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Research Note

Capacity Analysis of a CDMA Cellular System with Mixed Cell Sizes and Imperfect Power Control

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The demands for mobile communication services are growing rapidly. In heavily populated areas, cell splits are necessary to increase the capacity of cellular systems. Cell splitting causes a cellular system to have mixed cell sizes. Calculation of the reverse link capacity of CDMA cellular systems with mixed cell sizes and imperfect power control is presented in this paper. When a macro cell is split into micro cells, the capacity of a non-uniform cellular system has been calculated, together with the calculation of micro and neighboring macro cell capacity. The computer simulation results show that with the reduction in radius of a micro cell, its capacity increases, while that of the neighboring macro cells decreases.

INTRODUCTION

The use of the CDMA cellular system is growing rapidly with the increasing demands of mobile communication services. The capacity of CDMA systems is larger than FDMA and TDMA [1,2]. The level of interference, due to users working on the same frequency band, is the main reason for limitations in the capacity of CDMA systems. Cell size in a cellular system depends on the population and communication traffic of users in each area. Cell split is a good method for increasing capacity in a heavily populated area. However, cellular systems will become non-uniform using splitting. Then, the cellular system is configured with cells of mixed sizes. All cells have a similar condition and capacity in a uniform cellular system, but, in a non-uniform system, different neighboring cells and the interferences received by individual cells are different from each other. As a result, each cell will have a different reverse link capacity. Calculation of micro and macro cell capacity is the main step towards obtaining non-uniform cellular capacity. CDMA capacity calculations for uniform cellular systems have been presented in several references [1,3]. Hyoung-Goo Jeon [3] has calculated the capacity of non-uniform systems by considering perfect power control. However, the assumption of a perfect power control, due to non-accuracy of power measurement, delay, dynamic range of transmission power and step size of power adjustment, is not valid. As a practical result, there will be an imperfect power control. In this work, the capacity of a non-uniform cellular system with imperfect power control has been calculated. The remainder of this paper is organized as follows. In the next sections, first, the reverse link capacity of uniform and non-uniform systems is calculated, respectively. Then, the reverse link capacity of a non-uniform system with imperfect power control is presented and, finally, conclusions are presented.

REVERSE LINK CAPACITY IN UNIFORM CELL SIZE

In this section, the capacity of a uniform CDMA cellular system is obtained by computer simulation, with the assumption of a perfect power control. The power of each user in a cell is controlled in such a way that the received power of all users by BS are assumed equivalent. If there are N users in each cell, the total received power by BS is the summation of the power of desired users, plus the power of other users, which is considered as interference.

Interference received by the neighboring cell is I_0. With the assumption of a simple rake receiver, the

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reverse link capacity of this system can be calculated as follows [1]:

$$\Pr\ (\text{BER}{>}10^{-3}){=}\sum_{k=0}^{N-1}\binom{N-1}{k}\alpha^k(1-\alpha)^{N-1-k} \times Q\left(\frac{\delta-k-E(I_0/s)}{\sqrt{Var(I_0/s)}}\right), \tag{1}$$

where:

$$Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{(-y^2/2)}dy,$$

and:

$$\delta=\frac{W/R}{E_b/N_0}-\frac{\eta}{S}, \tag{2}$$

where W is the frequency bandwidth, S is the received power, K is the number of users that are in voice active state, R is the bit rate, $\frac{E_b}{N_0}$ is energy per bit to interference density ratio, η is background noise, α is voice activity factor and $E(\frac{I_0}{S})$ and var $(\frac{I_0}{S})$ are the mean and variance of $\frac{I_0}{S}$, respectively. The reverse link capacity, Nc, is defined as the maximum integer, N, satisfying Pr (BER$>10^{-3}$) <0.01. The reverse link capacity is easily calculated from Equation 1, if $E(\frac{I_0}{S})$ and var$(\frac{I_0}{S})$ are given. In order to calculate the above parameters, the propagation loss model should be considered. Usually, the propagation loss is modeled as μth power of distance and log-normal component for shadowing loss. For a user with the distance of r from BS, the propagation loss is:

$$L(\mu,\varepsilon)=r^{\mu}10^{\frac{\varepsilon}{10}}, \tag{3}$$

where ε is the dB loss due to shadowing and is usually a Gaussian random variable with zero mean and standard deviation σ dB. Since a perfect power control is assumed, the same power is received by BS from all users. As a result, for N users in a cell, the total interference cannot be more than $(N-1)S$. However, on average, this value decreases with voice activity factor α [1]. The transmitted power of the user can be given as:

$$P_t=Sr^{\mu}10^{\frac{\varepsilon}{10}}. \tag{4}$$

Interference generated by an interferer user at a distance of r_m from its own BS and r_o from the desired cell is:

$$\frac{I(r_o,r_m)}{S}=\left(\frac{r_m}{r_o}\right)^{\mu}10^{(\varepsilon_m-\varepsilon_0)/10}. \tag{5}$$

For M neighboring cells in a CDMA system with N users in each cell, the total interference to signal ratio is:

$$\frac{I_0}{S}=\sum_{j=1}^{M}\sum_{i=1}^{N}\frac{x_{i,j}I_{i,j}(r_o,r_m)}{S}, \tag{6}$$

where $I_{i,j}(r_o,r_m)$ is the generated interference by user i in cell j and $x_{i,j}$ is a random variable to represent the voice activity factor. If one has K samples, the average and variance of $\frac{I_0}{S}$ is obtained by:

$$E\left(\frac{I_0}{S}\right)=\frac{1}{L}\sum_{k=1}^{K}\left(\frac{I_0}{S}\right)_k, \tag{7}$$

$$\text{var}\left(\frac{I_0}{S}\right)=\frac{1}{L}\sum_{k=1}^{K}\left[\left(\frac{I_0}{S}\right)_k-E\left(\frac{I_0}{S}\right)\right]^2. \tag{8}$$

For the cellular system shown in Figure 1, with omni-directional antenna and with $K=40000, \sigma=8$ dB, $M=18, \alpha=\frac{3}{8}$ and $\mu=4, E(\frac{I_0}{S})$ and var$(\frac{I_0}{S})$ are calculated. Figure 1 shows the cell configuration used in the simulation. Substituting these two values in Equation 1 and with $W=1.25$ MHz, $R=8$ kbps, $\frac{E_b}{N_o}=7$ dB and $\frac{S}{\eta}=-1$ dB, Figure 2 is obtained. In this computer simulation, the capacity, defined in Equation 1 and with an outage probability of 0.01, is 34 ($N_c=34$).

REVERSE LINK CAPACITY IN NON-UNIFORM CELLULAR SYSTEM

In the previous section, the capacity for a uniform system was calculated. In that system, $E(\frac{I_0}{S})$ and var$(\frac{I_0}{S})$ are assumed equal for all cells.

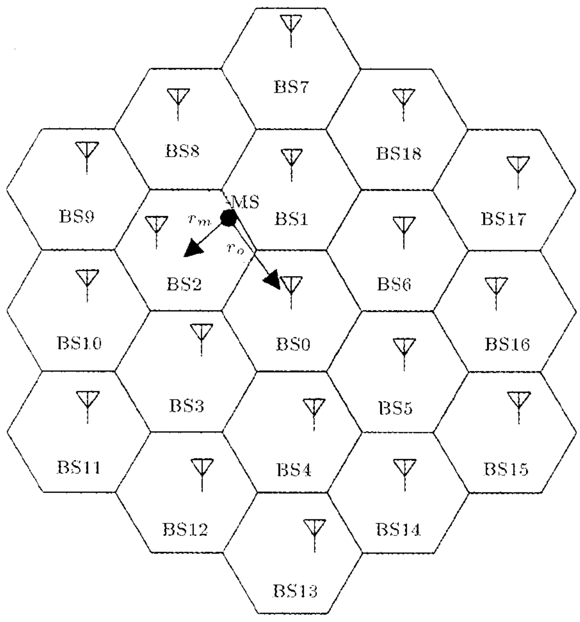

Figure 1. Uniform cellular system configuration.

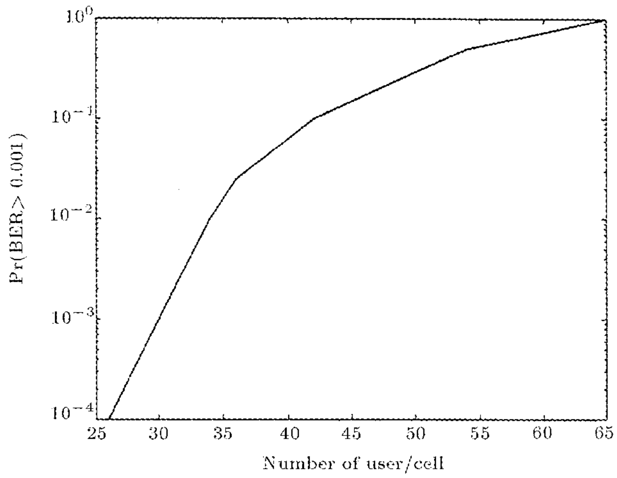

Figure 2. Reverse link capacity in uniform system.

In non-uniform cellular systems with mixed cell sizes, the level of interference received by each cell is different. Therefore, $E(\frac{I_0}{S})$ and $\text{var}(\frac{I_0}{S})$ are unique for each cell. As a result, each cell has an individual capacity and, in order to calculate it, one should consider $E(\frac{I_0}{S})$ and $\text{var}(\frac{I_0}{S})$ of that specific cell. Figure 3, which shows the split of one macro cell into three micro cells, has been used in this simulation. Since the second tiered of macro cells in Figure 1 have a longer distance from the micro cells compared to the first tiered cells and the capacity reduction in the first macro cells, cells 7 to 18 in Figure 3 have not been considered and it is assumed that their capacity does not change.

Micro cell BS0(1) receives interference from 18 outer macro cells and two neighboring micro cells, BS0(2) and BS0(3). If an interference user were located at a distance of r_m from its own site and r_o from BS0(1), the total interference to signal ratio would become:

$$\frac{I_o}{S_{\text{micro}}} = \sum_{j=7}^{18}\sum_{i=1}^{N_c} \frac{x_{i,B_{si}} I_{i,Bsj}(r_o,r_m)}{S\text{macro}}\left(\frac{R\text{micro}}{R\text{macro}}\right)^{\mu} + \sum_{j=1}^{6}\sum_{i=1}^{N\text{macro}} \frac{x_{i,B_{si}} I_{i,Bsj}(r_o,r_m)}{S\text{macro}}\left(\frac{R\text{micro}}{R\text{macro}}\right)^{\mu} + \sum_{k=2}^{3}\sum_{i=1}^{N\text{micro}} \frac{x_{i,B_{si}} I_{i,Bso(k)}(r_o,r_m)}{S\text{micro}}, \quad (9)$$

where Nc is the reverse link capacity in the uniform system and Nmacro and Nmicro are macro and micro cell capacity, respectively. In the same way, if an interferer user were located at a distance of r_m from its own site and r_o from BS1, the total interference-to-signal ratio would become:

$$\frac{I_o}{S_{\text{macro}}} = \sum_{j=7}^{18}\sum_{i=1}^{N_c} \frac{x_{i,B_{si}} I_{i,Bsj}(r_o,r_m)}{S\text{macro}} + \sum_{j=2}^{6}\sum_{i=1}^{N\text{macro}} \frac{x_{i,Bsi} I_{i,Bsj}(r_o,r_m)}{S\text{macro}} + \sum_{k=1}^{3}\sum_{i=1}^{N\text{micro}} \frac{x_{i,Bsi} I_{i,Bso(k)}(r_o,r_m)}{S\text{micro}}\left(\frac{R\text{micro}}{R\text{macro}}\right)^{-\mu}. \quad (10)$$

The results of the computer simulation are shown in Figures 4 and 5, where the following values are assumed:

$$\frac{R\text{micro}}{R\text{macro}} = \frac{\sqrt{3}}{2}, Nc = 34 \text{ and } K = 40000.$$

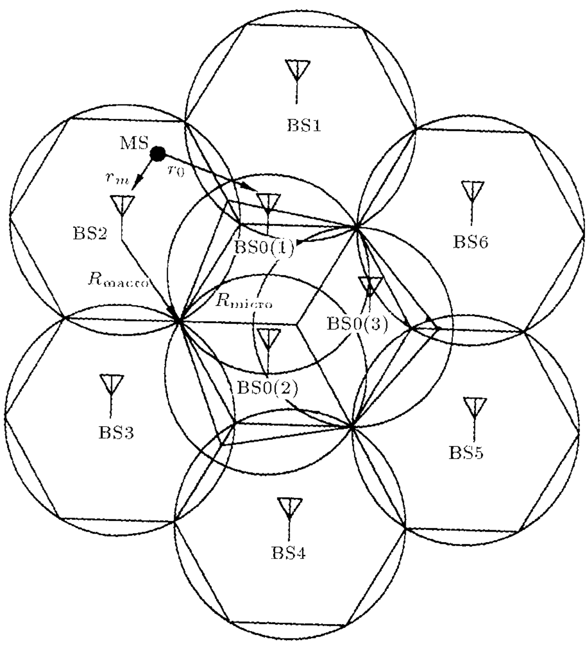

Figure 3. A non-uniform cellular system.

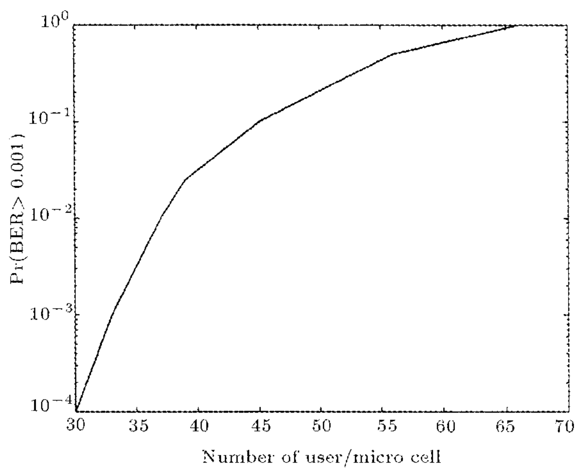

Figure 4. Capacity of micro cells.

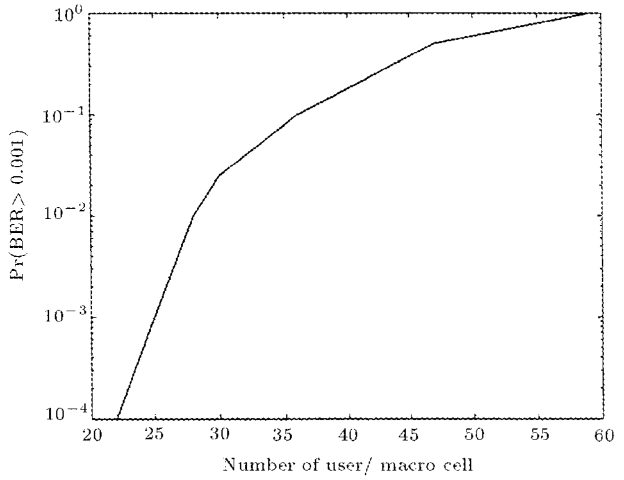

Figure 5. Capacity of macro cells.

The capacity of the macro and micro cells can be extracted from these figures. The capacity of 36 and 28 is obtained for the micro and macro cells, respectively, for an outage probability of 0.01. This will result in a split cell capacity of $3 * 36 = 108$.

REVERSE LINK CAPACITY IN NON-UNIFORM SYSTEM WITH IMPERFECT POWER CONTROL

The calculation of $E(\frac{I_0}{S})$ and $\mathrm{var}(\frac{I_0}{S})$ are the most important parts for calculating capacity. The power received by BS in a perfect power control condition is equal for all users and can be calculated using the following equation:

$$S = P_t . r^{-\mu} . r^{-\varepsilon/10}. \tag{11}$$

In the case of imperfect power control, depending on the amount of power control error (i.e., PCE), the received power will be different. Generally, power control error will be modeled by log-normal distribution [1,4,5]. Received power, in this case, becomes:

$$S = P_t . r^{-\mu} . r^{-\varepsilon/10} . r^{-\gamma/10}, \tag{12}$$

where γ is a Gaussian random variable with zero mean and variance σ dB. With an assumption of zero variance, one will have perfect power control. γ, with the range of 1 to 4 dB, is obtained with field measurements.

This range is used in this simulation for σ. Received power distribution in imperfect power control is given by:

$$f(s) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-[(10\log_{10}(s) - 10\log_{10}(s^{-})]^2}{2\sigma^2}\right], \tag{13}$$

$$\overline{s} = P_t . r^{-\mu}. \tag{14}$$

Power transmitted by each user can be calculated using the following:

$$10\log_{10} P_t = 10\mu 10\log_{10} r_j + 10\log_{10} s + \varepsilon_j + \gamma_j. \tag{15}$$

In fact, variation of transmitted power, given by Equation 15, causes the variation in interference, $E(\frac{I_0}{S})$ and $\mathrm{var}(\frac{I_0}{S})$. Therefore, the transmitted power is a random variable with the following mean and variance:

$$\mathrm{Var} : \sigma_{\varepsilon_j} + \sigma_{\gamma_j},$$

$$\mathrm{Mean} : 10\mu\log_{10} r_j + 10\log_{10} s. \tag{16}$$

As a result, Equation 5 should be modified as follows:

$$\frac{I(r_o, r_m)}{S} = \left(\frac{r_m}{r_o}\right)^{\mu} 10^{((\varepsilon_m + \gamma_m) - (\varepsilon_o + \gamma_o))}/10. \tag{17}$$

By substituting the above equations into Equations 9 and 10 and using Equations 7 and 8, $E(\frac{I_0}{S})$ and $\mathrm{var}(\frac{I_0}{S})$ can be calculated. By substituting these values in Equation 1, the capacity of the system is obtained, as shown in Figures 6 to 8. Furthermore, increasing power control error increases interference and causes a reduction in capacity.

It can be seen from Equations 9 and 10 that the capacity of micro and macro cells increases and decreases, respectively, with a reduction in $\frac{R_{\mathrm{micro}}}{R_{\mathrm{macro}}}$. The computer simulation for $\frac{R_{\mathrm{micro}}}{R_{\mathrm{macro}}} = \frac{\sqrt{3}}{4}$ is shown in Figures 9 and 10.

As can be seen, for an outage probability of 0.01, the capacity of micro cells increases from 36 to 41, but decreases from 28 to 21 in macro cells.

In non-uniform cellular systems, Cell-Splitting Gain (CSG) is defined as follows:

$$\mathrm{CSG} = \frac{\text{number of users in non-uniform system}}{\text{number of users in uniform system}}. \tag{18}$$

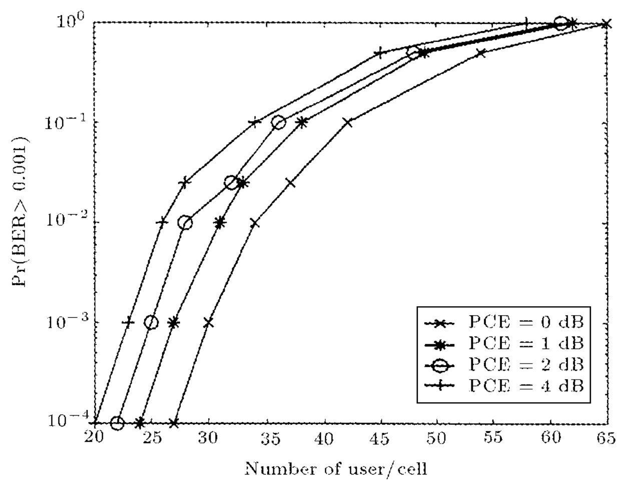

Figure 6. Capacity of uniform cellular system with imperfect power control.

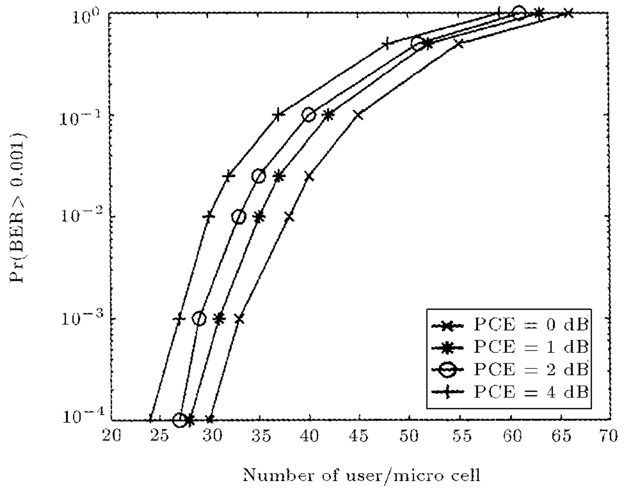

Figure 7. Capacity of micro cells with imperfect power control

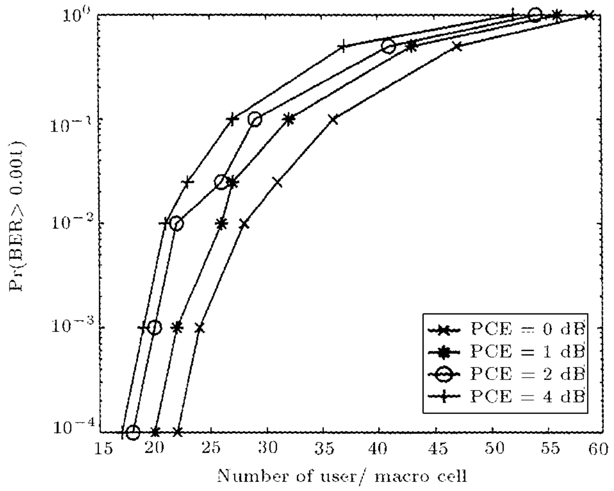

Figure 8. Capacity of macro cells with imperfect power control.

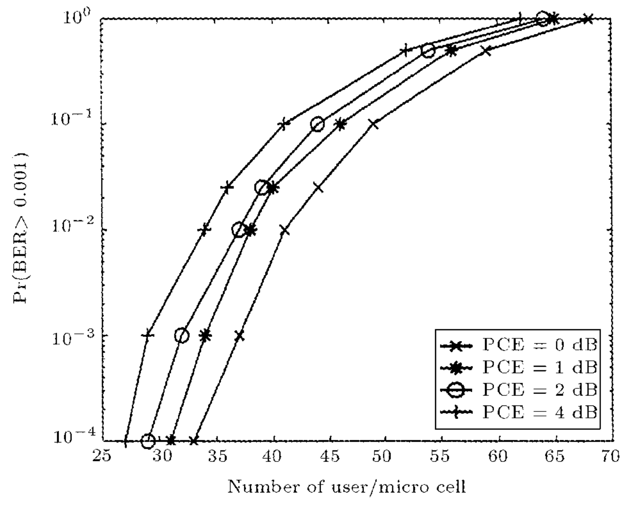

Figure 9. Capacity of micro cells with imperfect power control and $\frac{R_{\text{micro}}}{R_{\text{macro}}} = \frac{\sqrt{3}}{4}$.

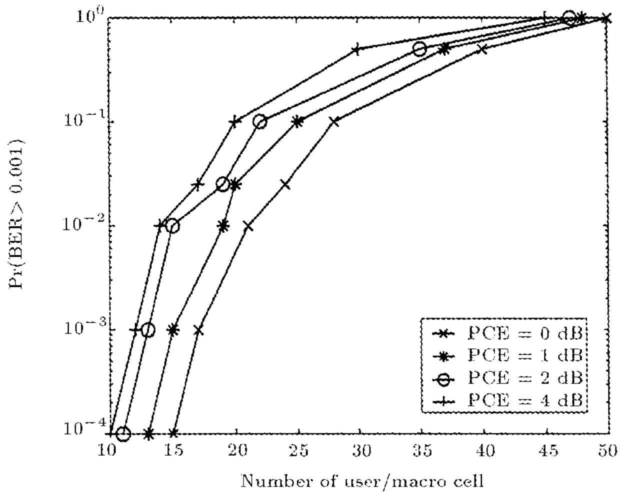

Figure 10. Capacity of macro cells with imperfect power control and $\frac{R_{\text{micro}}}{R_{\text{macro}}} = \frac{\sqrt{3}}{4}$

According to the simulation results, CSG is found to be 1.16 and 1.05 for $\frac{R_{\text{micro}}}{R_{\text{macro}}} = \frac{\sqrt{3}}{2}$ and $\frac{\sqrt{3}}{4}$, respectively. As shown, with a reduction in $\frac{R_{\text{micro}}}{R_{\text{macro}}}$, the capacity of the micro cells increases, which is suitable for a crowded area. However, this significantly decreases the capacity of neighboring macro cells, which, in turn, reduces the CSG. Interference received by neighboring macro cells increases by increasing the number of users in micro cells, which will result in a reduction in macro cell capacity. Level of interference directly depends on the level of power control perfection. Level of interference, due to micro cell users, can be reduced by increasing the power control perfection. Typically, power control error is about 2 dB. By using more complex algorithms and better technology one can obtain a relatively better power control error, as low as 1 dB in micro cells. According to the simulation results, in this case, CSG is found to be 1.18 and 1.01 for $\frac{R_{\text{micro}}}{R_{\text{macro}}} = \frac{\sqrt{3}}{2}$ and $\frac{\sqrt{3}}{4}$. Therefore, in non-uniform cellular system design, the amount of power control error in micro and macro cells is important in capacity and CSG. Capacity and CSG can be obtained from the results of this simulation in all conditions.

CONCLUSION

In densely populated areas, cell split is necessary to increase the capacity of the cellular system. Cell splitting makes a nonuniform cellular system. In this paper, a method was proposed to analyze the capacity of the CDMA cellular system with mixed cell sizes and imperfect power control. It was shown that the reverse link capacity of the micro cell increases as the cell size decreases. Then, however, the capacity of the neighboring macro cells decreases.

The results of this study will be useful for cell planning and the reverse link budget design of CDMA cellular systems.

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