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Cost optimum design of doubly reinforced high strength concrete T-beams

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Abstract. This paper presents a method for optimizing the design cost of a doubly reinforced High Strength Concrete (HSC) T-beam. The objective function used in the model includes the costs of HSC, steel, and formwork. The constraint functions are set to satisfy design requirements of Eurocode 2 (EC-2). The cost optimization process is developed by the use of the Generalized Reduced Gradient algorithm. An example problem is considered in order to illustrate the applicability of the proposed design model and solution methodology. It is concluded that the present approach is economically more effective when compared to conventional design methods and can be extended to deal with other sections without major alterations.

1. Introduction

Flanged sections are often used for medium span concrete beams, so that their structural efficiency is maximized. Within the same sectional area, a flanged section has better strength-ductility performance compared with a rectangular one. Structural elements with T-shaped sections are frequently used in an industrial construction. They are used repeatedly for large structures, because they are cost-effective, especially when using optimum cost design model, which is of great value to practicing designers and engineers. Compression reinforcement is often not required when designing the T-beams sections. One of the great advantages of T-beams sections is the economy of the amount of steel needed for reinforcement. For structures requiring reduced height such as parking spaces and bridge decks, beams with lower template are preferred [1-5].

High strength concrete T-beams are frequently used in an engineering practice. They are widely used for short to medium span highway bridges due to its moderate self weight, structural efficiency, ease of fabrication, fast construction, and low maintenance. The long span bridges and the tall buildings are highly considered in the conceptual design by HSC. The majority of research studies conducted so far have focused on the optimization of ordinary concrete T-beams, whose strength class is between 12 MPa and 50 Mpa; however, only few studies have been done with regard to the optimization of high strength concrete T-beams whose resistance class is between 50 MPa and 100 MPa. Compression reinforcement is not often required when designing the T-beams sections. It is also important to note that, in general, the use of top reinforcement in reinforced concrete T-beam sections is an indication of poor design. Thus, the doubly reinforced high-strength concrete T-beams are recommended for structures requiring low height such as parking spaces and bridge decks beams with lower

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template. The present optimization model is developed for HSC T-beams in flexure [6-9].

Recent developments in the technology of materials have led to the use of the high-strength concrete. This is mainly due to its efficiency and economy. The reduction in the quantities of construction materials has enabled both a gain in weight reduction and in foundation's cost [10].

HSC has a high compressive strength in the range of 50 to 100 MPa; it has not only the advantage of reducing member size and story height, but also the volume of concrete and the area of formwork. In terms of the amount of steel reinforcement, there is a substantial difference between the normal strength concrete structure compared to high strength concrete structures [11-14].

Advances in numerical optimization methods, computer-based numerical tools for analysis, and design of structures and availability of powerful computing hardware have significantly helped the design process to ascertain the optimum design. In the present research work, the Generalized Reduced Gradient (GRG) method is used for solving nonlinear programing. The GRG algorithm transforms inequality constraints into equality constraints by introducing slack variables. It is a very reliable and robust algorithm [15-20].

Consideration of serviceability conditions at working loads can be included without major alterations, but will be addressed elsewhere as it requires further attention in terms of restrictions on bending moment capacity, stress limitations in concrete and in steel as a function of cracking conditions, as well as limits on deflections. Such restrictions will have direct consequences on the boundaries of the design space and the feasible design solutions of the optimization problem.

In this paper, a model on how to calculate the optimum cost design of doubly reinforced High Strength Concrete (HSC) T-beams in flexure under Ultimate Limit State conditions (ULS) is presented. The objective function includes the costs of HSC, steel, and formwork. The constraint functions are set to satisfy design requirements used in Eurocode 2 (EC-2)-French annex. The cost optimization process is developed by the use of the Generalized Reduced Gradient (GRG) algorithm in the space of a reduced number of design variables.

For the cost optimization process, the objective cost function is taken as the cost of HSC concrete, steel, and forming of the T-beam, which is minimized while subjected to constraints and strength requirements.

The set of constraints includes restrictions in behavior constraints, compatibility conditions on strain for the steel and concrete, and geometric design variable constraints.

A typical example problem is considered in order to illustrate the applicability of the proposed design model and solution methodology. The optimized results are compared to traditional design solutions derived from conventional design methods in order to evaluate the performance of the developed cost model. It is shown through this work that optimal solutions achieved using the present approach can lead to substantial savings in the amount of construction materials to be used. In addition, the proposed approach is practically simple, reliable, and computationally effective compared to classical design procedures used by designers and engineers.

2. Model formulation

2.1. Ultimate design of doubly reinforced HSC T-section under bending

In accordance with EC-2-French annex [21], the assumptions used at the Ultimate Limit State (ULS) for strain and stress distributions in the typical reinforced HSC T-beam cross-section shown in Figure 1(a) are, respectively, illustrated in Figure 1(b) and (c).

Table 1 summarizes the input data for stress-strain diagrams of HSC T-section and constant parameters as prescribed by EC-2-French annex.

Table 1 shows input data considering different stress distributions for HSC under compression in accordance with EC2 and constant parameters.

In the present work, the three diagrams of stress distribution for HSC are used in order to obtain the optimal solutions, which are then compared to the classical solution. The elasto-plastic behavior for steel is considered. In addition, the steel strain is considered unlimited in accordance with EC-2 provisions.

2.2. Design variables

The design variables selected for the optimization are presented in Table 2.

2.3. Objective function

The objective function to be minimized in the present optimization problem is the total cost of construction material per unit length of the beam. This function can be defined as:

$$C_0 = C_c\,(b_w h + (b - b_w)h_f) + C_s(A_{s1} + A_{s2}) + C_f(b + 2h) \rightarrow \text{Minimum}, \tag{1}$$

where:

C_0/L	Total cost per unit length of HSC T-beam
C_s	Unit cost of reinforcing steel
C_c	Unit cost of HSC concrete
C_f	Unit cost of formwork

It should be noted that in a cost optimization problem, the optimal values of the design variables

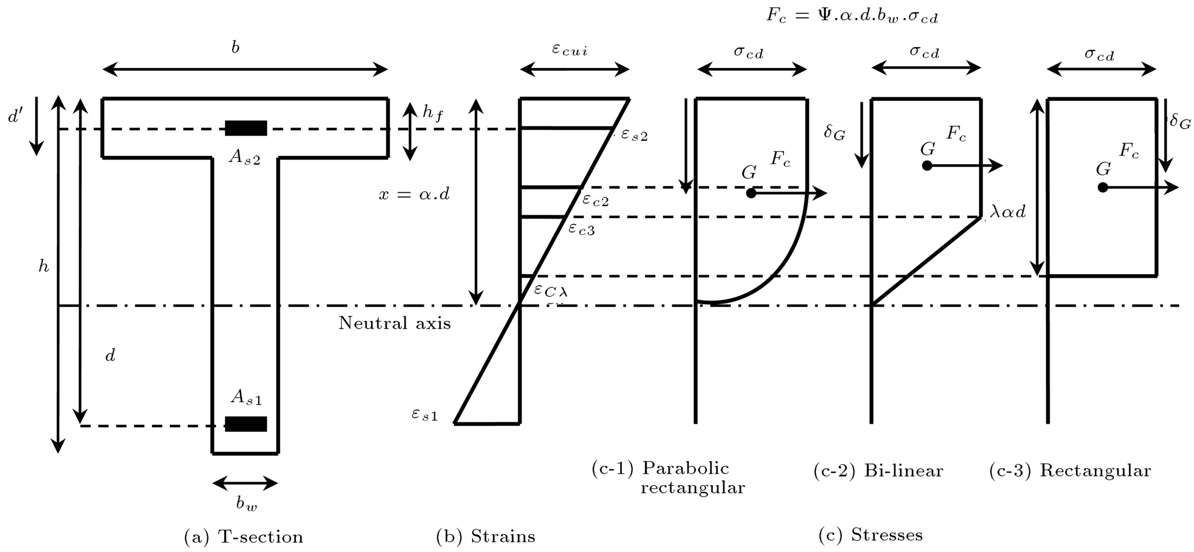

Figure 1. (a) Geometry of the doubly reinforced HSC T-section, (b) Strains distribution; and (c) Stresses distribution (c-1), (c-2) and (c-3).

Table 1. Input data considering different stress distributions for HSC under compression in accordance with EC2 and constant parameters.

C90/105 $f_{ck} = 90$ **MPa** $f_{cd} = 51$ **MPa**	**Parabolic-rectangle stress block** ε_{c2}, ε_{cu2}	**Rectangular stress block** $\varepsilon_{c\lambda}$, ε_{cu3} $\varepsilon_{c\lambda} = \varepsilon_{cu3}(1-\lambda)$	**Bilinear design stress-strain relation** ε_{c3}, ε_{cu3}
σ_{cd}	f_{cd}	ηf_{cd}	f_{cd}
Ψ	0.583	0.700	0.558
δ_G	0.353	0.350	0.337
λ	N/A	0.700	N/A
η	N/A	0.800	N/A
μ_{limit}	0.256	0.309	0.248
α_{limit}	0.5446	0.5446	0.5446
ε_{cui}	2.6%	2.6%	2.6%
ε_{ci}	2.6%	0.78%	2.3%

Table 2. Definition of design variables.

Number of variables	**Design variables**	**Defined variables**
01	b	Effective width of compressive flange
02	b_w	Web width
03	h	Total depth
04	d	Effective depth
05	h_f	Flange depth
06	d'	Depth from the top of the compression face to the centroid of the compression reinforcement
07	A_{s1}	Area of tension reinforcement
08	A_{s2}	Area of compression reinforcement
09	α	Relative depth of compressive concrete zone

are affected by the relative cost values of the objective function only, but not by the absolute cost values. In other words, the absolute cost values affect the final value of the objective function, but not the optimal values of the design variables.

The absolute cost, C_0, can then be recovered from the optimized relative cost, C, by using the following relation:

$$C_0 = C_c L C. \tag{2}$$

Thus, the objective function to be minimized can be written as follows:

$$C = b_w h + (b - b_w)h_f + (C_s/C_c)(A_{s1} + A_{s2}) + (C_f/C_c)(b + 2h) \rightarrow \text{Minimum}. \tag{3}$$

The values of the cost ratios C_s/C_c and C_f/C_c varied from one country to another and may eventually vary from one region to another for certain countries.

The values of these cost ratios can be estimated on the basis of data given in applicable unit price books of construction materials.

2.4. Formulation of design constraints

The following constraints for the HSC T-beams are defined in accordance with the design code specifications of the French Annex to EC-2:

a) Constraints for the ultimate flexural strength are as follows:

$$M_{Ed} \leq \sigma_{cd}(b-b_w)h_f(d - 0.5h_f) + \Psi b_w d^2 \sigma_{cd}\alpha(1-\delta_G\alpha)+f_{yd}A_{s2}(d-d'), \tag{4}$$

in which: External moment $\leq$ Resisting moment of the cross section.

$$\sigma_{cd}(b - b_w)h_f + \Psi b_w \alpha d \sigma_{cd} + f_{yd}A_{s2} - f_{yd}A_{s1} = 0, \tag{5}$$

which is in internal force equilibrium.

Conditions on strain compatibility in steel and concrete are as follows:

$$\frac{f_{yd}}{E_s} \leq \frac{\varepsilon_{cui}(1-\alpha)}{\alpha} \leq \infty, \tag{6}$$

in which elasto-plastic behavior for steel and the pivot point is B.

$$\frac{f_{yd}}{E_s} \leq \varepsilon_{cui}\left(1 - \frac{d'}{\alpha d}\right) \leq \varepsilon_{cui}, \tag{7}$$

in which elasto-plastic behavior for steel and the pivot point is B, and optimal use of steel requires that strains in steel must be limited to plastic region at the Ultimate Limit State (ULS).

$$\frac{M_{Ed} - \frac{(b-b_w)bh_f\sigma_{cd}(d-0.5h_f)}{b}}{b_w d^2 \sigma_{cd}} > \mu_{\text{limite}}, \tag{8}$$

for which compression reinforcement is required.

$$\mu_{\text{limite}} = \Psi\alpha_{\text{limite}}(1 - \delta_G\alpha_{\text{limite}}). \tag{9}$$

b) Constraint for minimum area of tension reinforcement is as follows:

$$p_{\min} \leq A_{s1}/b_w d, \tag{10}$$

which is minimum steel percentage.

c) Constraint for maximum area of tension reinforcement is as follows:

$$\frac{A_{s1}}{b_w h + (b - b_w)h_f} \leq p_{\max}, \tag{11}$$

which is maximum steel percentage.

d) Constraints for maximum area of compression reinforcement are as follows:

$$\frac{A_{s2}}{b_w h + (b - b_w)h_f} \leq p_{\max}, \tag{12}$$

which is maximum steel percentage.

$$f_{yd}A_{s2}(d - d') \leq 0.40 M_{Ed}. \tag{13}$$

e) Constraint for the ultimate shear strength is as follows:

$$V_{Ed} \leq V_{Rd,\max} = v_1 f_{cd} b_w z / \left(tg(\theta) + \cot g(\theta)\right). \tag{14}$$

f) Geometrical constraints including pre-design rules of the current practice are shown in Table 3.

2.5. Formulation of cost optimum design problem

Thus, the formulation of the optimum cost design of doubly reinforced HSC T-beams under ultimate loads can be mathematically stated as follows.

Given the characteristics of material, loading data, and constant parameters, find the design variables b, b_w, h, d, h_f, d', A_{s1}, A_{s2}, and α that minimize

Table 3. Design variables constraints including rules of current practice.

Design variables	Equation numbers
$L/30 \leq h \leq L/22$	(15)
$d/h = 0.90$	(16)
$0.05 \leq d' \leq 0.1$	(17)
$0.30 \leq b_w/d \leq 0.50$	(18)
$(b - b_w)/2 \leq L/10$	(19)
$b/h_f \leq 8$	(20)
$h_{f\min} \leq h_f$	(21)

the total cost of construction material per unit length of HSC T-beam, such that:

$$C = b_w h + (b - b_w)h_f + (C_s/C_c)(A_{s1} + A_{s2}) + (C_f/C_c)(b + 2h) \rightarrow \text{Minimum},$$

subjected to the design constraints.

2.6. Solution methodology

The objective function, Eq. (3), and the constraints equations, Eqs. (4) through (21), together, form a nonlinear optimization problem. This nonlinearity appears in the expression of the cost of the HSC concrete and in the constraints related to the ultimate flexural strength, the ultimate shear strength, and the remaining constraints. Both the objective function and the constraints functions are nonlinear in terms of the design variables involved. In order to solve this nonlinear optimization problem, the Generalized Reduced Gradient (GRG) algorithm is used.

3. Numerical results

A typical example problem is now considered. The step-by-step application of the HSC T-beam optimum cost design model is presented, followed by a comparison between the standard design solution using HSC and the optimal solution obtained from the use of the three diagrams of stress distribution for HSC, and then by a parametric study on optimum cost design solution under imposed conditions. These conditions are related to HSC T-beam dimensions, reinforcing steel, and relative depth of the neutral axis. Finally, a cost-sensitivity analysis was conducted for different values of the unit cost ratios.

3.1. Design example

As previously mentioned, the design constraints are defined in accordance with the code design specifications of the French Annex to EC-2. The optimal solutions are compared to the standard design solutions obtained in accordance with EC-2 design code. To further illustrate the variability of the optimal solutions with unit costs of materials, the optimal solutions are computed for given unit cost ratios. The results, in terms of the corresponding gains, are presented graphically.

The study of HSC T-beam corresponds to a T-beam simply supported at its ends and pre-designed in accordance with provisions of EC-2 design code.

The corresponding pre-assigned parameters are defined as follows:

- Beam span: $L = 22$ m;
- Ultimate bending moment capacity:

  $$M_{Ed} = 1.35M_G + 1.5M_Q = 5MNm;$$
- Ultimate design shear capacity:

  $$V_{Ed} = 1.35V_G + 1.5V_Q = 2MN;$$

Input data for HSC characteristics are as follows:

- Strength class of concrete: C90/105;
- Characteristic compressive cylinder strength of concrete at 28 days: $f_{ck} = 90$ MPa;
- Partial safety factor for concrete: $\gamma_c = 1.5$;
- Allowable compressive stress: $f_{cd} = 51$ MPa;
- $\lambda = 0.700$;
- $\eta = 0.800$;
- $\varepsilon_{cu3} = 2.6\%$ (in the case of pivot B, C90/105);
- $\mu_c = 0.309$ (steel HA class S500, concrete class C90/105);
- $h_{f\,\min} = 0.10$;
- $v_1 = 0.384$, $\theta = 45°$;
- $f_{ctm} = 5.0$ MPa.

Input data for steel characteristics are as follows:

- Steel class: S500;
- Yield strength: $f_{yk} = 500$ MPa;
- Partial safety factor for steel: $\gamma_s = 1.15$;
- Allowable tensile stress: $f_{yd} = f_{yk}/\gamma_s = 435$ MPa;
- Young's modulus: $E_s = 2 \times 10^5$ MPa;
- Yield strain:

  $$\varepsilon_{yd} = f_{yd}/E_s = f_e/E_s.\gamma_s, \quad \varepsilon_{yd} = 2.174\%$$

  (steel grade: S500);
- Minimum steel percentage:

  $$p_{\min} = 0.26 f_{ctm}/f_{yk} = 0.0026 = 0.26\%;$$
- Maximum steel percentage: $p_{\max} = 4\%$.

Input data for units costs ratios of construction materials are as follows:

- $C_s/C_c = 30$;
- $C_f/C_c = 0.01$.

3.2. Step-by-step procedure

The classical design solution based on the EC2 design code is briefly presented in Appendix.

The cost optimum design problem can be written as follows.

Find the design variables b, b_w, h, d, h_f, d', A_{s1}, A_{s2}, and α that minimize the total cost of construction material per unit length of HSC T-beam, such that:

$$C/C_c = b_w h + (b - b_w)h_f + 30(A_{s1} + A_{s2}) + 0.01(b + 2h) \rightarrow \text{Minimum}, \tag{22}$$

subjected to the design constraints as follows:

a) Constraints for the ultimate flexural strength:

$$5 \leq 40.80(b-b_w)h_f(d-0.5h_f) + 28.56b_w d^2\alpha(1-0.35\alpha)+435A_{s2}(d-0.05), \quad (23)$$

$$40.80(b-b_w)h_f + 28.56b_w\alpha d + 435A_{s2} - 435A_{s1} = 0, \quad (24)$$

$$0.002174 \leq \frac{0.0026(1-\alpha)}{\alpha} \leq \infty, \quad (25)$$

$$0.002174 \leq 0.0026\left(1-\frac{d'}{\alpha d}\right) \leq 0.0026, \quad (26)$$

$$\frac{5-\frac{40.80(b-b_w)bh_f(d-0.5h_f)}{b}}{40.80b_w d^2} > 0.309, \quad (27)$$

$$\mu_{\text{limit}} = 0.309. \quad (28)$$

b) Constraint for minimum area of tension reinforcement:

$$0.0026 \leq A_{s1}/b_w d. \quad (29)$$

c) Constraint for maximum area of tension reinforcement:

$$\frac{A_{s1}}{b_w h + (b-b_w)h_f} \leq 0.04. \quad (30)$$

d) Constraints for maximum area of compression reinforcement:

$$\frac{A_{s2}}{b_w h + (b-b_w)h_f} \leq 0.04. \quad (31)$$

$$435A_{s2}(d-d') \leq 2. \quad (32)$$

e) Constraint for the ultimate shear strength:

$$2 \leq 7.05b_w d. \quad (33)$$

f) Geometrical constraints including pre-design rules of the current practice:

$$0.73 \leq h \leq 1.00, \quad (34)$$

$$d/h = 0.90, \quad (35)$$

$$0.05 \leq d' \leq 0.1, \quad (36)$$

$$0.30 \leq b_w/d \leq 0.50, \quad (37)$$

$$(b-b_w)/2 \leq 2.20, \quad (38)$$

$$b/h_f \leq 8, \quad (39)$$

$$0.10 \leq h_f. \quad (40)$$

3.3. Comparison between the optimal cost design solutions for three idealized stress distributions and the standard design approach

The vector of design variables, including the geometric dimensions of the HSC T-beam cross-section and the area of tension and compression reinforcements as obtained from the conventional design solution, the optimal cost design solution using the proposed approach, and the list of binding constraints, is shown in Table 4.

From the above results, it is clearly seen from the values of the relative costs, $C_s/C_c = 30$ and $C_f/C_c = 0.01$, and those associated with the classical and optimal solutions that a significant cost saving of the order of 17% can be obtained by the proposed design formulation.

3.4. Parametric study

In this section, the optimal solution is obtained by considering:

(i) One of the dimensions of HSCT-section is imposed;

(ii) Imposed reinforcing steel;

(iii) Imposed relative depth of the neutral axis, $\alpha = \alpha_{\text{lim}}$.

Further practical requirements can also be implemented such as aesthetic, architectural, and limited authorized template.

Based on Table 5, it is clearly seen from the values of the relative costs, $C_s/C_c = 30$ and $C_f/C_c = 0.01$, and those associated with the classical and optimal solutions corresponding to rectangular stress block that the percentage saving is between 05% and 17%.

3.5. Cost-sensitivity analysis

Comparing the relative gains can be determined for the various values of the unit cost ratios. The corresponding results are reported in Tables 6 and 7 and illustrated graphically in Figures 2 and 3 for HSC class 90/105.

The relative gains can be determined for various values of the unit cost ratios, C_f/C_c, and for two given unit cost ratios, C_s/C_c. The corresponding results are reported in Table 6 and illustrated graphically in Figure 2 for $C_s/C_c = 30$ and $C_s/C_s = 60$.

In order to further illustrate the variability of optimal solution with the unit cost ratios C_f/C_c, for two sets of lower value and upper value of C_s/C_c, the optimal solution has been computed for various ratios of C_f/C_c taken to be between 0.01 and 0.1.

From Table 6, it is shown that the maximum percentage of saving corresponding to the value of C_f/C_c being equal to 0.01 is 17% and 26% for $C_s/C_c = 30$ and $C_s/C_c = 60$, respectively.

Table 4. Comparison of the classical solution and the optimal solution corresponding to the three diagrams' design of stress distribution for HSC under compression in accordance with EC2.

Vector solution for $C_s/C_c = 30$, $C_f/C_c = 0.01$	Classical solution	Optimal solution Parabolic-rectangle stress block	Optimal solution Rectangular stress block	Optimal solution Bilinear design stress-strain relation
b (m)	1.00	0.66	0.81	0.69
b_w (m)	0.30	0.34	0.32	0.34
h (m)	0.80	1.00	1.00	1.00
d (m)	0.72	0.90	0.90	0.90
h_f (m)	0.13	0.10	0.10	0.10
d' (m)	0.05	0.05	0.05	0.05
A_{s1} (m ^{2})	193.35×10^{-4}	148.75×10^{-4}	147.80×10^{-4}	149.04×10^{-4}
A_{s2} (m^2)	20.84×10^{-4}	5.71×10^{-4}	3.66×10^{-4}	1.67×10^{-4}
α	0.805	0.502	0.510	0.520
C	0.99958	0.86188	0.85448	0.851656
Gain		16%	17%	17%
List of binding constraints		(4); (5); (8); (11); (15); (16); (17); (21)	(4); (5); (8); (11); (15); (16); (17); (20)	(4); (5); (8); (11); (15); (16); (17); (21)

Table 5. Variation of relative gain with particular conditions imposed such as the HSC T-beam dimensions, reinforcing steel, and depth of the neutral axis.

Optimal solution with imposed variables	Gain
b imposed: $b = 0.90$ m	07%
b_w imposed: $b_w = 0.35$ m	14%
h imposed: $h = 0.90$ m	05%
h_f imposed: $h_f = 0.12$ m	17%
$(A_{s1} + A_{s2})$ imposed: $A_{s1} + A_{s2} \leq 0.01600$ m^2	17%
(A_{s2}/A_{s1}) imposed: $A_{s2}/A_{s1} = 0.40$	17%
A_{s2} imposed: $A_{s2} = 0.0015$ m^2	15%
α imposed: $\alpha = \alpha_{\text{lim}} = 0.5446$ ($\mu = \mu_{\text{limit}} = 0.309$)	17%

Table 6. Variation of relative gain in percent (%) versus unit cost ratio, C_f/C_c, of construction materials for $C_s/C_c = 30$ and $C_s/C_c = 60$.

$C_s/C_c = 30$	C_f/C_c	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
	Gain	17%	16%	15%	14.8%	14%	13.5%	13%	12.8%	12.5%	12%
$C_s/C_c = 60$	C_f/C_c	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
	Gain	26%	25%	24%	23.6%	23%	22%	21.9%	21%	21%	20%

Table 7. Variation of relative gain in percent (%) versus unit cost ratio C_s/C_c of construction materials for $C_f/C_c = 0.010$ and $C_f/C_c = 0.10$.

$C_f/C_c = 0.01$	C_s/C_c	10	15	20	25	30	35	40	45	50	60
	Gain	4%	9%	12%	15%	17%	19%	21%	22%	23%	26%
$C_f/C_c = 0.1$	C_s/C_c	10	15	20	25	30	35	40	45	50	60
	Gain	2%	5%	8%	9%	12%	14%	16%	17%	19%	20%

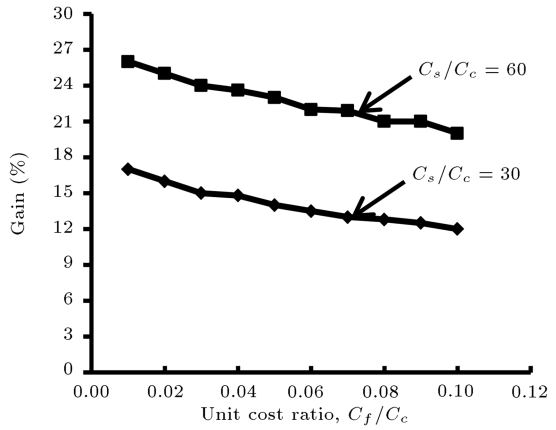

Figure 2. Variation of relative gain in percentage versus two unit cost ratios, C_s/C_c, for different values of C_f/C_c.

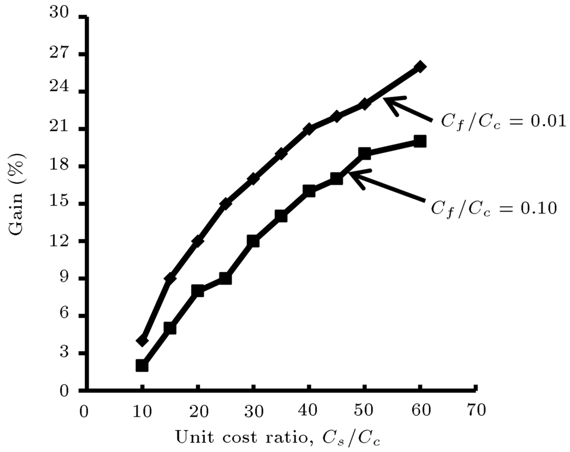

Figure 3. Variation of relative gain in percentage versus two unit cost ratios, C_f/C_c, for different values of C_s/C_c.

From Figure 2, the gain decreases monotonically with the increasing of the unit cost ratio, C_f/C_c.

The relative gains can be determined for various values of the unit cost ratios, C_s/C_c, for two given unit cost ratios, C_f/C_c. The corresponding results are reported in Table 7 and illustrated graphically in Figure 3 for $C_f/C_c = 0.01$ and $C_f/C_c = 0.10$.

In order to further illustrate the variability of the optimal solution with the unit cost ratio, C_s/C_c, for two sets of lower and upper values of C_f/C_c, the optimal solution has been computed for various ratios of C_s/C_c taken to be between 10 and 60.

From Table 7, it is shown that the maximum percentage of saving corresponding to the value of C_s/C_c being equal to 60 is 26% and 20% for $C_f/C_c = 0.01$ and $C_f/C_c = 0.1$, respectively.

From Figure 3, the gain increases with the increasing of the unit cost ratio, C_s/C_c, following a nonlinear relation.

4. Conclusions

The following important conclusions are drawn based on the present study:

1. The problem formulation of the optimal cost design of doubly reinforced HSC T-beams can be cast into a nonlinear programming problem, the numerical solution of which is efficiently determined using the GRG (Generalized Reduced Gradient) method in a space of only a few variables representing the concrete cross-section dimensions;
2. The space of feasible design solutions and the optimal solutions can be obtained using a reduced number of independent design variables;
3. Optimal values of the design variables are affected by the relative cost values of the objective function only, but not by the absolute cost values;
4. The optimal solutions are found to be insensitive to changes in the shear constraint. Shear constraint is usually not critical in the optimal design of reinforced HSC T-beams under bending and can thus be excluded from the problem formulation;
5. The observations of the optimal solutions' results reveal that the use of the optimization based on the optimum cost design concept may lead to substantial savings in the amount of the construction materials to be used in comparison to classical design solutions of doubly reinforced concrete HSC T-beams;
6. The objective function and the constraints considered in the present paper are illustrative in nature. The present approach based on nonlinear mathematical programming can be easily extended to other sections commonly used in structural design. More sophisticated objectives and considerations can be readily accommodated by suitable modifications of the present optimal cost design model;
7. It is also important to note that, in general, the use of top reinforcement in reinforced HSCT-beam sections is an indication of a poor design;
8. In this work, we have included the additional cost of formwork which makes a significant contribution to the total costs. This inclusion is important for an economical approach to design and manufacture;
9. The proposed methodology for optimum cost design is effective and more economical for the classical methods. The results of the analysis show that the optimization process presented here is effective and its application appears feasible.

Nomenclature

L	Beam span
M_{Ed}	Ultimate bending moment capacity including self-weight

M_G	Maximum design moments under dead loads
M_Q	Maximum design moments under live loads
V_G	Maximum design shears under dead loads
V_Q	Maximum design shears under live loads
V_{Ed}	Ultimate shear capacity including self-weight
$V_{Rd,\max}$	Maximum resistant shear force C80/105 Class of HSC
f_{ck}	Characteristic compressive cylinder strength of HSC at 28 days
f_{ctm}	Tensile strength of concrete
f_{cd}	Design value of concrete compressive strength
γ_c	Partial safety factor for concrete
η	Design strength factor
λ	Compressive zone depth factor
ε_{c2}	Strain at the maximum stress for the parabolic-rectangular stress distribution compressive concrete
ε_{cu2}	Ultimate strain for the parabolic-rectangular stress distribution compressive concrete
ε_{c3}	Strain at the maximum stress for the rectangular stress distribution compressive concrete
$\varepsilon_{c\lambda}$	Strain at the maximum stress for the bilinear design stress-strain relation
ε_{cu3}	Ultimate strain for the rectangular stress distribution compressive concrete and bilinear design stress-strain relation S500 grade of steel
f_{yk}	Characteristic elastic limit for steel reinforcement
γ_s	Partial safety factor for steel
f_{yd}	Design yield strength of steel reinforcement
ε_{yd}	Elastic limit strain
ε_1	Strain of steel under tension
ε_2	Strain of steel under compression
E_s	Young's elastic modulus
$p_{\min}$	Minimum steel percentage
$p_{\max}$	Maximum steel percentage
α_{limit}	Limit value of relative depth of compressive concrete zone
μ_{limit}	Limit value of reduced moment
θ	The angle between concrete compression struts and the main chord
v_1	A non dimensionnel coefficient
z	Lever arm
$h_{\min}$	Minimum depth of flange
C_0	Total cost per unit length of HSC T-beam
C	Total relative cost of HSC T-beam
C_s	Unit cost of reinforcing steel
C_c	Unit cost of HSC
C_f	Unit cost of formwork
δ_G	Coefficient used to calculate the centroid of the effective area of concrete in comparison to the most compressed fiber for concrete
ψ	Coefficient used to calculate the value of the resultant compressive force for HSC concrete F_c for the stress diagram considered
F_c	Resultant compressive force for HSC
σ_{cd}	Maximum design stress
G	Centroid of the effective area of concrete in compression

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Appendix

Classical design solution for design example

The study of high strength concrete T-beam corresponds to a T-beam simply supported at its ends and pre-designed in accordance with provisions of EC-2 design code.

One should determine the areas A_{s1} and A_{s2} of steel reinforcement for the reinforced HSC T-beam with the cross-section dimensions of $b = 1.00$ m, $b_w = 0.30$ m, $h = 0.80$ m, $d = 0.72$ m and $h_f = 0.13$ m, as obtained from a preliminary design of the study HSC T-beam.

Consider the following data:

- Beam span: $L = 22$ m;
- Ultimate bending moment capacity including self-weight: $M_{Ed} = 5$ MNm;
- Ultimate shear capacity including self-weight: $V_{Ed} = 2$ MN;
- Strength class of concrete: C90/105;
- Characteristic compressive cylinder strength of concrete at 28 days: $f_{ck} = 90$ MPa;
- Partial safety factor for concrete: $\gamma_c = 1.5$;
- Allowable compressive stress: $f_{cd} = 51$ MPa;
- Design strength factor: $\eta = 0.800$;
- Compressive zone depth factor: $\lambda = 0.700$;
- Steel class: S500;
- Elastic limit: $f_{yk} = 500$ MPa;
- Partial safety factor for steel: $\gamma_s = 1.15$;
- Allowable tensile stress: $f_{yd} = f_{yk}/\gamma_s = 435$ MPa;
- Unit cost ratios: $C_s/C_c = 30$; $C_f/C_c = 0.01$.

Moment of resistance of the flange, M_f, is obtained as follows:

- $M_f = \eta b h_f f_{cd}(d - 0.5h_f)$;
- $M_f = 3.474$ MNm;
- $M_f < M_{Ed}$, in which the stress block must extend below the flange;
- $\mu_{\lim} = \lambda\alpha_{\lim}(1 - 0.5\lambda\alpha_{\lim})$, which is limit value of reduced moment;
- $\alpha_{\lim} = 0.5446$;
- $\mu_{\lim} = 0.309$;
- $\mu = (M_{Ed} - M_f((b - b_w)/b)/b_w d2\eta f_{cd}) < \mu_{\lim}$;
- $\mu = 0.405$;
- $\mu > \mu_{\lim}$ for which compression reinforcement is required.

The area of compression reinforcement, A_{s2}, is obtained as follows:

$$A_{s2} = \left(M_{Ed} - \left(M_f(b - b_w)/b - \mu_{lim} b_w d^2 \eta f_{cd}\right)\right) / (d - d') f_{yd},$$

$$A_{s2} = 0.002084 \text{ m}^2.$$

The relative depth of compressive concrete zone, α, is obtained as follows:

$$\alpha = \left[1 - (1 - 2\mu)^{0.5}\right]/\lambda, \quad \alpha = 0.805.$$

Lever arm, z, is obtained as follows:

$$z = d(1 - 0.5\lambda\alpha), \quad z = 0.517 \text{ m}.$$

The area of tension steel, A_{s1}, is obtained as follows:

$$A_{s1} = \left[\mu_{lim} b_w d^2 \eta f_{cd}/z f_{vd}\right] + (b - b_w) h_f \eta f_{cd} / f_{yd} + A_{s2}$$

$$A_{s1} = 0.019335 \text{ m}^2.$$

Total cost per unit length of HSC T-beam, C, is obtained as follows:

$$C = 0.99958 C_c.$$

Biography

Ferhat Fedghouche has been a Public Works Engineer in National School of Built and Ground Works Engineering, Algiers, Algeria, since 1993. He obtained his MSc in National School of Built and Ground Works Engineering, Algiers, Algeria, in 2001, and his Doctorate in Science (PhD) in National Polytechnic School of Algiers in 2013. He is currently working as a full-time lecturer in the Department of Basic Infrastructures at the National School of Built and Ground Works Engineering, Algiers, Algeria. His areas of interest include engineering optimization, operational research in industry, software for operational research, pathologies of constructions and area bridges. He has published numerous papers in refereed national and international journals and conferences in these areas of expertise.

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