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Basic Issues in the Analytical Simulation of Unstiffened Extended End Plate Connection

M.R. Mohammadi-Shoreh[1] and M. Mofid*

In this paper, a new analytical method is presented that can be used to determine the behavior of a particular steel beam-to-column extended end plate connection in linear and nonlinear regions. A common means of forming a rigid joint between a universal beam and column is to weld an end plate to the beam end and then to bolt it to the column. A physically based analytical method that can predict the behavior of bolted and extended end plate eave connections, using the connection dimensions as input, is presented. This article demonstrates an analytical procedure for the establishment of elastic and plastic parts of the M θ curve of this form of connection. An analytical method is proposed for the extended end plate joints having four bolts in the tension region and without any stiffened plate. However, the presented technique can also properly be extended to other types of this form of connection. An extensive approach to estimate the plastic stiffness of the connection has also been performed. Comparison is made on a series of test results for a range of bolted end plate moment connections and good agreement is achieved. Furthermore, the authors believe that the method presented shall efficiently serve design engineers in real design conditions.

INTRODUCTION

Beam-to-column end plate connections have been the subject of increasing interest and study since 1960 [1,2]. Several researchers have been working to understand connection structural behavior, focusing, at first, on the hypothesis of similarity between the behavior of end plate and T-stub connections. According to this view, other works can be pointed out, such as those developed by Kato and McGuire [3] and Packer and Morris [4]. Krishnamurthy [5] conducted a finite element analysis of moment end plate connections and developed a practical design method for the beam-to-column end plate connections. Based on his analytical study of the end plates, as well as a series of experimental tests, Krishnamurthy developed a design procedure, which was eventually presented in the AISC Manual of Steel Construction (1980). Other recent studies have used the finite element method and multi-variable regression analysis developed by Gebboken et al. [6], Sherbourne and Bahaari [7-9], Bursi and Jaspart [10-12] and Nemati et al. [13].

1. *Department of Civil Engineering, Sharif University of Technology, Tehran, I.R. Iran.*
*. *Corresponding Author, Department of Civil Engineering, Sharif University of Technology, Tehran, I.R. Iran.*

Experimental investigations on the static and cyclic performance of end plate moment connections have also been conducted by Jenkins et al. [14], Cruz et al. [15], Riberio et al. [16], Adey et al. [17] and Yorgun et al. [18].

In addition, some other studies have introduced the standardized moment-rotation function or analytical approach for predicting the moment rotation behavior of extended end plate connections [9-26].

Experimental study of the connections is very expensive and it is impractical to experiment with all the different types and sizes of connections. On the other hand, the 3-D finite element models are fairly complicated for simulation of the connections, as well as the whole structure.

The objectives of this investigation are:

- To prepare a reliable, approximate model that can easily be constructed and be simple enough for use as a connecting element in the nonlinear analysis of steel structures;
- To study the behavior of extended end plate connections, which is widely used in high-rise buildings and industrial structures and to develop a step-by-step analytical technique to prepare the capacity for this type of connection;
- To verify the presented analytical solution by apply-

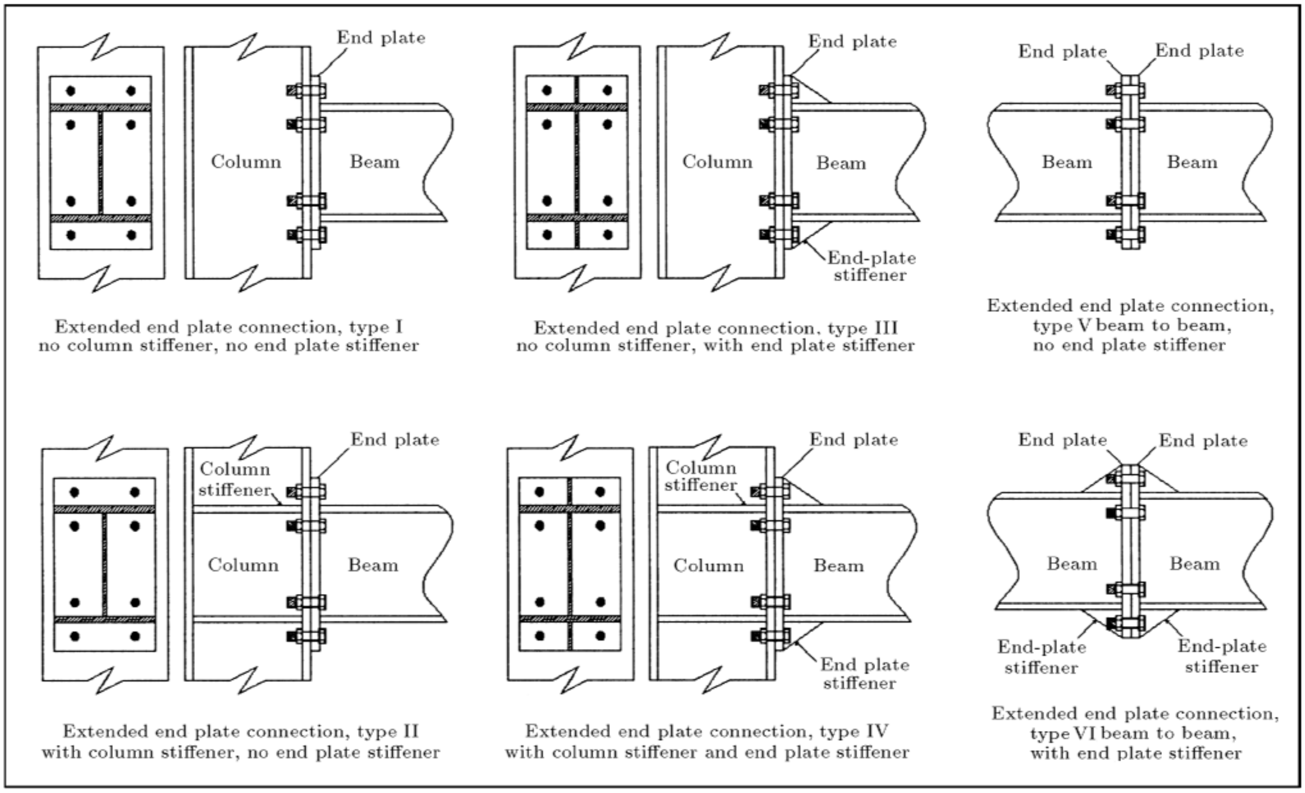

Figure 1. Common types of end plate connections.

ing the same test problems and by comparing the results with finite element models, as well as with experimental data;

- To inspect the important factors in the behavior of this type of connection, through on-parametric study.

THEORY OF THE ANALYTICAL MODEL

The popularity of moment resisting joints with extended connecting end plates can be attributed mainly to the simplicity and economy associated with their design, fabrication and erection. Four common types of tension bolted, extended, end plate connections are shown in Figure 1. These connections are used frequently to achieve a relatively rigid connection behavior. In this paper, the behavior of an extended end plate connection without column stiffeners and end plate, type I, has been studied.

Figure 2 schematically shows the moment rotation behavior of a variety of commonly used end plate connections. The initial slope of an M θ curve is defined as initial stiffness, K_e. The ultimate moment, M_u, is defined as the maximum moment that can be transmitted by the connection. If connection capacity is greater than the beam or column ultimate moment, then M_u^* is defined as the lowest ultimate moment capacity of the beam and column. However, if the connection capacity is smaller than the beam and column ultimate moment, then M_u^* shall be defined as M_u. Consequently, the slope of an M θ curve at ultimate moment M_u^*, is defined as plastic stiffness, K_p^*.

EVALUATION OF MODEL PARAMETERS

To study the behavior of the connection, the following assumptions are made.

- The end plate should be exactly attached to the beam;
- The effect of shear force on connection deformation

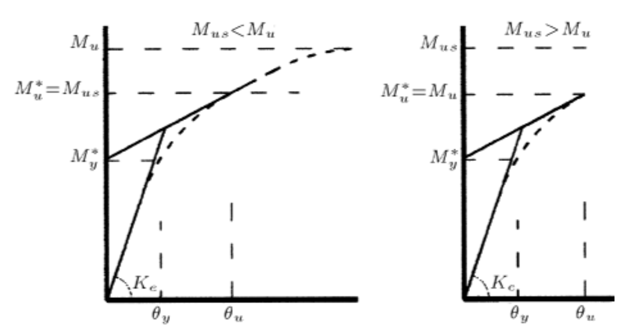

Figure 2. Schematically connection moment rotation curves.

is ignored. However, the effects of the shear deformation of the components of connections, caused by the connections moment, are determined;

- The material behavior of the beam, end plate and column is considered as perfect elastoplastic behavior, which is shown in Figure 3;
- All the bolts are preloaded and a pretension load, limited to 40 percent of the actual yield stress, was applied;
- In-plane deformations are negligible.

To overcome the implementation of a complex, nonlinear, finite element analysis in the prediction of the connection behavior, a simpler approach was developed in the form of a so called "Component Method" [25]. Briefly described, the method consists of modeling a joint as an assembly of extensional spring and rigid links, whereby the springs represent a specific part of a joint that, depending on the type of loading, makes an identified contribution to one or more of its structural properties. Considering the properties of a moment rotation curve, it is evident that in order to present a bilinear curve, the following requirements are needed:

$$M = 0 \quad \text{at} \quad \theta = 0, \tag{1}$$

$$\frac{dM}{d\theta} = K_e \quad \text{at} \quad \theta = 0, \tag{2}$$

$$M_y = K_e \times \theta_y, \tag{3}$$

$$\frac{dM}{d\theta} = K_p^* \quad \text{at} \quad M \to M_u^*, \quad \theta \to \theta_u^*. \tag{4}$$

In order to obtain a bilinear curve, the parameters K_e, M_y, M_u^* and K_p^* must be predicted. This can be done analytically, as described in the following sections. Figure 4 identifies the various geometric parameters, which have been used in the present study.

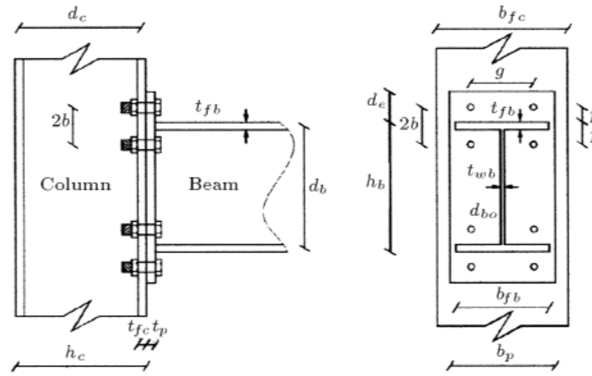

Figure 4. Unstiffened extended end plate connection geometry.

Evaluation of Initial Stiffness

To evaluate the initial stiffness of the connections, the following assumptions are made:

- The center of rotation of the connection should coincide with the middle of the beam lower flange;
- Applied force from the beam upper flange is considered for total plate width;
- Centerlines of bolts are assumed as a clamped edge for plate;
- The beams act as rigid members.

The rotational stiffness of a connection is directly related to the deformation of individual connection elements. The components, which contribute to the deformation of the connection, are: (1) End plate, (2) Tension bolts, (3) Column flange and (4) Column section. However, because of the complexity of these deformations, they have been separated into two parts. These parts contain deformation caused by beam flange and beam web and each of these deformations are made up of four parts. These divisions shall be considered as presented in Table 1.

The application of the method to a typical, bolted, end plate, beam-to-column joint is illustrated in Figure 5. A general expression for initial stiffness, K_e,

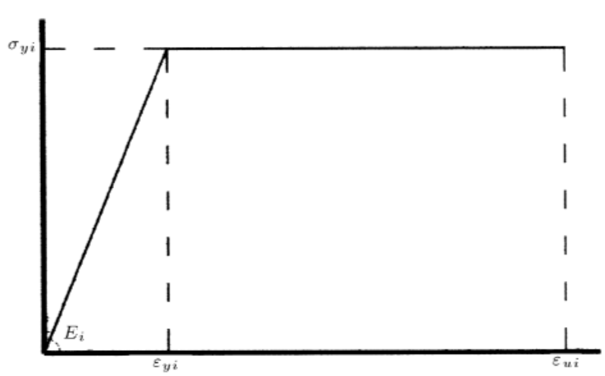

Figure 3. Connection material stress-strain curve.

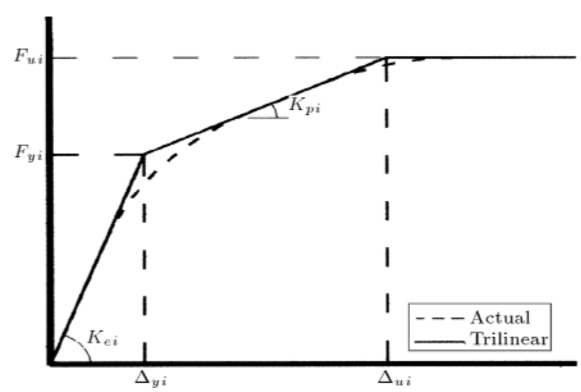

Figure 5. Characterization of component behavior.

Table 1. Considered deformations to calculate the initial stiffness.

Division	Subdivision	Deformation Symbol	Stiffness Symbol
Part 1	1.1 Deformation of end plate	δ_{fp}	K_{fp}
Beam	1.2 Deformation of bolts	δ_{fb}	K_{fb}
flange	1.3 Deformation of column flange	δ_{ff}	K_{ff}
effect	1.4 Deformation of column section	δ_{fs}	K_{fs}
Part 2	2.1 Deformation of end plate	δ_{wp}	K_{wp}
Beam	2.2 Deformation of bolts	δ_{wb}	K_{wb}
web	2.3 Deformation of column flange	δ_{wf}	K_{wf}
effect	2.4 Deformation of column section	δ_{ws}	K_{ws}

Table 2. Expressions for initial stiffness, K_e.

$K_e = \frac{1}{\frac{1}{K_{fp}}+\frac{1}{K_{fb}}+\frac{1}{K_{ff}}+\frac{1}{K_{fs}}+\frac{1}{K_{wp}}+\frac{1}{K_{wb}}+\frac{1}{K_{wf}}+\frac{1}{K_{ws}}} = \frac{E.b.h_b^2}{(1 \quad \lambda_e)\left(\frac{1}{R_o}+\frac{1}{2nR_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)+\frac{6\lambda_e}{R_2}\left(\frac{R_2}{R_1}+\frac{1}{R_3}+\frac{1}{R_4}+\frac{2}{R_5}\right)}$
$a^* = 2 \times b + g, \qquad \lambda_e = \frac{\frac{t_{wb}}{b_{fb}}}{\frac{t_{wb}}{b_{fb}}+6\frac{t_{fb}}{d_{wb}}+\left(\frac{t_{fb}}{d_{wb}}\right)^3} \cong \frac{1}{1+6\frac{A_{fb}}{A_{wb}}}$
$Q = 208(1+\frac{d_b}{b}) + 33.6 \times \frac{g^2}{b^2}(1+\frac{b}{d_b}) + 13 \times \frac{g^4}{b^4}(1+\frac{b^3}{d_b^3})$
$R_0 = \frac{t_p^3 b_{fc}/b^2}{0.455 \times b^2+1.56 \times t_p^2}, \qquad R_1 = \frac{8}{21(1 \quad v^2)} \times \frac{Q.t_p^3/g}{7g^2+6t_p^2\left(8+\frac{g^2}{b^2}+\frac{g^2}{d_b^2}\right)}$
$R_2 = \frac{64}{147}\frac{Q.b^2/d_b}{13 \times d_b+16 \times b \quad \frac{g^4}{b^3}}, \qquad R_3 = \frac{\pi}{4b.(t_p+t_{fc})}\left(1.33 \times d_{bo} + \frac{1}{2} \times \frac{t_p.t_{fc}}{t_p+t_{fc}}\right)^2$
$R_4 = \frac{4a^*.t_{fc}^3/(g \quad t_{wc} \quad d_{bo} \quad t_p)^3}{b.\left(1+3.12\frac{t_{fc}^2}{(g \quad t_{wc} \quad d_{bo} \quad t_p)^2}\right)}, \qquad R_5 = \frac{24I_c/b}{{a^*}^3+16 \times d_b^3+\frac{24}{7} \times d_c^2 a^*}$

is formulated in Table 2, which shows, in detail, the expressions for the initial stiffness of various components of unstiffened, extended, end plate eave connections.

Evaluation of Ultimate Moment

The various components contributing to the overall response of a generic, end plate, beam-to-column, steel joint include the following:

1. Column web in shear, compression and tension,
2. Column flange in bending,
3. End plate in bending,
4. Bolts in tension and shear,
5. Beam flange and beam web in tension and compression.

On the basis of these assumptions, the moment capacity of an unstiffened connection depends on the strength of the individual connection elements. Various investigations [4,27] have shown that an unstiffened connection will begin to lose its ability to sustain further loading when one or more of the following failure modes occurs:

1. Bolt failure (in tension),
2. Formation of the end plate plastic mechanism,
3. Formation of the column flange plastic mechanism,
4. Shear yielding, buckling or crippling of the column web,

5. Beam compression flange buckling.

Each of these failure modes will now be considered in terms of the tension and compression beam flange forces. In the elastic region of the beam bending behavior, using the bending theory, the simplified relationship between the moment acting on the connection, M, and beam flange force, F, can be given as follows:

$$F = \frac{M}{d_b} \times \frac{1}{1 + \frac{A_f}{6A_w}} = \frac{M}{\lambda_e d_b}. \tag{5}$$

If the moment of the connection increases until the plastic hinges occur at the beam, using the plastic theory, the beam flange force can be obtained as:

$$F = \frac{M}{d_b} \times \frac{1}{1 + \frac{A_f}{4A_w}} = \frac{M}{\lambda_p d_b}. \tag{6}$$

In seismic design, connections should bear beam moment capacity. Thus, the value of λ_p is considered as λ. Then, each of the possible failure modes shall be considered in terms of F and, consequently, the ultimate force for each mentioned component (end plate, tension bolt, column flange and column section) can be evaluated. Thus the lowest value corresponding to the following failure modes will present the amount of M_u^*. Besides, Table 3 summarizes the equations used to evaluate M_u^* for this type of extended end plate connection.

Evaluation of Yielding Moment

At first, for calculation of the connection yielding moment, the stress-strain curve of the beam material is considered perfectly elastoplastic, as shown in Figure 3. Then, using the plastic theory, the corresponding load-deformation curve will be calculated. According to extensive study in this area [28], the yielding moment of the end plate connection, M_y, can be evaluated as the following:

$$M_y^* = .7 \times \min(M_{ue}, M_{uf}, M_{us}), \tag{7}$$

$$M_y = \frac{1}{1 \quad K_p^*/K_e} \times M_y^*. \tag{8}$$

Evaluation of Plastic Stiffness

There is no exact applicable analytical method for calculation of the plastic stiffness of connections and, therefore, usually, test results are used to estimate the value of plastic stiffness. For example, the ratio of $K_p/K_e \approx .05$ for unstiffened end plate connections is suggested by Sherbourne et al. [23]. This ratio is resulted by testing and includes the effects of strain hardening and changes in geometry of the connection. Therefore, in this article, the equations used to evaluate K_p^* are formulated in Table 4.

Table 3. Considered mechanisms to calculate the ultimate moment.

End Plate Ultimate Moment	$M_{ue_1} = F_{yp} t_p^2 \times \left[\frac{b_p}{p_f} + 2\frac{d_b \quad p_t}{g \quad t_{wb}} + \frac{g \quad t_{wb}}{4d_b}\right] \times \lambda d_b$
	$M_{ue_2} = F_{yp} t_p^2 \times \left[\frac{g+b_p}{4} + \frac{d_b.b_p}{4d_e} + p_f.\left(\sqrt{\frac{2p_f}{g}} + \sqrt{\frac{b_p}{2p_f}}\right)^2\right] \times \lambda + nA_{bo} f_{ybo}\left(2 \quad \frac{p_f}{d_e} \quad \frac{p_t}{d_b}\right) \times \lambda d_b$
	$M_{ue_3} = F_{yp} t_p^2 \times \left[g + \frac{h.b_p}{2t_{fb}} + 2t_{fb}\frac{d_b \quad p_t}{g} + (d_b \quad t_{fb}).\left(\sqrt{\frac{2p_f}{g}} + \sqrt{\frac{b_p}{2p_f}}\right)^2\right] \times \lambda$
	$M_{ue} = \min(M_{ue_1}, M_{ue_2}, M_{ue_3})$
Column Flange Ultimate Moment	$M_{uf1} = F_{yc} t_{fc}^2 \times \left(\pi + \frac{p_f + p_t}{b_{fc} \quad t_{wc} \quad 2w_s}\right) \times \lambda d_b + \pi d_{bo}^2 F_{ybo}\left(\frac{b_{fc} \quad g}{b_{fc} \quad t_{wc} \quad 2w_s}\right) \times \lambda d_b$
	$M_{uf2} = F_{yc} t_{fc}^2 \times \left(\pi + 2\frac{b_{fc} \quad g+p_f+p_t \quad d_{bo}}{g \quad t_{wc} \quad 2w_s}\right) \times \lambda d_b$
	$M_{uf} = \min(M_{uf_1}, M_{uf_2})$
Column Web Ultimate Moment	$F_{pw1} = \frac{1}{\sqrt{3}} F_{yc} t_{wc} d_c, \qquad F_{pw2} = 10.765 \times t_{wc}^3 \sqrt{F_{yc}}/d_c, \qquad F_{pw3} = F_{yc} t_{wc}.(t_{bf} + 2t_p + 5k_c)$
	$M_{uw} = \min(F_{pw1}, F_{pw2}, F_{pw3}) \times \lambda d_b$
Tension Bolts Ultimate Moment	$F_{Pbo} = \frac{1}{\gamma} F_{ybo} A_{bo}$
	$M_{ubo} = 3F_{ybo} A_{bo} \times \lambda d_b$
Ultimate Moment	$\lambda = 1 + \frac{A_w}{4A_f}, \qquad M_u^* = \min(M_{ue}, M_{uf}, M_{uw}, M_{ubo})$

Table 4. Expressions for plastic stiffness, K_p^*.

Failure Mode	End Plate or Column Flange or Bolt Failure	Column Web Failure	Plastic Hinge in Beam or Column
Plastic Stiffness	$K_p^* = \frac{1}{\frac{1}{K_{p\ \ pfb}} + \frac{1}{K_{e\ \ web}} + \frac{1}{K_e}}$	$K_p^* = \frac{1}{\frac{1}{K_{e\ \ pfb}} + \frac{15.3}{K_e}}$	$K_p^* = \frac{K_e}{11}$

$$K_{p\ \ pfb} = \frac{40 \times E d_b^2}{16 \times \frac{d_e^3 + 6 d_e p_t^2 \quad 2p_t^3}{b_p . t_p^3} + \frac{b_{fc}^3 + 6 b_{fc} g^2 \quad 2g^3}{d_e . t_{wc}^3}}, \qquad K_e = \frac{1}{\frac{1}{K_{e\ \ pfb}} + \frac{1}{K_{e\ \ web}}}$$

$$K_{e\ \ web} = \frac{1}{\frac{1}{K_{fs}} + \frac{1}{K_{ws}}}, \qquad K_{e\ \ pfb} = \frac{1}{\frac{1}{K_{fp}} + \frac{1}{K_{fb}} + \frac{1}{K_{ff}} + \frac{1}{K_{wp}} + \frac{1}{K_{wb}} + \frac{1}{K_{wf}}}$$

Table 5. Schedule of different test problems, beam and column properties.

Ref.	Name	h_b	d_b	b_{fb}	t_{wb}	t_{fb}	F_{yb}	h_c	d_c	b_{fc}	t_{wc}	t_{fc}	k_{c1}	F_{yc}
[14]	Test 1	31	29.68	16.7	0.760	1.32	244	27.53	25.02	26.07	1.54	2.51	2.06	244
	Test 2	31	29.68	16.7	0.760	1.32	244	27.53	25.02	26.07	1.54	2.51	2.06	244
	Test 3	31	29.68	16.7	0.760	1.32	244	27.53	25.02	26.07	1.54	2.51	2.06	244
	Test 4	31	29.68	16.7	0.760	1.32	244	27.53	25.02	26.07	1.54	2.51	2.06	244
[23]	PAR3e	40.3	39.2	17.7	0.775	1.10	300	22.90	20.5	21	1.4	2.4	1.75	300
	PPAR3e	40.3	39.2	17.7	0.775	1.10	300	22.90	20.5	21	1.4	2.4	1.75	300
	PAR4	40.3	39.2	17.7	0.775	1.10	300	22.90	20.5	21	1.4	2.4	1.75	300
	PAR10	40.3	39.2	17.7	0.775	1.10	300	22.90	20.5	21	1.4	2.4	1.75	300
	PM5	45.9	44.38	15.4	0.910	1.52	250	25.30	23.89	25.4	0.88	1.41	1.75	250
	PM6	45.9	44.38	15.4	0.910	1.52	250	25.30	23.89	25.4	0.88	1.41	1.75	250

* All dimensions are in cm and MPa.

EXAMPLE PROBLEMS

To test the relative accuracy of the presented analytical method and, also, in order to consider various combinations of beams and columns in terms of load carrying capacity, ten unstiffened, extended, end plate examples from different studies are carefully selected. Beam and column properties are shown in Table 5 and properties of other components of the connections are shown in Table 6. The modulus of elasticity of all components is considered as $E = 210 \times 10^9$ N/m^2.

Using the presented analytical method, the initial stiffness, yielding moment and ultimate moments of each connection are calculated, respectively. Also, the ratio of K_p/K_e for every case is calculated and listed in Table 7. In addition, to show the accuracy of the results, the bilinear curve that is obtained with the presented technique and the experimental moment rotation curve for four cases is illustrated in Figure 6. Also, the presented bilinear curve and the finite element prediction curve for six specimens are illustrated in Figure 7. The presented bilinear curve passes through three points. The first point is origin, the second is (θ_y, M_y) and the third is (θ_u, M_u).

As noted, θ_y is obtained by "M_y/K_e" and θ_u is equal to "$\theta_y + (M_u \quad M_y)/K_p$".

CONCLUSION

In this investigation, an analytical procedure for estimation of the behavior of unstiffened extended end plate connections has been presented. The theory in this method is mainly based on the theory of a yield line mechanism. Substitution of the values of material property and the dimensions of the connection in the proposed formulas lead to calculation of the initial stiffness, yielding moment and ultimate moment stiffness of the connection, which demonstrates the connection behavior and which can be used by scientists and engineers. Comparison of the results in

Table 6. Schedule of different test problems, end plate and bolt properties.

Ref.	Test Designation	b_p	d_e	b	g	d_{bo}	P_f	P_t	w_s	t_p	F_{yp}	F_{ybo}
[14]	Test 1	20	11.0	6.0	13	2	4.34	6.66	1	1.27	244	627
	Test 2	20	11.0	6.0	13	2	4.34	6.66	1	1.52	244	627
	Test 3	20	11.0	6.0	13	2	4.34	6.66	1	2.03	244	627
	Test 4	20	11.0	6.0	13	2	4.34	6.66	1	2.54	244	627
[23]	PAR3e	21	12.6	7.15	12	2.4	5.6	7.7	1	3.0	300	627
	PPAR3e	21	12.6	7.15	12	2.4	5.6	7.7	1	2.4	300	627
	PAR4	21	12.6	7.15	12	2.4	5.6	7.7	1	1.5	300	627
	PAR10	21	12.6	7.15	12	2.4	5.6	7.7	1	4.0	300	627
	PM5	25.4	12.4	6.76	12	2.4	5.0	7.52	1	2.0	250	627
	PM6	25.4	12.4	6.76	12	2.4	5.0	7.52	1	3.0	250	627

* All dimensions are in cm and MPa.

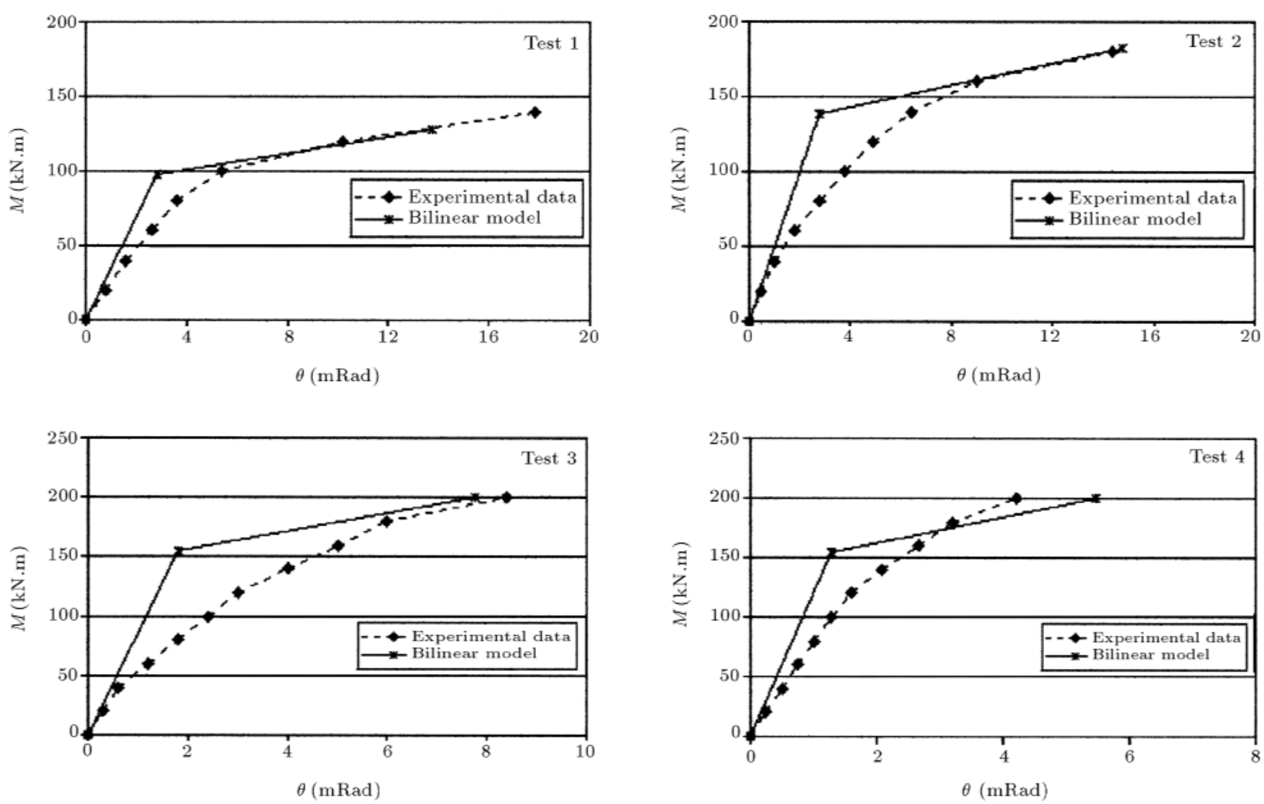

Figure 6. Comparison between test and prediction results.

Figure 6 shows a good agreement between experimental data and the presented technique in this investigation. Also, comparison of the results in Figure 7 shows an approximate agreement between finite element prediction and the presented technique in this investigation.

The significant points of this investigation can be summarized as follows:

- Ratio of K_p/K_e varies between 0.04 and 0.14 for end plate connections.
- If the connection ultimate moment, M_u, is greater than the beam or column ultimate moment, then, the ratio of K_p/K_e varies between 0.09 and 0.14.
- If the connection ultimate moment, M_u, is smaller

Table 7. Results of the presented analytical method.

Name	$K_y \times 10^7$	M_y	M_u^*	K_p^*/K_e	$K_p^* \times 10^7$
Test 1	3.41	97	128	0.081	0.278
Test 2	4.99	138	183	0.074	0.371
Test 3	8.53	154	200	0.091	0.776
Test 4	12.11	154	200	0.091	1.101
PAR3e	24.30	235	311	0.062	1.497
PPAR3e	17.72	235	311	0.062	1.093
PAR4	7.29	224	294	0.083	0.602
PAR10	32.57	235	311	0.061	2.002
PM5	6.65	184	211	0.063	0.419
PM6	12.17	184	211	0.063	0.762

*All dimensions are in kN.m and kN.m/rad.

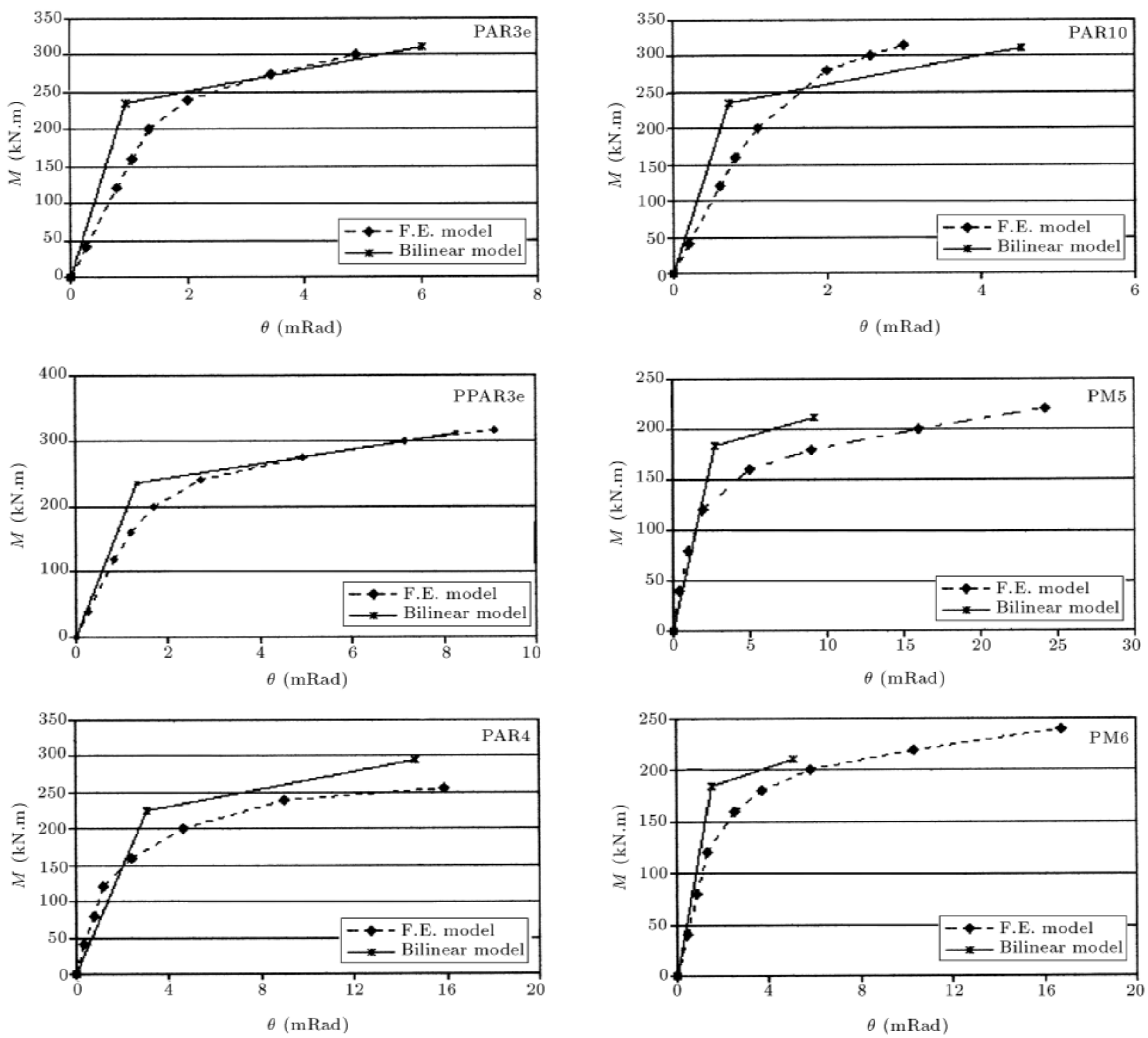

Figure 7. Comparison between F.E. and prediction results.

than the beam or column ultimate moment, then, the ratio of K_p/K_e varies between 0.04 and 0.09.

NOMENCLATURE

A_{bo}	area of each bolt
A_{fb}	area of beam flange
A_f	area of beam flange
A_w	area of beam web
E	modulus of elasticity
E_{bo}	modulus of elasticity of bolt
F	beam flange force
K_e	initial stiffness of the connection
K_p	plastic stiffness of the connection
M_y	yield moment applied to connection
M_u	total moment applied to connection
M_{ue}	total moment applied to end plate
M_{uf}	total moment applied to column flange
M_{uw}	total moment applied to column web
M_{ub}	total moment applied to bolts
$\lambda, \lambda_e, \lambda_p$	coefficients defined in Equations 5 and 6
Δ	displacement (general)
θ_y	total rotation in elastic zone
θ_u	total rotation

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