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Research Note

Extended Energy Approach to Propagation Problems in General Anisotropic Media

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In this article, a new general approach has been presented for exact and efficient extraction of eigenpolarizations in anisotropic electromagnetic media with arbitrary constitutive relations. It is shown that the plane wave propagation eigenpolarizations in a linear homogeneous time-independent anisotropic media without free sources, can be obtained through extremizing the difference between stored electric and magnetic energies as a variational functional. It is demonstrated that at these stationary points the wave equation is satisfied by showing that each of the Maxwell curl equations may be obtained by using the other equation as a constraint. Furthermore, it is proven that the theorem holds for extrema of the stored electric energy independently, when the medium is magnetically isotropic. It is concluded that when at least one of the permittivity and permeability tensors are scalar, both the total of and the difference between electric and magnetic energies are extremized simultaneously. As an example, the eigenpolarizations in a non-magnetic anisotropic medium with optical activity are obtained.

INTRODUCTION

With the advent of new optical materials, the theories dealing with the corresponding optical properties have become extremely complicated. Recent developments in composite technology have brought forth possibilities to fabricate artificial anisotropic media with desirable dielectric and magnetic characteristics and much research has been conducted over the past decade on the electromagnetic theory of such complex media [1-12]. Anisotropic photonic crystals have found applications in the switching of light [13] and new antennae [14]. Chiral and bianisotropic materials are now extensively studied for their important role in metamaterials and in the future generations of optical devices [15]. Metamaterials, as another type of complex media, have found numerous extraordinary potential applications [16]. Since all of these materials are anisotropic, the knowledge of eigenmodes or eigenpolarizations is essential for study of the effects associated with light propagation, including refraction, transmission, waveguiding and coupling phenomena.

The propagation of plane waves in an anisotropic medium usually leads to an eigenvalue problem, the solution of which determines the propagation eigenmodes or eigenpolarizations. Propagation eigenmodes are those modes which preserve their polarization during propagation across the anisotropic medium. The problem of light propagation in electrically anisotropic media has been considered previously in a number of reported works [17,18]. The dispersion relation for general anisotropic medium has been obtained for non-diagonal permittivity and permeability tensors [19,20]. The eigenmodes of uniaxial bianisotropic media have been found [21-24] and biorthogonal relations for electromagnetic eigenwaves in bianisotropic media have been studied [25]. In [26] the dispersion equation of a lossless anisotropic dielectric-magnetic medium in the principal system of coordinates, in which the permittivity and permeability tensors become diagonal, has been considered and some of its basic properties have been discussed. In another paper [27], a lossy bianisotropic medium has been investigated and conditions for occurrence of an optical axis have been derived. A variational approach has been reported to demonstrate the uniqueness of the solutions of Maxwell equations by using the difference between total-space stored energies [28-30]. The application of energy methods in electromagnetics is also investigated in another report [31].

Here, it is shown that the eigenpolarizations extremize an energy functional, given by the difference

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between stored electric and magnetic electromagetic energies. Therefore, the eigenpolarizations could be more efficiently found by extremizing a variational expression rather than solving an eigenvalue problem. Consequently, the eigenpolarizations characterize those directions along which a first order change in the direction vector makes a second order change in the difference between the stored electric and magnetic energies.

Furthermore, it is proven that this theorem holds for the extrema of the stored electric energy, as well as the stored magnetic energy, independently, when the medium is magnetically or electrically isotropic. In this case, not only the difference between stored electric and magnetic energies but, also, their sum, are extremized by propagation eigenpolarizations. Therefore, when at least one of the permittivity and permeability tensors are scalar, both the total of and difference between electric and magnetic energies are extremized simultaneously. Otherwise, energy would be the only functional extremized by the eigenpolarizations.

It should be pointed out that this theorem can be regarded as a reduced form of the Lagrangian formulation of the macroscopic electromagnetic theory [21] for a source free and space-time harmonic excitation. A similar form obtained by integration of the energy difference functional over the total space has been used in the complex Poynting theorem for time-harmonic electromagnetic fields [28,32,33] to show that the total-space time-average stored energy divides equally in the electric and magnetic parts. Thus, the extremum value of the energy difference functionals is expected to be zero, as discussed.

ENERGY FUNCTIONAL

The plane wave time-harmonic Maxwell equations, in a source-free time-independent linear homogeneous anisotropic media, are:

$$\mathbf{s}\times\mathbf{E}=\frac{c}{n}\mathbf{B}, \tag{1}$$

$$\mathbf{s}\times\mathbf{H}=-\frac{c}{n}\mathbf{D}, \tag{2}$$

in which $\mathbf{s}$ is a unit vector in the direction of propagation, c is the speed of light in a vacuum and n is the refraction index. Also, $\mathbf{D},\mathbf{E},\mathbf{B}$ and $\mathbf{H}$ are complex phasors related through constitutive relations [34].

The constitutive relations for the case of plane waves can be simplified as $\mathbf{D}=\varepsilon\mathbf{E}$ and $\mathbf{B}=\mu\mathbf{H}$, in which ε and μ, being constants of medium, are permittivity and permeability tensors, respectively. These tensors become Hermitian in the absence of loss. If the medium is supposed to be lossless, the direction vector, $\mathbf{s}$, would be pure real. As a matter of fact, effects such as optical activity and chirality may be incorporated into these tensors as antisymmetric purley imaginary terms, which do not affect its Hermitian property. If the medium is lossy and optically non-active, however, the corresponding tensors are symmetric and imaginary. Some of the fundamental properties of these tensors are discussed in detail in [35].

The eigenpolarizations can be obtained by combining Maxwell equations as:

$$\mathbf{s}\times\mu^{-1}(\mathbf{s}\times\mathbf{E})+\frac{c^2}{n^2}\varepsilon\mathbf{E}=\mathbf{0}, \tag{3}$$

$$\mathbf{s}\times\varepsilon^{-1}(\mathbf{s}\times\mathbf{H})+\frac{c^2}{n^2}\mu\mathbf{H}=\mathbf{0}, \tag{4}$$

which result in algebraic eigenvalue problems. In general, the above equations have non-trivial solutions for some values of the refraction index, n, and directions of field vectors $\mathbf{E}$ and $\mathbf{H}$, which are referred to as the eigenmodes or eigenpolarizations. The eigenpolarizations may be obtained by straightforward algebraic calculations, which take the form of simple expression when the medium is magnetically isotropic [17,18].

Here, it will be shown that the eigenpolarizations extremize an energy functional, given by the difference between stored electric and magnetic electromagetic energies U_e and U_m, respectively. That would mean that the eigenpolarizations characterize those directions, along which a first order change in the direction vector, $\mathbf{s}$, makes a second order change in the difference between U_e and U_m.

When the permittivity and permeability tensors ε and μ are symmetric, this functional is expressed by:

$$U=\frac{1}{2}\mathbf{E}.\mathbf{D}-\frac{1}{2}\mathbf{B}.\mathbf{H}. \tag{5}$$

If there are effects, such as optical activity, so that the permittivity and permeability tensors are Hermitian, time-average stored energies must be used. In this case, the functional takes the following form:

$$U=\Re\left\{\frac{1}{4}\mathbf{E}.\mathbf{D}^*-\frac{1}{4}\mathbf{B}.\mathbf{H}^*\right\}. \tag{6}$$

Furthermore, it is proven that this theorem holds for the extrema of the stored electric energy, U_e, as well as stored magnetic energy, U_m, independently, when the medium is magnetically or electrically isotropic. In this case, not only the difference between stored electric and magnetic energies but, also, their sum, are extremized by propagation eigenpolarizations. Therefore, when at least one of the permittivity and permeability tensors are scalar, both the total of and difference between electric and magnetic energies are extremized simultaneously. Otherwise, only the energy difference, as suggested by either Equation 5 or 6, is extremized by the eigenpolarizations.

As an application of this method, propagation eigenpolarizations, corresponding to an optically active, magnetically isotropic medium, are obtained in agreement with previously reported results [18].

EXTREMA OF ENERGY FUNCTIONAL

The process of extremizing the stored energy functional, U, is supposed to be subject to one of the Maxwell curl equations as a constraint. Here, Equation 1 is chosen. This special choice obviously does not destroy the generality of the problem, if someone considers the inherent duality of electric and magnetic fields in Maxwell equations. Firstly, it is supposed that the permittivity and permeability tensors are symmetric (they may have imaginary parts). Next, the situation is extended to the case of a Hermitian permittivity tensor.

Using the method of Lagrangian multipliers [36], the final functional would be:

$$U = \frac{1}{2}\mathbf{E}.\mathbf{D} - \frac{1}{2}\mathbf{B}.\mathbf{H} + \Lambda.\left(\mathbf{s}\times\mathbf{E} - \frac{c}{n}\mathbf{B}\right), \qquad (7)$$

in which Λ is a vector Lagrangian multiplier, to be determined. Also, the last terms in parentheses represent the constraint, which should be equal to zero at the extrema of the functional.

It is seen that the orthogonality of $\mathbf{s}$ and $\mathbf{B}$, as required by Maxwell divergence law, $\nabla.\mathbf{B}(\mathbf{r},t) = \mathbf{0}$, is maintained by Maxwell equation as the constraint. Therefore, since the vectors $\mathbf{E}$ and $\mathbf{H}$ can be, respectively, expressed in terms of $\mathbf{D}$ and $\mathbf{B}$, the above energy functional, U, is mainly a function of two independent variables. Moreover, this functional is second-order in its variables $\mathbf{D}$ and $\mathbf{B}$.

Setting the partial derivatives of the energy functional, U, with respect to $\mathbf{D}$ and $\mathbf{B}$, to zero results in:

$$\frac{\partial U}{\partial \mathbf{D}} = \mathbf{E} - \varepsilon^{-1}\mathbf{s}\times\Lambda = \mathbf{0}, \qquad (8)$$

$$\frac{\partial U}{\partial \mathbf{B}} = -\mathbf{H} - \frac{c}{n}\Lambda = \mathbf{0}. \qquad (9)$$

Here, the symmetric property of the permittivity and permeability tensors has been taken into account. From Equation 9, one has:

$$\Lambda = -\frac{n}{c}\mathbf{H}. \qquad (10)$$

Inserting the above value for the Lagrangian multiplier, Λ, in Equation 8 gives:

$$\mathbf{E} + \frac{n}{c}\varepsilon^{-1}\mathbf{s}\times\mathbf{H} = \mathbf{0}, \qquad (11)$$

which can be further simplified to:

$$\mathbf{D} = -\frac{n}{c}\mathbf{s}\times\mathbf{H}. \qquad (12)$$

However, this is the second Maxwell curl equation (Equation 2), as expected to be satisfied by plane wave propagation eigenpolarizations. It should be pointed out that Equation 12 also maintains $\mathbf{s}.\mathbf{D} = \mathbf{0}$, which is equivalent to Maxwell divergence law, $\nabla.\mathbf{D}(\mathbf{r},t) = \mathbf{0}$, for a source-free medium. This completes our assertion, as stated above.

The above statement may be easily extended to the case of lossless media with Hermitian permittivity and permeability tensors, by choosing the complex functional:

$$U = \frac{1}{4}\mathbf{E}.\mathbf{D}^* - \frac{1}{4}\mathbf{B}.\mathbf{H}^* + \Lambda.\left(\mathbf{s}\times\mathbf{E} - \frac{c}{n}\mathbf{B}\right). \qquad (13)$$

Here, the real part operator is omitted as introduced in Equation 6, since its extrema also characterize the eigenpolarizations as discussed below and, thus, this is so for its real part. Taking derivatives, with respect to $\mathbf{D}$ and $\mathbf{B}$, similarly results in:

$$\frac{\partial U}{\partial \mathbf{D}} = \frac{1}{2}\mathbf{E}^* - \varepsilon^{*-1}\mathbf{s}\times\Lambda = \mathbf{0}, \qquad (14)$$

$$\frac{\partial U}{\partial \mathbf{B}} = -\frac{1}{2}\mathbf{H}^* - \frac{c}{n}\Lambda = \mathbf{0}. \qquad (15)$$

Again, by noting $\mathbf{s} = \mathbf{s}^*$, the second Maxwell equation results through extremizing the complex functional. Therefore, the time-average energy difference functional Equation 6 lies at its stationary point when both Maxwell curl equations are applicable.

As an important final remark, it may be noticed that upon multiplying Equation 1 by $\mathbf{H}$ and Equation 2 by $\mathbf{E}$, the identity $\mathbf{E}.\mathbf{D} = \mathbf{B}.\mathbf{H}$ follows, which holds for any linear medium. It is pointed out that this is a direct result of the complex Poynting theorem for plane waves [28,32,33]. This means that the instantaneous electromagnetic energy of a traveling plane wave and, hence, also its time-average value is divided equally between the electric and magnetic parts. Therefore, at the stationary point of the energy functionals Equations 5 and 6, one has $U = 0$.

Finally, as may be observed in the functionals Equations 5 and 6, the symmetry between the electric and magnetic fields is preserved. This symmetry is responsible for this equal division of total stored energy. Of course, this symmetry breaks down in cases such as in the presence of free electrical charges. In such situations, one expects, therefore, that the extrema of the above mentioned functionals could not further characterize the eigenpolarizations, since no plane wave solutions could further exist.

NON-MAGNETIC MEDIA

Here, it is shown that a closely related theorem [37] still holds for a non-magnetic anisotropic medium with $\mu = \mu_0$, if one takes the extremizing energy functional as either:

$$U = \frac{1}{2}\mathbf{E}.\mathbf{D} + \lambda_1(\mathbf{s}.\mathbf{D}) + \lambda_2(\mathbf{D}.\mathbf{D} - 1), \tag{16}$$

or:

$$U = \frac{1}{4}\mathbf{E}.\mathbf{D}^* + \lambda_1(\mathbf{s}.\mathbf{D}) + \lambda_2(\mathbf{D}.\mathbf{D}^* - 1), \tag{17}$$

instead of Equation 5 or 6, respectively. Similarly, λ_1 and λ_2 are Lagrangian multipliers. The first constraint is imposed by the assumption of a source-free medium and the second is imposed to prevent the solution relaxing into a trivial case.

By following similar steps to the functional Equation 5, the extrema of Equation 16 are found through the following equation:

$$\mathbf{D} = \varepsilon_0 n^2[\mathbf{E} - (\mathbf{s}.\mathbf{E})\mathbf{s}]. \tag{18}$$

Here, the index of refraction, n, is obtained through either:

$$\varepsilon_0 n^2 = \frac{\mathbf{D}.\mathbf{D}}{\mathbf{E}.\mathbf{D}}, \tag{19}$$

or:

$$\varepsilon_0 n^2 = \frac{\mathbf{D}.\mathbf{D}^*}{\mathbf{E}.\mathbf{D}^*}. \tag{20}$$

Using the identity $\mathbf{s}\times(\mathbf{s}\times\mathbf{E}) = (\mathbf{s}.\mathbf{E})\mathbf{s} - \mathbf{E}$, it is deduced that Equation 18 is the same as the wave equation for the electric field (Equation 3). This equivalency holds only if the permeability tensor is scalar, otherwise the above functionals would lead to incorrect results. Therefore, the eigenpolarizations lie at the extrema of Equation 16 or 17. Moreover, it can be noticed that by Equation 19 or 20, the extremum values of the functionals (Equation 16 or 17) coincide with the extrema of $\varepsilon_0^{-1}n^{-2}$. Therefore, since there exists, generally, two distinct real eigenvalues for refraction index, n, one of the extrema should be a maximum while the other is a minimum.

Here, it should be remarked that the above theorem, with a real symmetric permittivity tensor, is mathematically equivalent to the method of index ellipsoid [17]. In this method, the eigenpolarizations are found by extremizing the functional

$$U = \mathbf{D}.\mathbf{D} + \lambda_1(\mathbf{s}.\mathbf{D}) + \lambda_2\left(\mathbf{E}.\mathbf{D} - \frac{1}{\varepsilon_0}\right). \tag{21}$$

This equivalency may be easily investigated by using Equation 19 and the functional Equation 16.

It is worth pointing out that the addition of constraints in Equation 16 or 17 to the functionals (Equation 7 or 13) has no effect, since the orthogonality of direction vector, $\mathbf{s}$, and field vector, $\mathbf{D}$, as the first constraint, is maintained by the extrema of Equation 7 or 13. Furthermore, the magnitude of $\mathbf{D}$ has no effect on their extremum values, which are equal to zero.

Hence, when the medium is non-magnetic, both the stored electric and magnetic energies are extremized at once and, therefore, this is so for their sum or difference. The equal of this theorem for electrically isotropic media with anisotropic magnetic properties is also true. Finally, if the medium is, at least, either magnetically or electricaly isotropic, the sum of the electric and magnetic energies is extremized, together with each of the independent energy functions and their difference.

EXAMPLE: OPTICAL ACTIVITY

In this section, the eigenpolarizations of a gyro-electric medium, that is, an optically active medium without magnetic anisotropy $\mu = \mu_0$, are obtained. In such media, the constitutive relations are given by [17,18]:

$$\mathbf{D} = \varepsilon\mathbf{E} + i\varepsilon_0\mathbf{G}\times\mathbf{E}, \tag{22}$$

$$\mathbf{B} = \mu_0\mathbf{H}, \tag{23}$$

in which $\mathbf{G} = G\mathbf{s}$ is referred to as the Gyration vector. The parameter G is a direction dependent parameter obtainable from the relation $G = g_{ij}S_iS_j$, where g_{ij} are the elements of the Gyration tensor, g being a constant of medium. The above relation may be rewritten as $\mathbf{D} = \varepsilon'\mathbf{E}$ where ε' is a Hermitian tensor with elements $\varepsilon'_{ik} = \varepsilon_{ik} + \varepsilon_0 G \in_{ijk} S_j$ with $\in_{ijk}$ being the permutation pseudo-tensor [36]. Therefore, the above theorem for the functional (Equation 17) applies.

To study eigenpolarization, the system of coordinates is rotated such that the propagation direction vector lies on the z-axis, that is, $\mathbf{s} = \hat{z}$. Both constraints in Equation 17 may be satisfied by choosing vector $\mathbf{D} = \cos\theta\hat{x}\exp(i\gamma)\sin\theta\hat{y}$. Here, θ and γ are two independent variables expressing the inclination and phase shift between components of vector $\mathbf{D}$. Therefore, by insertion of this value for $\mathbf{D}$ in Equation 17, both constraints may be dropped and the functional takes the following form:

$$U(\theta,\gamma) = \frac{\eta_{11}}{4\varepsilon_0}\cos^2\theta + \frac{\eta_{22}}{4\varepsilon_0}\sin^2\theta + \frac{(\eta_{12}+\eta_{21})\cos\gamma - i(\eta_{12}-\eta_{21})\sin\gamma}{4\varepsilon_0}\cos\theta\sin\theta, \tag{24}$$

where η_{ij} are components of the dimensionless inverse permittivity or impermeability tensor, $\eta = \varepsilon_0\varepsilon^{-1}$.

Upon differentiating with respect to γ and θ, setting the derivatives to zero and noting the Hermiticity of η, one has:

$$\gamma = \angle \eta_{12}, \tag{25}$$

$$\tan 2\theta = 2\frac{|\eta_{12}|}{\eta_{11} - \eta_{22}}. \tag{26}$$

The above equation has two distinct solutions for θ, differing by the amount of $\pi/2$. Therefore, the corresponding eigenpolarization are orthogonal, i.e. $\mathbf{D}_1.\mathbf{D}_2^* = 0$.

Now, the ellipticity of the eigenpolarizations, defined by $\epsilon = b/a$, is considered, where a and b are the radii of the ellipse traced by field vector $\mathbf{D}$ in time domain across the normal plane to direction vector $\mathbf{s}$. These radii are given by [18]:

$$\begin{aligned} a^2 =& \cos^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi \\ &+ 2\cos\theta\sin\theta\cos\gamma\cos\phi\sin\phi, \end{aligned} \tag{27}$$

$$\begin{aligned} b^2 =& \cos^2\theta\sin^2\phi + \sin^2\theta\cos^2\phi \\ &- 2\cos\theta\sin\theta\cos\gamma\cos\phi\sin\phi, \end{aligned} \tag{28}$$

Here, ϕ is the inclination angle of the ellipse, which may be obtained from:

$$\tan 2\phi = \tan 2\theta \cos\gamma = 2\frac{\Re\{\eta_{12}\}}{\eta_{11} - \eta_{22}}. \tag{29}$$

After some algebraic manipulations, one obtains the fairly simple equation:

$$\epsilon^2 = \tan(\theta + \phi)\tan(\theta - \phi). \tag{30}$$

Now, the triad $(\mathbf{D}_1^0, \mathbf{D}_2^0, \mathbf{s})$ is chosen as a system of coordinates, in which $\mathbf{D}_1^0$ and $\mathbf{D}_2^0$ are the eigenpolarizations, in absence of optical activity, with the eigenvalues n_1 and n_2, respectively. Then, in the limit of a small G, one has [18] $\eta_{11} = n_1^{-2}, \eta_{22} = n_2^{-2}, \eta_{12} = iGn_1^{-2}n_2^{-2}$ and $\eta_{12} = -iGn_1^{-2}n_2^{-2}$. Inserting these values results in $\epsilon = \pm\tan\theta$. Using the identity $\tan\theta = (\cot 2\theta \pm \sqrt{1 + \cot^2 2\theta})^{-1}$, the ellipticity, ϵ, is obtained as:

$$\epsilon = \frac{\pm G}{\frac{1}{2}(n_2^2 - n_1^2) \pm \sqrt{\frac{1}{2}(n_2^2 - n_1^2)^2 + G^2}}. \tag{31}$$

But this result is in perfect agreement with the known expression obtained by direct solution of the eigenvalue problem [18].

Finally, the exact closed form of the electric field, $\mathbf{E}$, eigenpolarizations for gyro-electric media with anisotropic permeability tensor, μ, can be shown either via this approach or via a direct algebraic method, to be [38,39]:

$$\begin{bmatrix} \frac{S_x}{\Delta_x} - G^2|\mu|\frac{S_x}{\Delta}\mathbf{s}.[\mu].\mathbf{s} + iG|\mu|\frac{\varepsilon_z\mu_y - \varepsilon_y\mu_z}{\Delta}S_yS_z \\ \frac{S_y}{\Delta_y} - G^2|\mu|\frac{S_y}{\Delta}\mathbf{s}.[\mu].\mathbf{s} + iG|\mu|\frac{\varepsilon_x\mu_z - \varepsilon_z\mu_x}{\Delta}S_zS_x \\ \frac{S_z}{\Delta_z} - G^2|\mu|\frac{S_z}{\Delta}\mathbf{s}.[\mu].\mathbf{s} + iG|\mu|\frac{\varepsilon_y\mu_x - \varepsilon_x\mu_y}{\Delta}S_xS_y \end{bmatrix} \tag{32}$$

where:

$$\begin{aligned} &|\mu| = \mu_x\mu_y\mu_z\,, \\ &\mathbf{s}.[\mu].\mathbf{s} = \mu_xS_x^2 + \mu_yS_y^2 + \mu_zS_z^2, \\ &\Delta = \Delta_x\Delta_y\Delta_z\,, \\ &\Delta_x = n^2\mathbf{s}.[\mu].\mathbf{s} - \varepsilon_x\mu_y\mu_z\,, \\ &\Delta_y = n^2\mathbf{s}.[\mu].\mathbf{s} - \varepsilon_y\mu_z\mu_x\,, \\ &\Delta_z = n^2\mathbf{s}.[\mu].\mathbf{s} - \varepsilon_z\mu_x\mu_y\,, \end{aligned} \tag{33}$$

CONCLUSIONS

A new variational method has been proposed for efficient extraction of eigenpolarizations in anisotropic media with arbitrary constitutive relations. It has been shown that the direction of eigenpolarizations for plane wave solutions of Maxwell equations in an anisotropic medium is determined by the extrema in an energy functional. The medium has been supposed to be linear, homogeneous and time-independent, with electric and magnetic anisotropy. This functional is equal to the difference between instantaneous or time-average energies of the electric and magnetic fields when the permittivity and permeability tensors are symmetric and pure real or Hermitian, respectively. It is also shown that its stationary value is zero, at which the total energy divides equally between the electric and magnetic parts. This equal partitioning of energy has been pointed out to be a result of a special case of a complex Poynting theorem, due to symmetry between the electric and magnetic fields in the source-free plane wave Maxwell equations. It is also proven that the theorem holds for extrema of the stored electric energy independently, when the medium is magnetically isotropic. In this case, not only the difference of stored electric and magnetic energies, but also their sum is extremized by propagation eigenpolarizations. It is concluded, therefore, that when at least one of the permittivity and permeability tensors are scalar, both the total of and difference between electric and magnetic energies are extremized simultaneously. Finally, the proposed variational approach is justified through application to a non-magnetic optically active medium.

REFERENCES

1. Saadoun, M.M.I. and Engheta, N. "A reciprocal phase shifter using novel pseudochiral or σ medium", *Microwave Opt. Technol. Lett.*, **5**, pp 184-188 (1992).
2. Weiglhofer, W.S., Lakhtakia, A. and Monzon, C.J., "Maxwell-Garnett model for composites of electrically small uniaxial objects", *Microwave Opt. Technol. Lett.*, **6**, pp 681-684 (1993).
3. Lindell, I.V., Sihvola, A.H., Tretyakov, S.A. and Viitanen, A.J., *Electromagnetic Waves in Chiral and Bi-Isotropic Media*, Artech House, Boston, USA (1994).
4. Whites, K.W. "Full-wave computation of constitutive parameters for lossless composite chiral materials", *IEEE Trans. Antennas Propagat.*, **43**, pp 376-384 (1995).
5. Theron, I.P. and Cloete, J.H. "The optical activity of an artificial nonmagnetic uniaxial chiral crystal at microwave frequencies", *J. Electromagn. Waves Applicat.*, **10**, pp 539-561 (1996).
6. Arnaut, L.R. "Interaction between bianisotropic particles", *J. Electromagn. Waves Applicat.*, **11**, pp 133-137 (1997).
7. Kristensson, G., Rikte, S. and Sihvola, A.H. "Mixing formulas in the time domain", *J. Opt. Soc. Amer. A.*, **15**, pp 1411-1422 (1998).
8. Auzanneau, F. and Ziolkowski, R.W. "Theoretical study of synthetic bianisotropic materials", *J. Electromagn. Waves Applicat.*, **12**, pp 353-370 (1998).
9. Moses, C.A. and Engheta, N. "An idea for electromagnetic 'feedforward feedbackward' media", *IEEE Trans. Antennas Propagation*, **47**(5), pp 918-928 (1999).
10. Sihvola, A.H., *Electromagnetic Mixing Formulas and Applications*, Institute of Electrical Engineers, London, UK (1999).
11. Weiglhofer, W.S., Lakhtakia, A. and Michel, B. "Correction to Maxwell Garnett and Bruggeman formalisms for a particulate composite with bianisotropic host medium", *Microwave Optical Tech. Lett.*, **22**, p 222 (1999).
12. Lindell, I.V. and Olyslager, F. "Potentials in bianisotropic media", *J. Electromagn. Waves Applicat.*, **15**, pp 3-18 (2001).
13. Schilling, J., Wehrspohn, R.B., Birner, A., Muller, F., Hillebrand, R., Gosele, U., Leonard, S.W., Mondia, J.P., Genereux, F., Van Driel, H.M., Kramper, P., Sandoghdar, V. and Busch, K. "A model system for two-dimensional and three-dimensional photonic crystals: macroporous silicon", *J. Opt. A: Pure Appl. Opt.*, **3**, pp S121-S132 (2001).
14. Jecko, B., Monediere, T., Thevenot, M., Serier, C. and Leger, L. "Anisotropic properties of PBG materials: applications to design a new kind of antenna", *NATO Advanced Research Workshop on Bianisotropics*, Morocco (2002).
15. Marquez, R., Medina, F. and Rafii-El-Idrissi, R. "Role of bianisotropic in negative permeability and left-handed materials", *Phys. Rev. B*, **65**, 144440 (2002).
16. Engheta, N. "An idea for thin subwavelength cavity resonators using metamaterials with negative permittivity and permeability", *IEEE Ant. Wireless Propag. Lett.*, **1**, pp 10-13 (2002).
17. Born, M. and Wolf, E., *Principles of Optics*, 6th Ed., Pergamon Press, Oxford, UK (1980).
18. Yariv, A. and Yeh, P., *Optical Waves in Crystals*, John Wiley & Sons, New York, USA (1984).
19. Damakos, N.J., Maffett, A.L. and Uslenghi, P.L.E. "Dispersion relation for general anisotropic media", *IEEE Trans. Ant. Propag.*, **30**, pp 991-993 (1982).
20. Graglia, R.D., Uslenghli, P.L.E. and Zich, R.E. "Dispersion relation for bianisotropic materials and its symmetry properties", *IEEE Trans. Ant. Propag.*, **39**, pp 83-90 (1991).
21. Kong, J.A., *Theory of Electromagnetic Waves*, Wiley, New York, USA (1975).
22. He, S. and Lindell, I.V. "Propagating eigenmodes for plane waves in a uniaxial bianisotropic medium and reflection from a planar interface", *IEEE Trans. Ant. Propag.*, **41**, pp 1659-1664 (1993).
23. Lindell, I.V. and Viitanen, A.J., in *Bianisotropic and Biisotropic Media and Applications*, **9**, Priou, A., Ed., EMW Publishing, Cambridge, UK (1994).
24. Engheta, N., Jaggard, D.L. and Kowarz, M.W. "Electromagnetic waves in Faraday chiral media", *IEEE Trans. Ant. Propag.*, **40**, pp 367-374 (1992).
25. Girgel, S.S. "Biorthogonal relations for electromagnetic eigenwaves in bianisotropic media", *Proc. CHIRAL'94*, pp 319-321 (1994).
26. Huber, C., Krumpholz, M. and Russer, P. "Dispersion in anisotropic media modeled by three-dimensional TLM", *IEEE Trans. Microwave Th. Tech.*, **43**, pp 1923-1934 (1995).
27. Weiglhofer, W.S. and Lakhtakia, A. "On electromagnetic waves in biaxial bianisotropic media", *Electromagnetics*, **19**, pp 351-362 (1999).
28. Smith, G.S., *An Introduction to Classical Electromagnetic Radiation*, Cambridge University Press, Cambridge, UK (1997).
29. Muller, C., *Foundations of the Mathematical Theory of Electromagnetic Waves*, Springer-Verlag, Berlin, Germany (1969).
30. Jones, D.S., *The Theory of Electromagnetism*, Pergamon Press, Oxford, UK (1964).
31. Hammond, P., *Energy Methods in Electromagnetism*, Clarendon Press, Oxford, UK (1981).
32. Jackson, J.D., *Classical Electrodynamics*, 2nd Ed., John Wiley & Sons, New York, USA (1975).
33. Tai, C.T. "On the presentation of Maxwell's theory", *Proc. IEEE*, **60**, pp 936-945 (1972).

34. Weiglhofer, W.S. "Constitutive relations", *Proc. SPIE*, **4806**, pp 67-80 (2002).

35. Lindell, I.V., *Methods for Electromagnetic Field Analysis*, Clarendon Press, Oxford, UK (1992).

36. Arfken, A., *Mathematical Methods for Physicists*, 3rd Ed., Academic Press, Orlando, USA (1985).

37. Khorasani, S. and Rashidian, B. "Energy approach to the propagation of light waves in anisotropic media", *Proc. SPIE*, **3896**, pp 602-607 (1999).

38. Khorasani, S. and Rashidian, B. "Addendum on generalized conditions for the existence of optical axes", *J. Optics A: Pure Appl. Opt.*, **4**, pp 111-113 (2002).

39. Khorasani, S. and Rashidian, B. "Pseudo-isotropic materials as reflectionlesss media", *Proc. SPIE*, **4580**, pp 399-406 (2001).