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Unified Multiple-Access Performance Analysis of Several Multirate Multicarrier Spread-Spectrum Systems

R. Nikjah[1] and M. Nasiri-Kenari*

A unified multiple-access performance analysis and comparison of three multicarrier spread-spectrum multiple-access schemes, namely, MC-CDMA (Multicarrier Code-Division Multiple-Access), MC-FH (Multicarrier Frequency Hopping) and a hybrid of the above systems, called DS-MC-FH (Direct-Sequence MC-FH), in a multirate environment, where each user can have several multirate services, is provided. In MC-CDMA and MC-FH systems, users and their diverse services are differentiated by means of only one kind of signature code. However, in a DS-MC-FH scheme, different users and different services of the same user are distinguished through the first and second signature codes, respectively. The performance of the above systems are evaluated and compared, using a unified structure in synchronous and asynchronous nonfading and synchronous correlated Rayleigh fading channels, with a Maximum-Ratio Combining (MRC) receiver. The near-far effect on the systems' performance is also investigated. The (second) signature in the MC-CDMA (DS-MC-FH) scheme is considered to be either a Pseudo-Noise (PN) sequence or a Walsh code. The authors analyses indicate that MC-CDMA systems with Walsh codes outperform the other schemes in different synchronous and asynchronous channels. DS-MC-FH systems with Walsh codes always surpass MC-FH systems. Furthermore, all of the schemes, except synchronous MC-CDMA systems with Walsh codes, are susceptible to a near-far effect with an MRC receiver

INTRODUCTION

In combination with Code-Division Multiple-Access (CDMA) techniques, multicarrier modulation has attracted a lot of attention in the past decade for future-generation wireless communications, on account of countering channel frequency selectivity and removing Inter-Symbol Interference (ISI), while supporting high-rate applications, providing frequency diversity, collecting the entire energy spread in the frequency domain and simple implementation through Fast-Fourier-Transformation (FFT) techniques [1].

On the one hand, different configurations of Multi-Carrier CDMA (MC-CDMA) schemes, as combinations of Direct-Sequence CDMA (DS-CDMA) and Orthogonal Frequency-Division Multiplexing (OFDM), were developed after 1993 [1,2]. The performance and design of such systems have been investigated extensively in different nonfading and fading channels since then [2-8]. However, limited research exists in the area of asynchronous MC-CDMA (for some results, see [6-8]). In this paper, MC-CDMA systems are considered as introduced in [2].

On the other hand, Frequency-Hopping Spread-Spectrum (FH-SS) techniques, in combination with OFDM or MC-CDMA, recently received considerable attention and, as a result, various Multi-Carrier Frequency-Hopping (MC-FH) systems were proposed [9-11,12]. MC-FH schemes, on account of fewer subcarriers transmitted in each symbol interval, have smaller Peak-to-Average-Power Ratio (PAPR) than MC-CDMA systems, making the implementation of MC-FH systems less complex than MC-CDMA schemes, especially in the uplink, where linear amplification with a large dynamic range at the transmitter side is not viable. The MC-FH system studied in this

1. *Department of Electrical Engineering, Sharif University of Technology, Tehran, I.R. Iran.*
*. *Corresponding Author, Department of Electrical Engineering, Sharif University of Technology, Tehran, I.R. Iran.*

paper is the one described in [12,13], wherein the frequency spacing between diversity hopping subcarriers in distinct frequency subbands is implemented in such a way as to diminish the correlation of fading gains on different subcarriers, while keeping the region of hopping for a single subcarrier so small that Phase-Shift Keying (PSK) modulation and coherent detection are practically feasible [12]. This scheme was developed from a frequency-diversity spread-spectrum system, called FD-SS [14], for countering band-limited jamming interference [12]. It has been examined in a single-user fading channel [13], as well as in multiuser nonfading and fading channels with and without coding [15].

In this paper, a multirate environment, depicted in Figure 1, is introduced, in which each user has two signature codes, the first of which serves to distinguish the user from the other $N_u - 1$ users and the second of which discriminates ξ services of the same user. The results of exploring such environments are also directly applicable to any system supporting independent groups of users, in which different groups can be viewed as virtual users and the users of each group can be interpreted as different services of the same virtual user.

In MC-CDMA and MC-FH systems, there is only one kind of signature code serving as both the first and second signatures. This explains the rationale for proposing a third multicarrier system as a hybrid of the previous two, called DS-MC-FH (Direct-Sequence MC-FH), possessing, as its first and second signature codes, the kinds of signature in MC-FH and MC-CDMA schemes, respectively. The first signature, as in a MC-FH system, determines the set of subcarriers allotted to a user, each belonging to one frequency subband. The second signature, the length of which is equal to the number of subcarriers in each transmission interval, modulates the subcarriers selected by the first signature, as in a MC-CDMA system, providing differentiation between services of the same user. Figure 2 shows the transmission frequency pattern in a DS-MC-FH system. The whole frequency band is partitioned into N_s subbands, each of which has exactly N_h subcarriers. $d_i^{(k,q)}$ is the ith data bit of the qth service of user k; T is the symbol duration excluding any guard interval; $\left\{h_{m,i}^{(k,q)}\right\}_{m=0}^{N_s-1}$ and $\left\{c_m^{(k,q)}\right\}_{m=0}^{N_s-1}$ are the first and second signatures of the qth service of user k at transmission interval i, respectively. Contrary to $h_{m,i}^{(k,q)}$'s, $c_m^{(k,q)}$'s are identical for different data bit intervals. Figure 2

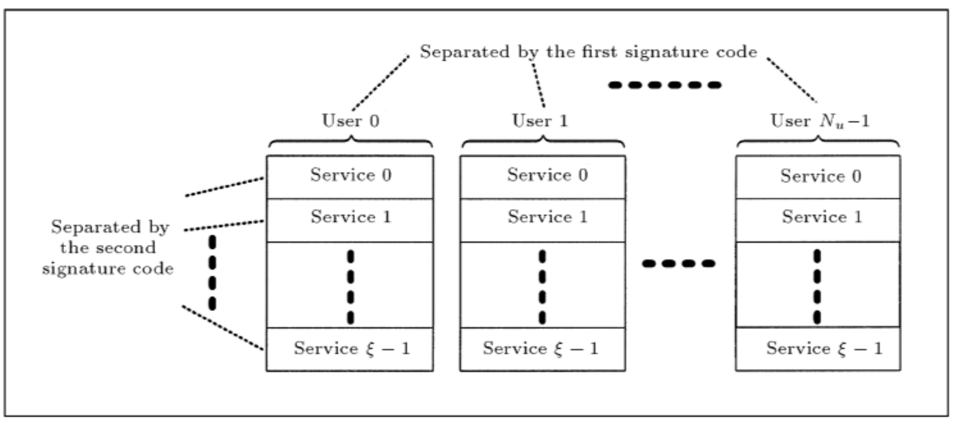

Figure 1. Distinguishing between different users and services by the first and second signature codes.

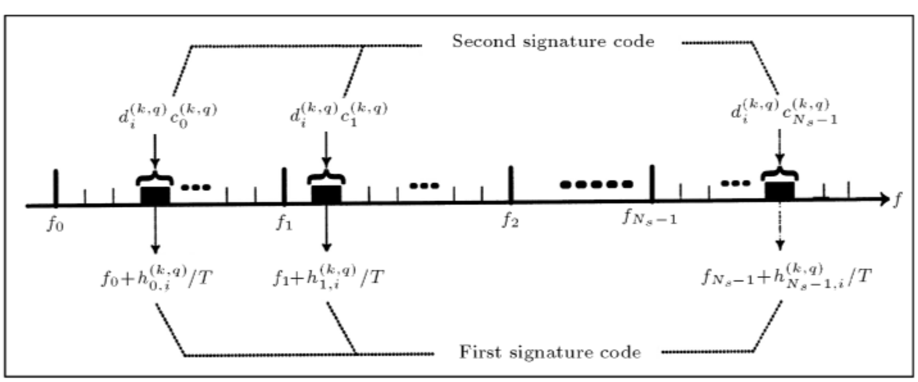

Figure 2. Transmission frequency pattern for service q of user k at the ith symbol interval.

is simply particularized to thoroughly describe MC-CDMA systems, if one sets all $h_{m,i}^{(k,q)}$'s to 0 and N_h to 1, and MC-FH schemes, if one sets all $c_m^{(k,q)}$'s to 1. Note that in contrast to a DS-MC-FH scheme, $h_{m,i}^{(k,q)}$'s are independent for different q's in a MC-FH system. In fact, in a MC-FH system, there is only one kind of signature code utilized for differentiating users and their different services alike, resulting in independent $h_{m,i}^{(k,q)}$'s for different k's and q's. However, in a DS-MC-FH scheme, the first signature is identical for all services of the same user and the services of each user are distinguished by the second signature code, $\{c_m^{(k,q)}\}$.

Figure 3 illustrates the differences between three systems having the same bandwidth, with two services for each user ($\xi = 2$). In MC-CDMA (Figure 3a), all services of all users share the entire frequency band, with the frequency spacing between adjacent subcarriers the same as that in OFDM. Different services are discriminated through distinct $c_m^{(k,q)}$'s. In MC-FH (Figure 3b), all the services carried in the system are differentiated by means of assigning to each an independent set $\{h_{m,i}^{(k,q)}\}$, called a hopping pattern that selects from different frequency subbands the subcarriers on which $d_i^{(k,q)}$ is transmitted. In the DS-MC-FH (Figure 3c) scheme, however, the hopping pattern, $\{h_{m,i}^{(k,q)}\}$, is common to all services of the same user.

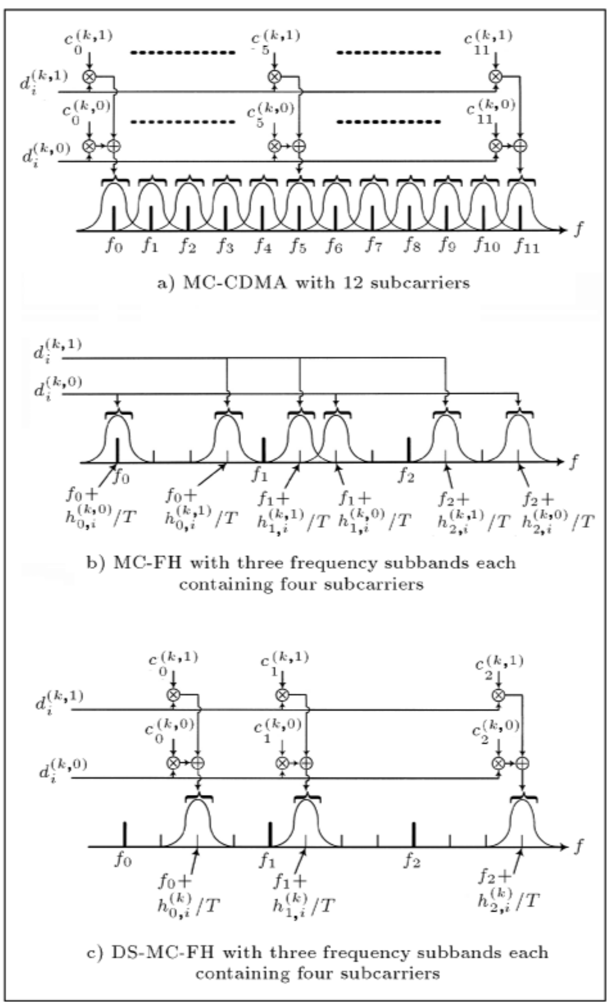

Figure 3. Transmission of two coded services over $12/T$ bandwidth.

It is worth noting that in all three schemes, the whole modulated subcarriers of the ith data bit of all the same user's services are added together, in order to be transmitted simultaneously in a one-bit interval. It is also assumed that each symbol, like OFDM symbols, has head and tail guard intervals with a cyclic extension in the intervals so as to avoid ISI problems.

In this paper, the multiple-access performance of the above three multicarrier schemes are evaluated in synchronous and asynchronous nonfading and in synchronous correlated Rayleigh fading channels, considering near-far effects and the results are compared. Even though only the use of Pseudo-Noise (PN) and Walsh codes for the signature, $\{c_m^{(k,q)}\}$, in MC-CDMA and DS-MC-FH systems is addressed, the authors analysis can be utilized to easily include any other code. The unified approach adopted for the performance analysis in this paper is based on the characteristic function method, applied previously to the performance evaluation of MC-FH, DS-CDMA, ultra-wideband impulse radio and MC-CDMA by [8,15-17], respectively. To the best of the authors knowledge, this paper is the first one considering and comparing the multiple-access performance of the two well-known schemes, namely MC-CDMA and MC-FH, and a generalization of these two schemes, called DS-MC-FH, in various multirate channels. The results indicate that MC-CDMA schemes with Walsh codes outperform other systems in all cases under consideration. Also, DS-MC-FH systems with Walsh codes always surpass MC-FH systems. Note that, although frequency hopping schemes do not show any performance superiority over MC-CDMA systems, in some applications, such as military and jamming environments, they are much more plausible.

The paper is organized as follows. In the following sections, first, the unified structure and expressions of the above schemes are described and then, performance analyses of the systems in nonfading and fading channels, respectively, are provided. After that, numerical results are presented and, finally, the conclusions are presented.

SYSTEM DESCRIPTION

MC-CDMA, MC-FH and DS-MC-FH systems can be described in a unified way, provided that, referring to

Figure 2, the following equalities are taken into account for each system:

$$\text{MC-CDMA:} \quad h_{m,i}^{(k,q)} = 0, \quad \text{for all } k, q, m \text{ and } i,$$

$$\text{and } N_h = 1, \tag{1a}$$

$$\text{MC-FH:} \quad c_m^{(k,q)} = 1, \quad \text{for all } k, q \text{ and } m, \tag{1b}$$

$$\text{DS-MC-FH:} \quad h_{m,i}^{(k,q)} = h_{m,i}^{(k)}, \quad \text{for all } k, q, m \text{ and } i. \tag{1c}$$

Note that $h_{m,i}^{(k,q)} = 0$ and $c_m^{(k,q)} = 1$ mean the lack of a frequency hopping signature and MC-CDMA signature codes, respectively. Also, it is noticeable that in a MC-CDMA scheme, each frequency subband is reduced to a single frequency subcarrier. $h_{m,i}^{(k,q)}$'s, for different k's, q's, m's and i's in MC-FH, and for different k's, m's and i's in DS-MC-FH, are assumed to be independent identically distributed (iid) random variables with uniform distribution over the integer interval $[0, N_h - 1]$. $\{c_m^{(k,q)}\}_{m=0}^{N_s-1}$'s in MC-CDMA and DS-MC-FH are supposed to be either PN or Walsh codes. In the former case, $c_m^{(k,q)}$'s are iid random variables taking on values -1 and $+1$ with equal probability. In the latter case, $\{c_m^{(k,q)}\}_{m=0}^{N_s-1}$'s, for different k's and q's, are different Walsh codes. Let f_0 through f_{N_s-1} be the base frequencies of non-overlapping frequency subbands (see Figure 2). For the authors' analysis, it is assumed that the whole frequency band is contiguous, i.e:

$$f_{m_2} - f_{m_1} = (m_2 - m_1)N_h/T,$$

where:

$$0 \leq m_1 < m_2 \leq N_s - 1. \tag{2}$$

The whole occupied bandwidth, by the system, is, thus, obtained as $B = BW/T$, where $BW = N_s N_h$ is the normalized bandwidth. Hence, considering Equation 1a, for a constant bandwidth, as can be seen in Figure 3, MC-CDMA systems have N_h times as many transmitted subcarriers in a symbol interval as the other two systems do, where it is supposed that both the hopping schemes have N_h subcarriers in each frequency subband.

In all of the following analyses, without any loss of generality, it is assumed that each service of each user has a bit rate of $1/T$. Multirate services, with bit rates higher than $1/T$, are assigned more than one signature code.

Transmitted Signal

The baseband Binary-Phase-Shift-Keying (BPSK) transmitted signal for the qth service of user k can be expressed as follows:

$$s_l^{(k,q)}(t) = \sum_i \sum_{m=0}^{N_s-1} \sqrt{2S_i^{(k)}} d_i^{(k,q)} c_m^{(k,q)} \exp\{j[2\pi (t - t_t^{(k)} - iT)(f_m + h_{m,i}^{(k,q)}/T) + \theta_i^{(k)}]\} p(t - t_t^{(k)} - iT), \tag{3}$$

where $d_i^{(k,q)}$ is the data bit of the qth service of user k at the ith symbol interval; $d_i^{(k,q)}$'s, for different k's, q's and i's, are iid random variables and take on values -1 and $+1$ with the same probability; $S_i^{(k)}$ is the transmission power for each subcarrier of user k at symbol interval i; $t_t^{(k)}$ and $\theta_i^{(k)}$ are the kth user's transmitter time origin and the kth user's transmission phase at the ith interval, respectively; and $p(t)$ is the unit pulse in the time interval $(0, T)$.

Model of Channel

An Additive-White-Gaussian-Noise (AWGN) nonfading or a slowly correlated Rayleigh fading channel is considered. The former case can be achieved from the latter by setting all fading coefficients to one. Therefore, fading channels are concentrated upon. The length of each symbol is conventionally considered to be so much greater than the maximum delay spread of the channel that the presumption of flat fading over each subcarrier is satisfied. As a result, the mth subcarrier of the qth service of user k at the ith symbol interval experiences a flat fading gain, $g_{m,i}^{(k,q)} = H^{(k)}(f_m + h_{m,i}^{(k,q)}/T)$, which is assumed to be constant over at least one symbol interval. $H^{(k)}(f)$ is the baseband frequency response of the transmission channel for the kth user. If $\mathbf{g}_i^{(k,q)}$ is defined as a column vector with components $\left[g_{m,i}^{(k,q)}\right]_{m=0}^{N_s-1}$, then, $\mathbf{g}_i^{(k,q)}$ is a Proper Complex Gaussian (PCG) random vector characterized by its probability-density function (pdf) $f_{\mathbf{g}_i^{(k,q)}}(\mathbf{g}) = \frac{\exp(-\mathbf{g}^H \mathbf{C}^{-1} \mathbf{g})}{\pi^{N_s} \det(\mathbf{C})}$ ("H" denotes Hermitian operation and "det" stands for determinant) [18], where $\mathbf{C}$, defined as $E\left\{\mathbf{g}_i^{(k,q)} \mathbf{g}_i^{(k,q)H}\right\}$ ($E\{.\}$ denotes expectation), is the frequency covariance matrix of the channel. Assuming that the channel is Wide-Sense Stationary Uncorrelated Scattering (WSSUS) and the frequency band of the system is contiguous, the entries of $\mathbf{C}$ are obtained as follows:

$$\mathbf{C}_{m_1,m_2} = \Phi_H(\Delta f_{m_1,m_2}) = \int_{-\infty}^{+\infty} \phi_h(\tau) \exp(-j2\pi \Delta f_{m_1,m_2} \tau) d\tau,$$

$$\text{with } \Delta f_{m_1,m_2} = N_h(m_1 - m_2)/T, \tag{4}$$

where $\phi_h(\tau)$ and $\Phi_H(f)$ are the power delay profile (pdp) and the frequency autocorrelation function of the channel, respectively. $\Delta f_{m_1,m_2}$ is the (averaged) frequency spacing between the m_1th and m_2th subcarriers. The following exponential model is adopted for the pdp [13]:

$$\phi_h(\tau) = \frac{\mu/T_m}{1 \quad (1+\mu)\exp(\quad \mu)}[\exp(\quad \mu\tau/T_m) \quad \exp(\quad \mu)]$$

$$\text{for} \quad 0 < \tau < T_m, \tag{5}$$

where μ is a decaying factor and T_m is the maximum delay spread of the channel.

Received Signal, Receiver Structure and Decision Variable

The baseband received signal can be written as follows:

$$r_l(t) = \sum_i \sum_{k=0}^{N_u \;\; 1} \sum_{q=0}^{\xi \;\; 1} \sum_{m=0}^{N_s \;\; 1} \sqrt{2R_i^{(k)}} g_{m,i}^{(k,q)} d_i^{(k,q)} c_m^{(k,q)}$$

$$\exp\{j[2\pi(t \quad t_r^{(k)} \quad iT)(f_m + h_{m,i}^{(k,q)}/T) + \theta_i^{(k)}]\}$$

$$p(t \quad t_r^{(k)} \quad iT) + z(t), \tag{6}$$

where N_u is the number of active users and $R_i^{(k)}$ denotes the received power at each subcarrier of user k at the ith symbol interval. $t_r^{(k)}$ is the receiver time origin and is equal to $t_t^{(k)} + t_d^{(k)}$, where $t_d^{(k)}$ is the reception delay for the kth user. $g_{m,i}^{(k,q)}$'s are assumed to be so normalized that $E\left\{\left|g_{m,i}^{(k,q)}\right|^2\right\} = 1$ and $z(t)$ is the complex zero-mean AWGN with double-sided power spectral density N_0.

A single-user Maximum-Ratio-Combining (MRC) receiver is considered, as shown in Figure 4, with $w_{m,i}^{(k,q)} = g_{m,i}^{(k,q)*} c_m^{(k,q)}$, where "*" stands for conjugation. Let the signal of the 0th service of user 0 at the 0th symbol interval be the signal of interest. Without any loss of generality, it is assumed that $\theta_0^{(0)} = 0$ and $t_r^{(0)} = 0$. Also, it is supposed that $t_r^{(k)}$ belongs to the interval $(0, T)$. With these assumptions, the following expression for the decision variable normalized by $\sqrt{2R_0^{(0)}}$ is attained:

$$D = d_0^{(0,0)} \sum_{m=0}^{N_s \;\; 1} \left|g_{m,0}^{(0,0)}\right|^2$$

$$+ \mathrm{Re}\Bigg\{ \sum_{q=1}^{\xi \;\; 1} \sum_{m=0}^{N_s \;\; 1} g_{m,0}^{(0,0)*} g_{m,0}^{(0,q)} d_0^{(0,q)} c_m^{(0,0)} c_m^{(0,q)}$$

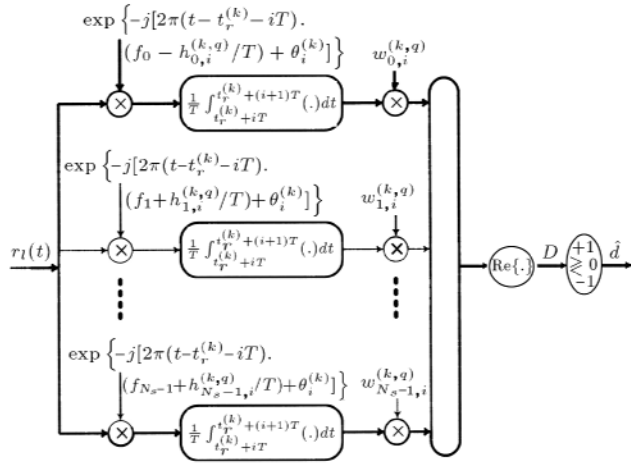

Figure 4. Baseband equivalent MRC receiver for the qth service of user k at the ith symbol interval.

$$\cdot \frac{1}{T}\int_0^T \exp\left[j2\pi\left(\frac{h_{m,0}^{(0,q)}}{T} \quad \frac{h_{m,0}^{(0,0)}}{T}\right)\right] dt \Bigg\}$$

$$+ \mathrm{Re}\Bigg\{ \sum_{k=1}^{N_u \;\; 1} \sum_{q=0}^{\xi \;\; 1} \sum_{m=0}^{N_s \;\; 1} \sum_{\nu=0}^{N_s \;\; 1}$$

$$\sqrt{R_{\;\;1}^{(k)}/R_0^{(0)}} g_{\nu,0}^{(0,0)*} g_{m, \;\; 1}^{(k,q)} d_{\;\;1}^{(k,q)} c_m^{(k,q)} c_\nu^{(0,0)}$$

$$\cdot \frac{1}{T}\int_0^{t_r^{(k)}} \exp\Bigg\{ j2\pi\left[\left(f_m + \frac{h_{m, \;\; 1}^{(k,q)}}{T}\right)(t \quad t_r^{(k)})\right.$$

$$\left.\left(f_\nu + \frac{h_{\nu,0}^{(0,0)}}{T}\right)t\right] + j\theta_{\;\;1}^{(k)}\Bigg\} dt$$

$$+ \sqrt{R_0^{(k)}/R_0^{(0)}} g_{\nu,0}^{(0,0)*} g_{m,0}^{(k,q)} d_0^{(k,q)} c_m^{(k,q)} c_\nu^{(0,0)}$$

$$\times \frac{1}{T}\int_{t_r^{(k)}}^T \exp\Bigg\{ j2\pi\left[\left(f_m + \frac{h_{m,0}^{(k,q)}}{T}\right)(t \quad t_r^{(k)})\right.$$

$$\left.\left(f_\nu + \frac{h_{\nu,0}^{(0,0)}}{T}\right)t\right] + j\theta_0^{(k)}\Bigg\} dt \Bigg\} + n, \tag{7}$$

where Re{.} stands for the real part. Also, n represents the noise term, which, conditioned on the fading coefficients $g_{m,0}^{(0,0)}$'s, is a zero-mean Gaussian random variable with variance equal to the following:

$$\sigma^2_{n|g_{m,0}^{(0,0)}} = \left(N_s \sum_{m=0}^{N_s \;\; 1} \left|g_{m,0}^{(0,0)}\right|^2\right) / 2\gamma_b,$$

$$\text{where}: \quad \gamma_b = N_s \gamma_{bs} \quad \text{with} \quad \gamma_{bs} = \frac{R_0^{(0)} T}{N_0}, \tag{8}$$

γ_{bs} is the received Signal-to-Noise Ratio (SNR) per bit per diversity branch and γ_b is the total received SNR per bit. In deriving Equation 7, it is assumed that $f_m.T$, for any m, is an integer.

Detection and Probability of Bit Error

Detection at the receiver is accomplished by comparing the decision variable, D, with the threshold, zero. Consequently, the Bit-Error Rate (BER) of the system is obtained as follows:

$$P_{be} = \Pr\left\{D < 0 | d_0^{(0,0)} = +1\right\} = F_U(0),$$

$$\text{where}: \quad U \triangleq D| + 1, \tag{9}$$

$F_U(u)$ stands for the cumulative-distribution function (cdf) of U and $D|+1$ denotes the decision variable, D, conditioned on transmitting $+1$.

PERFORMANCE EVALUATION IN NONFADING CHANNELS

Synchronous Systems

For a synchronous nonfading system, there is $g_{m,i}^{(k,q)} = 1$, $t_r^{(k)} = 0$ and $\theta_i^{(k)} = 0$, for any k, q, m and i. Now, from Equations 7 and 8, with the definition of U in Equation 9, one obtains the following:

$$U = N_s + \text{MAI} + n \quad \text{with} \quad \text{MAI} = \sum_{k=0}^{N_u \;\; 1} \text{MAI}^{(k)}, \tag{10a}$$

where:

$$\text{MAI}^{(0)} = \sum_{q=1}^{\xi \;\; 1} \text{MAI}^{(0,q)},$$

$$\text{MAI}^{(k)} = \sum_{q=0}^{\xi \;\; 1} \text{MAI}^{(k,q)} \quad \text{for} \quad k \geq 1,$$

and:

$$\text{MAI}^{(k,q)} = \sum_{m=0}^{N_s \;\; 1} \sqrt{R_0^{(k)}/R_0^{(0)}}\, d_0^{(k,q)} c_m^{(0,0)} c_m^{(k,q)} \eta_{m,0}^{(k,q)}$$

$$\text{for any } k \text{ and } q, \tag{10b}$$

in which:

$$\eta_{m,i}^{(k,q)} = \delta\left[h_{m,i}^{(k,q)} \;\; h_{m,0}^{(0,0)}\right], \tag{10c}$$

also,

$$\sigma_n^2 = N_s^2/(2\gamma_b). \tag{10d}$$

$\text{MAI}^{(k)}$ is the interference, due to user k, and $\text{MAI}^{(k,q)}$ is the interference term caused by the qth service of user k. n in Equation 10a is the Gaussian noise term with zero mean and variance that can be obtained from Equation 10d. δ in Equation 10c indicates the Kronecker delta. As the $h_{m,i}^{(k,q)}$'s are iid random variables, with uniform distribution over the integer interval $[0, N_h \;\; 1]$, from Equation 10c, the $\eta_{m,i}^{(k,q)}$'s are iid random variables with the following distribution:

$$\eta_{m,i}^{(k,q)} = \begin{cases} 0, & \text{with probability } \alpha \triangleq 1 \;\; 1/N_h \\ 1, & \text{with probability } \beta \triangleq 1/N_h \end{cases}. \tag{11}$$

From Equations 10, assuming that $R_0^{(k)} = R$ for any k, MAI is a discrete random variable with the pdf given by the following:

$$f_{\text{MAI}}(x) = \sum_{a= \;\; P}^{P} p_{\text{MAI}}[a]\delta(x \;\; a),$$

$$\text{with} \quad P = N_s(N_u\xi \;\; 1), \tag{12}$$

where $p_{\text{MAI}}[a]$ is the probability function of MAI. From Equations 9 and 10, one can write the following:

$$P_{be} = \Pr\{\text{MAI} + n < \;\; N_s\}. \tag{13}$$

The pdf of the "interference plus noise" is obtained as follows:

$$f_{(\text{MAI}+n)}(x) = f_{\text{MAI}}(x)^* f_n(x)$$

$$= \sum_{a= \;\; P}^{P} p_{\text{MAI}}[a] f_n(x \;\; a), \tag{14}$$

where $f_n(x)$ is the pdf of n. Now, the BER from Equations 13 and 14 can be acquired as follows:

$$P_{be} = \int_{\;\;\infty}^{\;\; N_s} f_{(\text{MAI}+n)}(x)dx$$

$$= \sum_{a= \;\; P}^{P} p_{\text{MAI}}[a] Q\left(\frac{N_s + a}{\sigma_n}\right), \tag{15}$$

in which $Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} \exp(\;\; w^2/2)dw$ is the Gaussian-tail function. In the Appendix, using the theorem proposed in [19], it is proven that:

$$p_{\text{MAI}}[a] = \frac{1}{L} \sum_{\substack{k=1 \\ k \text{ odd}}}^{2L \;\; 1} M_{\text{MAI}}(e^{jk\pi/L}) e^{\;\; jak\pi/L},$$

$$\text{with} \quad L = 2P + 1, \tag{16}$$

where $M_{\text{MAI}}(z)$ is the moment-generating function (mgf) of MAI. The exact BER is then obtained using Equations 15 and 16.

Now, for studying the near-far effect, it is assumed that $R_0^{(k)} = R$ for $k \geq 1$ and that $R_0^{(0)} < R$. In other words, all the interfering users' signals are received with the same power which is greater than the desired signal power. From Equations 10, supposing that $A \triangleq \sqrt{R/R_0^{(0)}}$ is an integer (note that this assumption, not affecting the final result, is made only for computational simplicity and does not impose any restriction on the subsequent analyses), one can follow the same above steps to reach Equations 15 and 16, except that $P = N_s[\xi - 1 + A(N_u - 1)\xi]$. Also, note that, in this case, $\mathrm{MAI}^{(0)}$ is the same as the $\mathrm{MAI}^{(0)}$ with $R_0^{(0)} = R$, but, $\mathrm{MAI}^{(k)}$ for $k \geq 1$ is A times the $\mathrm{MAI}^{(k)}$ with $R_0^{(0)} = R$.

For performance evaluation, one needs only to calculate $M_{\mathrm{MAI}^{(k)}}(z)$. As the $\mathrm{MAI}^{(k)}$'s are independent, $M_{\mathrm{MAI}}(z)$ is easily acquired from the product of all $M_{\mathrm{MAI}^{(k)}}(z)$'s. In the succeeding sections, assuming that $R_0^{(0)} = R$, $M_{\mathrm{MAI}^{(k)}}(z)$ is evaluated for MC-FH and DS-MC-FH systems. $M_{\mathrm{MAI}^{(k)}}(z)$ for MC-CDMA schemes is derived from that of DS-MC-FH systems, if $N_h = 1$ is set.

MC-FH System

For this scheme, combining Equations 1b and 10 results in:

$$\mathrm{MAI}^{(0)} = \sum_{q=1}^{\xi-1} \mathrm{MAI}^{(0,q)},$$

and:

$$\mathrm{MAI}^{(k)} = \sum_{q=0}^{\xi-1} \mathrm{MAI}^{(k,q)} \quad \text{for} \quad k \geq 1, \tag{17}$$

with:

$$\mathrm{MAI}^{(k,q)} = d_0^{(k,q)} \sum_{m=0}^{N_s-1} \eta_{m,0}^{(k,q)}.$$

$\mathrm{MAI}^{(k,q)}$, conditioned on $d_0^{(k,q)}$, is the sum of N_s iid random variables, i.e., the $d_0^{(k,q)}\eta_{m,0}^{(k,q)}$'s. Therefore, from Equations 10 and 11 and the fact that the $\mathrm{MAI}^{(k,q)}$'s are independent, one has following:

$$\begin{aligned}
M_{\mathrm{MAI}^{(0)}}(z) &= \prod_{q=1}^{\xi-1} M_{\mathrm{MAI}^{(0,q)}}(z) \\
&= \prod_{q=1}^{\xi-1} \Big\{ \Pr\{d_0^{(0,q)} = -1\} M_{\mathrm{MAI}^{(0,q)}|d_0^{(0,q)}=-1}(z) \\
&\quad + \Pr\left\{d_0^{(0,q)} = +1\right\} M_{\mathrm{MAI}^{(0,q)}|d_0^{(0,q)}=+1}(z) \Big\} \\
&= \prod_{q=1}^{\xi-1} \Big\{ \frac{1}{2} M_{\mathrm{MAI}^{(0,q)}|d_0^{(0,q)}=-1}(z) \\
&\quad + \frac{1}{2} M_{\mathrm{MAI}^{(0,q)}|d_0^{(0,q)}+1}(z) \Big\} \\
&= \left\{ \frac{1}{2}(\alpha + \beta z^{-1})^{N_s} + \frac{1}{2}(\alpha + \beta z)^{N_s} \right\}^{\xi-1},
\end{aligned}$$

and, in the same way,

$$M_{\mathrm{MAI}^{(k)}}(z) = \left\{ \frac{1}{2}(\alpha + \beta z^{-1})^{N_s} + \frac{1}{2}(\alpha + \beta z)^{N_s} \right\}^{\xi}$$

$$\text{for} \quad k \geq 1, \tag{18}$$

where $\alpha = 1 - \beta$ is equal to $1 - 1/N_h$.

DS-MC-FH System

In this system, from Equation 1c, $h_{m,i}^{(k,q)}$ and, thus, $\eta_{m,i}^{(k,q)}$ defined in Equation 10c, do not depend on q and are, thereby, represented by $h_{m,i}^{(k)}$ and $\eta_{m,i}^{(k)}$, respectively. From Equation 10c, it can easily be concluded that $\eta_{m,i}^{(0)}$'s are all equal to 1. Consequently, from Equations 10 one can write the following:

$$\mathrm{MAI}^{(0)} = \sum_{q=1}^{\xi-1} d_0^{(0,q)} \sum_{m=0}^{N_s-1} c_m^{(0,0)} c_m^{(0,q)}, \tag{19a}$$

and:

$$\begin{aligned}
\mathrm{MAI}^{(k)} &= \sum_{m=0}^{N_s-1} \mathrm{MAI}_m^{(k)} \\
&= \sum_{m=0}^{N_s-1} \eta_{m,0}^{(k)} \sum_{q=0}^{\xi-1} d_0^{(k,q)} c_m^{(0,0)} c_m^{(k,q)},
\end{aligned}$$

$$\text{for} \quad k \geq 1, \tag{19b}$$

PN Second-Signature Codes

In this case, $\mathrm{MAI}^{(0)}$, given in Equation 19a, consists of $(\xi - 1)N_s$ iid random variables taking on values -1 and $+1$ with the same probability. Therefore, one has the following:

$$M_{\mathrm{MAI}^{(0)}}(z) = \left(\frac{z^{-1}}{2} + \frac{z}{2} \right)^{(\xi-1)N_s}. \tag{20}$$

From Equations 11 and 19b, it is apparent that $\mathrm{MAI}_m^{(k)}$ is zero, with probability $\alpha = 1 - \beta = 1 - 1/N_h$; otherwise, it is equal to the sum of ξ independent uniformly distributed binary-valued (with values -1 and $+1$) random variables. Also, $\eta_{m,0}^{(k)}$'s are independent for different m's. Therefore, one easily obtains the following:

$$M_{\mathrm{MAI}^{(k)}}(z)=\left\{\alpha+\beta\left(\frac{z^{-1}}{2}+\frac{z}{2}\right)^{\xi}\right\}^{N_s},$$

$$\text{for}\quad k\geq 1. \tag{21}$$

Walsh Second-Signature Codes

For this case, as any two different Walsh codes are orthogonal, $\mathrm{MAI}^{(0)}$ (see Equation 19a) completely vanishes. Now, from Equation 19b, one has the following:

$$\mathrm{MAI}^{(k)}=\sum_{q=0}^{\xi-1}\mathrm{MAI}^{(k,q)},$$

with:

$$\mathrm{MAI}^{(k,q)}=\sum_{m=0}^{N_s-1}d_0^{(k,q)}c_m^{(0,0)}c_m^{(k,q)}\eta_{m,0}^{(k)},$$

$$\text{for}\quad k\geq 1. \tag{22}$$

Three cases are considered. First, if $N_u=1$, then, no multiple-access interference exists and, therefore,

$$P_{be}=\Pr\{U<0\}=\Pr\{n<-N_s\}=Q\left(\frac{N_s}{\sigma_n}\right). \tag{23}$$

Second, if $N_u>1$ and $\xi=1$, one only needs to calculate the mgf of $\mathrm{MAI}^{(k)}$ for $k\geq 1$, which is equal to $\mathrm{MAI}^{(k,0)}$, defined in Equation 22. Let E_1 be the set of k's ($k\geq 1$), for which $[c_m^{(k,0)}]_{m=0}^{N_s-1}=[c_m^{(0,0)}]_{m=0}^{N_s-1}$, with the cardinality of u and let E_2 be the complement of E_1 with respect to the set $\{1,2,\cdots,N_u-1\}$. On the one hand, $\mathrm{MAI}^{(k\in E_1)}$ has a form just similar to $\mathrm{MAI}^{(k,q)}$ for MC-FH schemes (see Equation 17), so, from Equation 18, one has the following:

$$M_{\mathrm{MAI}^{(k\in E_1)}}(z)=\frac{1}{2}(\alpha+\beta z^{-1})^{N_s}+\frac{1}{2}(\alpha+\beta z)^{N_s}. \tag{24}$$

On the other hand, for $k\in E_2$, $\{d_0^{(k,0)}c_m^{(0,0)}c_m^{(k,0)}\}_{m=0}^{N_s-1}$ has an equal number of -1's and $+1$'s. As $\eta_{m,0}^{(k)}$'s for different m's are independent, with the statistics given in Equation 11, from Equation 22, the following is obtained:

$$M_{\mathrm{MAI}^{(k\in E_2)}}(z)=(\alpha+\beta z^{-1})^{N_s/2}(\alpha+\beta z)^{N_s/2}. \tag{25}$$

Therefore, from Equations 24 and 25, one obtains the following:

$$M_{\mathrm{MAI}}(z)=\left\{\frac{1}{2}(\alpha+\beta z^{-1})^{N_s}+\frac{1}{2}(\alpha+\beta z)^{N_s}\right\}^{u}$$

$$\left\{(\alpha+\beta z^{-1})(\alpha+\beta z)\right\}^{\frac{N_s}{2}(N_u-1-u)}. \tag{26}$$

Now, substituting Equation 26 in Equation 15 and applying the result to Equation 16 leads to $P_{be}|u$ (the BER conditioned on u). Before investigating how to determine the unconditional bit-error rate, P_{be}, from the conditional bit-error rate, $P_{be}|u$, first, the remaining case, i.e., $N_u>1$ and $\xi>1$ is considered. For this case, as the exact analysis is very complicated and cumbersome, a Gaussian distribution approximation is utilized for the multiple-access-interference term, according to the Central-Limit Theorem (CLT). Under this assumption, using Equation 13 one has the following:

$$P_{be}=Q\left(\frac{N_s}{\sigma_{(\mathrm{MAI}+n)}}\right)\quad\text{with}\quad\sigma^2_{(\mathrm{MAI}+n)}=\sigma^2_{\mathrm{MAI}}+\sigma^2_n, \tag{27}$$

σ^2_{MAI} is the variance of MAI and σ^2_n is obtained from Equation 10d. In order to evaluate σ^2_{MAI}, it is assumed that there are u distinct pairs (k,q), for any of which, the qth service of user k has the same Walsh code as the desired service. For such pairs, the mgf of $\mathrm{MAI}^{(k,q)}$ has a form just the same as the one given in Equation 24. The variance of $\mathrm{MAI}^{(k,q)}$ is then obtained, via the second derivative of the mgf evaluated at $z=1$ as $N_s\beta(N_s\beta+\alpha)$. For the other services (with a different Walsh code from that of the desired service), the mgf of $\mathrm{MAI}^{(k,q)}$ has a form exactly similar to the mgf in Equation 25. Then, the variance of such $\mathrm{MAI}^{(k,q)}$'s is acquired as $N_s\beta\alpha$. Consequently, one can write:

$$\sigma^2_{\mathrm{MAI}}|u=u\{N_s\beta(N_s\beta+\alpha)\}+\{(N_u-1)\xi-u\}(N_s\beta\alpha). \tag{28}$$

Note that $\mathrm{MAI}^{(0)}$ is equal to zero and that one has a total of $(N_u-1)\xi$ interfering services. u services have the same code as the desired service and $(N_u-1)\xi-u$ services have different codes from the desired service. Equations 27 and 28 lead to $P_{be}|u$, i.e., the BER conditioned on u. Note that when taking the near-far effect into account, the variance in Equation 28 is easily multiplied by A^2, where $A=\sqrt{R_0^{(k)}/R_0^{(0)}}$.

For extracting P_{be} from $P_{be}|u$, it is assumed that the Walsh codes are assigned to different services in the system in a cyclic fashion. In other words, noting that there are totally N_s distinct Walsh codes used in the system, if e, defined as $k\xi+q$, is a counting index for the services in the system, the services with indices e_0 and e_0+N_s have the same Walsh codes and any N_s services with sequential indices have distinct Walsh codes. With this assignment and defining r as $\mathrm{mod}(N_u\xi,N_s)$, where "mod" denotes the remainder operation, three cases can be considered: 1) $N_u\xi\leq N_s$ implies that $u=0$ and $P_{be}=P_{be}|(u=0)$; 2) $N_u\xi>N_s$ and $r=0$ implies that $P_{be}=P_{be}|(u=N_u\xi/N_s-1)$; 3) $N_u\xi>N_s$ and $r\neq 0$ implies that

$u = u_1 = [N_u\xi/N_s] - 1$, with probability $1 - r/N_s$ and $u = u_2 = [N_u\xi/N_s]$, with probability r/N_s, where $[x]$ denotes the largest integer not greater than x; therefore, $P_{be} = (1 - r/N_s)P_{be}|u_1 + (r/N_s)P_{be}|u_2$.

Asynchronous Systems

For an asynchronous nonfading system, using Equations 2 and 7, and the definition of U in Equation 9, one obtains the following:

$$U = N_s + \mathrm{MAI} + n,$$

where:

$$\mathrm{MAI} = \sum_{k=0}^{N_u-1} \mathrm{MAI}^{(k)}, \tag{29a}$$

which:

$$\mathrm{MAI}^{(0)} = \sum_{q=1}^{\xi-1} \mathrm{MAI}^{(0,q)},$$

with:

$$\mathrm{MAI}^{(0,q)} = \sum_{m=0}^{N_s-1} d_0^{(0,q)} c_m^{(0,0)} c_m^{(0,q)} \eta_{(m,0)}^{(0,q)}.$$

Also,

$$\mathrm{MAI}^{(k)} = \mathrm{MAI}_{-1}^{(k)} + \mathrm{MAI}_0^{(k)} \quad \text{for } k \geq 1,$$

where:

$$\mathrm{MAI}_{-1}^{(k)} = \sum_{q=0}^{\xi-1} \mathrm{MAI}_{-1}^{(k,q)},$$

with:

$$\mathrm{MAI}_{-1}^{(k,q)} = \sum_{m=0}^{N_s-1} \sum_{v=0}^{N_s-1} \mathrm{MAI}_{-1,m,\nu}^{(k,q)},$$

which:

$$\mathrm{MAI}_{-1,m,\nu}^{(k,q)} = \sqrt{R_{-1}^{(k)}/R_0^{(0)}}\, d_{-1}^{(k,q)} c_m^{(k,q)} c_\nu^{(0,0)}$$

$$\cdot \begin{cases} \tau^{(k)} \cos\left\{\theta_{-1}^{(k)} - 2\pi\tau^{(k)}(Tf_m + h_{m,-1}^{(k,q)})\right\} & \text{for } \begin{cases} m = \nu, \text{ and} \\ h_{m,-1}^{(k,q)} = h_{\nu,0}^{(0,0)} \end{cases} \\ \dfrac{\sin\left\{\theta_{-1}^{(k)} - 2\pi\tau^{(k)}(Tf_\nu + h_{\nu,0}^{(0,0)})\right\} - \sin\left\{\theta_{-1}^{(k)} - 2\pi\tau^{(k)}(Tf_m + h_{m,-1}^{(k,q)})\right\}}{2\pi\left\{(m-\nu)N_h + h_{m,-1}^{(k,q)} - h_{\nu,0}^{(0,0)}\right\}} & \text{otherwise} \end{cases} \tag{29b}$$

Also, one has $\mathrm{MAI}_0^{(k)} = \sum_{q=0}^{\xi-1} \mathrm{MAI}_0^{(k,q)}$ with $\mathrm{MAI}_0^{(k,q)} = \sum_{m=0}^{N_s-1} \sum_{\nu=0}^{N_s-1} \mathrm{MAI}_{0,m,\nu}^{(k,q)}$, which:

$$\mathrm{MAI}_{0,m,\nu}^{(k,q)} = \sqrt{R_0^{(k)}/R_0^{(0)}}\, d_0^{(k,q)} c_m^{(k,q)} c_\nu^{(0,0)}$$

$$\cdot \begin{cases} (1-\tau^{(k)}) \cos\left\{\theta_0^{(k)} - 2\pi\tau^{(k)}(Tf_m + h_{m,0}^{(k,q)})\right\} & \text{for } \begin{cases} m = \nu, \text{ and} \\ h_{m,0}^{(k,q)} = h_{\nu,0}^{(0,0)} \end{cases} \\ \dfrac{\sin\left\{\theta_0^{(k)} - 2\pi\tau^{(k)}(Tf_m + h_{m,0}^{(k,q)})\right\} - \sin\left\{\theta_0^{(k)} - 2\pi\tau^{(k)}(Tf_\nu + h_{\nu,0}^{(0,0)})\right\}}{2\pi\left\{(m-\nu)N_h + h_{m,0}^{(k,q)} - h_{\nu,0}^{(0,0)}\right\}} & \text{otherwise} \end{cases} \tag{29c}$$

$\tau^{(k)}$ in Equation 29 is equal to $t_r^{(k)}/T$ and is assumed to be uniformly distributed in the interval $(0,1)$. Also, the $\tau^{(k)}$'s are independent. The $\theta_i^{(k)}$'s for different k's are independent uniformly distributed random variables in the interval $(-\pi,\pi)$. $\mathrm{MAI}_{-1}^{(k)}$ and $\mathrm{MAI}_0^{(k)}$ are parts of $\mathrm{MAI}^{(k)}$ related to -1th and 0th transmission intervals of user k, each consisting of $\mathrm{MAI}_{-1}^{(k,q)}$'s and $\mathrm{MAI}_0^{(k,q)}$'s (interference terms due to different services of user k), respectively. $\mathrm{MAI}_{-1,m,\nu}^{(k,q)}$ is the interference caused by the mth subcarrier of the qth service of user k at the -1th symbol interval in the νth subcarrier of the desired service. $\mathrm{MAI}_{0,m,\nu}^{(k,q)}$ denotes the same interference caused by the 0th symbol interval of user k. Note that the variance of the noise term in this case is the same as in the case of synchronous channels (see Equation 10d). As the exact BER calculation for the asynchronous case is very complicated, the BER is derived based on the Gaussian distribution for the MAI using Equation 27. Thus, one only needs to compute the variance of the MAI, which is obtained as follows:

$$\sigma_{\mathrm{MAI}}^2 = \sigma_{\mathrm{MAI}^{(0)}}^2 + \sum_{k=1}^{N_u-1} \sum_{q=0}^{\xi-1} (\sigma_{\mathrm{MAI}_{-1}^{(k,q)}}^2 + \sigma_{\mathrm{MAI}_0^{(k,q)}}^2). \tag{30}$$

Now, by variable changes, $\tau'^{(k)} = 1 - \tau^{(k)}$ and $\theta_0'^{(k)} = -\theta_0^{(k)}$ in Equation 29c and noting that $\tau'^{(k)}$ and $\theta_0'^{(k)}$ have the same statistics as $\tau^{(k)}$ and $\theta_0^{(k)}$, respectively and, assuming that $R_{-1}^{(k)} = R_0^{(k)}$ and Tf_m is an integer for any m, it can be easily realized that $\sigma_{\mathrm{MAI}_{-1}^{(k,q)}}^2$ and $\sigma_{\mathrm{MAI}_0^{(k,q)}}^2$ are equal. Hence, from Equation 30, in order to derive the BER of each system using Equation 27, one only needs to calculate $\sigma_{\mathrm{MAI}^{(0)}}^2$ and $\sigma_{\mathrm{MAI}_{-1}^{(k,q)}}^2$ for $R_{-1}^{(k)} = R_0^{(0)}$, which are computed in the succeeding sections. For considering the near-far effect, it is sufficient to multiply $\sigma_{\mathrm{MAI}_{-1}^{(k,q)}}^2$ (computed for $R_{-1}^{(k)} = R_0^{(0)}$) by $R_{-1}^{(k)}/R_0^{(0)}$.

MC-CDMA System

For this scheme, applying Equations 1a and 29b simplifies to:

$$\mathrm{MAI}^{(k,q)}_{1,m,\nu} = d^{(k,q)}_{1} c^{(k,q)}_{m} c^{(0,0)}_{\nu}$$

$$\cdot \begin{cases} \tau^{(k)} \cos(\theta^{(k)}_{1} \quad 2\pi\tau^{(k)} T f_m), & \text{for } m=\nu \\ \frac{\sin(\theta^{(k)}_{1} \ \ 2\pi\tau^{(k)} T f_\nu) \ \ \sin(\theta^{(k)}_{1} \ \ 2\pi\tau^{(k)} T f_m)}{2\pi(m \quad \nu)} & \text{otherwise} \end{cases}. \tag{31}$$

PN Signature Codes

Just as in the synchronous case, $\mathrm{MAI}^{(0)}$, in this case, is equal to the sum of $(\xi \quad 1)N_s$ independent uniformly distributed binary-valued random variables and, therefore, one has $\sigma^2_{\mathrm{MAI}^{(0)}} = (\xi \quad 1)N_s$. From Equation 31, it can simply be realized that $\mathrm{MAI}^{(k,q)}_{1,m,\nu}$'s, for different m's and ν's, are uncorrelated and, therefore, from Equation 29, $\sigma^2_{\mathrm{MAI}^{(k,q)}_{1}}$ is easily obtained as $\sum_{m=0}^{N_s \ 1}\sum_{\nu=0}^{N_s \ 1} \sigma^2_{\mathrm{MAI}^{(k,q)}_{1,m,\nu}}$. From Equation 31, after some simple statistical calculations, $\sigma^2_{\mathrm{MAI}^{(k,q)}_{1,m,\nu}}$ is acquired as follows:

$$\sigma^2_{\mathrm{MAI}^{(k,q)}_{1,m,\nu}} = \begin{cases} 1/6 & \text{for } m=\nu \\ \frac{1}{4\pi^2}\frac{1}{(m \quad \nu)^2}, & \text{otherwise} \end{cases}. \tag{32}$$

Walsh Signature Codes

In this case, just as in the synchronous case, $\mathrm{MAI}^{(0)}$ vanishes and $\sigma^2_{\mathrm{MAI}^{(0)}} = 0$. In contrast to the PN-signature case, $\mathrm{MAI}^{(k,q)}_{1,m,\nu}$'s in Equation 31, for different m's and ν's, are correlated. After some lengthy calculations, which are dropped here for briefness, the following result is easily attained:

$$\sigma^2_{\mathrm{MAI}^{(k,q)}_{1}} = \frac{N_s}{6} + \frac{1}{4\pi^2}\sum_{m=0}^{N_s \ 1}\sum_{\substack{v=0\\ v\neq m}}^{N_s \ 1}$$

$$\cdot\frac{c^{(k,q)}_{m} c^{(k,q)}_{\nu}(2c^{(0,0)}_{m} c^{(0,0)}_{\nu} \quad 1) \quad c^{(0,0)}_{m} c^{(0,0)}_{\nu} + 1}{(m \quad \nu)^2}$$

$$+\frac{1}{8\pi^2}\sum_{m=0}^{N_s \ 1}\sum_{\substack{\nu_1=0\\ \nu_1\neq m}}^{N_s \ 1}\sum_{\substack{\nu_2=0\\ \nu_2\neq m\\ \nu_2\neq\nu_1}}^{N_s \ 1}$$

$$\cdot\frac{c^{(0,0)}_{\nu_1} c^{(0,0)}_{\nu_2} + c^{(k,q)}_{\nu_1} c^{(k,q)}_{\nu_2} + 4c^{(k,q)}_{m} c^{(0,0)}_{m} c^{(k,q)}_{\nu_1} c^{(0,0)}_{\nu_2}}{(m \quad \nu_1)(m \quad \nu_2)}. \tag{33}$$

Hopping Systems (MC-FH and DS-MC-FH)

In hopping systems, contrary to MC-CDMA schemes, the frequency spacing between neighboring subcarriers is so large (this spacing in hopping systems on average is N_h times greater than that of MC-CDMA schemes) that the subcarrier spectra can well be assumed to be disjoint. In other words, $\mathrm{MAI}^{(k,q)}_{1,m,\nu}$ can well be assumed to be zero for $m \neq \nu$. For shortness, hereafter, $\mathrm{MAI}^{(k,q)}_{1,m,m}$ is denoted by $\mathrm{MAI}^{(k,q)}_{1,m}$. Now, from Equation 29b, one has the following:

$$\mathrm{MAI}^{(k,q)}_{1} \sum_{m=0}^{N_s \ 1} \mathrm{MAI}^{(k,q)}_{1,m}, \tag{34a}$$

with:

$$\mathrm{MAI}^{(k,q)}_{1,m} = d^{(k,q)}_{1} c^{(k,q)}_{m} c^{(0,0)}_{m}$$

$$\cdot\begin{cases} \tau^{(k)}\cos\left\{\theta^{(k)}_{1} \quad 2\pi\tau^{(k)}(Tf_m + h^{(k,q)}_{m,\ 1})\right\} \\ \qquad\qquad \text{for } h^{(k,q)}_{m,\ 1} = h^{(0,0)}_{m,0} \\ \frac{\sin\left\{\theta^{(k)}_{1} \ \ 2\pi\tau^{(k)}\left(Tf_m + h^{(0,0)}_{m,0}\right)\right\} \ \ \sin\left\{\theta^{(k)}_{1} \ \ 2\pi\tau^{(k)}\left(Tf_m + h^{(k,q)}_{m,\ 1}\right)\right\}}{2\pi\left(h^{(k,q)}_{m,\ 1} \ \ h^{(0,0)}_{m,0}\right)} \\ \qquad\qquad \text{otherwise} \end{cases} \tag{34b}$$

DS-MC-FH with PN Second-Signature Codes

For this scheme, $\mathrm{MAI}^{(0)}$ is just the same as in MC-CDMA systems with PN codes and, therefore, $\sigma^2_{\mathrm{MAI}^{(0)}}$ is obtained as $(\xi \quad 1)N_s$. As the $\mathrm{MAI}^{(k,q)}_{1,m}$'s are uncorrelated, $\sigma^2_{\mathrm{MAI}^{(k,q)}_{1}}$ is equal to the sum of the $\sigma^2_{\mathrm{MAI}^{(k,q)}_{1,m}}$'s over all m's. Knowing the statistics of the $d^{(k,q)}_{i}$'s, $h^{(k,q)}_{m,i}$'s, $c^{(k,q)}_{m}$'s, $\tau^{(k)}$'s and $\theta^{(k)}_{i}$'s and noting that $h^{(k,q)}_{m,\ 1}$ is equal to $h^{(0,0)}_{m,0}$ with probability $1/N_h$, it can easily be shown that the following is true:

$$\sigma^2_{\mathrm{MAI}^{(k,q)}_{1,m}} = \beta\left(\frac{1}{6}\right)$$

$$+\alpha\left\{\frac{1}{4\pi^2 N_h^2}\sum_{a=0}^{N_h \ 1}\sum_{\substack{b=0\\ b\neq a}}^{N_h \ 1}\frac{1}{(a \quad b)^2}\right\}. \tag{35}$$

From Equation 35, $\sigma^2_{\mathrm{MAI}^{(k,q)}_{1,m}}$ does not depend on m, so, $\sigma^2_{\mathrm{MAI}^{(k,q)}_{1}}$ is obtained as $N_s.\sigma^2_{\mathrm{MAI}^{(k,q)}_{1,m}}$.

DS-MC-FH with Walsh Second-Signature Codes

In this case, $\mathrm{MAI}^{(0)}$ vanishes like the synchronous case and one only needs to compute $\sigma^2_{\mathrm{MAI}^{(k,q)}_{1}}$, which, from Equation 34a, is acquired as follows:

$$\sigma^2_{\mathrm{MAI}^{(k,q)}_{1}} = \sum_{m=0}^{N_s \ 1}\sigma^2_{\mathrm{MAI}^{(k,q)}_{1,m}}$$

$$+\sum_{m_1=0}^{N_s \ 1}\sum_{\substack{m_2=0\\ m_2\neq m_1}}^{N_s \ 1} E\left\{\mathrm{MAI}^{(k,q)}_{1,m_1}.\mathrm{MAI}^{(k,q)}_{1,m_2}\right\}, \tag{36}$$

$\sigma^2_{\mathrm{MAI}^{(k,q)}_{1,m}}$ is obtained just the same as in Equation 35. However, in contrast to the PN-signature case, $\mathrm{MAI}^{(k,q)}_{1,m}$'s for different m's are correlated and, from Equation 34b, after some simple statistical calculations, one obtains the following:

$$E\left\{\mathrm{MAI}^{(k,q)}_{1,m_1}.\mathrm{MAI}^{(k,q)}_{1,m_2}\right\}=\frac{\beta c^{(k,q)}_{m_1}c^{(0,0)}_{m_1}c^{(k,q)}_{m_2}c^{(0,0)}_{m_2}}{4\pi^2N_h^3}$$

$$.\Big\{\sum_{a=0}^{N_h\ \ 1}\sum_{b=0}^{N_h\ \ 1}\frac{\beta N_h}{\{(m_1\ \ \ m_2)N_h+a\ \ \ b\}^2}$$

$$+\sum_{a=0}^{N_h\ \ 1}\sum_{b_1=0}^{N_h\ \ 1}\sum_{\substack{b_2=0\\ b_2\neq b_1}}^{N_h\ \ 1}$$

$$.\frac{\alpha}{\{(m_1\ \ \ m_2)N_h+\alpha\ \ \ b_1\}\{(m_1\ \ \ m_2)N_h+a\ \ \ b_2\}}\Big\}. \tag{37}$$

MC-FH

In this case, $\mathrm{MAI}^{(0)}$ is the same as the corresponding term in synchronous MC-FH and, thus, $\sigma^2_{\mathrm{MAI}^{(0)}}$ is obtained as $(\xi\ \ \ 1)N_s\beta(N_s\beta+\alpha)$ from Equation 18. Applying Equation 1b in Equation 34b, $\sigma^2_{\mathrm{MAI}^{(k,q)}_{1}}$ is attained, via Equation 35 to 37, with $c^{(k,q)}_m$'s all set to one.

PERFORMANCE EVALUATION IN FADING CHANNELS

For the performance evaluation in fading channels, synchronous single-service ($\xi=1$) systems are considered. From Equations 7 and 8, with $R_0^{(k)}=R_0^{(0)}$ for any k, one can write the following:

$$U=S+\mathrm{MAI}+n,\quad \text{where}\quad S=\mathbf{g}^{(0)H}\mathbf{g}^{(0)},$$

$$\mathrm{MAI}=\mathrm{Re}[I],\quad I=\mathbf{g}^{(0)H}\sum_{k=1}^{N_u\ \ 1}d^{(k)}\mathbf{V}^{(k)}\mathbf{g}^{(k)},$$

and:

$$\sigma^2_{n|\mathbf{g}^{(0)}}=\frac{N_s}{2\gamma_b}\mathbf{g}^{(0)H}\mathbf{g}^{(0)}, \tag{38a}$$

with:

$$\mathbf{g}^{(k)}\triangleq\left[g_0^{(k)}g_1^{(k)}\cdots g^{(k)}_{N_s\ \ 1}\right]^T,$$

$$\mathbf{V}^{(k)}\triangleq\mathrm{diag}\left\{\nu_0^{(k)},\nu_1^{(k)},\cdots,\nu^{(k)}_{N_s\ \ 1}\right\},$$

$$\nu_m^{(k)}\triangleq c_m^{(0)}c_m^{(k)}\eta_m^{(k)}. \tag{38b}$$

$\sigma^2_{n|\mathbf{g}^{(0)}}$ is the variance of n conditioned on $\mathbf{g}^{(0)}$. In Equation 38 and throughout this section, as ξ is equal to 1 and all the parameters are related to 0th symbol intervals, the indices, q and i, have been dropped. Note that $\eta_m^{(k)}$ has the same definition and statistics as given in Equations 10c and 11, with $q=0$ and $i=0$. From Equation 38, U conditioned on $\mathbf{g}^{(0)}$ and $\nu_m^{(k)}$'s is a Gaussian random variable with the following mgf:

$$\mathbf{M}_{U|\mathbf{g}^{(0)},\nu_m^{(k)}}(s)=\mathbf{M}_{S|\mathbf{g}^{(0)}}(s)\mathbf{M}_{\mathrm{MAI}|\mathbf{g}^{(0)},\nu_m^{(k)}}(s)\mathbf{M}_n(s). \tag{39}$$

Using the mgf of a Gaussian variable, from Equations 38 and 39 and having that MAI is a zero-mean variable, one has the following:

$$M_{U|\mathbf{g}^{(0)},\nu_m^{(k)}}(s)=\exp\left\{\mathbf{g}^{(0)H}\left[s\mathbf{I}_{N_s}+\frac{s^2}{4}\frac{N_s}{\gamma_b}\mathbf{I}_{N_s}\right]\mathbf{g}^{(0)}+\frac{s^2}{2}\sigma^2_{\mathrm{MAI}|\mathbf{g}^{(0)},\nu_m^{(k)}}\right\}, \tag{40}$$

wherein $\mathbf{I}_{N_s}$ is the identity matrix of order N_s. $\sigma^2_{\mathrm{MAI}|\mathbf{g}^{(0)},\nu_m^{(k)}}$ is, obviously, half of the variance of $I|\mathbf{g}^{(0)},\nu_m^{(k)}$, defined in Equation 38a. Now, using the fact that $\mathbf{g}^{(k)}$'s are independent, one can write as follows:

$$\sigma^2_{\mathrm{MAI}|\mathbf{g}^{(0)},\nu_m^{(k)}}=\frac{1}{2}E\left\{II^H|\mathbf{g}^{(0)},\mathbf{V}^{(k)}\right\}$$

$$=\frac{1}{2}\mathbf{g}^{(0)}\left[\sum_{k=1}^{N_u\ \ 1}d^{(k)}\mathbf{V}^{(k)}E\{\mathbf{g}^{(k)}\mathbf{g}^{(k)H}\}\mathbf{V}^{(k)}d^{(k)}\right]\mathbf{g}^{(0)H}$$

$$=\frac{1}{2}\mathbf{g}^{(0)}\left(\sum_{k=1}^{N_u\ \ 1}\mathbf{V}^{(k)}\mathbf{C}\mathbf{V}^{(k)}\right)\mathbf{g}^{(0)H}$$

$$=\frac{1}{2}\mathbf{g}^{(0)}(\mathbf{X}\bullet\mathbf{C})\mathbf{g}^{(0)H},$$

where:

$$\mathbf{X}=\sum_{k=1}^{N_u\ \ 1}X^{(k)}\quad\text{and :}$$

$$\mathbf{X}^{(k)}\triangleq\left[\nu_0^{(k)}\nu_1^{(k)}\cdots\nu^{(k)}_{N_s\ \ 1}\right]^T\left[\nu_0^{(k)}\nu_1^{(k)}\cdots\nu^{(k)}_{N_s\ \ 1}\right]. \tag{41}$$

$\mathbf{C}$ and $\mathbf{V}^{(k)}$ are defined in Equations 4 and 38b, respectively. "$\bullet$" is the matrix-matrix dot product operator and the superscript, "T", stands for transposition. Note that $\mathbf{X}^{(k)}\bullet\mathbf{C}$ is, in fact, the covariance matrix of $\mathbf{V}^{(k)}\mathbf{g}^{(k)}$, conditioned on $\mathbf{V}^{(k)}$. As for any given $\mathbf{V}^{(k)}$, $\mathbf{V}^{(k)}\mathbf{g}^{(k)}$ is a Gaussian random vector and the unconditional distribution of $\mathbf{V}^{(k)}\mathbf{g}^{(k)}$ as Gaussian distribution is well approximated. Hence, removing conditionality on $\nu_m^{(k)}$ from Equation 40, for calculating $M_{U|\mathbf{g}^{(0)}}(s)$, one needs only to compute $\sigma^2_{\mathrm{MAI}|\mathbf{g}^{(0)}}$. From, Equation 41 one easily obtains the following:

$$\sigma^2_{\mathrm{MAI}|\mathbf{g}^{(0)}} = \frac{1}{2}\mathbf{g}^{(0)}(E\{\mathbf{X}\}\bullet\mathbf{C})\mathbf{g}^{(0)H}. \tag{42}$$

From the definition of $\mathbf{V}^{(k)}$ and $\mathbf{X}$ in Equations 38b and 41, respectively, it is easy to show that:

$$E\{\mathbf{X}\} = (N_u - 1)\alpha\beta\mathbf{I}_{N_s} + \beta^2 E\{\mathbf{L}\mathbf{L}^T\}, \tag{43}$$

where $\beta = 1 - \alpha = 1/N_h$ and $\mathbf{L}$ is an $N_s \times (N_u - 1)$ matrix with its entry, (m,k), equal to $c_m^{(0)}c_m^{(k)}$. From Equations 40 (after removing conditionality on $v_m^{(k)}$), 42 and 43, one obtains the following:

$$M_{U|\mathbf{g}^{(0)}}(s) = \exp\left\{\mathbf{g}^{(0)H}\mathbf{Q}(s)\mathbf{g}^{(0)}\right\},$$

with:

$$\mathbf{Q}(s) = s\mathbf{I}_{N_s} + \frac{s^2}{4}\left\{\beta^2 E\{\mathbf{L}\mathbf{L}^T\}\bullet\mathbf{C} + \left[(N_u - 1)\alpha\beta + \frac{N_s}{\gamma_b}\right]\mathbf{I}_{N_S}\right\}. \tag{44}$$

As $\mathbf{g}^{(0)}$ is a PCG random vector, taking the expectation of Equation 44 results in the following [20]:

$$M_U(s) = \det\{\mathbf{I}_{N_s} - \mathbf{Q}(s)\mathbf{C}\}^{-1}. \tag{45}$$

In cases where Equation 45 leads to an explicit expression for the pdf of U, the BER can be computed by evaluating the cdf of U at zero. Otherwise, one can make use of the Beaulieu series [21] in the following manner, in order to obtain the BER directly from the mgf of U:

$$P_{be} = \Pr\{U < 0\} = \lim_{\substack{L\to\infty \\ N\to\infty}} F_U(0;L,N),$$

where:

$$F_U(0;L,N) = \frac{1}{2} - \sum_{\substack{n=1 \\ n\ \mathrm{odd}}}^{N} \frac{2}{n\pi}\mathrm{Im}\left\{M_U\left(j\frac{2n\pi}{L}\right)\right\}. \tag{46}$$

Im{.} denotes the imaginary part. From the error bounds given in [21], $F_U(0;L,100)$, with $L = |\mu_u| + 20\sigma_U$, where μ_U and σ_U are the mean and standard deviation of U, respectively, is a very good approximation of P_{be}.

In the subsequent sections, the mgf of U is evaluated for the hopping schemes. The pertinent expressions for MC-CDMA are derived from those of DS-MC-FH systems by substituting $N_h = 1$.

DS-MC-FH

PN Signature Code

In this case, from the statistics of $c_m^{(k)}$, it can simply be shown that $E\{\mathbf{L}\mathbf{L}^T\} = (N_u - 1)\mathbf{I}_{N_s}$. Therefore, from Equations 44 and 45 one has the following:

$$M_U(s) = \det\left\{\mathbf{I}_{N_s} - \left(s + \frac{1}{4}\left[\beta(N_u - 1) + N_s/\gamma_b\right]s^2\right)\mathbf{C}\right\}^{-1}. \tag{47}$$

Assuming that the eigenvalues of $\mathbf{C}$ are distinct, after some simplifications, one obtains the following:

$$M_U(s) = M_U^-(s) + M_U^+(s),$$

where:

$$M_U^-(s) = \prod_{m=0}^{N_s-1}\left(\frac{4\lambda_m^{-1}}{\beta(N_u - 1) + \frac{N_s}{\gamma_b}}\right)\sum_{m=0}^{N_s-1}\frac{a_{m,-1}}{s - p_{m,-1}},$$

and:

$$M_U^+(s) = \prod_{m=0}^{N_s-1}\left(\frac{4\lambda_m^{-1}}{\beta(N_u - 1) + \frac{N_s}{\gamma_b}}\right)\sum_{m=0}^{N_s-1}\frac{a_{m,+1}}{p_{m,+1} - s}.$$

λ_m's are eigenvalues of $\mathbf{C}$. $p_{m,-1}$, $p_{m,+1}$, $a_{m,-1}$ and $a_{m,+1}$ are given by:

$$p_{m,\pm 1} = 2\left(1 \pm \sqrt{1 + \left\{\beta(N_u - 1) + \frac{N_s}{\gamma_b}\right\}/\lambda_m}\right)^{-1},$$

and:

$$a_{m,\pm 1} = \frac{1}{p_{m,+1} - p_{m,-1}}\prod_{\substack{n=0 \\ n\neq m}}^{N_s-1}\frac{1}{(p_{m,\pm 1} - p_{n,-1})(p_{n,+1} - p_{m,\pm 1})}. \tag{48}$$

As $\mathbf{C}$ is positive definite, λ_m's are all positive. Therefore, $p_{m,-1}$'s are all negative and $p_{m,+1}$'s are all positive. Since the region of convergence of $M_U(s)$ includes the axis $s = j\omega$, $M_U^-(s)$ and $M_U^+(s)$ are the right-sided and left-sided parts of $M_U(s)$, respectively and, therefore, the BER is equal to $M_U^-(0)$. After some algebraic manipulations, the BER is obtained as follows:

$$P_{be} = \frac{1}{2}\left\{1 - \sum_{m=0}^{N_s-1}\left\{1 + \left[\beta(N_u - 1) + \frac{N_s}{\gamma_b}\right]/\lambda_m\right\}^{-\frac{1}{2}} \prod_{\substack{n=0 \\ n\neq m}}^{N_s-1}(1 - \lambda_n/\lambda_m)^{-1}\right\}. \tag{49}$$

For $N_h = 1$ and $N_u = 1$, the result in Equation 49 is in compliance with the one obtained in [3].

Walsh Signature Code

In this case, $\mathbf{L}$ is deterministic and $E\{\mathbf{LL}^T\} = \mathbf{LL}^T$. An explicit expression for the BER cannot be derived, so, the Beaulieu series is exploited (see Equation 46) for extracting the probability of error.

MC-FH

For this system, all the elements of $\mathbf{L}$ are equal to 1, so, from Equations 44 and 45, one obtains the following:

$$M_U(s) = \det\left\{\mathbf{I}_{N_s} \quad \left[s + \frac{1}{4}\left(\alpha\beta(N_u \quad 1) + \frac{N_s}{\gamma_b}\right)S^2\right]\mathbf{C} \quad \frac{\beta^2}{4}(N_u \quad 1)s^2\mathbf{C}^2\right\}^{\;1}, \tag{50}$$

which, following similar steps from Equations 47 to 49, gives the BER as:

$$P_{be} = \prod_{m=0}^{N_s \;\; 1} \frac{4}{\lambda_m\left[\alpha\beta(N_u \quad 1) + N_s/\gamma_b + \lambda_m\beta^2(N_u \quad 1)\right]} \sum_{m=0}^{N_s \;\; 1}\left(\quad \frac{a_{m,\;\;1}}{p_{m,\;\;1}}\right),$$

where $a_{m,\;\;1}$ is as given in Equation 48, with the parameters $p_{m,\;\;1}$ and $p_{m,+1}$ defined as:

$$p_{m,\pm 1} = \frac{2}{\lambda_m}\left\{1 \pm \sqrt{1 + \beta^2(N_u \quad 1) + \lambda_m^{\;\;1}\left[\alpha\beta(N_u \quad 1) + \frac{N_s}{\gamma_b}\right]}\right\}^{\;1}. \tag{51}$$

For studying the near-far effect, one needs only to multiply the variance of the multiple-access interference by $R/R^{(0)}$ (like the analyses in nonfading channels) and modify Equations 38 to 51, accordingly.

NUMERICAL RESULTS

In this Section, some numerical results are provided to compare the three multicarrier schemes, namely, MC-CDMA, MC-FH and DS-MC-FH, in nonfading and fading channels. Comparisons are made within a constant bandwidth. In all the graphs presented (Figures 5 to 13), N_s, N_h, N_u, ξ, BW and SNR signify the number of frequency subbands, the number of subcarriers in each frequency subband, the number of users, the number of services of each user, the normalized bandwidth (product of N_s and N_h) and γ_b (as defined in Equation 8), respectively. Also, T_r, defined as T_m/T (T_m is the maximum delay spread used in Equation 5), denotes the relative delay spread of the channel. μ is the decaying factor in Equation 5. (P) and (W) stand for the PN and Walsh schemes, respectively. The pair, (x, y), after the name of a hopping system, indicates N_s and N_h for that system ($N_s = x$ and $N_h = y$). In the cases where Walsh codes are utilized, the cyclic assignment of Walsh codes have been used, as described in the section of 'MS-MC-FH System'.

Figures 5 to 7 present plots of the BER versus the number of active users in different systems, separately. For comparison, in synchronous cases, the plots of the BER have also been included, based on the Gaussian distribution assumption for the multiple-access interference using Equation 27. The exact and Gaussian plots for MC-CDMA and DS-MC-FH systems with PN codes almost coincide. This uniformity is justified when noting that the multiple-access interference term generally contains sums of iid random variables and the CLT is, thus, applicable. The same is true for asynchronous environments and, therefore, the validity of the Gaussian approximation is retained. From

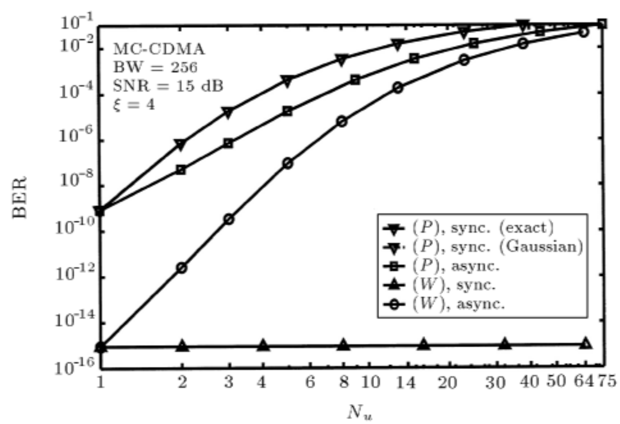

Figure 5. Performance of synchronous and asynchronous MC-CDMA systems in nonfading channels versus N_u with constant BW, SNR and ξ.

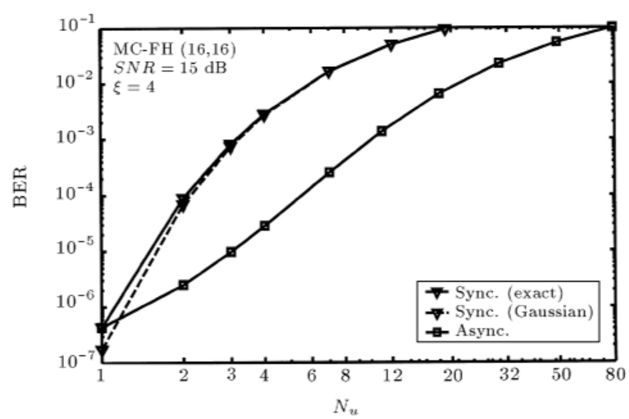

Figure 6. Performance of synchronous and asynchronous MC-FH systems in nonfading channels versus N_u with constant N_s, N_h, SNR and ξ.

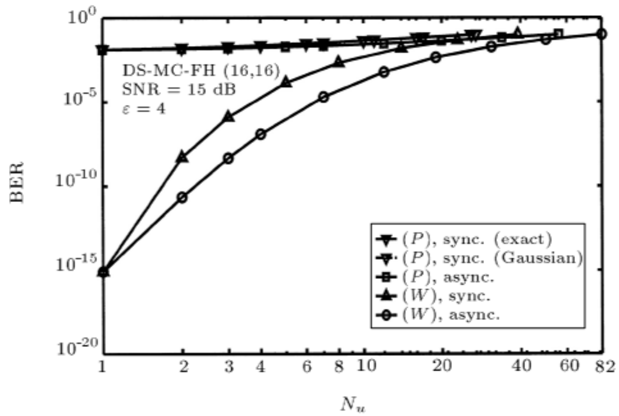

Figure 7. Performance of synchronous and asynchronous DS-MC-FH systems in nonfading channels versus N_u with constant N_s, N_h, SNR and ξ.

these figures, except in the case of MC-CDMA with Walsh codes (where the synchronous system, as a result of a complete orthogonality between signatures, outperforms the asynchronous system), an asynchronous scheme (because of the lower variance of the multiple-access interference term) has a better performance of up to two orders of magnitude than the synchronous scheme. Also, from Figure 7, both the synchronous and asynchronous DS-MC-FH systems with PN codes, perform much worse than the DS-MC-FH systems with Walsh codes, verifying the inappropriateness of PN codes as the second signature in DS-MC-FH schemes.

In Figure 8, the performance of MC-FH schemes versus N_s is evaluated when the bandwidth is constant. The phenomenon of having an optimum value for N_s is observed for synchronous and asynchronous cases alike. In fact, an increase in N_s, on the one hand, would enhance the frequency diversity of the system and, on the other hand, would augment the variance of the multiple-access interference dramatically as a result of modulating all the transmitted subcarriers

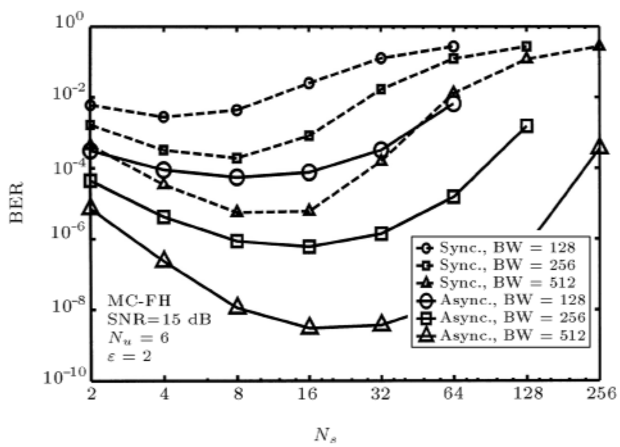

Figure 8. Performance of synchronous and asynchronous MC-FH systems in nonfading channels versus N_s with constant BW, SNR, N_u and ξ.

with an identical uncoded bit, as shown in Figure 2, thus, causing a great correlation among the signals carried by the transmitted subcarriers. For example, in the synchronous case, according to Equation 18, the variance induced, due to a single interfering service, is equal to $N_s\beta(N_s\beta + \alpha)$, which is a fourth-degree function of N_s, when taking the bandwidth constant. As a result of the above trade-off, the existence of an optimum N_s is foreseeable. Similar results in fading environments, dropped here due to space limitation, retain this phenomenon. An optimum N_s, however, will not be obtained for the other schemes and the performance is constantly improved by increasing N_s. In the other schemes, in contrast to MC-FH systems, the data stream is coded by a signature before being sent over transmitted subcarriers.

In Figures 9 and 10, the three schemes have been compared in asynchronous nonfading and synchronous fading channels, respectively. From these figures, it can be realized that the systems with Walsh codes perform much better than the systems with PN codes.

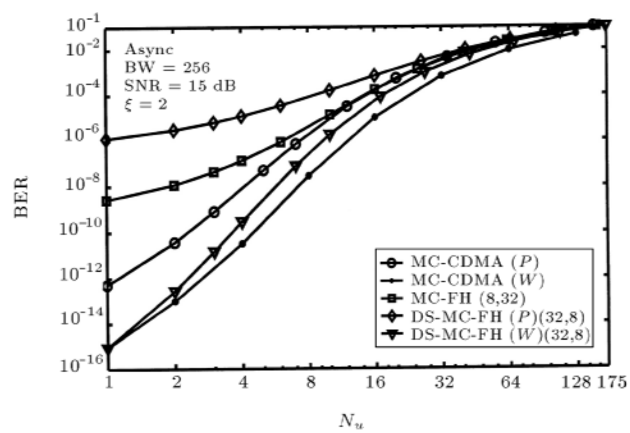

Figure 9. Performance of different asynchronous systems in nonfading channels versus N_u with constant BW, N_s, N_h, SNR and ξ.

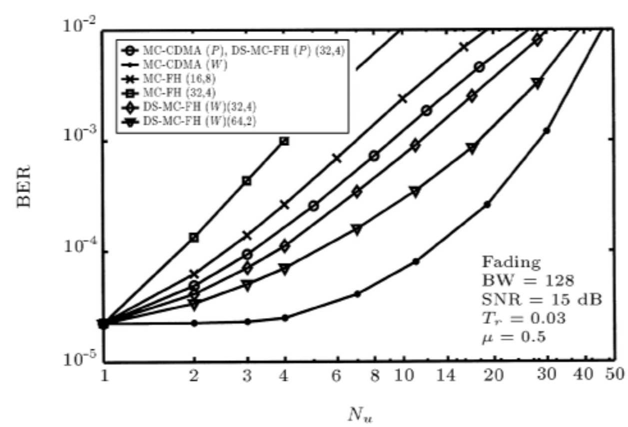

Figure 10. Performance of different synchronous systems in fading channels versus N_u with constant BW, N_s, N_h, SNR, T_r and μ.

Also, MC-CDMA schemes with PN codes outperform MC-FH schemes. MC-CDMA systems with PN codes always perform better than DS-MC-FH schemes with PN codes. However, the performance of the latter obviously approaches that of the former, as N_s increases for a given bandwidth. This can be seen in Figure 10, where the DS-MC-FH system with PN codes almost has the same performance as the MC-CDMA system with PN codes.

For studying the near-far effect, Figure 11 presents the plots of the BER versus $R_0^{(k)}/R_0^{(0)}$ in dB for different systems in a synchronous fading environment. From this figure, it is apparent that, for the single-user MRC detector under consideration, except for MC-CDMA schemes with Walsh codes, where the orthogonality among the codes is retained under the near-far effect but not in fading environments, the other systems are not near-far resistant. Note that in DS-MC-FH schemes with Walsh codes, the hopping itself rather distorts the orthogonality among the codes.

In Figures 12 and 13, the performances of different

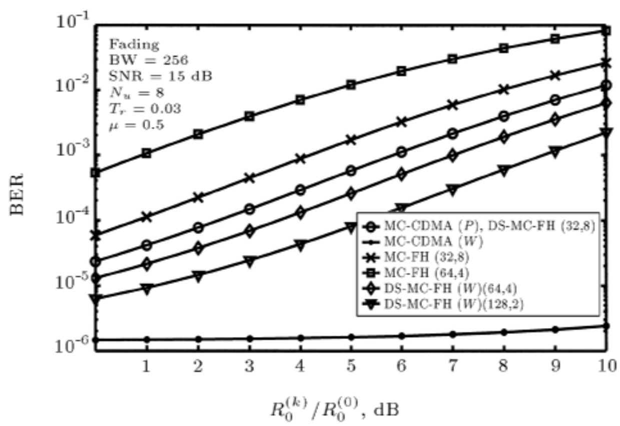

Figure 11. Performance of different synchronous systems in fading channels considering the near-far effect with constant BW, N_s, N_h, SNR, N_u, T_r and μ.

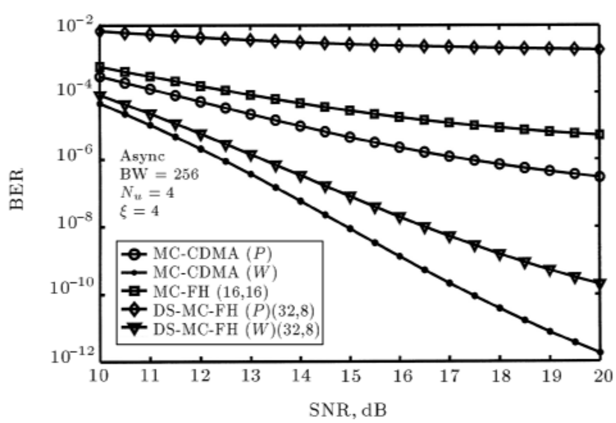

Figure 12. Performance of different asynchronous systems in nonfading channels versus SNR with constant BW, N_s, N_h, N_u and μ.

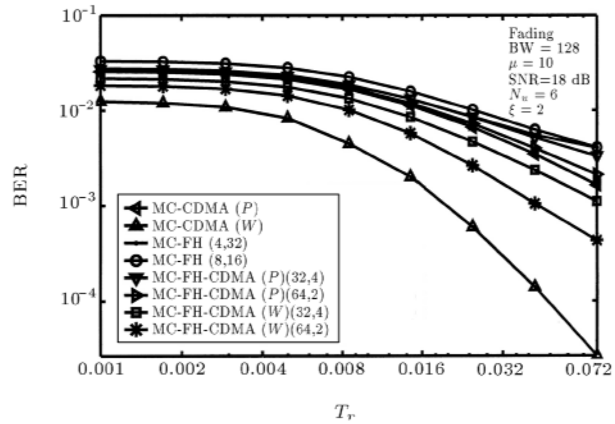

Figure 13. Performance of different systems in fading channels versus T_r with constant BW, μ, SNR, N_u and ξ.

systems are compared versus SNR and T_r in nonfading and fading channels, respectively. The performance priority order of different schemes is much clearer in these figures. These figures show a noticeable improvement in performance by augmenting SNR or T_r. In DS-MC-FH schemes with PN codes, the multiple-access interference term is dominant in relation to the noise term. The variance of the MAI term that is obtained from Equation 30 as $N_s(\xi \quad 1) + 2(N_u \quad 1)N_s\xi\sigma^2_{\mathrm{MAI}^{(k,q)}_{1,m}}$ with $\sigma^2_{\mathrm{MAI}^{(k,q)}_{1,m}}$ given in Equation 35, is quite large, with respect to the variance of the noise term, which is equal to $N_s^2/(2\gamma_b)$ (see Equation 10d). Therefore, as can be seen in Figure 12, the performance improvement with SNR in the DS-MC-FH system is not as evident as that of the other schemes. In Figure 13, an increase in T_r is simply translated into narrowing the spectrum of the pdp of the channel described in Equation 5, thereby, diminishing the correlation between the fading gains at adjacent transmitted subcarriers.

CONCLUSION

A unified multiple-access performance analysis of several multimedia multicarrier CDMA systems in synchronous and asynchronous nonfading and synchronous correlated Rayleigh fading channels, considering the near-far effect, was provided. The authors analysis has first indicated that MC-CDMA systems have the best performance. Furthermore, it has been found that the schemes with Walsh codes outperform the other systems in both nonfading and fading channels. Also, it has been realized that for the single-user MRC detector considered in this paper, MC-CDMA schemes with Walsh codes have considerable robustness against the near-far effect in synchronous channels. However, the other systems are vulnerable to the near-far effect.

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APPENDIX

In this Appendix, the result given in Equation 16 is derived. According to [19], if X is an integer-valued random variable with cdf $F_X[n] = E\{X \leq n\}$ and mgf $M_X(z) = E\{z^X\}$, then:

$$F_X[n] = \lim_{L\to\infty} F_X[n;L],$$

with:

$$F_X[n;L] = \frac{1}{2L}\sum_{k=0}^{2L\ \ 1} \mu_L[k] M_X\left(e^{jk\pi/L}\right) e^{\ \ jnk\pi/L}, \tag{A1}$$

where:

$$\mu_L[k] = \begin{cases} L & k=0 \\ 1 + j\cos[k\pi/(2L)] & k \text{ odd} \\ 0 & \text{otherwise} \end{cases}, \tag{A2}$$

and, furthermore, one has the following error bounds [19]:

$$\Pr\{X > L+n\} \leq F_X[n] \quad F_X[n;L] \leq \Pr\{X \leq \quad L+n\}. \tag{A3}$$

Now, if $p_X[n]$ is the probability function of X ($p_X[n] = \Pr\{X = n\}$), from $p_X[n] = F_X[n] - F_X[n-1]$ and Equations A1 to A3, one may easily conclude that:

$$p_X[n] = \lim_{L\to\infty} p_X[n;L],$$

where:

$$p_X[n;L] = \frac{1}{L}\sum_{\substack{k=1\\ k \text{ odd}}}^{2L-1} M_X\left(e^{jk\pi/L}\right) e^{-jnk\pi/L}, \qquad \text{(A4)}$$

with the following error bounds:

$$\Pr\{|X-n| > L\} \le p_X[n] - p_X[n;L] \le \Pr\{|X-n| \ge L\}. \qquad \text{(A5)}$$

Therefore, if $p_X[n]$ is nonzero for a finite set of integers, then, choosing a sufficiently large L results in $p_X[n;L] = p_X[n]$. Now, as $p_{\mathrm{MAI}}[a]$ is zero for $|a|$ greater than $P = N_s(N_u\xi - 1)$ (see Equation 12), one has $p_{\mathrm{MAI}}[a] = p_{\mathrm{MAI}}[a;2P+1]$ for any a with $|a| \le P$; thus, Equation 16 is derived.