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A Two-Stage Penalty Method for Discrete Optimization of Pipe Networks

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ϕA two-stage nonlinear programming penalty method for discrete optimization of pipe networks is presented in this paper. The problem of pipe network optimization is formulated as an unconstrained optimization problem via use of an iterative penalty method, which is then solved to get the continuous solution for the pipe diameters. In the second stage, a second optimization problem is defined to get the discrete solution starting from the already available continuous solution as a good initial guess. The search space for the discrete diameters is restricted to the upper and lower diameter limits of the optimal continuous solution. An all-purpose optimization toll, DOT, is used in both stages to obtain the solutions. The method is shown to be capable of producing results comparable to the existing algorithms with much less computational time. The method is used to find the optimal solution to some of the benchmark pipe networks and the results are presented. The results obtained for the networks consisting of pipes are encouraging. Further research is underway to extend the method for the optimization of pumped networks.

INTRODUCTION

The problem of network optimization requires the determination of pipe sizes from a set of commercially available diameters, ensuring a feasible least cost solution. Various methods, with different degrees of success, have been devised by different researchers to solve this problem. These methods can be divided into three classes, namely: Enumeration, mathematical programming and random search methods. Mathematically speaking, a pipe network optimization problem is known to possess many local optima hindering the search towards the desired global optimum. Amongst all the methods developed so far, only the enumeration methods have been proved able in locating the global optimum. These methods, however, suffer from limited practical application, due to an extraordinary wide search space and, consequently, the enormous computational time required when applied to real world size networks, where such a least cost solution is mostly desired [1,2]. Therefore, as a global optimum solution to the pipe network optimization problem is unattainable at the present time, the logical objective, from an engineering point of view, could be defined as discovering a proper balance between design cost and the time required to obtain the solution.

This paper addresses the determination of optimal diameters of pipes in a network with a pre-determined layout, in order to provide the required pressure and quantity of water at every demand node. Here, a two-stage penalty method is used to formulate the optimal design of a pipe network as an unconstrained optimization problem, which is solved by a general purpose optimization package, DOT (Vanderplaats, Miura and Associates, 1994, http://www.vrand.com), for the continuous and then discrete solution.

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CONTINUOUS SOLUTION STAGE

The optimal design of a pipe network with a pre-specified layout in its standard form is described as the following:

$$\min C = \sum_{l=1}^{m} L_l C_l. \qquad (1)$$

Subject to:

1. Hydraulic constraints:

$$\sum_{l\in k} q_l = Q_k \qquad k=1,2,\cdots,n,$$

$$\sum_{l\in p} J_l = 0 \qquad p=1,2,\cdots,P,$$

$$q_l = K ch_l d_l^{\alpha}(J_l/L_l)^{\beta}. \tag{2}$$

2. Head and flow constraints:

$$H_k \geq H_{\min} \quad k=1,2,\cdots,n,$$

$$q_l \geq q_{\min} \quad l=1,2,\ldots,m. \tag{3}$$

3. Pipe size constraints:

$$d_{\min} \leq d_l \leq d_{\max} \quad l=1,2,\cdots,m, \tag{4}$$

where, L_l = length of the lth pipe; C_l = per unit cost of the lth pipe; d_l = diameter of the lth pipe; q_l = flow in the lth pipe; J_l = head loss in the lth pipe; H_k = nodal head at the kth node; Q_k = consumption at node k; $H_{\min}$ = minimum allowable hydraulic head; $q_{\min}$ = minimum allowable pipe flow; $d_{\min}$ and $d_{\max}$ = minimum and maximum allowable pipe diameter, respectively; n, p, m = total number of nodes, loops and links in the network, respectively; K = Hazen-Williams constant (whose value depends on the system of units used); ch = Hazen-Williams roughness coefficient (whose value is dependent on the pipe characteristics); $\alpha = 2.63$ and $\beta = 0.54$. The first set of constraints describes the flow continuity at the nodes, the head loss balance in the loops and the Hazen-Williams equation. The second set refers to the minimum nodal head and pipe flow requirements and, finally, the last constraint requires that the optimal pipe diameters should be between the maximum and minimum available pipe diameters, respectively. Equation 1 describes the total cost of the pipes in the network.

In this work, the hydraulic constraints are satisfied via the use of an element-by-element simulation program, which explicitly solves the set of hydraulic constraints for nodal heads [3]. The second set of constraints is included in the objective function via the use of an exterior penalty method resulting in the following penalized problem:

$$\min C_p = \sum_{l=1}^{m} C_l L_l + \sum_{l=1}^{m} \alpha_l \left(q_l - q_{\min}\right)^2 + \sum_{k=1}^{n} \alpha_k \left(H_k - H_{\min}\right)^2, \tag{5}$$

where α_l and α_k are the pipe flow and nodal head penalty parameters, respectively, with large values when corresponding constraints are violated and zero values otherwise. The pipe size constraints are handled by the optimization package, DOT, as box constraints, since the pipe diameters are taken as decision variables. In this work, an iterative setting of the penalty parameter is chosen, whereby the minimization of the penalized cost function in Equation 5 is substituted with a series of minimization problems with differing values of penalty parameter values. The procedure starts with a value of unity for penalty parameters and, then, their value is increased by an order of magnitude if corresponding constraints are not satisfied. This procedure is continued until all the constraints are satisfied, i.e., all the penalized terms are equal to zero [4].

Use of mathematical programming methods often requires a continuous representation of the tabular cost per unit length of the commercial pipes. It is a usual practice to use an analytical function of the form αD^{β}, where the values of the unit cost parameters, α and β, are usually obtained via a least squares fit of the above function to the discrete pipe cost data. It is obvious, however, that this fit would not be exact in terms of both function value and its gradient. Most of the common mathematical search methods use function value or its derivative to find the optimum of the objective function. Thus, it is expected that improving the accuracy of the approximate cost function would improve the optimization results. In this work, a piecewise cubic spline is used to approximate the cost per unit length of the pipes and the results are compared with the usual least-squares form. Cubic splines are, by definition, third-order functions enjoying zero and first derivative continuity. A piecewise cubic spline fit to a set of N discrete data can, therefore, be easily constructed by assuming the continuity of $N-1$ piecewise third-order functions defined in $N-1$ reach and their first derivative at N discrete points. This leads to $2(N-1)$ equations, stemming from the fact that each of the $N-1$ piecewise functions at their two end points should equate with the corresponding discrete data, plus $2(N-2)$ equations, stemming from the continuity of the first derivative at $N-2$ internal points, summing up to $4(N-1)-2$ equations in terms of $4(N-1)$ unknowns defining $N-1$ piecewise third-order functions. Two boundary conditions regarding the slope of the spline at the two extreme points of the set are used to close the system of simultaneous linear equations. The solution of this system yields the value of the parameters required to define each of the cubic splines at $N-1$ reach. This function can then be used to approximate the value of the discrete function at any arbitrary points, including discrete data points. It is obvious that cubic spline approximation always yields exact values at discrete data points and, therefore, is a more accurate representation of the approximated function compared to a least-squares fit. A more detailed description of the iterative setting of the penalty parameter and use of cubic spline fit, including their effect on the final solution, is presented elsewhere [4].

DISCRETE SOLUTION STAGE

An iterative penalty method is used at this stage to search for the optimal discrete solution of the problem, once the continuous solution is known. In this work, the search space is restricted to the lower and upper diameters of the continuous solution obtained at the first stage. The solution so obtained, of course, could not be guaranteed of global optimality. The penalized form of the objective function to be minimized in this phase is of the following form:

$$\min \overline{C}_p = \sum_{l=1}^{m} C_l L_l + \sum_{l=1}^{m} \alpha_l (q_1 - q_{\min})^2 + \sum_{k=1}^{n} \alpha_k (H_k - H_{\min})^2 + \sum_{l=1}^{m} \overline{\alpha}_l f(d_i), \qquad (6)$$

where the last term is a penalty term used to enforce the satisfaction of the discrete diameter constraints. The function $f(d_i)$, therefore, should have the value of zero at upper d_U and lower d_L discrete diameters. Here, $\overline{\alpha}_i$ is a penalty parameter whose value increases iteratively until the corresponding constraints are satisfied; ie., the value of the last term is zero. The values of the parameters, α_l and α_k, are fixed at this stage to the last values used at the first stage. Different forms of function $f(d_i)$, with differing degrees of success, have been used in the literature [5-7]. Some of these forms, which are used for structural optimization, are as follows:

1. Sinusoidal form:

$$f(d_l) = \frac{1}{2}\left\{\sin\frac{2\pi[d_l - 0.25(d_l^U + 3d_l^L)]}{d_l^U - d_l^L} + 1\right\}, \qquad (7)$$

2. Quadratic form:

$$f(d_l) = 4[(\frac{d_l - d_l^L}{d_l^U - d_l^L})(\frac{d_l - d_l^U}{d_l^U - d_l^L})]^\beta \quad \beta \geq 1, \qquad (8)$$

3. Exponential form:

$$f(d_l) = [(\exp(q(d_l))(\exp(q(d_l) - 1)]^\beta \quad \beta \geq 1,$$

$$q(d_l) = [(d_l - d_l^L)(d_l - d_l^U)]. \qquad (9)$$

Here, all the above functions have been tried to find the proper form of the function for pipe network optimization. The authors' experience shows that the exponential form with:

$$q(d_l) = 4\frac{(d_i - d_L)(d_i - d_U)}{(d_U - d_L)(d_L - d_U)} \quad \text{and} \quad \beta = 1, \qquad (10)$$

is the most useful one in terms of the convergence of the method and optimality of the final solution.

Test Problems

The fist problem to be considered is a two-loop network with 8 pipes, 7 nodes and one reservoir (Figure 1) taken from [8]. All the pipes are 1000^m long and the Hazen-Williams coefficient is assumed to be 130 for all the pipes. The minimum nodal head requirement for all demand nodes is 30^m and there are 14 commercially available pipe diameters, as listed in Table 1. Table 2 shows the continuous results produced by the presented method, along with some of the cheapest split solutions obtained with some LP and NLP methods. Table 3 compares the discrete results produced by the presented method to some of the results obtained with some random search methods [9] and the best solution ever achieved for this problem [10]. It can be seen that the continuous solution is much better than the best split solution, while the discrete solution is marginally more expensive than the best solution. It should be noted that these solutions are obtained with 2 and 5 seconds of CPU time on a 366 Hz P.C. for continuous and discrete solutions, respectively.

The second test problem is that of the Hanoi network (Figure 2) with 34 pipes, 31 demand nodes and one reservoir. The network data is taken from the

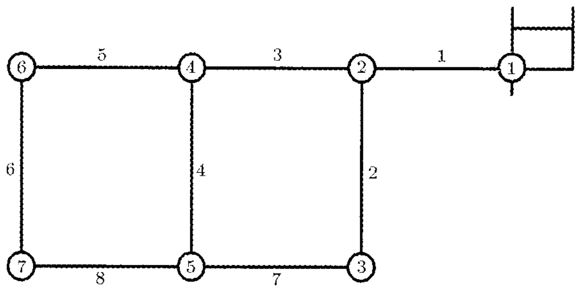

Figure 1. Two-loop network.

Table 1. Cost data for the two-loop network.

Diameter (Inch)	1	2	3	4	6	8	10	12	14	16	18	20	22
Cost (Units)	2	5	8	11	16	23	32	50	60	90	130	170	300

Table 2. Optimal continuous pipe diameters and corresponding nodal heads obtained with the method along with some of the split results obtained in previous research for two-loop networks.

Pipe Diameters											Nodal Heads	
Pipe	Present Work	Alperovits & Shamir [8]		Goulter et al. [12]		Kessler & Shamir [13]		Eiger et al. [14]			Node	Present Work
1	18.92	256.00	20	383.00	20	1000.00	18	1000.00	18		1	—
		744.00	18	617.00	18							
2	10.04	996.38	8	1000.00	10	66.00	12	238.0	12		2	54.69
		3.63	6			934.00	10	761.98	10			
3	15.41	1000.00	18	1000.00	16	1000.00	16	1000.00	16		3	30.34
4	2.14	319.38	8	687.00	6	713.00	3	1000.00	1		4	44.31
		680.62	6	313.00	4	287.00	2					
5	14.86	1000.00	16	1000.00	16	836.00	16	628.86	16		5	30.00
						164.00	14	371.14	14			
6	9.96	784.94	12	98.00	12	109.00	12	989.0	10		6	30.00
		215.6	10	902.00	10	891.00	10	10.95	8			
7	9.50	1000.00	6	492.0	10	819.00	10	921.86	10		7	30.00
				508.00	8	181.00	8	78.14	8			
8	1.00	990.93	6	20.00	2	920.00	3	1000.00	1			
		9.07	4	980.00	1	80.00	2					
Cost (Units)	392000	497525		435015		417500		402352				

Table 3. Optimal discrete pipe diameters and corresponding nodal heads obtained with the method along with some of the discrete results obtained in previous research for two-loop networks.

Pipe Diameters						Nodal Heads	
Pipe	Present Work	Abeb & Solomatine [9]		Savic & Walters [15]		Node	Present Work
		GA	ACCOL	GA2	GA1		
1	20	18	22	20	18	1	–
2	10	14	18	10	10	2	55.95
3	16	14	20	16	16	3	31.23
4	2	1	3	1	4	4	46.49
5	14	14	16	14	16	5	33.11
6	10	1	4	10	10	6	30.73
7	10	14	18	10	10	7	30.83
8	1	12	16	1	1		
Cost (Units)	423000	424000	447000	420000	419000		

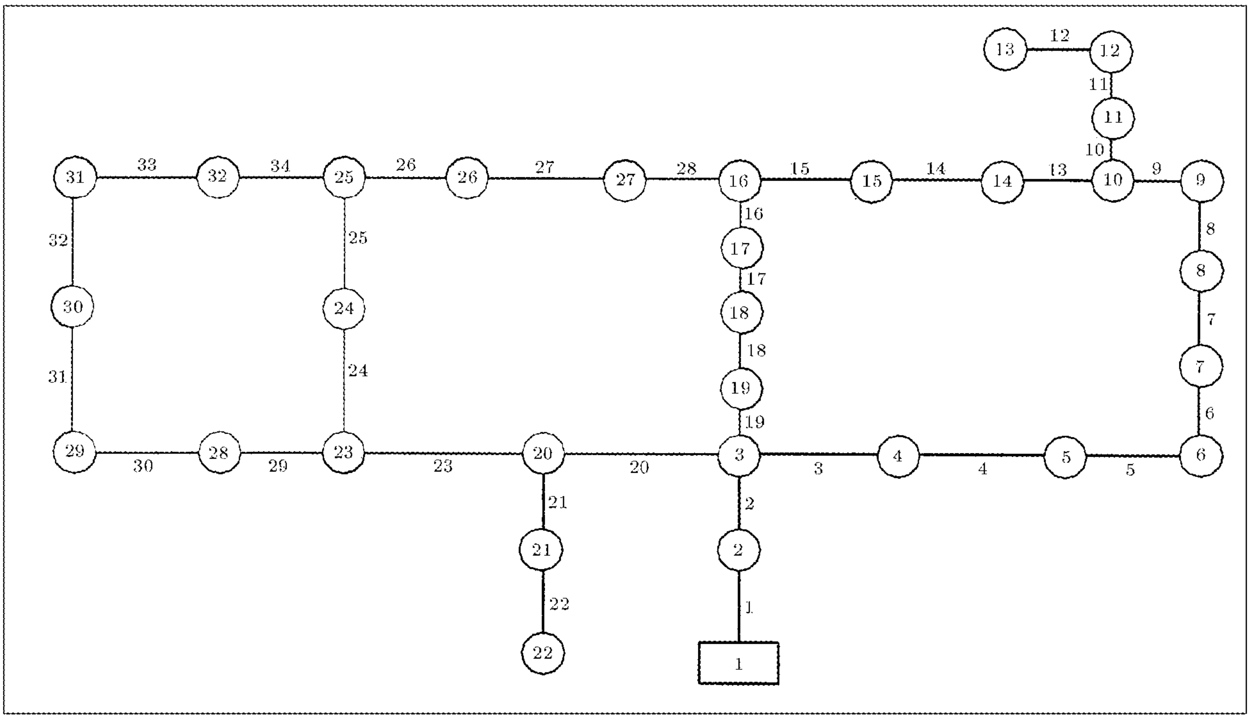

Figure 2. Hanoi network.

Table 4. Cost data for the Hanoi network.

Diameter (Inch)	12	16	20	24	30	40
Cost ($)	45.73	70.40	98.39	129.33	180.75	278.28

literature [11]. The minimum nodal head requirement at all demand nodes is 30^m. Various diameters of pipe are commercially available and their cost is calculated, based on the analytical cost function 1.1 $D^{1.5}$ used in references (Table 4). Table 5 compares the results, continuous and discrete, produced by the proposed method with the solution of [11] (though infeasible) and two of the solutions obtained by [9]. These solutions are obtained with 60 and 90 seconds of CPU time on the same machine used for a two-loop network.

The third test problem concerns the rehabilitation of the New York City water supply network (Figure 3) with 21 pipes, 20 demand nodes and one reservoir. The network data is taken from [10]. The commercially available pipe diameters and their respective costs are listed in Table 6. As seen from the table, a set of pipes with 12 inch increments is used in this study, while some other authors assumed the availability of diameters in 4-inch [10] and 20-inch increments [16]. The pipe and nodal data of the existing network are shown in Table 7. The solution to this problem is shown in Table 8, along with some of the continuous and discrete pipe solutions. It should be remarked that the solution of [17] does not even correspond to a local minimum, the solution of [18] is reported infeasible by [10] and the solution of [16] is obtained under some additional constraints for reasons of easier construction. It can be seen that the continuous solution is the cheapest solution ever achieved. The discrete solution, however, is marginally more expensive than the solution obtained by [10]. These solutions are obtained with 20 and 50 seconds of CPU time on the same machine used before, compared to 50 minutes of CPU time on a Sun sparc 1+ station used by [10].

CONCLUDING REMARKS

A two-stage iterative penalty method for the discrete optimization of pipe networks is presented. The method first calculates the continuous optimal solution for a pipe network using a penalty formulation. The continuous solution is, later, converted to discrete diameters, exploiting another penalty formulation in the second stage. In the discrete solution phase, the search space is restricted to the upper and lower diameters of the continuous diameters available from the first phase. The method is shown to be capable of producing comparable results to the existing methods with much less computational effort. Further research is underway to extend the method for the optimization of pumped networks and early results are encouraging.

Table 5. Optimal pipe diameters and nodal heads obtained by different methods for the Hanoi network.

Pipe Diameters						Nodal Heads		
Pipe	Present Work		Fujiwara & Chang [11]	Abeb & Solomatine [9]		Node	Present Work	
	Cont.	Disc.		GA	ACCOL		Cont.	Disc.
1	40.00	40	40.00	40	40	1	–	–
2	39.98	40	40.00	40	40	2	97.14	97.14
3	39.98	40	38.8	40	40	3	61.53	61.63
4	39.98	40	38.7	40	40	4	57.62	57.68
5	39.98	40	37.8	30	40	5	52.79	52.82
6	36.84	40	36.3	40	30	6	47.80	47.48
7	34.61	40	33.8	40	40	7	46.12	46.65
8	33.05	30	32.8	30	40	8	43.62	45.39
9	31.29	30	31.5	30	24	9	41.24	41.51
10	28.12	30	25.0	30	40	10	39.10	38.81
11	25.94	30	23.0	30	30	11	36.96	37.24
12	23.57	24	20.2	30	40	12	34.60	36.09
13	12.01	12	19.0	16	16	13	30.00	31.87
14	12.00	12	14.5	24	16	14	33.63	32.34
15	12.24	12	12.0	30	30	15	34.90	33.25
16	31.22	30	19.9	30	12	16	40.50	38.62
17	33.18	30	23.1	30	20	17	49.33	48.57
18	35.55	40	26.6	40	24	18	55.88	58.67
19	34.95	40	26.8	40	30	19	59.52	60.63
20	36.16	40	35.2	40	40	20	51.96	55.59
21	17.60	20	16.4	20	30	21	34.53	46.23
22	12.34	16	12.0	20	30	22	30.02	44.96
23	29.14	30	29.5	30	40	23	42.34	46.75
24	18.00	16	19.3	16	40	24	35.76	34.84
25	12.01	12	16.4	20	40	25	33.88	32.76
26	21.17	20	12.0	12	24	26	36.46	35.67
27	24.73	24	20.0	24	30	27	37.75	37.03
28	27.21	30	22.0	20	12	28	37.43	42.80
29	18.44	20	18.9	24	16	29	32.05	35.29
30	16.21	16	17.1	30	40	30	30.00	30.86
31	12.01	12	14.6	30	16	31	30.01	30.86
32	25.66	24	12.0	30	20	32	31.74	31.40
33	14.12	16	12.0	30	30			
34	22.51	24	19.5	12	24			
Cost ($M)	6.11	6.35	5.354	7.0	7.836			

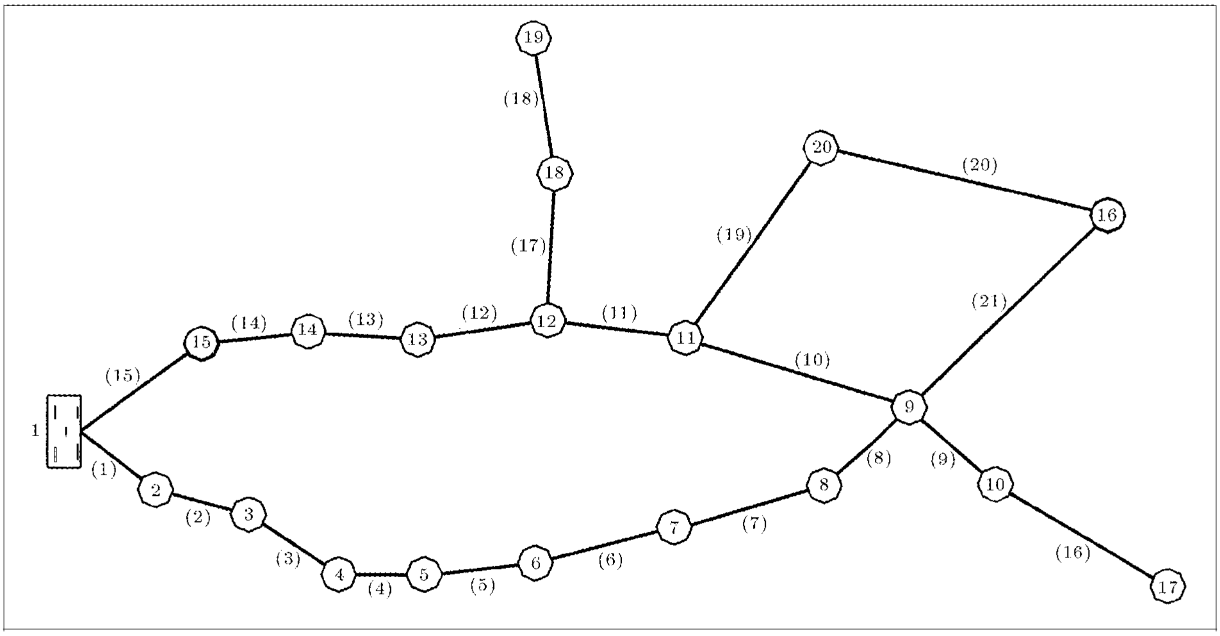

Figure 3. New York tunnel network.

Table 6. Pipe cost data for the New York network.

Diameter (Inch)	0	36	48	60	72	84	96	108
Cost ($/ft)	0	93.5	134.0	176.0	221.0	267.0	316.0	365.0
Diameter (Inch)	120	132	144	156	168	180	192	204
Cost($/ft)	417.0	469.0	522.0	577.0	632.0	689.0	746.0	804.0

Table 7. Pipe and nodal data for the New York tunnel network.

Pipe Data					**Nodal Data**		
Pipe	**Start Node**	**End Node**	**Length (ft)**	**Existing Diameter (Inch)**	**Node**	**Demand (Cft/s)**	**Min Total Head (ft)**
1	1	2	11600	180	1	reservoir	300
2	2	3	19800	180	2	92.4	255
3	3	4	7300	180	3	92.4	255
4	4	5	8300	180	4	88.2	255
5	5	6	8600	180	5	88.2	255
6	6	7	19100	180	6	88.2	255
7	7	8	9600	132	7	88.2	255
8	8	9	12500	132	8	88.2	255
9	9	10	9600	180	9	170	255
10	11	9	11200	204	10	1	255
11	12	11	14500	204	11	170	255
12	13	12	12200	204	12	117.1	255
13	14	13	24100	204	13	117.1	255
14	15	14	21100	204	14	92.4	255
15	1	15	15500	204	15	92.4	255
16	10	17	26400	72	16	170	260
17	12	18	31200	72	17	57.5	272.8
18	18	19	24000	60	18	117.1	255
19	11	20	14400	60	19	117.1	255
20	20	16	38400	60	20	170	255
21	9	16	26400	72			

Table 8. Optimal pipe diameters and nodal heads obtained by different methods for the New York network.

Pipe Diameters								Nodal Heads		
Pipe	Present Work		Dandy et al. [10]	Morgan & Goulter [14]	Bhave [19]	Gessler [12]	Quindry et al. [13]	Node	Present Work	
	Cont.	Disc.							Cont.	Disc.
1	0	0	0	0	0	0	0	1	300	300
2	0	0	0	0	0	0	0	2	294.56	294.59
3	0	0	0	0	0	0	0	3	287.06	287.11
4	0	0	0	0	0	0	0	4	284.88	284.95
5	0	0	0	0	0	0	0	5	282.97	283.06
6	0	0	0	0	0	0	0	6	281.52	281.61
7	0	0	0	144	0	100	0	7	279.28	279.39
8	0	0	0	0	0	100	0	8	276.05	276.20
9	0	0	0	0	0	0	0	9	273.75	273.94
10	0	0	0	0	0	0	0	10	273.72	273.91
11	0	0	0	0	0	0	119.02	11	273.87	274.06
12	0	0	0	0	0	0	134.39	12	275.30	275.50
13	0	0	0	0	0	0	0132.49	13	278.47	278.69
14	0	0	0	0	0	0	0132.87	14	286.40	286.64
15	99.86	108	120	0	136.43	0	131.37	15	294.60	294.88
16	94.51	96	84	96	87.37	100	19.26	16	260.04	261.88
17	95.79	96	96	96	99.23	100	91.71	17	272.80	273.03
18	83.70	84	84	84	78.17	80	72.76	18	261.22	261.52
19	81.85	84	72	60	54.40	80	72.64	19	255.00	255.38
20	0	0	0	0	0	0	0	20	263.93	265.14
21	67.47	72	72	84	81.50	80	54.97			
Cost ($M)	38.60	39.94	38.80	39.20	40.18	41.8	63.58			

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