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Research Note

Investigation of Temperature Dependencies of Thermophysical Properties of Solids and Liquids

H.G. Hassanov[1]

Under strong laser radiation action on solids and liquids, all the thermophysical parameters which characterize these media become dependent on the medium temperature. Assuming for the coefficients $\lambda(T)$ and $c\rho(T)$ to be linearly changed by the temperature, the non-linear inverse problem of heat conductivity is resolved. The problem is resolved for two cases: 1) For solids and fixed liquids and 2) For heat conductivity with liquid laminar convection due to laser radiation action. The parameter γ, describing the gradient of coefficient of the heat conductivity, is calculated and the influence of liquid convection on γ is estimated. It is considered how the Bi number affects the final result. A developed algorithm for solving inverse problem may be used for finding the exact analytical solution of some problems of diffusion and fluid mechanics.

INTRODUCTION

As an effect on solids and liquids, laser radiation leads to an essential heating of the media. As a result, most of the thermophysical parameters characterizing these media behaviors change, such as coefficient of heat conductivity, heat capacity, optical absorption and so on. Currently, temperature dependencies of the above mentioned properties of solids and liquids are investigated using experimental methods. Existing theoretical approaches do not allow the acquisition of accurate information, because the approaches, principally speaking, are tentative (a detailed review of these methods can be found in [1]). However, methods of solving the inverse problems of the heat conductivity theory, gives one an opportunity to study the above dependencies theoretically as well. In this technique, it is given that the laws $\lambda(T)$ and $c\rho(T)$ are certain functions of temperature and the coefficients introducing them should be found by means of solving the inverse problem. The main difficulty of the approach is in solving the nonlinear differential equation.

There are various methods for solving nonlinear differential equations of heat conductivity, for instance, linearization of nonlinear component, using approximation and/or numerical methods, simplification of conditions etc. [2,3]. A perfectly new variant of the asymptotic functions of non-linear equations has been proposed in [4]. However, recently, the exact analytical solution of a non-linear equation of heat conductivity may be obtained for some limited cases only. In this paper, the algorithm of an exact solution for one class of non-linear equations of heat conductivity is developed and, in using it, one may investigate the temperature dependencies of the thermophysical properties of solids and liquids.

1. *Azerbaijan State Oil Research and Project Institute (AzNSETLI), Aga-Neymatullah Str. 39, Baku 370033-Azerbaijan.*

MATHEMATICAL FORMULATION OF PROBLEM

Let one have a sample of a solid in cylindrical form (or liquid in a cylindrical pipe). The lateral side of the cylinder is thermo-isolated. The laser radiation falls into the beginning point of the cylinder, $z = 0$ and, naturally, at a certain time later, the temperature wave comes to the end point of the cylinder, $z = l$ (l = length of the cylinder). By means of thermocoupling, the temperature alteration laws at both sections are measured. It is assumed that the temperature changes by law $f(t)$ at the section $z = 0$ and the law $\varphi(t)$ at section $z = l$.

One assumes that the radius of the cylinder, R, is greatly less than its length, so that $R << l$. In this case, the temperature field within the cylinder may be taken as one-dimensional. The thermophysical parameters of the solid and/or liquid change by the temperature. The temperature coefficients of heat conductivity and volume heat capacity are considered to be the same :

$$\lambda(T) = \lambda_0(1+\gamma T),$$

$$c\rho(T) = c_0\rho_0(1+\gamma T). \qquad (1)$$

The differential equation of heat conductivity for an involved case has the following form:

$$c(T)\rho(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\lambda(T)\frac{\partial T}{\partial z}\right). \qquad (2)$$

Such a statement of the problem has been described in [1], where γ is assumed to be a small constant. However, obtained solutions are limited to some values of variables z and t only. For Equations 1 and 2, the initial and boundary conditions are given as:

$$T(z,0) = T_0 = \text{const.},$$

$$T(0,t) = f(t),$$

$$T(l,t) = \varphi(t). \qquad (3)$$

It is worthwhile to note that Equations 1 are empirically observed in a wide region of temperature changes, therefore, it might be considered as the closest to experimental data. The author's task is to determine the temperature coefficient, γ, by solving Equation 2 under the given conditions (Equations 3). For determining the parameter γ, it is necessary to take additional boundary conditions into consideration:

$$\frac{\partial T(l,t)}{\partial z} = 0, \qquad (4)$$

in other words, section $z = l$ is thermo-isolated and heat flow through the section is absent.

METHOD OF SOLVING THE PROBLEM

For solving the non-linear Equations 2 and 3, one introduces the non-linear auxiliary function:

$$\theta(T) = T + \frac{1}{2}\gamma T^2. \qquad (5)$$

This substitution has been considered for the first time in [5] for solving an inverse problem of electrothermodynamics. There are obvious relationships:

$$\frac{\partial\theta}{\partial t} = \frac{\partial\theta}{\partial T}\frac{\partial T}{\partial t}, \quad \frac{\partial\theta}{\partial z} = \frac{\partial\theta}{\partial T}\frac{\partial T}{\partial z}, \quad \frac{\partial\theta}{\partial T} = 1+\gamma T. \qquad (6)$$

Taking into account the last formulae, Equation 2 is transformed into:

$$c_0\rho_0(1+\gamma T)\frac{\frac{\partial\theta}{\partial t}}{\frac{\partial\theta}{\partial T}} = \frac{\partial}{\partial z}\left(\lambda_0(1+\gamma T)\frac{\frac{\partial\theta}{\partial z}}{\frac{\partial\theta}{\partial T}}\right),$$

or ultimately into:

$$\frac{\partial\theta}{\partial t} = a_0\frac{\partial^2\theta}{\partial z^2}, \quad a_0 = \frac{\lambda_0}{c_0\rho_0} = \text{const.} \qquad (7)$$

For function $\theta(z,t)$, the initial and boundary conditions will be represented as:

$$\theta(z,0) = T_0 + \frac{1}{2}\gamma T_0^2 = \text{const},$$

$$\theta(0,t) = f(T) + \frac{1}{2}\gamma f^2(t),$$

$$\theta(0,t) = \varphi(T) + \frac{1}{2}\gamma\varphi^2(t). \qquad (8)$$

The additional boundary condition for function $\theta(z,t)$ will be the following:

$$\frac{\partial\theta(l,t)}{\partial z} = 0. \qquad (9)$$

For the already linear problem in Equations 7 and 9, the Laplace transform is used. In images, Equation 7 may be written as the following:

$$\frac{d^2\theta^*}{dz^2} = \frac{s}{a_0}\theta^* \quad \frac{1}{a_0}\left(T_0 + \frac{1}{2}\gamma T_0^2\right),$$

$$\theta^* = \theta^*(z,s) = \int_0^\infty \theta(z,t)e^{\;st}dt, \qquad (10)$$

with appropriate boundary conditions:

$$\theta^*(0,s) = f^*(s) + \frac{1}{2}\gamma f_1^*(s),$$

$$\theta^*(l,s) = \varphi^*(s) + \frac{1}{2}\gamma\varphi_1^*(s),$$

$$\frac{d\theta^*(l,s)}{dz} = 0, \qquad (11)$$

where the following notations are used:

$$\begin{Bmatrix} f^*(z,s) \\ \varphi^*(z,s) \end{Bmatrix} = \int_0^\infty \begin{Bmatrix} f(z,t) \\ \varphi(z,t) \end{Bmatrix} e^{\;st}dt,$$

$$\begin{Bmatrix} f_1^*(z,s) \\ \varphi_1^*(z,s) \end{Bmatrix} = \int_0^\infty \begin{Bmatrix} f^2(z,t) \\ \varphi^2(z,t) \end{Bmatrix} e^{\;st}dt,$$

and s is the parameter of Laplace transform.

The solution of Equation 10 may be given as:

$$\theta^*(z,s) = \frac{1}{s}\left(T_0 + \frac{1}{2}\gamma T_0^2\right) + C_1 ch\sqrt{\frac{s}{a_0}}z$$

$$+ C_2 sh\sqrt{\frac{s}{a_0}}z. \qquad (12)$$

For determining three constants, γ, C_1 and C_2 , there are three conditions (Equations 11). Not concerned with intermediate calculations, the following expression is used for defining the required value of parameter γ:

$$\gamma = 2\frac{f^*(s) \quad \frac{T_0}{s}\left(1 \quad ch\sqrt{\frac{s}{a_0}}l\right) \quad \varphi^*(s)ch\sqrt{\frac{s}{a_0}}l}{\varphi_1^*(s)ch\sqrt{\frac{s}{a_0}}l + \frac{T_0^2}{s}\left(1 \quad ch\sqrt{\frac{s}{a_0}}l\right) \quad f_1^*(s)}. \qquad (13)$$

From Equation 13 it is shown that the value γ depends greatly on the input and output information of the temperature changes by time, $f(t)$ and $\varphi(t)$. Experiments carried out in [6] revealed the following laws for the functions $f(t)$ and $\varphi(t)$:

$$f(t) = T_0 + T_{01}(1 \quad e^{\quad \delta_1 t}),$$

$$\varphi(T) = T_0 + T_{02}(1 \quad e^{\quad \delta_2 t}).$$

Hereafter, T_{01}, T_{02}, δ_1 and δ_2 are empirical permanent magnitudes. The Laplace images of these functions have the view:

$$f^*(s) = \frac{T_0}{s} + \frac{T_{01}\delta_1}{s(s+\delta_1)},$$

$$\varphi^*(s) = \frac{T_0}{s} + \frac{T_{01}\delta_2}{s(s+\delta_2)},$$

$$f_1^*(s) = \frac{(T_0+T_{01})^2}{s} \quad \frac{2T_{01}(T_0+T_{01})}{s+\delta_1} + \frac{T_{01}^2}{s+2\delta_1},$$

$$\varphi_1^*(s) = \frac{(T_0+T_{02})^2}{s} \quad \frac{2T_{02}(T_0+T_{02})}{s+\delta_2} + \frac{T_{02}^2}{s+2\delta_2}.$$

Inserting the values of the above listed functions $f^*(s), \varphi^*(s), f_1^*(s)$ and $\varphi_1^*(s)$ into Equation 13, one can calculate the value γ. Assuming $s = 1/t_0$, where t_0 is the temperature relaxation time, one could transform Equation 13 into :

$$\gamma = 2\frac{f^*\left(\frac{1}{t_0}\right) \quad T_0 t_0\left(1 \quad ch\sqrt{\frac{1}{a_0 t_0}}l\right) \quad \varphi^*\left(\frac{1}{t_0}\right) ch\sqrt{\frac{1}{a_0 t_0}}l}{\varphi_1^*\left(\frac{1}{t_0}\right) ch\sqrt{\frac{1}{a_0 t_0}}l + T_0^2 t_0\left(1 \quad ch\sqrt{\frac{1}{a_0 t_0}}l\right) \quad f_1^*\left(\frac{1}{t_0}\right)}. \qquad (14)$$

It is obvious that:

$$f^*\left(\frac{1}{t_0}\right) \quad T_0 t_0 \quad \left\{\varphi^*\left(\frac{1}{t_0}\right) \quad T_0 t_0\right\}$$

$$= t_0^2\left\{F_{11}(t_0) \quad F_{12}(t_0)\right\},$$

where:

$$F_{11}(t_0) = \frac{T_{01}\delta_1}{1+\delta_1 t_0},$$

$$F_{12}(t_0) = \frac{T_{02}\delta_2}{1+\delta_2 t_0} ch\sqrt{\frac{1}{a_0 t_0}}l.$$

In the same way, one may yield, respectively:

$$\left\{f_1^*\left(\frac{1}{t_0}\right) \quad T_0^2 t_0\right\} \quad \left\{\varphi_1^*\left(\frac{1}{t_0}\right) \quad T_0^2 t_0\right\}$$

$$= t_0\left\{F_{21}(t_0) \quad F_{22}(t_0) ch\sqrt{\frac{1}{a_0 t_0}}l\right\},$$

$$F_{21}(t_0) = 2T_0T_{01} + T_{01}^2 \quad \frac{2(T_0T_{01} + T_{01}^2)}{1+\delta_1 t_0} + \frac{T_{01}^2}{1+2\delta_1 t_0},$$

$$F_{22}(t_0) = 2T_0T_{02} + T_{02}^2 \quad \frac{2(T_0T_{02} + T_{02}^2)}{1+\delta_2 t_0} + \frac{T_{02}^2}{1+2\delta_2 t_0}.$$

Under these permissions, the value γ is determined by the following relationship:

$$\frac{\gamma}{t_0} = \frac{F_{11}(t_0) \quad F_{12}(t_0)}{F_{21}(t_0) \quad F_{22}(t_0) ch\sqrt{\frac{1}{a_0 t_0}}l}.$$

The left side of this relationship is a linear dependence on factor $1/t_0$. Naturally, the right side of the last equation should be linear in dependence on $1/t_0$ also. Having drawn the graph of the right side vs $1/t_0$ and compared the angle coefficient of this line with γ, one can find the magnitude γ. It is shown that Equation 14 is sufficiently difficult, so let a partial case be studied when:

$$f(t) = T_1 = \text{const.}, \quad \varphi(t) = T_2 = \text{const.}$$

Then, one obviously has:

$$f^*(s) = \frac{T_1}{s}, \quad \varphi^*(s) = \frac{T_2}{s},$$

$$f_1^*(s) = \frac{T_1^2}{s}, \quad \varphi_1^*(s) = \frac{T_2^2}{s}.$$

Using the last expression in Equation 14, after non-complex calculations, it yields:

$$\gamma = \quad \frac{2}{T_1 + T_2}.$$

In this case, the temperature dependencies of the thermophysical parameters of solids and liquids may be represented as:

$$\lambda(T) = \lambda_0\left(1 \quad \frac{2}{T_1+T_2}\right),$$

$$c\rho(T) = c_0\rho_0\left(1 \quad \frac{2}{T_1+T_2}\right). \qquad (15)$$

THE PROBLEM WITH HEAT EXCHANGE

Absence of heat exchange between solid/liquid and the surrounding medium during laser radiation action is greatly idealized. In practice, this aspect is always valid and essentially changes the final result. Let one consider how heat exchange affects the temperature dependencies of basic parameters. With an account of heat exchange, the differential equation of heat conductivity has a view of:

$$c(T)\rho(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\lambda(T)\frac{\partial T}{\partial z}\right) \quad \alpha(T)\frac{P}{S_0}(T \quad T_0), \tag{16}$$

where $\alpha(T)$ is the coefficient of the heat exchange, T_0 is temperature of ambient medium that is accepted to be constant during the process and P and S_0 are perimeter and square of a cross-section of the cylinder, respectively. The initial and boundary conditions for Equation 16 will be the same as the condition in Equations 3, but the additional boundary condition in Equation 4 is changed into:

$$\lambda(T)\frac{\partial T(l,t)}{\partial z} = \alpha(T).(T \quad T_0), \tag{17}$$

that is usually taken for problems with heat exchange. For solving Equation 16, under the conditions in Equations 3 and 17, one introduces an auxiliary function as above (Equation 5). The temperature dependencies of the coefficients λ and $c\rho$ are taken as in Equation 1. The temperature dependence of the heat exchange coefficient, α, is expressed by the following equation:

$$\alpha(T) = \alpha_0\frac{T + \frac{1}{2}\gamma T^2)}{T \quad T_0},$$

where α_0 is the initial value of the coefficient of heat exchange before the laser radiation action. For the auxiliary function, $\theta(z,t)$, there is the following equation:

$$\frac{\partial\theta}{\partial t} = a_0\frac{\partial^2\theta}{\partial z^2} \quad \alpha_0\frac{P}{S_0}\theta, \tag{18}$$

and the following conditions:

$$\theta(z,0) = T_0 + \frac{1}{2}\gamma T^2 = \text{const.},$$

$$\theta(0,t) = f(T) + \frac{1}{2}\gamma f^2(t),$$

$$\theta(0,t) = \varphi(T) + \frac{1}{2}\gamma\varphi^2(t),$$

$$\lambda_0\frac{\partial\theta(l,t)}{\partial z} = \alpha_0\theta(l,t). \tag{19}$$

For solving Equations 18 and 19, one may also use the Laplace transform. In images, the following differential equation appears as:

$$\frac{d^2\theta^*}{dz^2} = \frac{s+w}{a_0}\theta^* \quad \frac{1}{a_0}\left(T_0 + \frac{1}{2}\gamma T_0^2\right),$$

$$w = \alpha_0\frac{P}{S_0}. \tag{20}$$

In images, the existing boundary conditions are represented as:

$$\theta^*(0,s) = f^*(s) + \frac{1}{2}\gamma f_1^*(s),$$

$$\theta^*(l,s) = \varphi^*(s) + \frac{1}{2}\gamma\varphi_1^*(s),$$

$$\lambda_0\frac{d\theta^*(l,s)}{dz} = \alpha_0\theta^*(l,s). \tag{21}$$

Solution of Equations 20 can be written in a form formally close to that of Equation 12, obtained for the problem without the heat exchange. The only difference is that instead of parameter s, now, in the final solution, the index $(s+w)$ takes place:

$$\theta^*(z,s) = \frac{1}{s+w}\left(T_0 + \frac{1}{2}\gamma T_0^2\right) + C_1 ch\sqrt{\frac{s+w}{a_0}}z + C_2 sh\sqrt{\frac{s+w}{a_0}}z. \tag{22}$$

The value of parameter γ is found by solving the system of three Equations 21 relative to three unknown magnitudes C_1, C_2 and γ. The final result for required magnitude γ is noted without intermediate calculations:

$$\gamma = \quad 2\frac{Bi.\varphi^*(s)sh\chi l \quad \chi l[P_1 \quad \varphi^* ch\chi l] \quad \frac{P_3}{\lambda_0}lsh\chi l}{Bi.\varphi_1^*(s)\,ch\chi l \quad \chi l[P_2 \quad \varphi_1^* sh\chi l]}. \tag{23}$$

Herein, the next notations are introduced

$$Bi = \frac{\alpha_0 l}{\lambda_0}, \quad \chi = \sqrt{\frac{s+w}{a_0}}, \quad P_1 = f^*(s) \quad \frac{T_0}{s+w},$$

$$P_2 = f_1^*(s) \quad \frac{T_0^2}{2(s+w)}, \quad P_3 = \frac{\lambda_0\chi T_0}{s+w}\left(1 + \frac{T_0}{2}\right).$$

Analyzing Equation 23 for every interesting variant of the laser radiation influence on solid/liquid, one is able to define parameter γ of the appropriate solid/liquid, for each case, with an account of the heat exchange with the surrounding medium. Although Equation 23 is difficult, for some cases it can be simplified. In the first case, realizing the algorithm with relaxation time $s = 1/t_0$ for a steady temperature regime at both sections, when T_1 and T_2 are permanent, at small values of the Bi number, one has, for parameter γ, the value equivalent to that obtained for the previous problem, namely:

$$\gamma = \quad \frac{2}{T_1 + T_2}.$$

So, in this case, coefficients λ and $c\rho$ could be represented by the known Equations 15. However, the last law is not always valid. The second case, which is of practical interest, corresponds to sufficiently great values of the Bi number. For this process variant, a form of parameter γ is changed and, according to the authors calculations, becomes equal to the following magnitudes that have been obtained from Equation 23;

$$\gamma = \quad \frac{2}{T_2}, \tag{24}$$

with corresponding dependencies for λ and $c\rho$, respectively;

$$\lambda(T) = \lambda_0 \left(1 \quad \frac{2}{T_2}\right), \quad c\rho(T) = c_0\rho_0 \left(1 \quad \frac{2}{T_2}\right). \tag{25}$$

The physical matter of the result (Equation 25) is, for intensive heat exchange, available between the solid/liquid and ambient medium. Temperature T_1 at the point $z = 0$ does not play a sufficient role in temperature distribution along the pipe. On the other hand, the temperature of section $z = l$ is much more important, since it characterizes the temperature as well as all the thermophysical parameter gradients.

ACCOUNT OF LIQUID CONVECTION DUE TO LASER RADIATION

On the basis of the theoretical calculations in [7], it has been established that if the laser radiation energy density is greatly less than the liquid internal energy density (in other words, there is the condition $I << C\rho c_p T$ (herein C = sound velocity in the medium, other indications are well-known)), then the medium is weakly absorbing and the thermophysical coefficient may be accepted to be constant. In this case, the mathematical model of the heat transfer is described by a linear differential equation. However, if $I \approx c\rho C_p T$, then the temperature of the liquid already changes over a wide area and the thermophysical parameters of the liquid involved above essentially alter by temperature. Hence, for determining the kinetic coefficients, it is necessary to use the non-linear models, one of which has been successfully considered above.

During non-resonance interaction of great laser radiation with non-transparent liquids, various opto-thermophysical and opto-thermodynamic processes take place, which are connected with strong compression and the heating of the liquids. In propagating laser radiation in absorbing the medium (liquid), there are convection flows due to its heating. Increasing laser generator power leads to a transformation of the laminar flow to a turbulent one [7]. It is of great interest to elucidate what changes in the temperature dependence of the liquid thermophysical parameters leads to the liquid convection.

The non-linear differential equation of the heat conductivity of liquid with a convective laminar flow may be represented as :

$$c(T)\rho(T)\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial z}\right) = \frac{\partial}{\partial z}\left(\lambda(T)\frac{\partial T}{\partial z}\right). \tag{26}$$

Hereafter, v is the velocity of the liquid convection. Here, for simplicity of mathematical calculations, one considers the problem of liquid convection without heat exchange, although, in solving the more general problem of heat conductivity, the last circumstance can be taken into account without any principal difficulties. For solving this variant of the equation, it is necessary to introduce the same non-linear auxiliary function $\theta(z,t)$. Subsequently, Equation 26 is transformed into the appropriate linear equation:

$$\frac{\partial\theta}{\partial t} + v\frac{\partial\theta}{\partial z} = a_0\frac{\partial^2\theta}{\partial z^2}, \tag{27}$$

with the initial and boundary conditions valid for the problem without the convection of Equations 8 and 9. Then, in full accordance with the already considered algorithm of problem solving, it is necessary to realize a Laplace transform. Now there is the following equation, relative to the function $\theta^*(z,s)$:

$$\frac{d^2\theta^*}{dz^2} \quad \frac{v}{a_0}\frac{d\theta^*}{dz} \quad \frac{s}{a_0}\theta^* = \quad \frac{1}{a_0}\left(T_0 + \frac{1}{2}\gamma T_0^2\right), \tag{28}$$

which is the equation of forced vibrations. Solution of Equation 28 is well-known. The form of its solution depends on parameter σ:

$$\sigma = \left(\frac{v}{a_0}\right)^2 + 4\frac{s}{a_0}. \tag{29}$$

A general solution of Equation 28 may be represented as:

$$\theta^*(z,s) = C_1 e^{\quad \xi_1 z} + C_2 e^{\xi_2 z} + \frac{1}{s}\left(T_0 + \frac{1}{2}\gamma T_0^2\right), \tag{30}$$

therefore:

$$\xi_1 = \frac{v}{a_0} + \sigma, \quad \xi_2 = \frac{v}{a_0} \quad \sigma.$$

For finding the three magnitudes C_1, C_2 and γ, one should apply Equations 8 and 9. The following is a system of three equations relative to three unknown magnitudes:

$$C_1 + C_2 = a_1 + \frac{1}{2}\gamma a_2,$$

$$C_1 e^{\quad \xi_1 l} + C_2 e^{\xi_2 l} = b_1 + \frac{1}{2}\gamma b_2,$$

$$\xi_1 C_1 e^{\quad \xi_1 l} + \xi_2 C_2 e^{\xi_2 l} = 0.$$

After simple computations, one can find any constant. Herein, one is interested in the value of parameter γ. Let this be described without the bulky intermediate calculations:

$$\gamma = \quad 2\frac{a_5 e^{\xi_1 l} + a_3 e^{\quad \xi_2 l} \quad a_1(\xi_1 + \xi_2)}{a_6 e^{\xi_1 l} + a_4 e^{\quad \xi_2 l} \quad a_2(\xi_1 + \xi_2)}, \tag{31}$$

where the following abbreviations are used:

$$\begin{array}{ll} a_1 = f^*(s) \quad \frac{1}{s}T_0, & a_2 = f_1^*(s) \quad \frac{1}{s}T_0^2, \\ a_3 = \xi_1 b_1, & a_4 = \xi_1 b_2, \\ b_1 = \varphi^*(s) \quad \frac{1}{s}T_0, & b_2 = \varphi_1^*(s) \quad \frac{1}{s}T_0^2, \\ a_5 = \xi_2 b_1, & a_6 = \xi_2 b_2. \end{array} \tag{32}$$

Equation 31 is the most general form of parameter γ. For every variant of the problem, which is of practical interest, one can find concrete values of the parameter using the conditions of the problem. For example, in a steady case of the radiation action at both sections (the values of T_1 and T_2 are constant) Equations 32 are simplified. As a result, one has for parameter γ:

$$\gamma = \quad \frac{2}{T_1 + T_2}, \tag{33}$$

with appropriate dependencies for basic thermophysical magnitudes.

$$\lambda(T) = \lambda_0 \left(1 \quad \frac{2}{T_1 + T_2}\right),$$

$$c\rho(T) = c_0\rho_0 \left(1 \quad \frac{2}{T_1 + T_2}\right). \tag{34}$$

Comparison of Equations 33 and 34 with Equations 15 previously obtained, allows one to reach the conclusion that, in cases of liquid convection with permanent velocity of flow, the coefficient of heat conductivity is absolutely the same as for fixed liquid. However, this situation will be realized only if one considers stabilized liquid flow. For heat conductivity with convection, the period of time considered has great meaning, because, as may be concluded from Equation 30, temperature distribution is highly sensitive to the duration of this investigation. At the beginning of the heat conductivity process with laminar convection under radiation, the dependence $\lambda(T)$ is not already determined by Equation 33. For finding the exact value of parameter γ at the beginning of the process, it is necessary to resolve Equation 31 at great values of magnitude s corresponding to small values of t.

As follows from Equation 31, the laminar convection of the liquid caused by laser radiation is not able to sufficiently change the value of parameter γ. In other words, the presence of laminar convection with permanent velocity, v, does not change the character of the thermophysical behavior of the studied liquid in full accordance with experimental research [8]. The dependencies (Equations 33 and 34) are valid only if there is the condition v = const. Unfortunately, one is not able to describe here all the specifications of the thermophysical behavior of liquid moving under laser radiation action, for example, flow with altered velocity, $v(z)$, due to the real liquid viscous tensions [9], as well as the influence of turbulence flow that takes place under the great values of radiation intensity as has been experimentally observed in [8]. All these questions are proposed to be the subjects of separate investigation.

CONCLUSION

In the present paper, the algorithm of an exact analytical solution of the non-linear problem of heat conductivity is developed. The dependence of basic thermophysical parameters on temperature are assumed as linear (see Equations 1). In practice, the existing experimental data is the most available in support of this class of problem. The procedure developed here is not to be considered as a linearization of the non-linear equation, since linearization means a certain approximation (in other words, average) and, ultimately, leads to an inaccuracy in the final result. The author's technique is free of this defect and yields to a correct solution of the problem. Such an approach gives an opportunity to acquire useful information about the thermophysical properties of studied solids and liquids, in particular when realizing the experiment is complicated by the conditions of the problem involved. For example, on the basis of the authors' research, it is deduced that the dependence:

$$\gamma = \quad \frac{2}{T_1 + T_2},$$

is the most available for processes of heat conductivity caused by laser radiation action on solids or liquids. It may be seen from this ratio that the closer the values of the temperature at the initial and final sections of the body involved (T_1 and T_2), the more influence the