Scientia Iranica, Vol. 12, No. 2, pp 190–198

SCIENTIA IRANICA

Optimal Synthesis of Planar and Spatial Mechanism for Path Generation Using Regression Deviation

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This method introduces the structural error of regression deviation, which is an effective method for the path generation of a vast type of planar and spatial mechanism. The proposed method avoids point-by-point comparison and requirement of timing and reflects the difference between the two curves very effectively in the objective function. By decreasing the number of the design variables, this method would help considerably in decreasing CPU time. The objective function that is based on regression error would converge to a global minimum by a genetic algorithm. At the end, the effectiveness of the method is shown by two numerical examples.

INTRODUCTION

In dimensional synthesis of a mechanism, it is required to determine the linkage dimensions so that a point on the coupler link traces the desired curve. This type of synthesis is called path generation and there are two different methods for this work. In the first, a limited number of points are specified, then, a mechanism is designed, so that the coupler point can pass exactly through these points. For instance, in a four-bar linkage, the number of design variables is 9. So, the maximum number of points that could be specified equals 9. Many attempts have been made to increase this number. Jensen has increased this number to even 12 points [1]. The other limitation of this method is having no control over the path between these points. This method leads to the exact solving of a system of equations [2]. In the other method, the whole path or many points on the path are specified, so that the method is called continuous path generation. Since the number of points is more than the number of design variables, the problem would lead to an optimization problem. There are various methods of optimal synthesis to minimize the difference between the desired and the generated paths. Constraints, such as full rotation of crank, proper links dimension, pressure angle, etc. can be added to the minimization problem. The objective function, that is generally called structural error, equals the square of the difference between the two paths. Usually, for facilitating the selection of points, timing would be imposed on the mechanism. That is, when the coupler point moves from one point to another, the crank should rotate through a specified angle. As can be seen in the first view, the timing process would just impose an excess constraint on the problem and, as a result, the solution space would be limited. To avoid this limitation, some methods without timing requirements have been innovated. Fox and Willmert take the difference between the y-coordinate of two points, which have the same x-coordinate as the objective function [3]. Angeles compares each point on the desired path with the nearest point on the generated path, so, he needs an optimization procedure in order to find the nearest points [4]. Watanabe uses the curvature of two curves for comparing the difference between them [5]. In this method, the curvature function is expressed as an equation of the curve length. Firstly, changing the comparison points would minimize the curvature difference between the two curves, then, this difference would be considered as the objective function. This method is independent of the size and orientation of the curve.

Another method that is independent of timing, includes the work of Cheung and Zhou [6]. They have introduced a simple and interesting way for expressing the difference between two curves by introducing the

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concept of the orientation structural error of the fixed link. In this method, the fixed link would be unconstrained and the coupler point is constrained to move on the desired path. As the coupler point moves, the fixed link would have some orientation. The objective function is regarded as the difference of the maximum and minimum orientation.

Structural error is a highly nonlinear function, so, in addition to an efficient method, one needs a proper optimization procedure. Ramstein and Chedmail used a genetic algorithm to select a robot, which should trace a given path among some obstacles [7]. Cheung and Zhou also used a genetic algorithm to minimize the structural error [6].

In this paper, a kind of continuous synthesis method is proposed that is independent of timing requirements. In this method, the structural error is equal to the regression deviation for some specified points. Depending on the type of mechanism, circular or linear regression is used.

STRUCTURAL ERROR OF REGRESSION DEVIATION

Crank-Rocker Linkage

A crank rocker linkage is shown in Figure 1. When link AB rotates a full turn around point A, point M traces a closed path, K. In the full rotation, link AB would be collinear with link BM twice. These cases are shown in Figure 2. It can be seen that in a case where two links are collinear, point M is located at a maximum distance from point A and, in the other case, it is at a minimum distance from point A.

So, if curve K and point A are definite, the length of links L_1 and L_2 can be determined by the following equations:

$$L_1 + L_5 = R_{\max}, \tag{1}$$

$$|L_1 - L_5| = R_{\min}, \tag{2}$$

where $R_{\max}$ and $R_{\min}$ are the maximum and minimum distances from point A to curve K, respectively. If point A lies outside the curve $K, L_1 < L_5$ and, if point A lies inside the curve, then $L_5 < L_1$.

In the proposed method, the independent variables are the x and y coordinates of point A, the length of link L_2 and the angle β. If these variables are definite, by moving point M on the specified points on the curve, corresponding point, C, can be determined. If the linkage is the one that has drawn the desired curve, then points C_i should lie on the circumference of a circle whose center is point D and whose radius equals L_3. So, the deviation of points C from a circle obtained by circular regression can be regarded as the objective function. The corresponding necessary calculations for determining point C, corresponding to point M, are as follows:

$$\theta_1 = \tan^{-1}\left(\frac{y_M - y_A}{x_M - x_A}\right) \pm \cos^{-1}\left(\frac{L_1^2+(x_M-x_A)^2+(y_M-y_A)^2-L_5^2}{2L_1\sqrt{(x_M-x_A)^2+(y_M-y_A)^2}}\right), \tag{3}$$

$$\theta_5 = \tan^{-1}\left(\frac{y_M - y_A - L_1\sin\theta_1}{x_M - x_A - L_1\cos\theta_1}\right). \tag{4}$$

The $\pm$ in Equation 3 corresponds to the rotational direction of link AB and, also, to the distance of point A to point M that is increasing or decreasing.

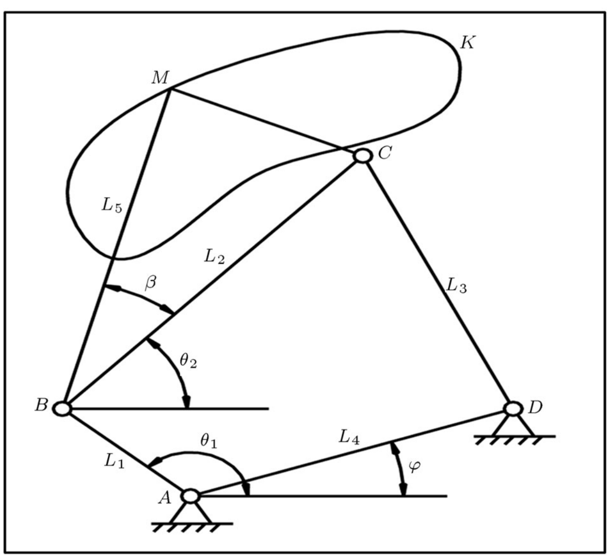

Figure 1. Crank-rocker linkage.

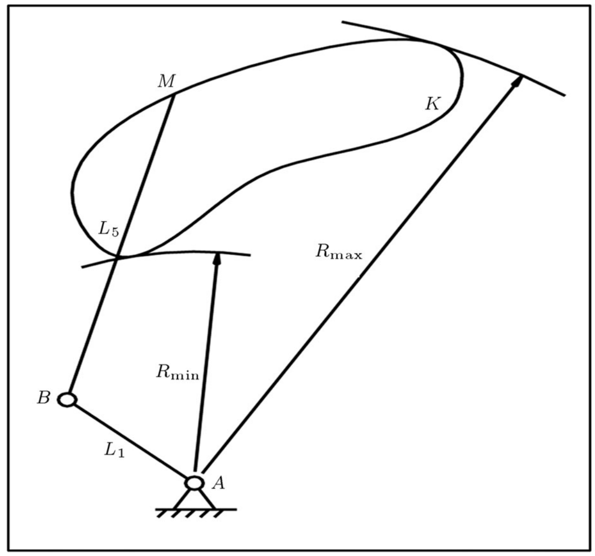

Figure 2. Minimum and maximum distances.

Obtaining the value of θ_5, one has:

$$\theta_2 = \theta_5 - \beta. \tag{5}$$

So, the coordinates of point C can be obtained as follows:

$$x_C = x_A + L_1 \cos\theta_1 + L_2 \cos\theta_2, \tag{6}$$

$$y_C = y_A + L_1 \sin\theta_1 + L_2 \sin\theta_2. \tag{7}$$

Now, there are a number of points C that are determined and it is necessary to obtain the best circle that passes through them (circular regression). Assume that the coordinate of the center of the desired circle is (a, b) and the radius is R. By the best circle, it means that the sum of the distances of the points from the circle is minimum. So, the following function should be minimized:

$$\text{dist} = \sum_{i=1}^{n} \left(\sqrt{(x_i - a)^2 + (y_i - b)^2} - R \right)^2, \tag{8}$$

where x_i and y_i are the coordinates of the desired points. There is no analytical method to obtain the unknown a, b and R. So, one should use an optimization procedure. After obtaining a, b and R for the best circle, the value of the objective function is equated to the value of function dist.

It is obvious that by finding an analytical method for circular regression, the CPU time of the main program should decrease considerably.

Four independent variables are needed to obtain a mechanism, which are $[x_A, y_A, \beta, L_2]$. After obtaining the mechanism, one should be confident that point B rotates uniformly on a circle, so, if the crank rotates clockwise, the following constraint should be satisfied:

$$g_1(i) = \theta_{1.i+1} - \theta_{1.i} \leq 0, \qquad i = 1, \cdots, n-1. \tag{9}$$

Otherwise, if the crank rotates counterclockwise, one should have:

$$g_2(i) = \theta_{1.i} - \theta_{1.i+1} \leq 0, \qquad i = 1, \cdots, n-1. \tag{10}$$

Another constraint is for the full turn of the crank, that is

$$g_3 = L_1 + L_4 - L_2 - L_3 < 0. \tag{11}$$

Slider-Crank Linkage

A slider-crank linkage is shown in Figure 3. If the points of curve K and point A are definite, as discussed in the previous section, the length of the links, L_1 and L_2, can be determined easily by Equations 1 and 2.

In this method, independent variables are x and y coordinates of point A, angle β and the length of link L_2. If these variables are known, by moving point M on the specified points on the curve, K, the corresponding points, C_i, can be obtained. If the linkage is the one that has drawn the desired curve, then points C_i should lie on a straight line, that is, line d. If the deviation of C_i from the curve is less, the curve that is generated by the linkage is closer to the desired curve. So, the objective function is defined as the deviation of points C_i from the best straight line that passes through points C_i.

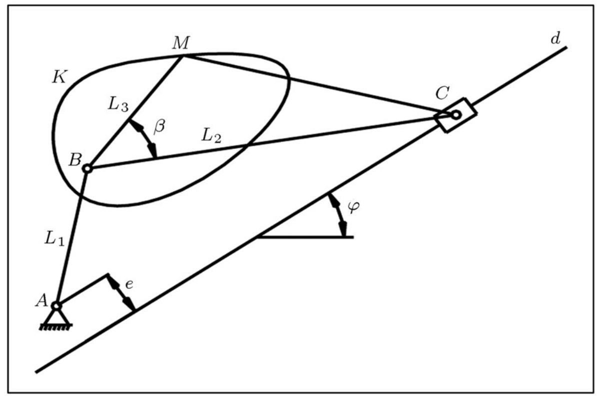

Figure 3. Slider-crank linkage.

In the previous section, it was illustrated how to determine points C_i that are correspondent to points M_i. These calculations are similar for slider-crank linkage and are omitted. Now, it is necessary to find the best straight line that passes through points C_i. This line can be obtained easily by an analytical method. Suppose the equation of the best line is $y = ax + b$. The best line means the line that minimizes the following function dist:

$$\text{dist} = \sum_{i=1}^{n} (ax_i + b - y_i)^2. \tag{12}$$

Since this line is supposed to be the best line, one should have:

$$\frac{\partial \text{dist}}{\partial a} = \sum_{i=1}^{n} [2a(ax_i + b - y_i)] = 0, \tag{13}$$

$$\frac{\partial \text{dist}}{\partial b} = \sum_{i=1}^{n} [2(ax_i + b - y_i)] = 0. \tag{14}$$

Unknowns are a and b; solving the above equations, they can be determined as follows:

$$a = \frac{\overline{xy} - \overline{xy}}{\overline{x^2} - (\overline{x})^2}, \tag{15}$$

$$b = \frac{\overline{x^2}\overline{y} - \overline{xxy}}{\overline{x^2} - (\overline{x})^2}. \tag{16}$$

Determining the unknown a and b, one can put them in Equation 12 and, then, one can consider the value of function dist as the objective function.

After obtaining the linkage, some constraints should be checked. The main constraint is the full rotation of the crank that can be obtained by satisfying the following equation:

$$g_1 = e - L_1 + L_2 < 0.$$

The other constraint is on the sequence of the obtained angles, θ_i. Because of the similarity to Equations 9 and 10, the constraint is not expressed here. Some upper and lower limits can also be considered for the length of the links.

RSSP Spatial Mechanism

An RSSP spatial mechanism ($R \equiv$ revolute joint, $S \equiv$ spherical joint, $P \equiv$ prismatic joint) is shown in Figure 4. As point $\mathbf{a}$ rotates a full turn around $\mathbf{a}_0$ and perpendicular to the vector, $\mathbf{u}_a$, point $\mathbf{c}$ traces a closed curve, K. In a full turn of point $\mathbf{a}$, links $\mathbf{aa_0}, \mathbf{ac}$ and vector $\mathbf{u_a}$, lie in a plane. In these cases, the distance of point $\mathbf{a_0}$ from curve K reaches its maximum and minimum. This can be used in order to obtain the lengths of links L_1 and L_2 as follows.

Two expressed cases are shown in Figure 5. In one of the cases, the distance of point $\mathbf{a_0}$ to point $\mathbf{c}$ is maximum ($d_{\max}$) and, in the other case, the distance is minimum ($d_{\min}$). Having points $\mathbf{a_0}$ and vector $\mathbf{u_a}$ as the input independent variables, plane M can be determined and, then, the distances $h_{\max}$ and $h_{\min}$ can be obtained as follows.

$$h_{\max} = (\mathbf{c}_{\max} - \mathbf{a}_0).\mathbf{u}_a, \qquad (17)$$

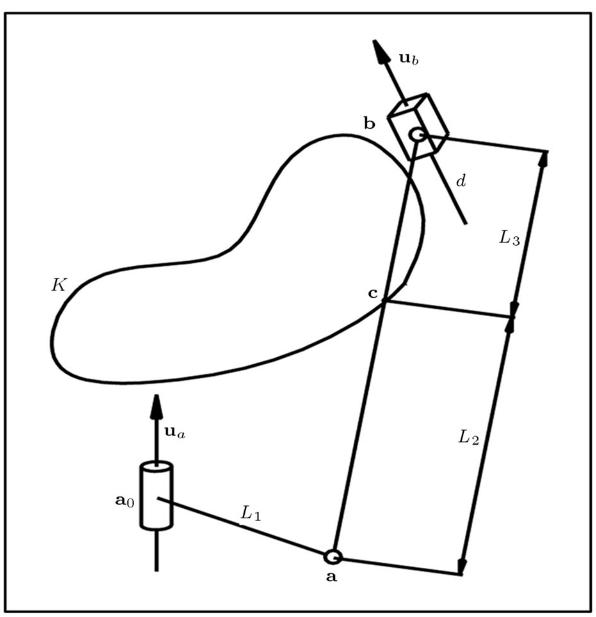

Figure 4. RSSP mechanism.

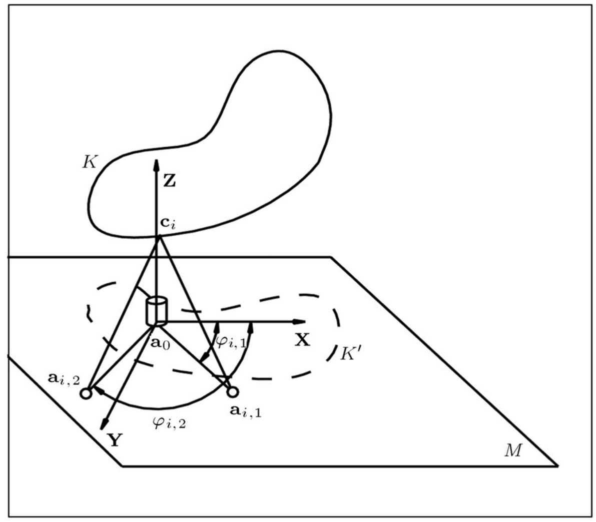

Figure 5. Two possible solutions for $\mathbf{a_i}$.

$$h_{\min} = (\mathbf{c}_{\min} - \mathbf{a}_0).\mathbf{u}_a, \qquad (18)$$

where $h_{\max}$ and $h_{\min}$ are the distances of $\mathbf{c}_{\max}$ and $\mathbf{c}_{\min}$ from the plane M, respectively. Then, $\varphi_{\max}$ and $\varphi_{\min}$ are obtained as follows:

$$\varphi_{\max} = \cos^{-1}(h_{\max}/d_{\max}), \qquad (19)$$

$$\varphi_{\min} = \cos^{-1}(h_{\min}/d_{\min}). \qquad (20)$$

Equation 15 can be used, when point $\mathbf{a_0}$ lies inside curve K' (the projection of curve K on plane M), otherwise, when point $\mathbf{a_0}$ lies outside curve $K', \varphi_{\min}$ should be obtained by Equation 16:

$$\varphi_{\min} = \pi - \cos^{-1}(h_{\max}/d_{\max}). \qquad (21)$$

Now, L_1 and L_2 can be calculated:

$$L_1 = \frac{d_{\max}^2 - d_{\min}^2}{2(d_{\max}\cos\phi_{\max} - d_{\min}\cos\phi_{\min})}, \qquad (22)$$

$$L_2 = \sqrt{L_1^2 - d_{\max}^2 - 2d_{\max}\cos\phi_{\max}}. \qquad (23)$$

Having point $\mathbf{a_0}$, vector $\mathbf{u_a}$ and length L_3 as input variables and obtaining length L_1 and L_2 for each point, $\mathbf{c_i}$, on curve K, the corresponding points, $\mathbf{b_i}$, can be determined. If the input variables are so selected that the obtained mechanism draws curve K exactly, then, points $\mathbf{b_i}$ should lie on a straight line, that is, line d, otherwise, the deviation of the linear regression of points $\mathbf{b_i}$ is regarded as the objective function. Points $\mathbf{b_i}$ that are corresponding to points $\mathbf{c_i}$ on curve K are determined as follows.

Before obtaining these points it can be seen in Figure 6 that except for points $\mathbf{c}_{\max}$ and $\mathbf{c}_{\min}$, for each point $\mathbf{c_i}$, there exist two possible solutions for $\mathbf{a_i}$. For determining point $\mathbf{a_i}$, both solutions are required and, then, in comparison with point $\mathbf{a_{i-1}}$, the proper

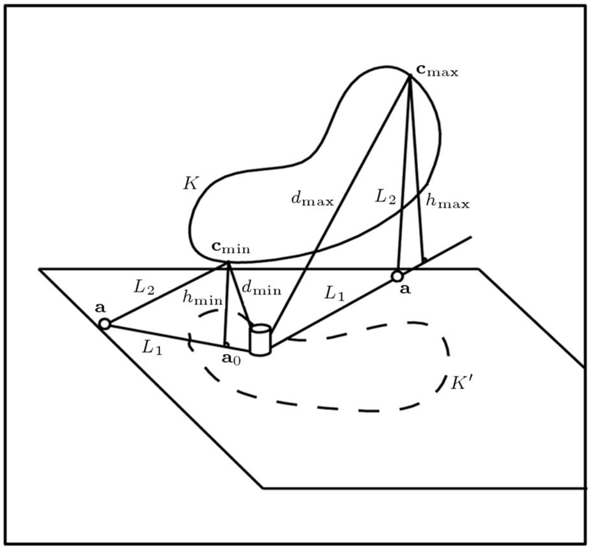

Figure 6. The positions of the minimum and maximum distances.

solution is selected. So, it should be noted that both solutions should be obtained.

$\mathbf{a_i}$ should satisfy the following equations [8]:

$$(\mathbf{a}_i-\mathbf{a}_0).\mathbf{u_a}=0, \tag{24}$$

$$(\mathbf{a_i}-\mathbf{a_0}).(\mathbf{a_i}-\mathbf{a_0})^T=L_1^2, \tag{25}$$

$$(\mathbf{a_i}-\mathbf{c_i}).(\mathbf{a_i}-\mathbf{c_i})^T=L_2^2. \tag{26}$$

Solving the above system of equations, $\mathbf{a_i}$ would be obtained. Then, by the following equation, points $\mathbf{b_i}$ are determined.

$$\mathbf{b_i}=[(L_1+L_3)\mathbf{c_i}-L_3\mathbf{a_i}]/L_2. \tag{27}$$

Up to now, for each point, $\mathbf{c_i}$, the corresponding point, $\mathbf{b_i}$, is obtained.

At this stage, one needs to determine the best line passing through points $\mathbf{b_i}(x_i,y_i,z_i)$. The best line means a line with the following equation:

$$x=\alpha t,$$

$$y=\beta t+b,$$

$$z=\gamma t+c, \tag{28}$$

that minimizes the function dist that is defined as follows:

$$\text{dist}=\sum\left[(dx_i+b-y_i)^2+(ex_i+c-y_i)^2\right], \tag{29}$$

where d and e can be obtained as follows:

$$d=\beta/\alpha, \qquad e=\gamma/\alpha. \tag{30}$$

For the function dist being minimized, the following equations should be satisfied:

$$\frac{\partial \text{dist}}{\partial d}=0, \tag{31}$$

$$\frac{\partial \text{dist}}{\partial b}=0, \tag{32}$$

$$\frac{\partial \text{dist}}{\partial c}=0, \tag{33}$$

$$\frac{\partial \text{dist}}{\partial e}=0. \tag{34}$$

Solving the above system of equations, one has:

$$d=\frac{n\overline{xy}-\overline{y}\overline{x}}{n\overline{x^2}-(\overline{x})^2}, \tag{35}$$

$$b=\frac{\overline{y}\overline{x^2}-\overline{xy}\overline{x}}{n\overline{x^2}-(\overline{x})^2}, \tag{36}$$

$$e=\frac{n\overline{xz}-\overline{z}\overline{x}}{n\overline{x^2}-(\overline{x})^2}, \tag{37}$$

$$c=\frac{\overline{z}\overline{x^2}-\overline{xz}\overline{x}}{n\overline{x^2}-(\overline{x})^2}. \tag{38}$$

Putting the values of b,d,e and c in Equation 24, the value of function dist, that is, the desired objective function, can be obtained. Similar to the previous section, after obtaining the mechanism, one should be confident of the order of the obtained angles and the full rotation of the crank. For satisfying the former, Equations 9 and 10 can be used as constraints. But, for the latter, it is still impossible to have a constraint independent of time. So, a time-dependent constraint was used as follows.

Point $\mathbf{a}$ should be rotated around vector $\mathbf{u_a}$ and by every small change in the angle of rotation, the distance of point $\mathbf{a}$ from line d is calculated. This distance should be always larger than L_2+L_3. The changes in angle of rotation should be selected so small that they will cover all possible angles. If all the distances are larger than L_2+L_3, the condition of full rotation is satisfied.

OPTIMIZATION PROCEDURE

Conventional search techniques, such as hill-climbing, are often incapable of optimizing a non-linear function. In such cases, a random search method might be required. However, undirected search techniques are extremely inefficient for large domains. A Genetic Algorithm (GA) is a directed random search technique, which can find the global optimal solution in search space [9]. A GA is modeled on natural evolution in that the operators it employs are inspired by the natural evolution process. These operators, known as genetic operators, manipulate individuals in a population over several generations to improve their fitness gradually.

There are three common genetic operators: Selection, crossover and mutation. An additional reproduction operator, inversion, is sometimes also applied. Some of these operators were inspired by nature of which many versions can be found. Each operator functions independently and it is not necessary to use all of them in a GA.

There are two common representation methods for individuals, a binary string and a vector of real number representation. In this paper, the latter is employed. In the following, the operators used in this paper are briefly illustrated.

Selection

The aim of the selection procedure is to reproduce more copies of individuals whose fitness values are high, as compared to those whose fitness values are low. This procedure has a significant influence on driving the search toward a promising area and finding a good solution in a short time. In this paper, a normalized geometric ranking selection method is used. The probability for an individual to be selected is:

$$P_r = \frac{P_b}{1-(1-P_b)^{N_p}}(1-P_b)^{r-1}, \tag{39}$$

where P_b is a constant and is proportional to the probability of selecting the best individual; r is the rank of individual, where 1 is the best and N_p is the population size.

Crossover

This operation is considered the one that makes GA different from other algorithms, such as dynamic programming. It is used to create two new individuals (children) from two existing individuals (parents) picked from the current population by a selection operation. There are several ways of doing this. The method used in this paper is as follows.

It is assumed that two individuals, X_1 and X_2, are to be crossed and that X_1 is better than X_2 in terms of fitness. The new individuals, X_1^* and X_2^*, are calculated as follows:

$$X_1^* = X_1 + n_r(X_1 - X_2), \tag{40}$$

$$X_2^* = X_1, \tag{41}$$

where n_r is a random number in the range (0,1). if X_1^* is infeasible, then, it generates a new random number, n_r. If it were not successful for N_T times, let the children be equal to the parents and stop.

Mutation

Unlike crossover, this is a monadic operation. That is, a child string is produced from a single parent string. The mutation operator forces the algorithm to search new areas. Eventually, it helps the GA avoid premature convergence and finds the global optimal solution. Assume the individual to be mutated is $X = (x_1, x_2, \cdots, x_n)$. Generate a random integer, i_r, here, $1 \leq i \leq m$. Then, the new value of the i_rth component after mutation is:

$$x_i' = \begin{cases} x_i + (u_i - x_i)f(G) & \text{if } x_i < 0.5 \\ x_i - (x_i - l_i)f(G) & \text{if } x_i \geq 0.5 \end{cases}, \tag{42}$$

where u_i and l_i are the upper and lower bounds of the i_rth component, respectively. Moreover:

$$f(G) = \left[n_2\left(1 - \frac{G}{G_{\max}}\right)\right]^{b_s}, \tag{43}$$

where n_1 and n_2 are two random numbers in the range $(0,1)$ and G is the current generation number. $G_{\max}$ is the maximum generation number and b_s is the shape parameters of mutation.

GA is generally innovated for applying on an unconstrained function, but, the constrained function can also be considered by defining a penalty function.

For example, if the following functions are the desired constraints:

$$g_1 = 0, \tag{44}$$

$$g_2 \leq 0. \tag{45}$$

one has:

$$F_1 = \begin{cases} 0 & g_1 = 0 \\ cf_1 + |g_1| & g_1 \neq 0 \end{cases}, \tag{46}$$

$$F_2 = \begin{cases} 0 & g_2 \leq 0 \\ cf_2 + g_2 & g_2 > 0 \end{cases}, \tag{47}$$

where cf_1 and cf_2 are two large constants and $cf_1 < cf_2$. Only when $F_1 = 0$, would F_2 be calculated.

APPLYING THE PROPOSED METHOD ON NUMERICAL EXAMPLES

Example 1

In this example, a curve is generated that is drawn by a crank-rocker linkage whose design parameters are given in Table 1. The coordinates of the specified points on the curve are given in Table 2 and the design parameters of the generated linkage are given in Table 3.

For the generated linkage, the value of the function dist equals 0.003. The desired curve and the generated curve are shown in Figure 7.

Table 1. Design parameters of the desired linkage in Example 1.

x_A	y_A	L_1	L_2	L_3	L_4	L_5	β	φ
0.00	0.00	1.50	4.00	3.00	3.00	3.00	30.00°	30.00°

Table 2. Coordinates of points on the curve in Example 1.

i	1	2	3	4	5	6	7	8	9	10	11	12	13
$M_{i,x}$	3.85	4.11	3.75	2.89	2.21	1.73	1.84	1.50	1.82	2.35	2.95	3.45	3.72
$M_{i,y}$	1.86	1.81	0.85	0.54	0.58	0.58	0.46	0.26	0.12	0.17	0.050	1.08	1.58

Table 3. Design parameters of the generated linkage in Example 1.

x_A	y_A	L_1	L_2	L_3	L_4	L_5	β	φ
0.38	1.44	1.20	4.14	6.11	3.26	3.30	32.51°	-52.91°

Table 4. Design parameters of the desired linkage in Example 2.

x_A	y_A	L_1	L_2	L_3	L_4	L_5	β	φ
0.00	0.00	1.80	1.90	3.90	4.80	5.15	40.00°	00.00°

Table 5. Coordinates of points on the curve in Example 2.

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$M_{i,x}$	-0.6	1.2	1.7	1.8	1.6	1.4	0.8	-0.2	-1.2	-1.6	-1.9	-2.1	-1.9	-1.7	-1.5
$M_{i,y}$	4.8	6.4	6.7	6.4	5.6	4.9	4.1	3.8	3.4	3.1	2.9	2.6	2.9	3.2	3.5

Example 2

In this example, a curve is generated that is drawn by a crank-rocker linkage whose design parameters are given in Table 4. The desired curve has a sudden change in path and is more complicated in comparison to Example 1.

The coordinates of the specified points on the curve are given in Table 5 and the design parameters of the generated linkage are given in Table 6.

For the generated linkage, the value of the function dist equals 0.0365. The desired curve and the generated curve are shown in Figure 8.

Example 3

In this example, the coupler curve of a RSSP mechanism is generated. The specifications of the main mechanism are given in Table 7. In Table 8, the coordinates of the specified points on the desired curve are given. Finally the specifications of the generated mechanism are given in Table 9. For the generated

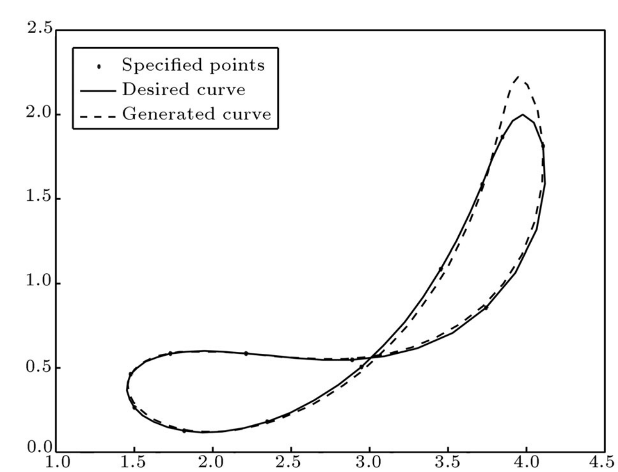

Figure 7. Generated and desired curves in Example 1.

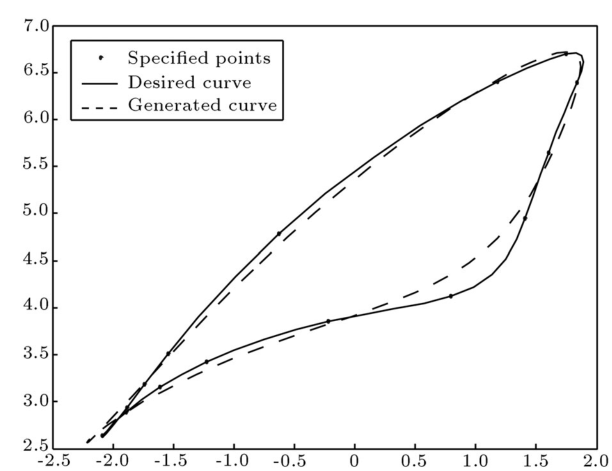

Figure 8. Generated and desired curves in Example 2.

Table 6. Design parameters of the generated linkage in Example 2.

x_A	y_A	L_1	L_2	L_3	L_4	L_5	β	φ
-0.2	-1.5	1.97	3.92	6.7	8.03	6.48	43.86°	-9.68°

Table 7. Design parameters of the desired mechanism in Example 3.

$a_{0,x}$	$a_{0,y}$	$a_{0,z}$	a_x	a_y	a_z	b_x	b_y	b_z	$u_{1,x}$	$u_{1,y}$	$u_{1,z}$	$u_{2,x}$	$u_{2,y}$	$u_{2,z}$
3	0	0	0	0	0	-1	5	10	0.31	0.31	0.9	0	0	1

Table 8. Coordinates of the points in Example 3.

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$M_{i,x}$	-0.5	-0.4	-0.2	0.15	0.61	1.07	1.47	1.74	1.99	1.94	1.64	1.26	0.74	0.28	-0.1
$M_{i,y}$	2.5	1.81	1.13	0.70	0.50	0.55	0.86	1.42	2.33	3.07	3.65	4.00	4.05	3.86	3.49
$M_{i,z}$	5.0	4.70	4.30	3.91	3.55	3.20	3.01	3.10	3.61	4.16	4.66	5.04	5.27	5.34	5.3

Table 9. Design parameters of the generated mechanism in Example 3.

$a_{0,x}$	$a_{0,y}$	$a_{0,z}$	a_x	a_y	a_z	b_x	b_y	b_z	$u_{1,x}$	$u_{1,y}$	$u_{1,z}$	$u_{2,x}$	$u_{2,y}$	$u_{2,z}$
8.67	-5.9	-5.2	4.80	-6.2	-6.2	-3.6	7.68	11.7	0.17	0.27	0.95	-0.2	0.27	0.93

mechanism, the value of the function dist equals 0.047. The desired curve and the generated curve are shown in Figure 9.

CONCLUSION

The ordinary method of synthesis regards structural error as the objective function, which equals the sum of the difference between two curves in some specified points. For simplification, in the selection of these points, a concept of timing is introduced that has no practical value. Timing imposes unnecessary constraints on the problem. So, it would cause some limitation on solution space. In ordinary methods,

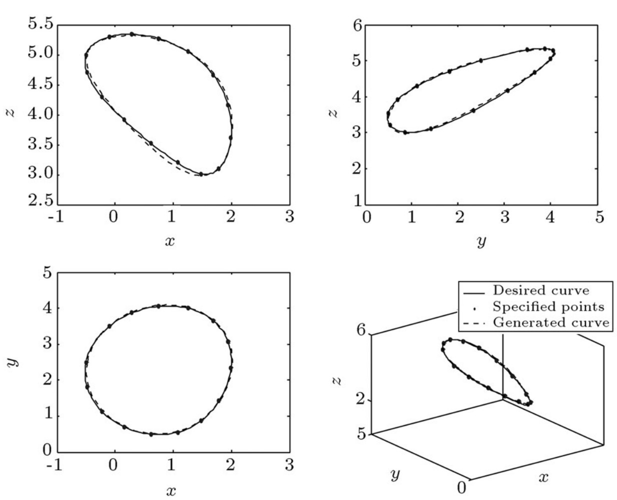

Figure 9. Generated and desired curves in Example 3.

some optimization procedures are used that lead to a local optimal point and is highly dependent on the starting solution.

In the proposed method, an effective way for optimal synthesis is introduced that is applicable to a vast type of planar and spatial mechanisms. The objective function is based on the deviation of some specific points from a curve that is obtained by regression. It can be a circle in a plane for a crank-rocker mechanism, a straight line in a plane for a slider-crank mechanism, a line in space for RSSP mechanisms, a circle in space for a mechanism where one end of its coupler link traces a circle, such as a RSSR mechanism and a cylinder for a mechanism where one end of its coupler link traces a cylinder, such as a RRSC mechanism, etc. So, as can be seen, the method is easily applicable to most well known mechanisms.

This method is independent of timing and reflects, effectively, the difference between two curves as the objective function. The dominant characteristics of this method are the decrease in the number of input design variables and the applicability to different types of planar and spatial mechanisms. For example, for the crank-rocker linkage, four design variables are needed and for the RSSP mechanism, six design variables are needed. So, it would result in a decrease in the number of the input variables of the optimization procedure and, so, a considerable decrease in CPU time of the program. For optimization, a genetic algorithm is used that leads to a global optimal solution. The objective function is somehow defined so that, at the end, one is confident of the satisfied constraints.

Numerical examples for a crank-rocker linkage, a slider-crank linkage and a RSSP mechanism illustrate the effectiveness of the method.

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