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Research Note

Cost Evaluation of a Two-Echelon Inventory System with Lost Sales and Approximately Normal Demand

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The inventory system under consideration consists of one central warehouse and an arbitrary number of retailers controlled by a continuous review inventory policy (R, Q). Independent Poisson demands are assumed with constant transportation times for all retailers and a constant lead time for replenishing orders from an external supplier for the warehouse. Unsatisfied demands are assumed to be lost in the retailers and unsatisfied retailer orders are backordered in the warehouse. An approximate cost function is developed to find optimal reorder points for given batch sizes in all installations and the related accuracy is assessed through simulation.

INTRODUCTION

A two-echelon inventory system is considered consisting of one central warehouse and an arbitrary number of retailers with identical ordering batch sizes. The inventory control policy is assumed to be a continuous review (R, Q) policy in all installations, which means that when the inventory position reaches a predetermined value of R, an order of size Q is placed. The demand processes for a consumable (not repairable) item are assumed to be independent Poisson and unsatisfied demands to be lost in all retailers. The transportation time of each order placed by the retailers is assumed to be constant. A constant lead time is assumed for replenishing the warehouse orders from an external supplier and unsatisfied retailer orders to be backordered in the warehouse and all backordered orders are filled according to a FIFO-policy. The reorder point and batch size of the warehouse are assumed to be integer multiples of the retailers identical batch size.

One of the oldest papers in the field of continuous review multi-echelon inventory systems is a basic and famous one written by Sherbrooke [1] in 1968. He assumed $(S - 1, S)$ policies in a Depot-Base system for repairable items in the American air force and could approximate the average inventory and stockout level in the bases. The result of this paper has been used by many subsequent researchers because it uses an efficient approximation for the lead time of the bases (which is usually one of the complexities of multi-echelon systems). However, other papers, like [2], studied Sherbrooke's model by changing some of its critical assumptions and gained some more interesting results.

Continuous review models of multi-echelon inventory systems in the 1980's concentrated more on repairable items in a Depot-Base system than on consumable items. For example, Graves [3] worked on the determination of the stocking levels in such a system, Moinzadeh and Lee [4] considered the issue of determining the optimal order batch size and stocking levels at the stocking locations by using a power approximation and Lee and Moinzadeh [5] generalized previous models on multi-echelon repairable inventory systems to cover the cases of batch ordering and batch shipment. On consumable items, Deuermeyer and Schwarz [6] proposed a simple approximation for a complex multi-echelon system (one warehouse and multiple retailers) assuming the backordering of stockouts in all installations with a batch ordering policy. Svoronos and Zipkin [7] proposed several refinements by considering second-moment information (standard deviation as well as mean) in their approximations.

In the 1990's, Axsäter [8] provided a simple recursive procedure for determining the holding and stockout costs of a system, consisting of one central warehouse and multiple retailers with an $(S - 1, S)$

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policy, independent Poisson demands in the retailers, a backordered demand during stockouts in all installations and constant lead times. Axsäter [9] proposed exact and approximate methods for evaluating the previous system for the case of a general batch size in all installations but with identical retailers. For the case of non-identical retailers and a general batch size, Axsäter [10] proposed methods for the exact evaluation of two retailer cases and an approximate evaluation for the case of more than two retailers. Forsberg [11] presented a method for exactly evaluating the costs of a system with one warehouse and a number of different retailers using another approach. The common assumption of the above papers is that demands during stockout in the retailers are backordered. However, on some conditions, demands may be lost. Andersson and Melchiors [12] have proposed an approximate method for the case of lost sales when the inventory control policy is $(S \quad 1, S)$ in all installations (one warehouse and multiple retailers) and unsatisfied demands are lost in the retailers. They also introduced the cost evaluation of such a system in case of a general batch ordering policy as a future field of research. This is what is being considered in this paper.

The contents of this paper are now outlined. First, a detailed problem formulation is given and the review of two special single echelon problems that are referred to later are presented. Then, it will be explained how to overcome the two important complexities of the model. After that, the approximate total cost of the system is presented and discussed for finding reorder points. Finally, numerical results and some conclusions and further research opportunities are presented.

PROBLEM FORMULATION

It is assumed that Q (the identical batch size of all retailers) is determined through a deterministic model with a known replenishment cost at both warehouse and retailers, as many similar papers such as [6,7,9,10] have done before, to simplify the problem. The objective is to find the optimal reorder points by minimizing the total holding costs of the warehouse and retailers and the stockout costs of the retailers. Let the following notation be introduced:

- N number of retailers,
- λ_i demand rate at retailer $i, i = 1, 2, ..., N$,
- λ_o demand rate at the warehouse,
- L_i transportation time for deliveries from the warehouse to retailer $i, i = 1, 2, ..., N$,
- L_o lead time of the warehouse orders,
- Q identical batch size of all retailers,
- Q_o batch size of the warehouse,
- R_i reorder point of retailer i (integer value, since demand is one at a time), $i = 1, 2, ..., N$,
- R_o reorder point of the warehouse (an integer multiple of Q),
- h_i holding cost per unit time at retailer $i, i = 1, 2, ..., N$,
- h_o holding cost per unit time at the warehouse,
- π_i penalty cost per unit of lost sale at retailer $i, i = 1, 2, ..., N$,
- C_i cost per unit time of retailer i in steady state, $i = 1, 2, ..., N$,
- C_o cost per unit time of the warehouse in steady state,
- TC total cost of the inventory system per unit time in steady state.

REVIEW OF TWO SPECIAL CASES

Review of Exact Solution for Backordering Problem with Normal Demand

Considering a single-echelon inventory system with a continuous review control policy, a reorder point of R and batch size of Q, a constant lead time for replenishing orders, demand (per unit time) as a normal distribution with mean λ and standard deviation δ and backordered unsatisfied demand, Axsater [13] presents formulae for the average stock level ($D(Q, R)$) and the average stockout level ($B(Q, R)$). Assuming the linear unit costs of holding and stockout, the corresponding annual costs can be obtained. The results are briefly reviewed and the parameters are introduced since they will be used later in the authors approximation.

$$B(Q,R)=\frac{\delta'^2}{Q}\left[H\left(\frac{R \quad \mu'}{\delta'}\right) \quad H\left(\frac{R+Q \quad \mu'}{\delta'}\right)\right], \tag{1}$$

where:

$$H(x)=\int_x^\infty G(v)d(v)=\frac{1}{2}\left[(x^2+1)(1 \quad \phi(x)) \quad x\varphi(x)\right], \tag{2a}$$

and:

$$\phi(x)=\int_\infty^x \frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}du, \quad \varphi(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}, \tag{2b}$$

$$D(Q,R)=\frac{Q}{2}+R \quad \mu'+B(Q,R). \tag{3}$$

The definition of the parameters in the above formulae is as follows:

- Q ordering batch size of a continuous review policy,
- R reorder point of a continuous review policy,
- L the constant lead time of each order,
- μ' average of lead time demand, $\mu' = \lambda.L$,
- δ' standard deviation of lead time demand, $\delta'=\delta\sqrt{L}$.

Review of Exact Solution for Lost Sale Problem with Poisson Demand

Considering a single-echelon inventory system with a continuous review control policy, a reorder point of R and batch size of Q, a constant lead time for replenishing orders, demand generated by a Poisson process, a lost demand during a stockout and $R < Q$ (to make sure of not having more than one order outstanding at a time), Hadley and Whitin [14] developed formulae for the average stock level, (D), and for the average number of lost sales incurred per unit time, (E). Assuming the linear unit costs of holding and stockout, they obtained the corresponding annual cost. Here, their results are briefly reviewed and the parameters they have used in their formulae are introduced, since they will be used in the presented approximation.

$$E = \frac{\lambda}{Q + \lambda\hat{T}}.\lambda\hat{T}, \tag{4}$$

$$D = \frac{\lambda}{Q + \lambda\hat{T}}\left[\frac{Q(Q+1)}{2\lambda} + \frac{QR}{\lambda} \quad QL\right] + \frac{Q}{\lambda}E, \tag{5}$$

where:

$$\hat{T} = L\,P(R;\lambda L) \quad \frac{R}{\lambda}P(R+1;\lambda L), \tag{6}$$

and:

$$T = \frac{Q + \lambda\hat{T}}{\lambda}. \tag{7}$$

The definition of the parameters in the above formulae is as follows:

- Q ordering batch size of a continuous review policy,
- R reorder point of a continuous review policy,
- λ demand rate (mean of Poisson demand distribution),
- L constant lead time,
- $\hat{T}$ the expected length of time per cycle that the system is out of stock,
- T time per cycle.

and:

$$P(x;\lambda L) = \sum_{i=x}^{\infty} e^{\ \lambda L}\frac{(\lambda L)^i}{i!} \quad x = 0, 1, 2, 3, ...$$

COMPLEXITIES OF THE PROBLEM

In many multi-echelon systems that which makes the problem difficult is how to exactly, or approximately, determine the type of demand in higher echelons and, also, the real replenishment time of orders, from the downstream echelons to higher ones, because of possible stockouts in the higher ones. In the authors problem, the same problem occurs and some approximations are used to tackle them. The approximations seem to be efficient and reasonable, but they will be tested through some numerical problems in the next section. Here, how to analyze the demand in the warehouse and, also, the lead time of the retailers are explained.

Demand Analysis in the Warehouse

The average number of cycles per unit time in a continuous review inventory system when demand is lost during a stockout is $T^{\ 1} = \frac{\lambda}{Q+\lambda.\hat{T}}$ [14], without any special assumption concerning the nature of the stochastic processes generating demands and lead times except to assume that they do not change with time and that units are demanded one at a time. Equation 7 is just a special case of this relation when the stochastic process generating demand is Poisson and the lead time is constant. Since a batch size of Q is ordered in each cycle, the mean rate of demand (from this inventory system to a higher echelon) will be $T^{\ 1}$ in terms of the batch size of Q.

As Moinzadeh and Lee [4] mention, when the stockout is backordered in the retailers and the demand process at each retailer is Poisson, the arrival process of orders at the warehouse (higher echelon) is a superposition of N arrival processes in the case of one warehouse and N retailers, so that each inter-arrival time is Erlang distributed with shape parameter Q. When the number of retailers in the model is large, the arrival process can be well approximated by a Poisson process with mean rate of $\sum_{i=1}^{N}\frac{\lambda_i}{Q}$. Moinzadeh and Lee also stress that such an approximation has been used or suggested by Muckstadt [15], Deuermeyer and Schwarz [6], Albin [16] and Zipkin [17]. However, this classic Poisson approximation has also been used in some recent papers, like [18]. The main difference between the authors model and theirs is that demand during a stockout is lost instead of backordering in the retailers.

As Axsäter [13] pointed out, for items with high demand, it is usually more convenient to model the demand over a time period by a continuous distribution and the discrete Poisson demand will become approximately normally distributed.

Using the spirit of the two mentioned approximations and extending it for the case of lost sales, one can assume that when the number of retailers in the model is large, the arrival process can be well approximated by a continuous normal demand process with a mean rate of $\lambda_o = \sum_{i=1}^{N}\frac{\lambda_i}{Q+\lambda_i\hat{T}_i}$ per unit and the standard deviation of such a normal demand process can be approximated by $\sqrt{\lambda_o}$ (both in terms of the identical batch size of

Q, since the retailers' ordering batch size was identical and equal to Q).

Approximating the Retailers Lead Time

As mentioned before, retailers at the first echelon of the model experience independent Poisson demand processes. Demand during a stockout is assumed to be lost. Each order that is placed on the warehouse by each retailer will have a minimum lead time equal to the transportation time. Since some of the orders are placed when there is a stockout at the warehouse, the lead time may be more than just the transportation time. The real lead time of each retailer order consists of two components: First, the transportation time of the orders from the warehouse into the retailer, and second, an additional waiting time, which results from a stockout in the warehouse. This waiting time does not have any clear distribution and it is just known that it is zero when the orders do not incur stockouts in the warehouse and has a positive value when they are backordered in the warehouse.

Based on the approximation of demand at the warehouse described in the previous section, the stock in the warehouse behaves just like an inventory system of the type described before. From Little's famous formula in the queuing theory (as Andersson and Melchiors [12] use it in their approximation), one can use the expression for the average stock level given by Equation 1 to obtain the average waiting time of each retailer order, as given by Equation 8.

It should be noticed that Equation 1 is valid when customer demands occur one at a time. Since each retailer orders a batch size, Q, Equation 1 can still be used if one makes the additional assumption that the batch size and reorder point of the warehouse, Q_o and R_o, are integer multiples of the identical batch size of the retailers, Q.

$$\overline{W} = \frac{B_o(\frac{Q_o}{Q}, \frac{R_o}{Q})}{\lambda_o}, \tag{8}$$

where:

$$\lambda_o = \sum_{i=1}^{N} \frac{\lambda_i}{Q + \lambda_i \hat{T}_i}. \tag{9}$$

In the above formula, $\overline{W}$ is the average waiting time of the orders placed by retailers. Based on an approximation, $\overline{W}$ is added to the transportation time of each retailer to make the approximate constant lead time of the orders. This can be used for evaluating the retailers costs (holding and stockout costs). λ_o is the mean rate of demand in the warehouse. Equation 9 follows directly from the result in the previous section.

APPROXIMATE TOTAL COST AND OPTIMIZATION METHOD

Total Cost Function of the System

Based on the results of the previous sections, the total cost of holding and shortage in the retailers and holding in the warehouse is as follows:

$$TC = C_o + \sum_{i=1}^{N} C_i. \tag{10}$$

The warehouse cost consists of just the holding cost, as follows:

$$C_o = h_o.D_o(\frac{Q_o}{Q}, \frac{R_o}{Q}).Q. \tag{11}$$

In the above formula, $D_o(\frac{Q_o}{Q}, \frac{R_o}{Q})$ is the average stock level in the warehouse and is as follows, using Equation 3 and noting that Q_o should be an integer multiple of Q:

$$D_o(\frac{Q_o}{Q}, \frac{R_o}{Q}) = \frac{(\frac{Q_o}{Q})}{2} + \frac{R_o}{Q} \quad \lambda_o L_o + B_o(\frac{Q_o}{Q}, \frac{R_o}{Q}). \tag{12}$$

In the above equation, λ_o is obtained through Equation 13 (as explained before) and $B_o(\frac{Q_o}{Q}, \frac{R_o}{Q})$ is the average backorder level in the warehouse in terms of Q, which is obtained in Equation 14 using Equations 1 and 2, noting, again, that Q_o should be an integer multiple of Q:

$$\lambda_o = \sum_{i=1}^{N} \frac{\lambda_i}{Q + \lambda_i \hat{T}_i}, \tag{13}$$

$$B_o\left(\frac{Q_o}{Q}, \frac{R_o}{Q}\right) = \frac{(\sqrt{\lambda_o L_o})^2}{\left(\frac{Q_o}{Q}\right)} \left[H\left[\frac{\frac{R_o}{Q} \quad \lambda_o L_o}{\sqrt{\lambda_o L_o}}\right] \quad H\left[\frac{\frac{R_0}{Q} + \frac{Q_o}{Q} \quad \lambda_o L_o}{\sqrt{\lambda_o L_o}}\right]\right], \tag{14}$$

where:

$$H(x) = \int_{x}^{\infty} G(\nu) d\nu = \frac{1}{2}\left[(x^2+1)(1 \quad \phi(x)) \quad x\varphi(x)\right], \tag{15a}$$

and:

$$\phi(x) = \int_{\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{\quad \frac{u^2}{2}} du,$$

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{\quad \frac{x^2}{2}}. \tag{15b}$$

Knowing λ_o and $B_o(\frac{Q_o}{Q}, \frac{R_o}{Q})$, one can determine $\overline{W}$ from the following equation, as explained before:

$$\overline{W} = \frac{B_o(\frac{Q_o}{Q}, \frac{R_o}{Q})}{\lambda_o}. \tag{16}$$

An important point in our approximation is that $\overline{W}$ is dependent on λ_o, which, itself, is dependent on $\hat{T}_i$. Since the stochastic distribution of lead time for the retailer, i, is not clear, $\hat{T}_i$ does not have any exact form in this complicated model. It is, therefore, approximated by the constant lead time, L_i, as given in Equation 17. The authors experience shows that the effect of this approximation is negligible on λ_o. However, the numerical tests will show how accurate this approximation is:

$$\hat{T}_i = L_i P(R_i; \lambda_i L_i) \quad \frac{R_i}{\lambda_i} P(R_i + 1; \lambda_i L_i). \tag{17}$$

Each retailer cost consists of shortage and holding costs as follows:

$$C_i = \pi_i.E_i + h_i.D_i,$$

$$i = 1, 2, \cdots, N, \quad 0 \leq R_i \prec Q. \tag{18}$$

In the above formulae, E_i is the average number of lost sales incurred per unit time in retailer i and D_i is the average stock level in retailer i. Based on the approximation shown in the section of "Approximating the Retailers Lead Time", $L_i + \overline{W}$ is assumed to be constant. Using Equations 4 to 6, for the case of a constant lead time, one has the following:

$$E_i = \frac{\lambda_i}{Q + \lambda_i \hat{T}_i'} [\lambda_i \hat{T}_i'], \tag{19}$$

$$D_i = \frac{\lambda_i}{Q + \lambda_i \hat{T}_i'} \left[\frac{Q(Q+1)}{2\lambda_i} + \frac{QR_i}{\lambda_i} \quad Q(L_i + \overline{W}) \right] + \frac{Q}{\lambda_i} E_i, \tag{20}$$

$$\hat{T}_i' = (L_i + \overline{W}) P(R_i; \lambda_i(L_i + \overline{W}))$$

$$\quad \frac{R_i}{\lambda_i} P(R_i + 1; \lambda_i(L_i + \overline{W})), \tag{21}$$

$\hat{T}_i'$ is the average length of time per cycle, for which retailer i is out of stock when the lead time is the constant value, $L_i + \overline{W}$.

Optimization Method

It is clear that one should find the optimal values of the reorder points of all installations through minimizing TC. Since it is a complicated cost function (even C_i by itself is very complicated as Hadley and Whitin mention [14]), it is not easy to find the optimal reorder points of the warehouse and retailers. Here, by defining some new notation, a method is presented for minimizing the cost function. It is necessary to state that the reorder point of retailer $i(R_i)$ is bounded by 0 and Q, $0 \leq R_i \prec Q$, since there should not be more than one order outstanding in each retailer at any time and this constraint satisfies this condition for a continuous review inventory system with lost sales [14]. Furthermore, since there are N retailers in this model and none of them can have more than one order outstanding, one has $R_o \geq (\quad NQ)$. This is because, if $R_o \prec \quad NQ$, then, the reorder point is never reached in the warehouse.

Since, in the numerical problems, one only considers the identical retailers case (like many other papers in the area of multi-echelon modeling), the optimization method is based on this assumption. In short, R_o is increased from its lower limit and the optimal, R_i, for all retailers is found. This continues until a local minimum is reached. Because of the complexity of the cost function one is not able to prove its convexity. However, by considering a logical upper limit for the reorder point of the warehouse (R_o), one can constrain the solution space. The warehouse reorder point can be limited by $R_o \leq \lambda_o + 3\sqrt{\lambda_o}$ with the confidence coefficient of 99.7%. In the numerical tests that will be presented in the next section, both the optimization method has been used and the total solution space has been searched (since both the reorder points are limited). The results have been the same and this strongly suggests that the authors local minimum is the global minimum. The notations used in the algorithm are as follows:

$R_o(n)$	reorder point of the warehouse in stage n,
R_o^*	optimal reorder point of the warehouse,
R	reorder point of the identical retailers in each stage ($0 \leq R \prec Q$),
$R^*(n)$	optimal reorder point of the identical retailers in stage n,
R^*	optimal reorder point of the identical retailers (through all stages),
$TC(n)$	total system cost in stage n (assuming $R(0 \leq R \prec Q)$ and $R_o(n)$),
$TC^*(n)$	optimal total system cost in stage n,
TC^*	optimal total system cost.

Here, the algorithm for finding the optimal reorder points of the warehouse and the retailers is presented as follows:

Step 0: Set $n = 0$,
Step 1: Set $R_o(n) = (\quad N + n).Q$,
Step 2: Set $R = 0, TC^*(n) =$ a large enough number,
Step 3: Determine $TC(n)$ using Equation 10, assuming that n is a counter of stages,

Step 4: If $TC(n) \leq TC^*(n)$, then, $R^*(n) = R$ and $TC^*(n) = TC(n)$,
Step 5: If $R \prec Q - 1$, then, $R = R + 1$ and go to Step 3,
Step 6: If $n = 0$, then, $n = n + 1$ and go to Step 1,
Else: If $TC^*(n) \succ TC^*(n - 1)$, then, $TC^* = TC^*(n - 1)$, $R_o^* = R_o(n - 1)$, $R^* = R^*(n - 1)$ and stop,
Else: If $TC^*(n) \leq TC^*(n - 1)$, then, $n = n + 1$ and go to Step 1.

NUMERICAL RESULTS

In order to determine the power of the authors approximation, a set of 36 numerical problems have been designed with the assumption of identical retailers. To the best of the authors knowledge and, as Andersson and Melchiors [12] mention, no work has been done on the case of lost sales in retailers with the policy of batch ordering in all installations. Since there were no previous numerical problems as a reference with which to compare our approximation, a problem set was developed, which offered a reasonable range of model parameters. The optimal reorder points of all installations were found using the optimization method described in the previous section. As already mentioned, the same optimal reorder points were found through searching the total solution space by setting an upper limit on the warehouse reorder point.

Each numerical problem was, also, simulated 10 times (considering 10 runs for each problem), for the optimal reorder points obtained from the approximate model, using GPSS/H simulation software. The simulation time length of each run is 110000 unit times with 10000 unit times as a "run in" period. Different starting random number seeds were employed for each problem. All of the results show that this length of time is sufficient for the system to reach a steady state. This is also clear from the standard deviation of the total system cost. The cost error is obtained by the following relation:

$$\text{Cost Error} = \frac{|\text{simulated total cost} - \text{approximated total cost}|}{\text{simulated total cost}}. \tag{22}$$

The identical retailers' service level is also reported as their ready rate (the fraction of time with positive stock on hand). This can be obtained through Hadley and Whitin [14] as:

$$\text{Service Level} = 1 - \frac{\text{average number of lost sales per unit time}}{\text{average number of demands per unit time}}. \tag{23}$$

The above relation has been employed in the approximation and, also, in the simulation model to find the service levels.

The numerical problems are as in Table 1. The number of retailers is 20 (a large enough number to approximate the demand distribution as normal in the warehouse), the holding costs of the warehouse and the identical retailers per unit per unit time are assumed to be 1, $h_o = h_i = 1$ and the transportation time for the identical retailers and, also, the lead time of the warehouse are assumed to be 1, $L_o = L_i = 1$. The total cost and service level results are shown in Table 2.

As can be seen from Table 2, the errors in the approximate total cost and approximate service levels are small in comparison with simulated values. The error levels are consistent with those obtained for similar approximations used by other researchers in this area.

CONCLUSION AND FUTURE RESEARCH

In this paper, an approximate cost function is developed for a two-echelon inventory system with one warehouse and several retailers, where unsatisfied demand in the retailers is lost and the control policy is continuous review. The main point of this paper is to assume lost sales during a stockout in the retailers, since most of the previous papers had assumed demand during a stockout to be backordered. Only Andersson and

Table 1. Designed numerical problems.

No	Q_o	Q	λ_i	π_i	No	Q_o	Q	λ_i	π_i
1	16	8	0.5	100	19	32	16	0.5	100
2	16	8	0.5	200	20	32	16	0.5	200
3	16	8	1	100	21	32	16	1	100
4	16	8	1	200	22	32	16	1	200
5	16	8	1.5	100	23	32	16	1.5	100
6	16	8	1.5	200	24	32	16	1.5	200
7	16	16	0.5	100	25	64	8	0.5	100
8	16	16	0.5	200	26	64	8	0.5	200
9	16	16	1	100	27	64	8	1	100
10	16	16	1	200	28	64	8	1	200
11	16	16	1.5	100	29	64	8	1.5	100
12	16	16	1.5	200	30	64	8	1.5	200
13	32	8	0.5	100	31	64	16	0.5	100
14	32	8	0.5	200	32	64	16	0.5	200
15	32	8	1	100	33	64	16	1	100
16	32	8	1	200	34	64	16	1	200
17	32	8	1.5	100	35	64	16	1.5	100
18	32	8	1.5	200	36	64	16	1.5	200

Table 2. Total cost and service level results.

No	Approximation			Simulation		Cost Error %	Approximation	Simulation	Service Level Error %
	R_o	R_i	Total Cost	Mean of Total Cost	Standard Deviation of Total Cost		Service Level %	Service Level %	
1	0	2	124.49	126.82	0.07	1.8%	99.38	99.54	0.2%
2	0	2	130.68	131.31	0.15	0.5%	99.38	99.54	0.2%
3	8	3	143.35	144.53	0.17	0.8%	99.15	99.29	0.1%
4	16	3	154.56	156.40	0.25	1.2%	99.49	99.54	0.1%
5	24	4	159.82	162.21	0.12	1.5%	99.41	99.47	0.1%
6	16	5	170.63	173.57	0.19	1.7%	99.72	99.74	0.0%
7	0	1	196.43	197.80	0.13	0.7%	98.39	98.97	0.6%
8	0	2	206.26	210.64	0.13	2.1%	99.61	99.78	0.2%
9	0	3	216.96	218.02	0.14	0.5%	99.11	99.43	0.3%
10	16	3	227.66	232.81	0.20	2.2%	99.67	99.74	0.1%
11	16	4	231.87	236.14	0.15	1.8%	99.43	99.55	0.1%
12	16	5	245.03	250.92	0.18	2.3%	99.82	99.85	0.0%
13	-8	2	125.90	128.38	0.06	1.9%	99.25	99.39	0.1%
14	-8	2	133.45	134.37	0.16	0.7%	99.25	99.40	0.2%
15	0	3	145.21	147.33	0.18	1.4%	99.06	99.15	0.1%
16	0	4	155.80	159.57	0.16	2.4%	99.76	99.76	0.0%
17	16	4	161.37	164.34	0.23	1.8%	99.36	99.40	0.0%
18	8	5	172.09	176.53	0.23	2.5%	99.70	99.69	0.0%
19	0	1	199.45	204.09	0.15	2.3%	98.83	99.12	0.3%
20	-16	2	208.44	207.11	0.26	0.6%	99.14	99.57	0.4%
21	0	3	218.10	222.75	0.15	2.1%	99.43	99.59	0.2%
22	0	3	229.51	231.18	0.18	0.7%	99.43	99.58	0.2%
23	16	4	234.62	240.80	0.12	2.6%	99.60	99.65	0.1%
24	0	5	245.81	249.63	0.26	1.5%	99.68	99.74	0.1%
25	-24	2	130.86	137.52	0.16	4.8%	98.77	98.52	0.3%
26	-16	2	140.55	146.09	0.16	3.8%	99.29	99.21	0.1%
27	-8	3	150.89	155.87	0.10	3.2%	99.17	99.13	0.0%
28	-16	4	160.51	170.32	0.19	5.8%	99.64	99.50	0.5%
29	-16	5	165.95	176.28	0.18	5.9%	99.34	99.13	0.2%
30	0	5	176.76	184.77	0.31	4.3%	99.75	99.69	0.1%
31	-32	2	202.73	209.61	0.18	3.3%	98.88	98.97	0.1%
32	-32	2	213.95	219.68	0.11	2.6%	98.88	98.98	0.1%
33	-16	3	221.34	226.50	0.12	2.3%	99.28	99.42	0.1%
34	-16	4	234.61	241.51	0.22	2.9%	99.79	99.82	0.0%
35	0	4	237.38	244.29	0.18	2.8%	99.51	99.54	0.0%
36	-16	5	249.22	255.54	0.28	2.5%	99.62	99.65	0.0%
Mean						2.3%	99.36	99.44	0.1%
Standard Deviation						1.4%	0.33	0.30	0.1%

Melchiors [12] have developed an approximate solution for the case of lost sales using an $(S \quad 1, S)$ policy in all installations. The warehouse arrival process has been approximated by a continuous normal demand process and each retailer lead time by a constant lead time, obtaining the average waiting time of the retailers orders using Little's formula from the queuing theory. The approximations were compared with simulation results for 36 numerical problems. The mean error of the cost is 2.3 %, which seems to be good. In future research, the retailers lead time could be approximated by other distributions and the mean error reduced. Another future research field is to use a service level objective for determining the optimal control policy.

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REFERENCES

1. Sherbrooke, C.C. "METRIC: A multi-echelon technique for recoverable item control", *Operations Research*, **16**, pp 122-141 (1968).
2. Wang, Y. and Cohen, M.A. and Zheng, Y.-S. "A two-echelon repairable inventory system with stocking-center-dependent depot replenishment lead time", *Management Science*, **46**, pp 1441-1453 (2000).
3. Graves, S.C. "A multi-echelon inventory model for a repairable item with one-for-one replenishment", *Management Science*, **31**, pp 1247-1256 (1985).
4. Moinzadeh, K. and Lee, H.L. "Batch size and stocking levels in multi-echelon repairable systems", *Management Science*, **32**, pp 1567-1581 (1986).
5. Lee, H.L. and Moinzadeh, K. "Operating Characteristics of a two-echelon inventory system for repairable and consumable items under batch ordering and shipment policy", *Naval Research Logistics*, **34**, pp 365-380 (1987).
6. Deuermeyer, B. and Schwarz, L.B. "A model for the analysis of system service level in warehouse/retailer distribution system: the identical retailer case", *Multilevel Production/Inventory Control Systems*, Chap. 13 (TIMS Studies in Management Science 16), L. Schwarz, Ed., Elsevier, New York, USA (1981).
7. Svoronos, A.P. and Zipkin, P. "Estimating the performance of multi-level inventory systems", *Operations Research*, **36**, pp 57-72 (1988).
8. Axsäter, S. "Simple solution procedures for a class of two-echelon inventory problems", *Operations Research*, **38**, pp 64-69 (1990).
9. Axsäter, S. "Exact and approximate evaluation of batch-ordering policies for two-level inventory systems", *Operations Research*, **41**, pp 777-785 (1993).
10. Axsäter, S. "Evaluation of installation stock-based (R, Q) policies for two-level inventory systems with Poisson demand", *Operations Research*, **46**, pp 135-145 (1998).
11. Forsberg, R. "Exact evaluation of (R, Q) policies for two-level inventory systems with Poisson demand", *European Journal of Operational Research*, **96**, pp 130-138 (1996).
12. Andersson, J. and Melchiors, P. "A two echelon inventory model with lost sales", *International Journal of Production Economics*, **69**, pp 307-315 (2001).
13. Axsäter, S., *Inventory Control*, Kluwer's International Series (2000).
14. Hadley, G. and Whitin, T.M. "Analysis of inventory systems", Prentice-Hall, Englewood Cliffs, N.J., (1963).
15. Muckstadt, J.A. "Analysis of a two-echelon inventory system in which all locations follow continuous review (s, S) policy", *Technical Report*, (337), School of Operations Research and Industrial Engineering, Cornell University, USA (1997).
16. Albin, S. "On Poisson approximation for superposition arrival processes in queues", *Management Science*, **28**, pp 126-137 (1982).
17. Zipkin, P.H. "Models for design and control of stochastic, multi item batch production systems", *Operations Research*, **34**, pp 91-104 (1986).
18. Cheung, K.L. and Hausman, W.H. "An exact performance evaluation for the supplier in a two-echelon inventory system", *Operations Research*, **48**, pp 646-653 (2000).