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Development and Calibration of a Resistivity Probe for Measurement of Air Concentration and Bubble Count in High-Speed Air-Water Flows

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In this paper, the development of a resistivity probe for measurement of air concentration and bubble count in high-speed air-water flow is described. One advantage of this type of probe is in its ability for real time measurement. The sampling frequency of the new probe is increased up to 250 kHz with 72 seconds sampling time. Polarization of the probe tip was noticed in this research work and the probe circuit was designed to avoid it. Sensitive microampere meters were installed on the probe to detect possible weak leakage currents. Electronic tests were performed to check the probe circuit, as well as more than 30 laboratory tests in air-water flow to set the threshold voltage of the probe circuit and to test its accuracy.

INTRODUCTION

Two phase gas-liquid flows are studied in many fields of engineering, including chemical, mechanical, civil and environmental engineering. In civil engineering hydraulics, for example, the aeration of high-speed flows on dam spillways is studied as a method for the prevention of cavitation. It is known that with 8% air in water, the danger of a cavitation attack is completely prevented [1,2]. Experimental studies of air-water mixtures may also be necessary in physical models to check the design of hydraulic structures, or to collect data for basic research and/or mathematical model validation. Several methods are available for measuring bubble size and void fraction in gas-liquid flows, for example, photography, hot wire anemometery, gamma-ray absorption, ultrasonic, optical fiber and resistivity (needle) probes. Herringe and Davis used a resistivity probe in bubbly flows and compared its accuracy with an infrared probe, hot film and hot wire anemometers and concluded that the resistivity probe is the most suitable method for detecting phase change in the air-water flow [3]. With air concentration more than 10-15% and with the liquid fraction larger than about 5-10%, the most reliable probes are found to be the optical fiber and the resistivity probe [4-7]. It is also reported that the operating range of conductivity probes is very large [4]. Resistivity probes involve a needle-shape tip, which can detect which of the two-phases is instantaneously present by using electrical conductivity, while the optical fiber probe detects the phase changing by using total internal reflection. Optical fiber probes are usually more expensive and more fragile than resistivity probes [8]. Therefore, among all the methods, the resistivity probe is attractive for measurements in bubbly flows because of its relative simplicity and wide applicability.

The resistivity probe was first introduced by Neal and Bankoff for measurement of nitrogen bubbles in mercury [9]. Application of this probe was extended to air-water flows by Nassos and Bankoff [10]. Cain and Wood developed a needle probe for measurement of air-water flow characteristics in dam spillways [11]. Needle probes were also successfully used for measurement at high speed flows, up to 25 and 36 m/s by Cain [12] and Heraud [13], respectively. Sandullah et al. [8], Zarrati et al. [14] and Jalili et al. [15], reported the development of resistivity probes with 20 kHz, 100 kHz and 250 kHz sampling rates, res pectively. Liu and Bankoff [16], Leung [17] and Cummings and Chanson [18] developed miniature probes, with the size of probe tips less than 0.03 mm. By means of such probes, bubbles as small as 0.1 mm in diameter were detectable [18].

In the present work, a resistivity probe introduced in [14] was further developed and tested. An advantage of the new probe is the real time measurement of

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air concentration and bubble count. Frequency of sampling is adjustable and can be set up to 250 kHz. This means that the probe can detect 25 points when a 1 mm bubble, traveling with a speed of 10 m/s, impacts the probe sensor. Despite previous research work, there is still a need for calibration of these probes, in different ranges of air concentration and flow velocity and, also, rates of sampling. The principles of this probe, its calibration and accuracy are discussed in the following sections.

PRINCIPLES OF PROBE DESIGN

The probe works based on the high difference in conductivity between liquid and gas. The resistance of water is one thousand times lower than that of air. The tip of the probe consists of a needle with an isolated wire passing through it (Figure 1) and a potential difference across them. In an air-water mixture, when the probe is in water, the tip of the wire and the needle are connected through the water media and the current passes through the circuit. When the wire tip pierces a bubble, resistivity increases sharply until the bubble is swept away. The air concentration at a point of flow can be determined as follows [19,20]:

$$c = \frac{t_a}{t_t}, \tag{1}$$

where t_a is the time of circuit disconnection (probe tip in air) and t_t is the total time of sampling. The number of times the circuit is disconnected shows the number of bubbles. If flow velocity is known and the signal is saved, the bubble chord spectrum can, also, be determined.

As bubbles pass and are pierced by the probe tip, the output voltage of the probe changes between a minimum (probe tip in water with full current in the circuit) and a maximum (probe tip in air and circuit disconnected). Ideally the probe output signals should have a square wave shape (Figure 2).

In practice, as the probe tip approaches a bubble, it imposes a local pressure surge and some liquid is entrained along the direction of the flow velocity. This

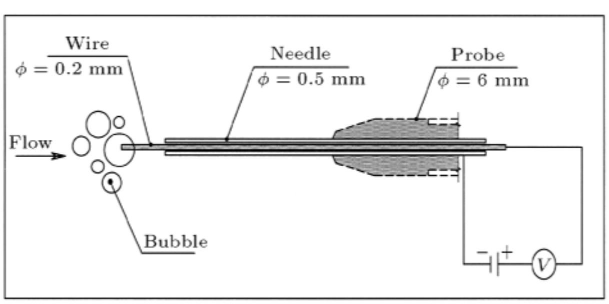

Figure 1. Configuration of needle probe tip.

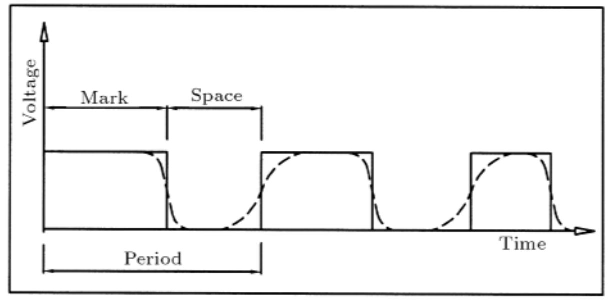

Figure 2. The typical output signal from a probe.

causes deformation of the bubble-water interface. Also, with the breakup of the water layer, owing to its wetability, a water layer remains attached to the probe tip. Due to this phenomenon, the output voltage of the probe does not rise immediately. This effect is, however, less for the transition from a gas to liquid phase because of the low gas inertia [21-25]. Therefore, the probe output signal is schematically represented by a dashed line in Figure 2.

Noises are also caused in the probe signals by bubbles passing the neighboring of the probe, and/or breakup of bubbles as they hit the probe tip. Ideally, the circuit must distinguish, from the input signal, between the effect of the tip actually piercing the bubble and the near miss of an air bubble. To generate the corresponding square wave, a Schmidt trigger should be incorporated in the circuit [24-27]. There are several techniques to process the raw output signals of the probe, such as single threshold technique, double-threshold technique and differentiation technique.

The first method is based on a single threshold level whose intersections with the raw signal determine the starts and the ends of rectangular waves, producing, thus, a succession of gas residence times. In the double-threshold technique, two distinct threshold levels could be imposed. In the differentiation method, the raw signal is, first, clipped to eliminate the noise and, then, it is differentiated. A positive trigger applied to the rising slope defines the start of the rectangular wave and a negative level acts, similarly, on the falling slope.

Among the methods mentioned above, the single-threshold level is the most simple and common method to use [22,25] and this technique was also adopted for the present study. In the single threshold method, the probe tip is assumed to be in air if the voltage rises above a predetermined voltage set for the Schmidt trigger (threshold level). The threshold level is usually set as a percentage of the maximum voltage of the probe circuit. The choice of this setting affects the accuracy of the probe for air concentration measurement. There have been a number of studies to find the optimum value of the threshold level in resistivity probes. Sekoguchi et al. found 40% of the maximum voltage as the optimum threshold level for an air concentration range of 1% to 6% and a flow velocity between 0.5 m/s and 2 m/s [28]. In the same range of flow velocity, but with a higher air concentration of 5% to 50%, Galaup recommended a threshold level equal to 30% [29]. Herringe and Davis [3], with a range of velocity between 2.0 m/s to 4 m/s, found that a threshold level between 20% and 70% did not affect their results considerably, but recommended a value of 28% [3]. Reimann and John did experiment with bubbly and slug flows in a wide range of flow velocity between 1 m/s to 10 m/s, with air concentrations of 25% and 95% and reported a threshold level of 30% [30]. However, Banerjee et al., in their experiments with mist and bubbly to slug flows, with air concentration between 10% and 90%, reported a value of 50% for the threshold level [31]. In another research work, Kobri and Terada, with 1 m/s to 2 m/s flow velocity, found the optimum threshold level to be 20%, 40% and 50% for 20%, 30% and 90% air concentrations, respectively [32].

In the present work, a probe was developed for deaeration studies in a range of flow velocities between 2 m/s to 5 m/s, which is usually encountered in a hydraulic laboratory. Despite previous studies, there is not yet a general agreement on the threshold level corresponding to different flow conditions and probe geometry. It was, therefore, planned to conduct a comprehensive study on the threshold level in the present work. Moreover, a high sampling frequency of 250 kHz in the present probe, allowed the interface between air and water phases to be reproduced more accurately. Comparison of the optimum threshold found in this study with previous works also leads to the application of the resistivity probe, in a wider range of flow conditions and probe design.

If the rate of sampling is high enough, the size of the smallest bubbles, which can be detected, depends on the size of the needle tip in diameter. Although the ratio between the smallest size of a detectable bubble and probe tip has not yet been established, Herrings and Davis [26] concluded from their experiments that this ratio is five. Cain and Wood [11] and Zarrati and Hardwick [33] also assumed the same ratio for design of their probes. Cummings and Chanson [18], reported a detectable bubble of 0.1 mm for a probe tip diameter of 0.025 mm, which shows a ratio of four. Obviously, smaller bubbles would also be detected by the probe tip but, as this ratio becomes less, the possibility of extreme deformations or diversion of the bubbles from their path increases. Therefore, for detecting bubbles as small as 1 mm, the probe dimensions were selected as shown in Figure 1.

ELECTRICAL CIRCUIT AND DATA ACQUISITION SYSTEM

The analog signals from the probe were processed in the probe circuit. The sampling rate must be high enough to provide a reliable representation of analog signals. It is known that to record a signal correctly, at least three points should be measured on it. That is, the frequency of data acquisition should be at least twice the input wave frequency [34]. If a 1 mm bubble travels with a 10 m/s velocity, the minimum frequency for data collection is 20 kHz. Higher frequencies could, however, produce more correctly the interface between air and water. It is advised to measure at least 10 points of a signal wave [34]. In the new probe system, the circuit is designed so that the frequency of sampling is adjustable up to 250 kHz.

A block diagram of the probe circuit is shown in Figure 3. This circuit is installed in a computer similar to an interface board. In this circuit, two 24 bit counters are utilized for measuring total sampling and disconnection time. In addition, a 20

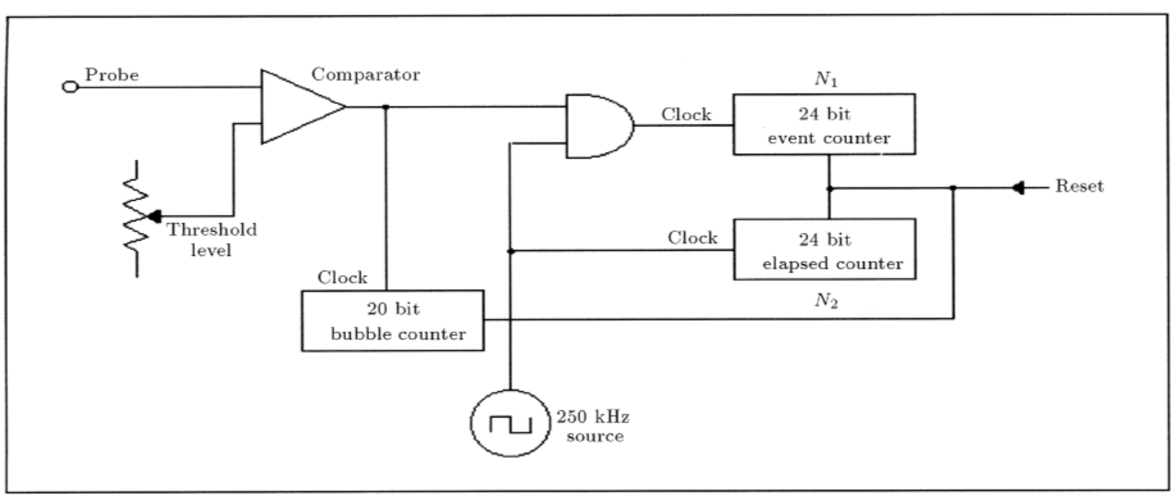

Figure 3. Needle probe block diagram.

bit counter is employed for counting the number of bubbles. The maximum frequency of sampling is 250 kHz for which the maximum time of sampling in this frequency is 72 seconds. The frequency of sampling can be adjusted in the tests and, with lower scanning frequencies, the time of sampling increases proportionally.

The time of sampling depends on the non-uniformity of bubble size and the fluctuations in flow characteristics. Many tests were carried out to find the longest time of sampling necessary for an independent result. A sample of recorded air concentration in these tests is shown in Figure 4. This test is performed with a 50 kHz frequency and with a maximum of 30 seconds of sampling time. In some of the tests, small fluctuations in measured air concentration could be seen for 45 seconds [35]. Based on these tests, a maximum of 45 seconds was considered enough for all tests of the present study and the electronic circuit was designed for a maximum of 72 seconds data collection with the highest frequency. However, one of the advantages of the new probe is its ability for real time measurements. Therefore, the last results of bubble count and air concentration is shown on a computer monitor in the form of graphs, simultaneously, with sampling. Therefore, the operator could stop the measurement as soon as the air concentration value showed no variation in time.

One of the possible problems with this kind of probe is the leakage of a very weak current in the circuit, for example, as the result of a penetration of water into the probe or a deficiency in the isolation of the wire or electrical connections. This problem may affect the accuracy of the probe. To detect this problem, very sensitive microampere meters were installed on the probe so that, if such current occurs the user will come to know. A LED displayer was also installed, where its light changed according to voltage (air concentration), so that the operator could realize the case of a short circuit.

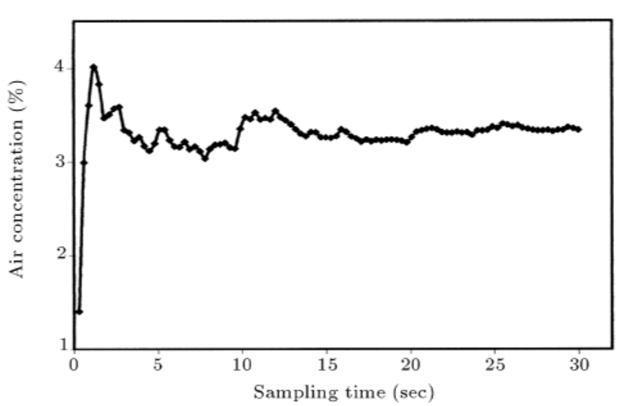

Figure 4. Variation of measured air concentration with sampling time.

POLARIZATION OF THE PROBE TIP

Since the probe works with DC current, polarization of the needle tip (deposition of ions on the probe tip) occurs as a result of electrolysis, which can significantly affect the results [3,14,22,36]. A high serial resistor was used in the circuit to reduce the current and, therefore, the polarization effect. By increasing the serial resistor, the sensitivity of the probe increased and it became necessary to design the Schmidt trigger with a high entrance resistance. In the present work, a 330 kΩ serial resistor was employed. The probe was tested in still water and a variation of voltage was monitored with a sensitive voltmeter. If polarization reduced the conductive area of the wire section, the current would drop. After 2 hours, no change in current voltage was observed. This means that the polarization effect was postponed. On the other hand, the air concentration variation in time and on line bubble count could be seen on the computer monitor and, therefore, any problem in data collection could be noticed by the user. Owing to small suspended material in the water flow, which may stick to the probe tip and disconnect or lower the current, it is recommended to clean the probe tip periodically with a fine file.

CALIBRATION OF THE PROBE

For calibration purposes, the probe was employed in the air-water flow and its measurements were compared with other instruments. However, before such experiments, the probe circuit was tested electronically.

ELECTRONIC TEST OF THE PROBE CIRCUIT

In these tests, signals with square and triangular shapes and with different frequencies were generated by a function generator and sent to the probe as input signals. Performance of the circuit could then be evaluated from examining its output.

If a square wave, as shown in Figure 2, is assumed, it represents an ideal signal from the air-water flow, with its air concentration equal to the duty cycle of the wave, as defined as the following:

$$\text{duty cycle} = \frac{\text{mark}}{\text{period}}. \tag{2}$$

Waves with different frequencies were generated and air concentration, as given by the probe, was compared with the known duty cycle of the wave. Relative error, as defined by the following relation, was then plotted against the input signal frequency (Figure 5):

relative error =(air concentration as given by the

probe duty cycle)/duty cycle.

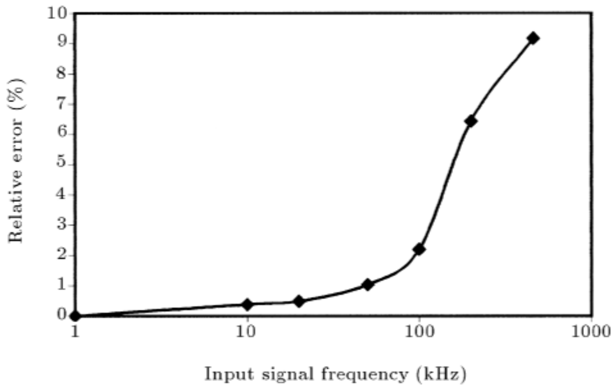

Figure 5. Relative error versus input signal frequency.

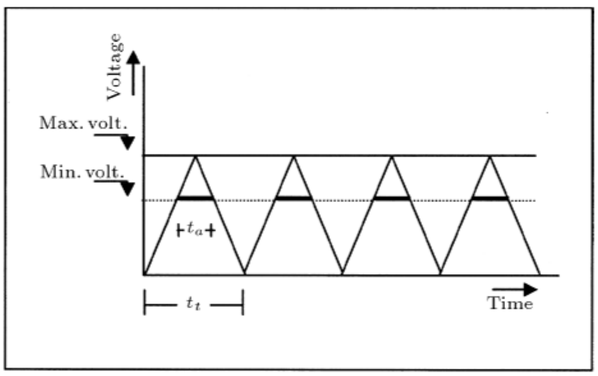

Figure 6. Triangular signals.

These tests were done with the frequency of data collection set to a maximum of 250 kHz. It was concluded that relative error increases with input signal frequency and, for input frequencies (more than half of the sampling frequencies), errors increase sharply as expected and stated before.

Adjustments of the Schmidt trigger could be examined with triangular waves. Known triangular waves were generated and introduced to the circuit. The threshold voltage of the circuit was changed, which caused different air concentrations to be measured by the probe, as shown in Figure 6. All tests in this section confirmed the correct performance of the circuit.

CALIBRATION OF THE PROBE IN AIR-WATER FLOW

Calibration tests were performed in a 20 cm-wide flume [35]. Different air discharges were injected through a slot in the bed of the flume, filled with porous material, to distribute air evenly through the width (Figure 7). Air concentration profiles were then measured over 1 mm spacing in the flow depth. Velocity profiles were also measured at the same sections using a pilot tube when air injection was stopped. Air discharge in the flow was calculated from known velocity and air concentration profiles by the following equation and compared with the injected air discharges, which were measured with a rotameter:

$$Q_{\text{air}} = \int_0^h b.c.u.dy, \tag{3}$$

where c is air concentration measured by the probe, b is flume width, u is velocity and h is the flow depth. The flow characteristics of these tests are given in Table 1.

Counter lines of air concentration measured along the flume in two different velocities are also given in Figure 8. These figures clearly show the rising of bubbles towards the surface in the downstream direction.

DETERMINING THE THRESHOLD LEVEL

Not all changes in the circuit current are recorded as the passing of a bubble and only the rise in the voltage of more than a predetermined value, called the threshold level, is taken as the gas phase. Measurements in air-water flow were repeated with 28%, 35%, 50% and 72% of the maximum voltage. More than 30 calibration tests were carried out with 4 threshold levels in 3 flow velocities and 3 air discharges.

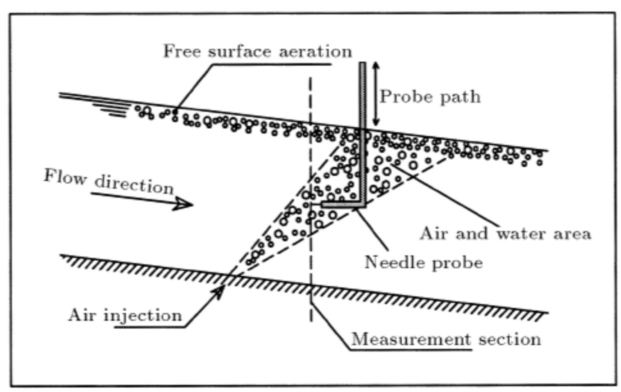

Figure 7. Testing the probe in air-water mixture.

Table 1. Experiments specifications.

Air Discharge (lit/sec)	Water Discharge (lit/sec)	Flow Velocity (m/sec)	Range of Air Concentration Measured	Flow Depth (mm)
0.2-0.5	6-30	2-5	0-15%	25-45

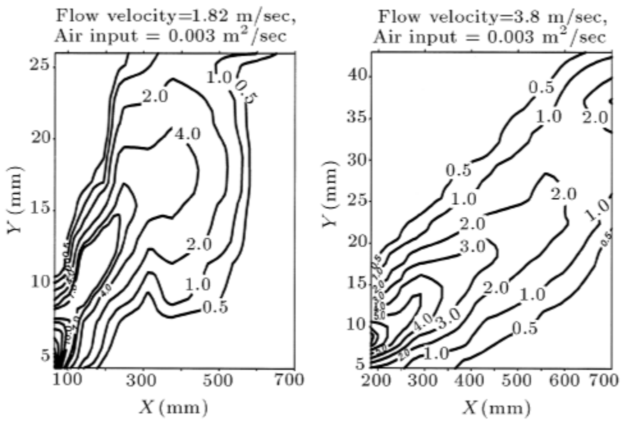

Figure 8. Measured air concentrations along the flume in various conditions.

Comparison of calculated air discharge, using Equation 3 and what was measured by the rotameter, is shown in Figure 9. Air flow rates measured by the probe increased with a decrease in the threshold level and the best result is for 35% threshold level, with most of the measured air discharge within ±15% difference with rotameter measurements.

The best fit line for each threshold level was drawn and shown in Figure 10. According to this figure, a threshold level of 35% shows an average zero difference with rotameter readings. Regarding the good performance of the probe circuit after electronic tests, it can be said that other factors, such as fluctuation in water surface elevation and errors in locating the tip of the probe in position throughout the depth, could be responsible for the major part of the ±15% difference observed between needle probe and rotameter measurements. It is interesting to notice that, despite the different probe designs, flow conditions and calibration methods, the value of the threshold obtained in the present study is in the range of 28% to 40%, reported by most of the other investigators. Therefore, it can be concluded that, for velocities up to 10 m/s and air

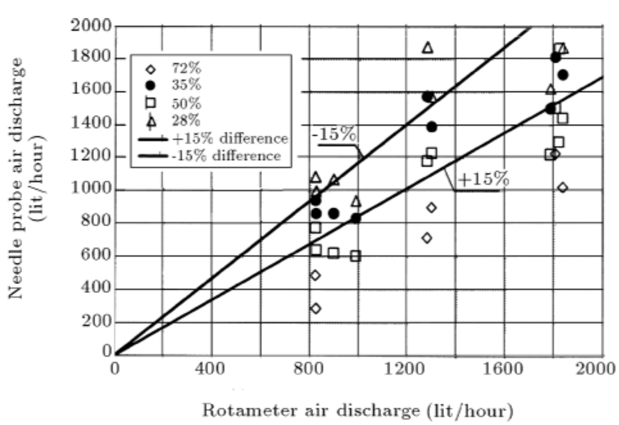

Figure 9. Comparison of air discharge measured by needle probe and rotameter.

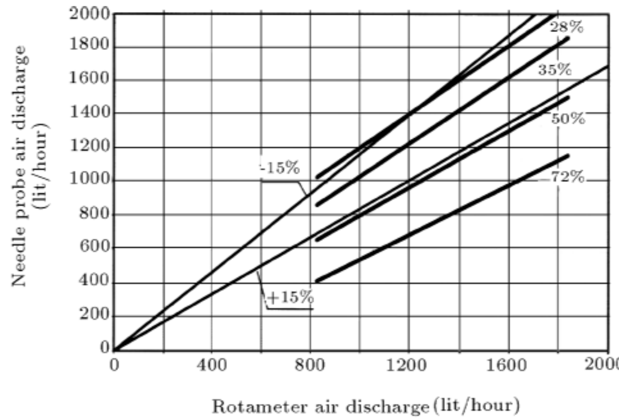

Figure 10. Comparison of measured results for various threshold voltages.

discharges up to 50%, a threshold level between 28% and 40% seems to be appropriate.

SUMMARY AND CONCLUSIONS

A needle probe for measurement of air concentration and bubble count in high-speed air-water flow was developed. One advantage of this method is in its ability for the real time measurement of local void fraction and bubble count. The sampling frequency of the new probe is adjustable up to 250 kHz for a sampling time of 72 seconds.

Polarization of the probe tip was treated using a high serial resistor in the circuit. Electronic test of the probe circuit ensured its correct performance and use in air-water flow showed that this probe is suitable and easy to use. Sensitive microampere meters were installed on the probe to detect possible weak leakage currents. More than 30 laboratory tests in air-water flow were carried out to set the threshold voltage of the probe circuit and to test its accuracy.

Laboratory studies showed that the optimum threshold level for the probe is 35%. The maximum difference in air discharge, calculated by the probe measurements, and those measured by a rotameter, was ±15% at the optimum threshold level. Despite a different probe design, flow condition and calibration methods, the optimum level found in the present study was in the range of 28% to 40%, reported by most of the other investigators.

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