Cost optimum design of doubly reinforced high strength concrete T-beams

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Abstract. This paper presents a method for optimizing the design cost of a doubly reinforced High Strength Concrete (HSC) T-beam. The objective function used in the model includes the costs of HSC, steel, and formwork. The constraint functions are set to satisfy design requirements of Eurocode 2 (EC-2). The cost optimization process is developed by the use of the Generalized Reduced Gradient algorithm. An example problem is considered in order to illustrate the applicability of the proposed design model and solution methodology. It is concluded that the present approach is economically more effective when compared to conventional design methods and can be extended to deal with other sections without major alterations.

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1. Introduction

Flanged sections are often used for medium span concrete beams, so that their structural efficiency is maximized. Within the same sectional area, a flanged section has better strength-ductility performance compared with a rectangular one. Structural elements with T-shaped sections are frequently used in an industrial construction. They are used repeatedly for large structures, because they are cost-effective, especially when using optimum cost design model, which is of great value to practicing designers and engineers. Compression reinforcement is often not required when designing the T-beams sections. One of the great advantages of T-beams sections is the economy of the amount of steel needed for reinforcement. For structures requiring reduced height such as parking spaces and bridge decks, beams with lower template are preferred [1-5].

High strength concrete T-beams are frequently used in an engineering practice. They are widely used for short to medium span highway bridges due to its moderate self weight, structural efficiency, ease of fabrication, fast construction, and low maintenance. The long span bridges and the tall buildings are highly considered in the conceptual design by HSC. The majority of research studies conducted so far have focused on the optimization of ordinary concrete T-beams, whose strength class is between 12 MPa and 50 MPa; however, only few studies have been done with regard to the optimization of high strength concrete T-beams whose resistance class is between 50 MPa and 100 MPa. Compression reinforcement is not often required when designing the T-beams sections. It is also important to note that, in general, the use of top reinforcement in reinforced concrete T-beam sections is an indication of poor design. Thus, the doubly reinforced high-strength concrete T-beams are recommended for structures requiring low height such as parking spaces and bridge decks beams with lower

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Recent developments in the technology of materials have led to the use of the high-strength concrete. This is mainly due to its efficiency and economy. The reduction in the quantities of construction materials has enabled both a gain in weight reduction and in foundation’s cost [10].

HSC has a high compressive strength in the range of 50 to 100 MPa; it has not only the advantage of reducing member size and story height, but also the volume of concrete and the area of formwork. In terms of the amount of steel reinforcement, there is a substantial difference between the normal strength concrete structure compared to high strength concrete structures [11-14].

Advances in numerical optimization methods, computer-based numerical tools for analysis, and design of structures and availability of powerful computing hardware have significantly helped the design process to ascertain the optimum design. In the present research work, the Generalized Reduced Gradient (GRG) method is used for solving nonlinear programming. The GRG algorithm transforms inequality constraints into equality constraints by introducing slack variables. It is a very reliable and robust algorithm [15-20].

Consideration of serviceability conditions at working loads can be included without major alterations, but will be addressed elsewhere as it requires further attention in terms of restrictions on bending moment capacity, stress limitations in concrete and in steel as a function of cracking conditions, as well as limits on deflections. Such restrictions will have direct consequences on the boundaries of the design space and the feasible design solutions of the optimization problem.

In this paper, a model on how to calculate the optimum cost design of doubly reinforced High Strength Concrete (HSC) T-beams in flexure under Ultimate Limit State conditions (ULS) is presented. The objective function includes the costs of HSC, steel, and formwork. The constraint functions are set to satisfy design requirements used in Eurocode 2 (EC-2)-French annex. The cost optimization process is developed by the use of the Generalized Reduced Gradient (GRG) algorithm in the space of a reduced number of design variables.

For the cost optimization process, the objective cost function is taken as the cost of HSC concrete, steel, and forming of the T-beam, which is minimized while subjected to constraints and strength requirements.

The set of constraints includes restrictions in behavior constraints, compatibility conditions on strain for the steel and concrete, and geometric design variable constraints.

A typical example problem is considered in order to illustrate the applicability of the proposed design model and solution methodology. The optimized results are compared to traditional design solutions derived from conventional design methods in order to evaluate the performance of the developed cost model. It is shown through this work that optimal solutions achieved using the present approach can lead to substantial savings in the amount of construction materials to be used. In addition, the proposed approach is practically simple, reliable, and computationally effective compared to classical design procedures used by designers and engineers.

2. Model formulation

2.1. Ultimate design of doubly reinforced HSC T-section under bending

In accordance with EC-2-French annex [21], the assumptions used at the Ultimate Limit State (ULS) for strain and stress distributions in the typical reinforced HSC T-beam cross-section shown in Figure 1(a) are, respectively, illustrated in Figure 1(b) and (c).

Table 1 summarizes the input data for stress-strain diagrams of HSC T-section and constant parameters as prescribed by EC-2-French annex.

Table 1 shows input data considering different stress distributions for HSC under compression in accordance with EC2 and constant parameters.

In the present work, the three diagrams of stress distribution for HSC are used in order to obtain the optimal solutions, which are then compared to the classical solution. The elasto-plastic behavior for steel is considered. In addition, the steel strain is considered unlimited in accordance with EC-2 provisions.

2.2. Design variables

The design variables selected for the optimization are presented in Table 2.

2.3. Objective function

The objective function to be minimized in the present optimization problem is the total cost of construction material per unit length of the beam. This function can be defined as:

\[ C_0 = C_s (b_w h + (b - b_w) h_f) + C_c (A_{x1} + A_{x2}) + C_f (b + 2 h) \rightarrow \text{Minimum}, \]

where:

- \( C_0 / L \) Total cost per unit length of HSC T-beam
- \( C_s \) Unit cost of reinforcing steel
- \( C_c \) Unit cost of HSC concrete
- \( C_f \) Unit cost of formwork

It should be noted that in a cost optimization problem, the optimal values of the design variables...
$F_e = \Psi \cdot \alpha \cdot d \cdot b_w \cdot \sigma_{cd}$

(a) T-section  (b) Strains  (c) Stresses

(c-1) Parabolic rectangular  (c-2) Bi-linear  (c-3) Rectangular

Figure 1. (a) Geometry of the doubly reinforced HSC T-section, (b) Strains distribution; and (c) Stresses distribution (c-1), (c-2) and (c-3).

Table 1. Input data considering different stress distributions for HSC under compression in accordance with EC2 and constant parameters.

<table>
<thead>
<tr>
<th>C90/105</th>
<th>Parabolic-rectangle stress block</th>
<th>Rectangular stress block</th>
<th>Bilinear design stress-strain relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ck} = 90 \text{ MPa}$</td>
<td>$\varepsilon_{c,e}$, $\varepsilon_{c,\text{mu2}}$</td>
<td>$\varepsilon_{c,\lambda} = \varepsilon_{c,\text{mu2}}(1 - \lambda)$</td>
<td>$\varepsilon_{c,\lambda}$, $\varepsilon_{c,\text{mu2}}$</td>
</tr>
<tr>
<td>$f_{cd} = 51 \text{ MPa}$</td>
<td>$f_{cd}$</td>
<td>$\eta f_{cd}$</td>
<td>$f_{cd}$</td>
</tr>
<tr>
<td>$\sigma_{cd}$</td>
<td>$0.583$</td>
<td>$0.700$</td>
<td>$0.558$</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>$0.353$</td>
<td>$0.350$</td>
<td>$0.337$</td>
</tr>
<tr>
<td>$\delta_{G}$</td>
<td>$0.700$</td>
<td>$0.800$</td>
<td>$0.700$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$N/A$</td>
<td>$N/A$</td>
<td>$N/A$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$0.256$</td>
<td>$0.309$</td>
<td>$0.248$</td>
</tr>
<tr>
<td>$\beta_{\text{limit}}$</td>
<td>$0.5416$</td>
<td>$0.5416$</td>
<td>$0.5416$</td>
</tr>
<tr>
<td>$\varepsilon_{c,\mu1}$</td>
<td>$2.6%$</td>
<td>$2.6%$</td>
<td>$2.6%$</td>
</tr>
<tr>
<td>$\varepsilon_{c,\mu2}$</td>
<td>$0.78%$</td>
<td>$0.78%$</td>
<td>$2.3%$</td>
</tr>
</tbody>
</table>

Table 2. Definition of design variables.

<table>
<thead>
<tr>
<th>Number of variables</th>
<th>Design variables</th>
<th>Defined variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>$b$</td>
<td>Effective width of compressive flange</td>
</tr>
<tr>
<td>02</td>
<td>$b_w$</td>
<td>Web width</td>
</tr>
<tr>
<td>03</td>
<td>$h$</td>
<td>Total depth</td>
</tr>
<tr>
<td>04</td>
<td>$d$</td>
<td>Effective depth</td>
</tr>
<tr>
<td>05</td>
<td>$h_f$</td>
<td>Flange depth</td>
</tr>
<tr>
<td>06</td>
<td>$d' = d_h$</td>
<td>Depth from the top of the compressive face to the centroid of the compression reinforcement</td>
</tr>
<tr>
<td>07</td>
<td>$A_{11}$</td>
<td>Area of tension reinforcement</td>
</tr>
<tr>
<td>08</td>
<td>$A_{12}$</td>
<td>Area of compression reinforcement</td>
</tr>
<tr>
<td>09</td>
<td>$\alpha$</td>
<td>Relative depth of compressive concrete zone</td>
</tr>
</tbody>
</table>
are affected by the relative cost values of the objective function only, but not by the absolute cost values. In other words, the absolute cost values affect the final value of the objective function, but not the optimal values of the design variables.

The absolute cost, \( C_0 \), can then be recovered from the optimized relative cost, \( C \), by using the following relation:

\[
C_0 = C_cLC.
\]

Thus, the objective function to be minimized can be written as follows:

\[
C = b_w h + (b - b_w) h_f + (C_s/C_c) (A_{s1} + A_{s2}) + (C_f/C_c)(b + 2h) \rightarrow \text{Minimum}.
\]

The values of the cost ratios \( C_s/C_c \) and \( C_f/C_c \) varied from one country to another and may eventually vary from one region to another for certain countries.

The values of these cost ratios can be estimated on the basis of data given in applicable unit price books of construction materials.

### 2.4. Formulation of design constraints

The following constraints for the HSC T-beams are defined in accordance with the design code specifications of the French Annex to EC-2:

a) Constraints for the ultimate flexural strength are as follows:

\[
M_{Ed} \leq \frac{\sigma_{cd}(b-b_w)h_f(d-0.5h_f)}{\Psi_{cd}d}\leq \frac{\Psi_{cd}(1-\delta_G\alpha)+f_{yd}A_{s2}}{\delta_G\alpha} (d-d') \leq \Psi_{cd}d\leq p_{max},
\]

in which: \( \Psi_{cd}d\leq \text{Resisting moment of the cross section} \)

\[
\sigma_{cd}(b-b_w)h_f + \Psi_{cd}d\leq f_{yd}A_{s2} = 0,
\]

which is in internal force equilibrium.

Conditions on strain compatibility in steel and concrete are as follows:

\[
\frac{f_{yd}}{E_s} \leq \frac{\varepsilon_{cut}(1-\alpha)}{\alpha} \leq \infty,
\]

in which elasto-plastic behavior for steel and the pivot point is B.

\[
\frac{f_{yd}}{E_s} \leq \frac{\varepsilon_{cut}(1-\frac{d'}{\alpha d})}{\alpha} \leq \varepsilon_{cut},
\]

in which elasto-plastic behavior for steel and the pivot point is B, and optimal use of steel requires that strains in steel must be limited to plastic region at the Ultimate Limit State (ULS).

\[
M_{Ed} = \frac{(b-b_w)h_f\sigma_{cd}(d=0.5h_f)}{b_w d^2\sigma_{cd}} > \mu_{\text{limite}},
\]

for which compression reinforcement is required.

\[
\mu_{\text{limite}} = \Psi_{\alpha_{\text{limite}}}(1-\delta_G\alpha_{\text{limite}}).
\]

b) Constraint for minimum area of tension reinforcement is as follows:

\[
p_{\text{min}} \leq A_{s1}/b_wd,
\]

which is minimum steel percentage.

c) Constraint for maximum area of tension reinforcement is as follows:

\[
\frac{A_{s1}}{b_w h + (b - b_w) h_f} \leq p_{\text{max}},
\]

which is maximum steel percentage.

d) Constraints for maximum area of compression reinforcement are as follows:

\[
\frac{A_{s2}}{b_w h + (b - b_w) h_f} \leq p_{\text{max}},
\]

which is maximum steel percentage.

e) Constraint for the ultimate shear strength is as follows:

\[
V_{Ed} \leq V_{Rd,\text{max}} = v_1 f_{cd} b_w z/ (\theta g(\theta) + \cot g(\theta))
\]

f) Geometrical constraints including pre-design rules of the current practice are shown in Table 3.

### 2.5. Formulation of cost optimum design problem

Thus, the formulation of the optimum cost design of doubly reinforced HSC T-beams under ultimate loads can be mathematically stated as follows.

Given the characteristics of material, loading data, and constant parameters, find the design variables \( b, b_w, h, d, h_f, d', A_{s1}, A_{s2}, \) and \( \alpha \) that minimize

<table>
<thead>
<tr>
<th>Design variables</th>
<th>Equation numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L/30 \leq h \leq L/22 )</td>
<td>(15)</td>
</tr>
<tr>
<td>( d/h = 0.90 )</td>
<td>(16)</td>
</tr>
<tr>
<td>( 0.05 \leq d' \leq 0.1 )</td>
<td>(17)</td>
</tr>
<tr>
<td>( 0.30 \leq b_w/d \leq 0.50 )</td>
<td>(18)</td>
</tr>
<tr>
<td>( (b-b_w)/2 \leq L/10 )</td>
<td>(19)</td>
</tr>
<tr>
<td>( b/h_f \leq 8 )</td>
<td>(20)</td>
</tr>
<tr>
<td>( h_f/\min \leq h_f )</td>
<td>(21)</td>
</tr>
</tbody>
</table>
the total cost of construction material per unit length of HSC T-beam, such that:
\[
C = b_w h + (b - b_w) h_f + \left( C_s/C_e \right) (A_{s1} + A_{s2}) \\
+ (C_f/C_e)(b + 2h) \rightarrow \text{Minimum},
\]
subjected to the design constraints.

2.6. Solution methodology
The objective function, Eq. (3), and the constraints equations, Eqs. (4) through (21), together, form a nonlinear optimization problem. This nonlinearity appears in the expression of the cost of the HSC concrete and in the constraints related to the ultimate flexural strength, the ultimate shear strength, and the remaining constraints. Both the objective function and the constraints functions are nonlinear in terms of the design variables involved. In order to solve this nonlinear optimization problem, the Generalized Reduced Gradient (GRG) algorithm is used.

3. Numerical results
A typical example problem is now considered. The step-by-step application of the HSC T-beam optimum cost design model is presented, followed by a comparison between the standard design solution using HSC and the optimal solution obtained from the use of the three diagrams of stress distribution for HSC, and then by a parametric study on optimum cost design solution under imposed conditions. These conditions are related to HSC T-beam dimensions, reinforcing steel, and relative depth of the neutral axis. Finally, a cost-sensitivity analysis was conducted for different values of the unit cost ratios.

3.1. Design example
As previously mentioned, the design constraints are defined in accordance with the code design specifications of the French Annex to EC-2. The optimal solutions are compared to the standard design solutions obtained in accordance with EC-2 design code. To further illustrate the variability of the optimal solutions with unit costs of materials, the optimal solutions are computed for given unit cost ratios. The results, in terms of the corresponding gains, are presented graphically.

The study of HSC T-beam corresponds to a T-beam simply supported at its ends and pre-designed in accordance with provisions of EC-2 design code.

The corresponding pre-assigned parameters are defined as follows:

- Beam span: \(L = 22\) m;
- Ultimate bending moment capacity:
  \[M_{Ed} = 1.35 M_G + 1.5 M_Q = 5 MNm;\]
- Ultimate design shear capacity:
  \[V_{Ed} = 1.35 V_G + 1.5 V_Q = 2 MN;\]

Input data for HSC characteristics are as follows:

- Strength class of concrete: C90/105;
- Characteristic compressive cylinder strength of concrete at 28 days: \(f_{ck} = 90\) MPa;
- Partial safety factor for concrete: \(\gamma_c = 1.5;\)
- Allowable compressive stress: \(f_{cd} = 51\) MPa;
- \(\lambda = 0.700;\)
- \(\eta = 0.800;\)
- \(\varepsilon_{cd} = 2.6\%\) (in the case of pivot B, C90/105);
- \(\mu_c = 0.300\) (steel HA class S500, concrete class C90/105);
- \(h_{fmin} = 0.10;\)
- \(v_1 = 0.384, \theta = 45^\circ;\)
- \(f_{ctm} = 5.0\) MPa.

Input data for steel characteristics are as follows:

- Steel class: S500;
- Yield strength: \(f_{yk} = 500\) MPa;
- Partial safety factor for steel: \(\gamma_s = 1.15;\)
- Allowable tensile stress: \(f_{yd} = f_{yk}/\gamma_s = 435\) MPa;
- Young’s modulus: \(E_s = 2 \times 10^5\) MPa;
- Yield strain:
  \[
  \varepsilon_{yd} = f_{yd}/E_s = f_{ec}/E_s \gamma_s, \quad \varepsilon_{yd} = 2.174\% \\
  \text{(steel grade: S500)};
  \]
- Minimum steel percentage:
  \[
  p_{min} = 0.26 f_{ctm}/f_{yk} = 0.0026 = 0.26\%;
  \]
- Maximum steel percentage: \(p_{max} = 4\%\).

Input data for units costs ratios of construction materials are as follows:

- \(C_s/C_c = 30;\)
- \(C_f/C_c = 0.01.\)

3.2. Step-by-step procedure
The classical design solution based on the EC2 design code is briefly presented in Appendix.

The cost optimum design problem can be written as follows.

Find the design variables \(b, b_w, h, d, h_f, d_f, A_{s1}, A_{s2},\) and \(\alpha\) that minimize the total cost of construction material per unit length of HSC T-beam, such that:

\[
C/C_c = b_w h + (b - b_w) h_f + 30(A_{s1} + A_{s2}) \\
+ 0.01(b + 2h) \rightarrow \text{Minimum},
\]
subjected to the design constraints as follows:
a) Constraints for the ultimate flexural strength:
\[
5 \leq 40.80(b - b_w)h_f(d - 0.5h_f) + 28.56bh wd^2\alpha(1 - 0.35\alpha) + 435A_{x2}(d - 0.05),
\]
(23)
\[
40.80(b - b_w)h_f + 28.56bh wd + 435A_{x2} - 435A_{x1} = 0,
\]
(24)
\[
0.002174 \leq \frac{0.0026(1 - \alpha)}{\alpha} \leq \infty,
\]
(25)
\[
0.002174 \leq 0.0026\left(1 - \frac{d'}{\alpha d}\right) \leq 0.0026,
\]
(26)
\[
5 - \frac{40.80(b - b_w)h_f(d - 0.5h_f)}{40.80bw d^2} > 0.309,
\]
(27)
\[
\mu\lim = 0.309.
\]
(28)

b) Constraint for minimum area of tension reinforcement:
\[
0.0026 \leq A_{x1}/bw d.
\]
(29)

c) Constraint for maximum area of tension reinforcement:
\[
\frac{A_{x1}}{bw h + (b - b_w)h_f} \leq 0.04.
\]
(30)

d) Constraints for maximum area of compression reinforcement:
\[
\frac{A_{x2}}{bw h + (b - b_w)h_f} \leq 0.04.
\]
(31)
\[
435A_{x2}(d - d') \leq 2.
\]
(32)

e) Constraint for the ultimate shear strength:
\[
2 \leq 7.05bh_w d.
\]
(33)

f) Geometrical constraints including pre-design rules of the current practice:
\[
0.73 \leq b \leq 1.00,
\]
(34)
\[
d/h = 0.90,
\]
(35)
\[
0.05 \leq d' \leq 0.1,
\]
(36)
\[
0.30 \leq bw/d \leq 0.50,
\]
(37)
\[
(b - b_w)/2 \leq 2.20,
\]
(38)
\[
b/h_f \leq 8,
\]
(39)
\[
0.10 \leq h_f.
\]
(40)

3.3. Comparison between the optimal cost design solutions for three idealized stress distributions and the standard design approach

The vector of design variables, including the geometric dimensions of the HSC T-beam cross-section and the area of tension and compression reinforcements as obtained from the conventional design solution, the optimal cost design solution using the proposed approach, and the list of binding constraints, is shown in Table 4.

From the above results, it is clearly seen from the values of the relative costs, \(C_f/C_r = 30\) and \(C_f/C_r = 0.01\), and those associated with the classical and optimal solutions that a significant cost saving of the order of 17% can be obtained by the proposed design formulation.

3.4. Parametric study

In this section, the optimal solution is obtained by considering:

(i) One of the dimensions of HSCT-section is imposed;

(ii) Imposed reinforcing steel;

(iii) Imposed relative depth of the neutral axis, \(\alpha = \alpha_{\lim}\).

Further practical requirements can also be implemented such as aesthetic, architectural, and limited authorized template.

Based on Table 5, it is clearly seen from the values of the relative costs, \(C_f/C_r = 30\) and \(C_f/C_r = 0.01\), and those associated with the classical and optimal solutions corresponding to rectangular stress block that the percentage saving is between 05% and 17%.

3.5. Cost-sensitivity analysis

Comparing the relative gains can be determined for the various values of the unit cost ratios. The corresponding results are reported in Tables 6 and 7 and illustrated graphically in Figures 2 and 3 for HSC class 90/105.

The relative gains can be determined for various values of the unit cost ratios, \(C_f/C_r\), and for two given unit cost ratios, \(C_s/C_r\). The corresponding results are reported in Table 6 and illustrated graphically in Figure 2 for \(C_f/C_r = 30\) and \(C_f/C_r = 60\).

In order to further illustrate the variability of optimal solution with the unit cost ratios \(C_f/C_r\), for two sets of lower value and upper value of \(C_f/C_r\), the optimal solution has been computed for various ratios of \(C_f/C_r\) taken to be between 0.01 and 0.1.

From Table 6, it is shown that the maximum percentage of saving corresponding to the value of \(C_f/C_r\) being equal to 0.01 is 17% and 26% for \(C_f/C_r = 30\) and \(C_f/C_r = 60\), respectively.
Table 4. Comparison of the classical solution and the optimal solution corresponding to the three diagrams’ design of stress distribution for HSC under compression in accordance with EC2.

<table>
<thead>
<tr>
<th>Vector solution for</th>
<th>Classical solution</th>
<th>Optimal solution</th>
<th>Optimal solution</th>
<th>Optimal solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_s/C_e = 30$, $C_f/C_e = 0.01$</td>
<td></td>
<td>Parabolic-rectangle</td>
<td>Rectangular stress</td>
<td>Bilinear design</td>
</tr>
<tr>
<td>$b$ (m)</td>
<td>1.00</td>
<td>0.66</td>
<td>0.81</td>
<td>0.69</td>
</tr>
<tr>
<td>$b_w$ (m)</td>
<td>0.30</td>
<td>0.34</td>
<td>0.32</td>
<td>0.34</td>
</tr>
<tr>
<td>$h$ (m)</td>
<td>0.80</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$d$ (m)</td>
<td>0.72</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>$h_f$ (m)</td>
<td>0.13</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$d'$ (m)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$A_{11}$ (m^2)</td>
<td>$193.35 \times 10^{-4}$</td>
<td>$148.75 \times 10^{-4}$</td>
<td>$147.80 \times 10^{-4}$</td>
<td>$149.04 \times 10^{-4}$</td>
</tr>
<tr>
<td>$A_{12}$ (m^2)</td>
<td>$20.84 \times 10^{-4}$</td>
<td>$5.71 \times 10^{-4}$</td>
<td>$3.66 \times 10^{-4}$</td>
<td>$1.67 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.805</td>
<td>0.502</td>
<td>0.510</td>
<td>0.520</td>
</tr>
<tr>
<td>$C$</td>
<td>0.99958</td>
<td>0.86388</td>
<td>0.85448</td>
<td>0.851656</td>
</tr>
<tr>
<td>Gain</td>
<td>16%</td>
<td>17%</td>
<td>17%</td>
<td></td>
</tr>
<tr>
<td>List of binding constraints</td>
<td>(4); (5); (8); (11); (15);</td>
<td>(4); (5); (8); (11);</td>
<td>(4); (5); (8); (11);</td>
<td>(4); (5); (8); (11);</td>
</tr>
<tr>
<td></td>
<td>(16); (17); (21)</td>
<td>(16); (17); (20)</td>
<td>(15); (16); (17); (21)</td>
<td>(15); (16); (17); (21)</td>
</tr>
</tbody>
</table>

Table 5. Variation of relative gain with particular conditions imposed such as the HSC T-beam dimensions, reinforcing steel, and depth of the neutral axis.

<table>
<thead>
<tr>
<th>Optimal solution with imposed variables</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$ imposed: $b = 0.90$ m</td>
<td>07%</td>
</tr>
<tr>
<td>$b_w$ imposed: $b_w = 0.35$ m</td>
<td>14%</td>
</tr>
<tr>
<td>$h$ imposed: $h = 0.90$ m</td>
<td>05%</td>
</tr>
<tr>
<td>$h_f$ imposed: $h_f = 0.12$ m</td>
<td>17%</td>
</tr>
<tr>
<td>$(A_{11} + A_{12})$ imposed: $A_{11} + A_{12} \leq 0.1600$ m^2</td>
<td>17%</td>
</tr>
<tr>
<td>$(A_{12}/A_{11})$ imposed: $A_{12}/A_{11} = 0.40$</td>
<td>17%</td>
</tr>
<tr>
<td>$A_{12}$ imposed: $A_{12} = 0.0015$ m^2</td>
<td>15%</td>
</tr>
<tr>
<td>$\alpha$ imposed: $\alpha = \alpha_{\text{min}} = 0.5446$ ((\mu = \mu_{\text{limit}} = 0.309))</td>
<td>17%</td>
</tr>
</tbody>
</table>

Table 6. Variation of relative gain in percent (%) versus unit cost ratio, $C_f/C_e$, of construction materials for $C_s/C_e = 30$ and $C_s/C_e = 60$.

<table>
<thead>
<tr>
<th>$C_s/C_e = 30$</th>
<th>$C_f/C_e$</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>0.04</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>0.07</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_s/C_e = 60$</td>
<td>$C_f/C_e$</td>
<td>Gain</td>
</tr>
<tr>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>0.04</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>0.07</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>0.10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Variation of relative gain in percent (%) versus unit cost ratio, $C_s/C_e$, of construction materials for $C_f/C_e = 0.010$ and $C_f/C_e = 0.10$.

<table>
<thead>
<tr>
<th>$C_f/C_e = 0.01$</th>
<th>$C_s/C_e$</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>25</td>
<td>30</td>
<td>35</td>
</tr>
<tr>
<td>40</td>
<td>45</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_f/C_e = 0.1$</td>
<td>$C_s/C_e$</td>
<td>Gain</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>25</td>
<td>30</td>
<td>35</td>
</tr>
<tr>
<td>40</td>
<td>45</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The problem formulation of the optimal cost design of doubly reinforced HSC T-beams can be cast into a nonlinear programming problem, the numerical solution of which is efficiently determined using the GRG (Generalized Reduced Gradient) method in a space of only a few variables representing the concrete cross-section dimensions.

The space of feasible design solutions and the optimal solutions can be obtained using a reduced number of independent design variables.

Optimal values of the design variables are affected by the relative cost values of the objective function only, but not by the absolute cost values.

The optimal solutions are found to be insensitive to changes in the shear constraint. Shear constraint is usually not critical in the optimal design of reinforced HSC T-beams under bending and can thus be excluded from the problem formulation.

The observations of the optimal solutions’ results reveal that the use of the optimization based on the optimum cost design concept may lead to substantial savings in the amount of the construction materials to be used in comparison to classical design solutions of doubly reinforced concrete HSC T-beams.

The objective function and the constraints considered in the present paper are illustrative in nature. The present approach based on nonlinear mathematical programming can be easily extended to other sections commonly used in structural design. More sophisticated objectives and considerations can be readily accommodated by suitable modifications of the present optimal cost design model.

It is also important to note that, in general, the use of top reinforcement in reinforced HSCT-beam sections is an indication of a poor design.

In this work, we have included the additional cost of formwork which makes a significant contribution to the total costs. This inclusion is important for an economical approach to design and manufacture.

The proposed methodology for optimum cost design is effective and more economical for the classical methods. The results of the analysis show that the optimization process presented here is effective and its application appears feasible.

**Nomenclature**

$L$ Beam span

$M_{Ed}$ Ultimate bending moment capacity including self-weight
Maximum design moments under dead loads

Maximum design moments under live loads

Maximum design shears under dead loads

Maximum design shears under live loads

Ultimate shear capacity including self-weight

Maximum resistant shear force C80/105 Class of HSC

Characteristic compressive cylinder strength of HSC at 28 days

Tensile strength of concrete

Design value of concrete compressive strength

Partial safety factor for concrete

Design strength factor

Compressive zone depth factor

Strain at the maximum stress for the parabolic-rectangular stress distribution compressive concrete

Ultimate strain for the parabolic-rectangular stress distribution compressive concrete

Strain at the maximum stress for the rectangular stress distribution compressive concrete

Strain at the maximum stress for the bilinear design stress-strain relation

Ultimate strain for the rectangular stress distribution compressive concrete and bilinear design stress-strain relation S500 grade of steel

Characteristic elastic limit for steel reinforcement

Partial safety factor for steel

Design yield strength of steel reinforcement

Elastic limit strain

Strain of steel under tension

Strain of steel under compression

Young’s elastic modulus

Minimum steel percentage

Maximum steel percentage

Limit value of relative depth of compressive concrete zone

Limit value of reduced moment

The angle between concrete compression struts and the main chord

A non-dimensional coefficient

Lever arm

Minimum depth of flange

Total cost per unit length of HSC T-beam

Total relative cost of HSC T-beam

Unit cost of reinforcing steel

Unit cost of HSC

Unit cost of formwork

Coefficient used to calculate the centroid of the effective area of concrete in comparison to the most compressed fiber for concrete

Coefficient used to calculate the value of the resultant compressive force for HSC concrete $F_c$ for the stress diagram considered

Resultant compressive force for HSC

Maximum design stress

Centroid of the effective area of concrete in compression

References


Appendix

**Classical design solution for design example**

The study of high strength concrete T-beam corresponds to a T-beam simply supported at its ends and pre-designed in accordance with provisions of EC-2 design code.

One should determine the areas $A_{s1}$ and $A_{s2}$ of steel reinforcement for the reinforced HSC T-beam with the cross-section dimensions of $b = 1.00$ m, $b_w = 0.30$ m, $h = 0.80$ m, $d = 0.72$ m and $h_f = 0.13$ m, as obtained from a preliminary design of the study HSC T-beam.

Consider the following data:

- Beam span: $L = 22$ m;
- Ultimate bending moment capacity including self-weight: $M_{Ed} = 5$ MNm;
- Ultimate shear capacity including self-weight: $V_{Ed} = 2$ MN;
- Strength class of concrete: C90/105;
- Characteristic compressive cylinder strength of concrete at 28 days: $f_{ck} = 90$ MPa;
- Partial safety factor for concrete: $γ_c = 1.5$;
- Allowable compressive stress: $f_{ck} = 51$ MPa;
- Design strength factor: $ν = 0.800$;
- Compressive zone depth factor: $λ = 0.700$;
- Steel class: S500;
- Elastic limit: $f_{yk} = 500$ MPa;
- Partial safety factor for steel: $γ_s = 1.15$;
- Allowable tensile stress: $f_{yld} = f_{yk}/γ_s = 435$ MPa;
- Unit cost ratios: $C_s/C_c = 30$; $C_f/C_c = 0.01$.

Moment of resistance of the flange, $M_f$, is obtained as follows:

- $M_f = νbb_f f_{ck}(d - 0.5h_f)$;
- $M_f = 3.474$ MNm;
- $M_f < M_{Ed}$, in which the stress block must extend below the flange;
- $μ_{lim} = λ_{lim}(1 - 0.5λ_{lim})$, which is limit value of reduced moment;
- $λ_{lim} = 0.5446$;
- $μ_{lim} = 0.300$;
- $μ = (M_{Ed} - M_f(b - b_w)/b)/b_w d^2 f_{ck} < μ_{lim}$;
- $μ = 0.405$;
- $μ > μ_{lim}$ for which compression reinforcement is required.

The area of compression reinforcement, $A_{s2}$, is obtained as follows:

$$A_{s2} = \left(\frac{M_{Ed} - (M_f(b - b_w)/b - μ_{lim} b_w d^2 f_{ck})}{(d - d_f)f_{yld}}\right)$$

$A_{s2} = 0.002084$ m$^2$. 

The relative depth of compressive concrete zone, $\alpha$, is obtained as follows:

$$\alpha = \left[1 - (1 - 2\mu)^{0.5}\right] / \lambda, \quad \alpha = 0.805.$$

Lever arm, $z$, is obtained as follows:

$$z = d(1 - 0.5\lambda \alpha), \quad z = 0.517 \text{ m}.$$

The area of tension steel, $A_{s1}$, is obtained as follows:

$$A_{s1} = \left[\mu_{C}\frac{b_{d}d^{2}f_{cd}}{z f_{yd}} + (b - b_{w})h f_{yd}ight] / f_{yd} + A_{s2}$$

$$A_{s1} = 0.019335 \text{ m}^2.$$

Total cost per unit length of HSC T-beam, $C$, is obtained as follows:

$$C = 0.99058C_c.$$

**Biography**

Ferhat Fedghouche has been a Public Works Engineer in National School of Built and Ground Works Engineering, Algiers, Algeria, since 1993. He obtained his MSc in National School of Built and Ground Works Engineering, Algiers, Algeria, in 2001, and his Doctorate in Science (PhD) in National Polytechnic School of Algiers in 2013. He is currently working as a full-time lecturer in the Department of Basic Infrastructures at the National School of Built and Ground Works Engineering, Algiers, Algeria. His areas of interest include engineering optimization, operational research in industry, software for operational research, pathologies of constructions and area bridges. He has published numerous papers in refereed national and international journals and conferences in these areas of expertise.

His previous experience includes teaching as well as engineering consultancy for the National Organization of Technical Control of Control of Public Works, Algiers, Algeria, 1995-2010.