi}}{\mathrm{tg}\varphi} \times (e^{\theta \mathrm{tg}\varphi} - 1). \tag{27}$$

As a result, the ratio of the rupture surface length, converting from shallow to deep conditions, which was defined as $\alpha_1$, is:

$$\alpha_1 = \frac{L_o}{L_{\theta=(\pi+\pi/4-\phi/2)}} = \frac{e^{\theta \mathrm{tg}\phi} - 1}{e^{(5\pi/4 - \phi/2)} - 1}. \tag{28}$$

In addition to the length of the rupture surface effect, the influence of surcharge around the penetrometer (depth) can be considered as area ($A$) of the surcharge in unit length and calculated as follows:

$$A_\theta = m^2/4\mathrm{tg}\varphi(e^{2\theta \mathrm{tg}\varphi} - 1). \tag{29}$$

Similarly, the ratio of the rupture surface area, converting from a shallow to deep condition of rupture, which was defined as $\alpha_2$, is:

$$\alpha_2 = \frac{A_o}{A_\theta = (\pi + \pi/4 - \varphi/2)} = \frac{e^{2\theta \mathrm{tg}\varphi} - e^{2(3\pi/4 - \varphi/2)}}{e^{2(5\pi/4 - \varphi/2)} - e^{2(3\pi/4 - \varphi/2)}}. \tag{30}$$

Figures 7a and 7b illustrate values of $\alpha_1$ and $\alpha_2$ versus $\varphi$ and $D/B$ parameters.

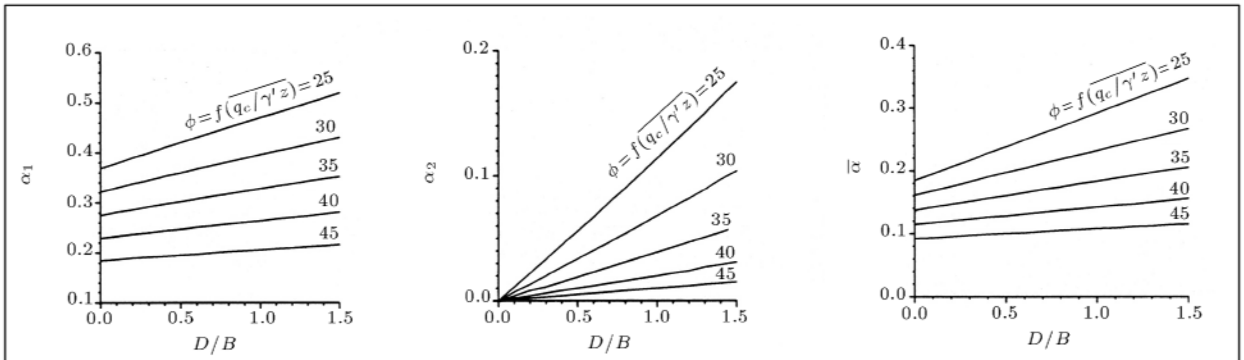
Based on Equation 19, the variation of $\overline{\alpha}$, as an average of $\alpha_1$ and $\alpha_2$, is illustrated in Figure 7c as a function of equivalent $\varphi$ and $D/B$ values, in which the equivalent $\varphi$ is dependent on effective stress and measured cone resistance, $q_c$, at depth.

Figure 8 illustrates a simplified correlation for estimating the equivalent $\varphi$-angle from $q_c$ and the effective stress level, which has been calculated from the following equation, based on the bearing capacity theory on a deep seated failure zone.

$$\varphi = \frac{\log\left(\frac{\overline{q}_c}{\gamma' z}\right) + 0.5095}{0.0915}. \tag{31}$$

This is obtained from Equation 1 by $q_c$ replaced instead of $q_{ult}$ and by using standard cone penetrometer geometry.

Finally, a summary of the proposed method, in a step by step procedure, is described as follows:




(a) Modification factor for length effect

(b) Modification factor for depth (area) effect

(c) Combined length and area modification factor

**Figure 7.** Bearing capacity correlation factor for relating $q_{ult}$ to $q_{c.g}$.

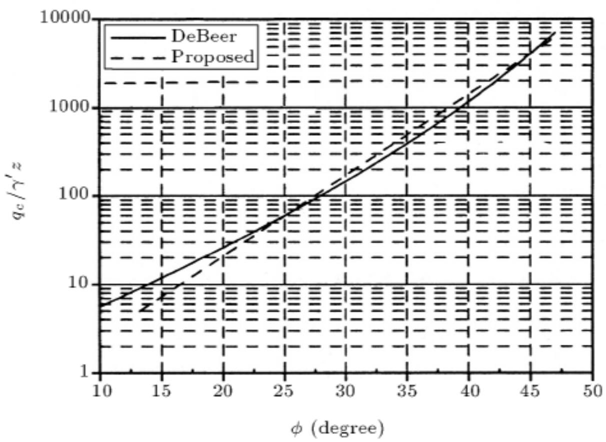



**Figure 8.** Correlation between friction angle and ratio of $q_c$ to effective stress.

1. The zone located between foundation base to $2B$ beneath can be divided in sublayers. The values of $\overline{q}_{c.g}$ and $(\overline{q/\gamma' z})_{c.g}$ in this interval are calculated;
2. The average $\varphi$ angle is obtained by using $(\overline{q/\gamma' z})_{c.g}$ from Equation 31 or Figure 8;
3. Based on $D/B$ and $\varphi$ values, from Figure 7, $\overline{\alpha}$ can be obtained;
4. Finally, the ultimate bearing capacity is calculated as: $q_{ult} = \overline{\alpha} \times \overline{q}_{c.g}$.

## EVALUATION OF DIRECT CPT METHODS

A database has been compiled from six sites, including 28 full scale footings and/or plate load tests accompanying CPT soundings performed close to foundation locations. The brief site specifications of cases are summarized as follows:

Site No. I. It is located in Texas, USA and reported by Briaud and Gibbens [22]. Five square spread footings, with footing width $B$ ranging from 1 to 3m and footing embedment $D_f$ from 0.7 to 0.9 m, were constructed on uniform sand (SP). The measured $q_{ult}$ was about 15 kg/cm$^2$ and the $q_c$ values vary from 40 to 110 kg/cm$^2$.

Site No. II. Amar [23] reported that four square 1m surface footings were loaded on silty (ML) soil. The measured $q_{ult}$ ranges from 3 to 3.75 kg/cm$^2$ and $q_c$ values range from 17 to 28 kg/cm$^2$.

Site No. III. It is located in Texas, USA (Tand et al.) [17]. Four circular footings with diameters of 1.75 to 2 m were located at depths of 2.16 m to 2.35 m. The subsoil type is silty sand (SM&SP). The $q_{ult}$ values vary from 9.4 to 13.5 kg/cm$^2$ and the $q_c$ values range from 10-180 kg/cm$^2$.

Site No. IV. Three load tests were performed on 0.6 m width surface plates in Tabriz, Northwest Iran [24]. The site is formed of silty and clayey sands, in which the measured $q_{ult}$ values range from 12.6 to 12.8 kg/cm$^2$ and the $q_c$ values vary from 30 to 100 kg/cm$^2$.

Site No. V. Consoli et al. [25] reported that five plate load tests had been done on 0.3 to 0.6 m diameter plates. The subsoil of the footing is a mixture of clay and sand. The $q_{ult}$ obtained from PLT was about 1.7 kg/cm$^2$ and the $q_c$ values were in the range of 5 to 20 kg/cm$^2$.

Site No. VI. According to Tand et al. [26], seven 0.6 m plate load tests were performed on silty clay and stiff clay deposits at a 1.5 m depth. The measured bearings were in the range of 3.1 to 7 kg/cm$^2$ and the $q_c$ values varied from 10 to 60 kg/cm$^2$.

**Table 1.** Case records summary.

| Site No. | Reference | Location | Soil Type | Footing Shape | Footing Size (m) $B$ & $D_f$ | | Relative Settlement (%) | $q_u$ (kg/cm$^2$) |
|---|---|---|---|---|---|---|---|---|
| I | Briaud & Gibbens [22] | USA | Sand, silty sand | Square | 1 | 0.71 | 10 | 15 |
| I | Briaud & Gibbens [22] | USA | Sand, silty sand | Square | 1.5 | 0.76 | 10 | 15 |
| I | Briaud & Gibbens [22] | USA | Sand, silty sand | Square | 2.5 | 0.76 | 10 | 15 |
| I | Briaud & Gibbens [22] | USA | Sand, silty sand | Square | 3 | 0.76 | 10 | 15 |
| I | Briaud & Gibbens [22] | USA | Sand, silty sand | Square | 3 | 0.89 | 10 | 13 |
| II | Amar [23] | France | Silt | Square | 1 | 0 | 6 | 3.75 |
| II | Amar [23] | France | Silt | Square | 1 | 0 | 6 | 3.7 |
| II | Amar [23] | France | Silt | Square | 1 | 0 | 6 | 3 |
| II | Amar [23] | France | Silt | Square | 1 | 0 | 6 | 3.25 |
| III | Tand et al. [17] | USA | Sand, silty sand | Circular | 1.75 | 2.35 | 10 | 11.5 |
| III | Tand et al. [17] | USA | Sand, silty sand | Circular | 1.75 | 2.35 | 10 | 11.5 |
| III | Tand et al. [17] | USA | Sand, silty sand | Circular | 2 | 2.2 | 10 | 7.5 |
| III | Tand et al. [17] | USA | Sand, silty sand | Circular | 1.88 | 2.16 | 10 | 9.5 |
| IV | Saniee [24] | Iran | Silty sand | Square | 0.6 | 0 | 8 | 12.6 |
| IV | Saniee [24] | Iran | Silty sand | Square | 0.6 | 0 | 8 | 12.6 |
| IV | Saniee [24] | Iran | Silty sand | Square | 0.6 | 0 | 8 | 12.8 |
| V | Consoli et al. [25] | Brazil | Silty clay | Circular | 0.3 | 0 | 1 | 1.7 |
| V | Consoli et al. [25] | Brazil | Silty clay | Circular | 0.3 | 0 | 1 | 1.7 |
| V | Consoli et al. [25] | Brazil | Silty clay | Circular | 0.45 | 0 | 1 | 1.7 |
| V | Consoli et al. [25] | Brazil | Silty clay | Circular | 0.6 | 0 | 1 | 1.7 |
| V | Consoli et al. [25] | Brazil | Silty clay | Circular | 0.6 | 0 | 1 | 1.7 |
| VI | Tand et al. [26] | USA | Stiff clay | Circular | 0.6 | 1.5 | 12 | 6 |
| VI | Tand et al. [26] | USA | Stiff clay | Circular | 0.6 | 1.5 | 12 | 6 |
| VI | Tand et al. [26] | USA | Stiff clay | Circular | 0.6 | 1.5 | 12 | 6 |
| VI | Tand et al. [26] | USA | Silty clay | Circular | 0.6 | 1.5 | 11 | 5.2 |
| VI | Tand et al. [26] | USA | Silty clay | Circular | 0.6 | 1.5 | 15 | 3.1 |
| VI | Tand et al. [26] | USA | Silty clay | Circular | 0.6 | 1.5 | 15 | 3.1 |
| VI | Tand et al. [26] | USA | Stiff clay | Circular | 0.6 | 1.5 | 10 | 6.9 |

Table 1 presents the case histories general specifications. A typical load-settlement diagram and $q_c$ profile is shown in Figure 9.

For ultimate bearing capacity, $q_{ult}$, when the trend of failure has not been achieved in the testing process, the bearing capacity corresponding to 10% of the relative settlement was chosen as recommended by current methods. Utilizing 28 footing case histories, the predicted bearing capacity was compared to the measured one from test results. For validation of the predictive method outputs, the numerical and graphical approaches were used to evaluate the accuracy of the results.

The numerical approach employs a cumulative probability procedure to quantify the comparison of the estimated and measured footing capacity for all of the case records in a linear plot. A comparison of the methods will provide more insight when the values are plotted versus a cumulative average called "cumulative probability". For the current set of data, the ratio of calculated to measured footing capacity, $q_{ult,\text{Cal.}}/q_{ult,\text{Mes.}}$ is arranged in ascending order (numbered, $1, 2, \cdots, i, \cdots, n$) and a cumulative probability, $P$, is determined for each capacity value as:

$$P = \frac{i}{n+1}, \tag{32}$$

where:

$q_{ult,\text{Cal.}}$ calculated ultimate bearing capacity,
$q_{ult,\text{Mes.}}$ measured bearing capacity,

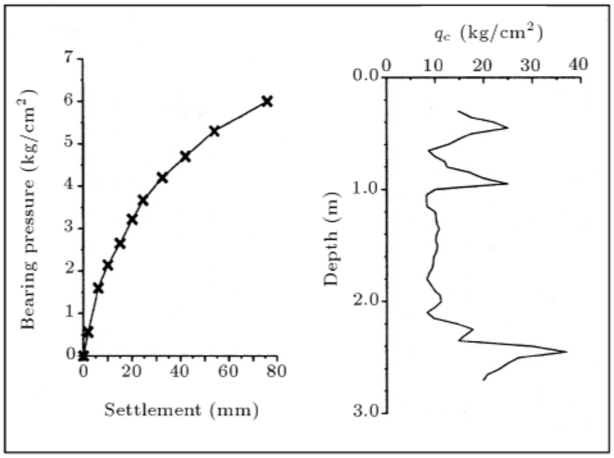



**Figure 9.** Typical full scale footing load test results and $q_c$ profile for Site No. VI [26].

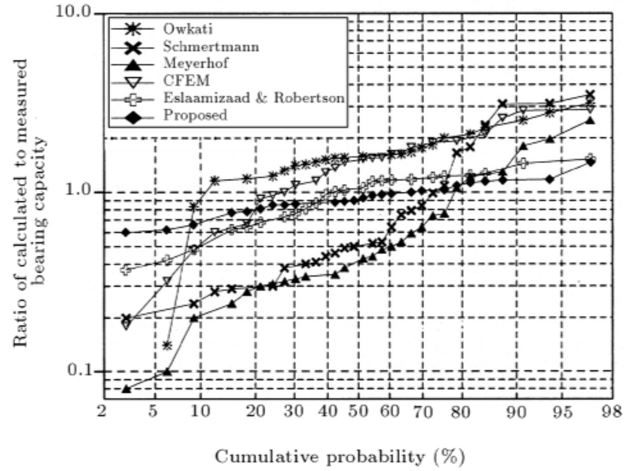



**Figure 10.** Comparison of predictive methods by cumulative probability approach.

$P$ cumulative probability,
$n$ number of total cases,
$i$ case number.

To assess the bias and dispersion associated with a particular predictive method, the following is useful [27]:

- The value of $q_{ult,\mathrm{Cal.}}/q_{ult,\mathrm{Mes.}}$ at $P = 50\%$ probability is a measure of the tendency to overestimate or underestimate the bearing capacity. The closer to a ratio of unity, the better the agreement,
- Log-normal distributed data will plot on a straight line,
- The slope of the line through the data points is a measure of the dispersion or "standard deviation" The flatter the line, the better the general agreement.

The results of comparison indicate that the $q_{ult,\mathrm{Cal.}}/q_{ult,\mathrm{Mes.}}$ at a probability of 50% is closer to unity for the proposed method than the others, as shown in Figure 10. It means that the new method predicts the foundation capacity with less overestimation or underestimation than the other methods. Besides, the slope of the line through the points for the current method exhibits higher dispersion than that of the proposed method. Therefore, the results are closer to a log-normal distribution for the proposed method.

As show in Figure 11, the graphical approach presents the plot of the estimated against the measured footing capacity. For reference, a solid diagonal line is presented in each diagram indicating a perfect agreement between calculated and measured capacity. A deviation of $\pm 20\%$ is illustrated by the dashed lines. As illustrated in Figure 11, the Meyerhof, Schmertmann and Eslamizad-Robertson methods tend to underestimate, while the Owkati and CFEM methods overestimate the bearing capacity of the compiled cases. For the Eslamizad and Robertson method, nine cases are excluded from comparison because this method was calibrated based on these cases. The results of the comparison indicate better accuracy and less scatter for the proposed method than other current methods. The Tand et al. method is excluded in the comparison of methods, because this method was developed based on an extrapolation of the load-settlement curve and the criteria of 5% relative settlement, as opposed to 10% criteria for other methods.

## DISCUSSIONS

Among major aspects for analysis and design of foundations, the bearing capacity and settlement aspects are interactive and commonly realized by geotechnical engineers. In spite of the modifications for a bearing capacity basic formula, developed by Terzaghi, there are still some delusions, due to application and output. The bearing capacity 3N formula involves a rather approximate $\varphi$ $N$ relationship coupled with the difficulty of determining a reliable and representative in-situ value of the $\varphi$-angle. To overcome some problems, other design and analysis approaches must be realized. In-situ soil testing techniques can be considered as a solution which provide continuous records of strength and stiffness with depth and measure the basic soil parameters in real situations, where the load directly transfers to the subgrade soil and subsequent deformation occurs.

Full-scale tests on footings, supported by theoretical analysis based on soil response to a stress increase, show that bearing capacity failure rarely occurs for footings. Even for settlements much in excess of 10% of the footing diameter or width, the soil response is not that of the footing approaching or reaching an

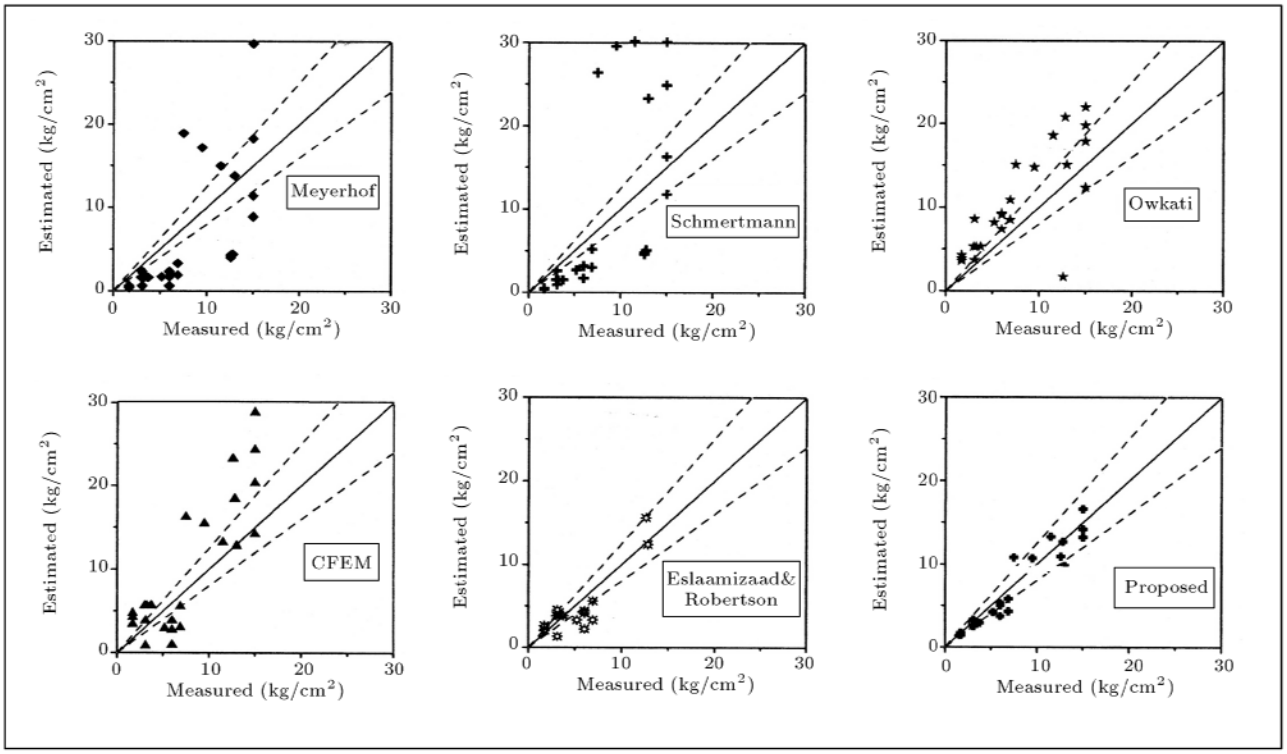



**Figure 11.** Graphical comparison of bearing capacity prediction by six direct CPT methods.

ultimate failure mode, but of soil stiffness [28]. For this reason, the conventional bearing capacity formula does not correctly represent foundation behavior.

Moreover, to apply an ultimate resistance approach (whether in a global factor of safety or to limit state conventions) is fundamentally not true. Instead, the approach should calculate the settlement of the footing considering the serviceability of the foundation unit. Therefore, geotechnical practice should place the emphasis on determining the foundation capacity and settlements based on ex-situ and in-situ records to satisfy foundation design and safety requirements. In this way, the CPT or CPTu data, due to providing a variety of continuous soil strength and stiffness records, can de realized as a sophisticated tool to overcome conventional ex-situ testings problems and static analysis limitations.

## CONCLUSIONS

For determining the bearing capacity of shallow foundations from CPT data, six direct methods and a new one have been presented and compared. The main advantages of utilizing the in-situ methods for determining the bearing capacity of shallow foundations are those which overcome the problems of providing representative undisturbed samples and so determining the correct values of required soil parameters by conventional laboratory testing will be eliminated. Moreover, there is no need for estimating intermediate $N_c$, $N_q$ and $N_\gamma$ coefficients. The new method is based on an analytical model to relate the deep failure surface of cone points to the shallow rupture surface around footings.

The ultimate bearing capacity of foundations, $q_{ult}$, is equated to cone point resistance, $q_c$, by a correlation factor in a direct approach. The correlation factor is a function of equivalent $\varphi$-angle and footing relative depth, in which the $\varphi$-angle can be obtained from the geometric average of the effective stress level and measured $q_c$ values at each depth interval from CPT or CPTu.

A database has been compiled, including 28 footings, with the full scale loading test results of $q_{ult}$ and CPT soundings which performed close to foundation locations. Validation of the methods for prediction of the ultimate bearing capacity of foundations, by graphical and probability approaches, shows less scatter and more accuracy for the new method than for the current methods.

Among the current methods, the Meyerhof, Shmertmann and Eslamizad-Robertson methods underestimate the bearing capacity of cases, while the Owkati and CFEM methods involve overestimation. Because of the promising results, simplicity and routine procedure, the proposed method can be considered in geotechnical practice for the safe and cost-effective design of shallow foundations.

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