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# Seismic wave scatter study in valleys using coupled 2D finite element approach and absorbing boundaries

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## KEYWORDS

Ground response;  
Finite element;  
Absorbing boundary;  
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irregularity.

**Abstract.** Topographical and mechanical properties of soil layers can lead to amplification or attenuation of seismic waves. Such a phenomenon can be theoretically explained by means of ground response analysis. Definition of boundaries is of great concern in modeling ground response, and application of boundaries with any constrain can lead to the so-called “trap box” effect on seismic waves in the model, and hence to fictitious results. In the present study, two-dimensional Finite Element Method (FEM) is applied in which boundaries, known as “absorbing boundaries”, are used to study the effect of wave scatter on valleys with different forms of the amplification or attenuation of SV waves. Comparison of the results is conducted for the current approach and those of the coupling Finite Element and the Infinite Element (sometimes called as FE-IFE) method. The results are also presented in non-dimensional diagrams of  $A_u$  and  $A_v$  for horizontal and vertical displacement amplitudes, respectively, through the valley span and its surrounding area. Comparison of the results indicate that the proposed boundaries can improve the seismic analysis when coupled with the FEM. Also, because of topographic irregularities, variations of displacement are seen inside the valley and around it.

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## 1. Introduction

In common seismic events, body waves travel from the source mostly across a bedrock, and finally end in soil layers, while most of the changes in the characteristics of ground motions occur in the soil layers [1]. Due to the drastic variation of the nature of seismic waves passing through soil layers, it is very important to incorporate realistic and precise seismic excitation models into the analysis of the seismic response of structures. Ground response analyses can properly satisfy the needs for seismic excitation in the analysis of structures and the soil-structure interaction. Hence, vulnerability of structures can be a function

of an important factor known as the site seismic response.

Site effects are generally divided into two categories: effects of local deposits and those of topography. In this regard, the effect of topographic irregularities on ground motions is of great importance. Recent studies have indicated that topographic irregularities (e.g., mountain ridges or valley notches) have caused significant changes to strong ground motions during earthquakes. Investigations on many earthquakes occurred in the past indicated the effect of surface topographic changes on ground response. The September 19, 1985 Michoacan earthquake ( $M_s = 8.1$ ) only resulted in moderate damages around the epicenter (near the Pacific coast of Mexico). However, it caused extensive damages to a site located as far as 350km away from Mexico City. Another example of the effects of topography was seen in records taken by a seismograph

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installed on the piers of the Pacoima dam in southern California. The seismograph recorded horizontal peak accelerations as high as 1.25 g in both directions during the 1971 San Fernando earthquake ( $M_L = 6.4$ ). These values recorded by this seismograph were significantly larger than those expected of an earthquake of such magnitude [1]. Other pieces of evidence of topographic effects are also available in Alaska 1964 [2], Canal Beagle Chile 1985 [3,4], Northridge 1994 [5,6], and Athens 1999 [7]. The significance of local site effects is shown by the fact that earthquake causes vast damages to some certain regions and only slight damages to others.

Extensive researches have been conducted to optimize ground seismic response analysis (e.g., [8–14]). For example, Lermo and Chávez-García [15] referred to the limitations of the classic method of spectral analysis, especially limitations imposed on field operation and the process of recording data. They also proposed a new method for studying site effects. Following the well-known Nakamura method, they used the ratio of the spectral amplitude of the horizontal to the vertical component of minor earthquakes. They also compared their results with those of the classic spectral approach. LeBrun et al. [16] carried out an experimental study on the topographic effects of a large hill with a height of 700 m, width of 3 km, and length of 6 km on seismic analysis. In the course of investigation, seven seismographs were installed on a hill and a total of 58 earthquake records were obtained. They studied the ground motion with different methods: the Classical Spectral Ratios (CSR) and the horizontal to vertical spectral ratios calculated both on noise, so-called Nakamura's method (HVNR), and then on earthquake data, so-called Receiver Function technique (RF). The comparison between these two methods showed that the H/V method is able to suggest the fundamental frequencies of a hill. Fu [17] investigated effect of surface waves scatter by comparing different theories. He also examined these waves by studying on the propagation of SH waves through two-dimensional models. He compared accuracy of different theories according to dimension of model and incident wavelength. Bouckovalas and Papadimitriou [18] analyzed the effect of topography on seismic waves using the Finite Difference Method (FDM). Their study was based on a site with uniform slope and a visco-elastic soil. Study was also conducted on the effects of different parameters on seismic ground motion and based on the vertical propagation of SV waves. Kamalian et al. [19] studied the effects of topography on a medium with heterogeneous materials. They used a two-dimensional modeling method based on Finite Element coupled with Boundary Element (FEM-BEM). Also, they modeled distant boundaries using confining elements. They showed that their proposed method needs smaller time step compared with the

BEM scheme. They also discussed effective dimension of irregularities on seismic ground motion. Gatmiri et al. [9] investigated the effect of alluvial valleys on the amplification or attenuation of seismic waves. They used a coupled model of FEM-BEM in which nearby field was modeled using FEM, while the far field was modeled using BEM. They proved the accuracy of their method by numerical study and discussed that artificial waves developing at the truncation points of the model would vanish easier if the optimized method was used. Asgari and Bagheripour [20] performed a non-linear one-dimensional analysis of ground response using the HFTD method. They used the advantages of both time domain and frequency domain methods to optimize the solution procedure. They also proved the accuracy of their method by different illustrative examples. Di Fiore [21] considered the effect of incident wave frequency and gradient of slopes as a form of topographic irregularities on the amplification or attenuation of seismic waves using FEM. Investigations showed that amplification of seismic wave is increased with increasing the gradient of slope. Also, analysis of 0.5 to 32 Hz incident wave frequencies revealed that the largest amplification of seismic waves was seen at frequencies 4 to 12 Hz. Bazrafshan Moghaddam and Bagheripour [22] proposed a new method for a non-linear analysis of ground response based on the time-frequency hybrid approach. This method was based on a non-repetitive process and used a matrix-notation. Comparison of the results of the proposed method and results of SHAKE and NERA softwares with records obtained on the site of several actual earthquakes showed the accuracy and efficiency of the proposed method. Tripe et al. [23] conducted a time domain study on the contribution of slopes of homogeneous and linear-elastic soils to the amplification or attenuation of seismic waves using FEM.

It should be noted that one-dimensional ground response analysis methods are suitable for horizontal or gently sloping grounds, and perhaps for soil profiles having parallel set of layers. However, other problems such as slopes, non-linear ground surfaces, topographic irregularities, presence of heavy and stiff structures, buried structures, and tunnels require two- or even three-dimensional analysis [1].

One of the important problems in ground response analysis is the limitation or inability of numerical methods to simulate infinite boundaries and the development of mathematical models for the passage of seismic waves and reduction of reflected waves from the boundaries into the models. Such deficiency greatly influences the results. In 1993, the introduction of coupling FE and IFE aimed to overcome these drawbacks [24]. Position of IFEs at far field of the model prevents inward reflection of seismic waves and develops conditions for the decay of their amplitudes

at far distances as happens in real circumstances. For geotechnical problems and specially geotechnical earthquake engineering studies, it is a realistic approach. However, the problem with this method is that no clear definition for the interface between FEs and IFEs is available. On the other hand, shifting the interface drastically influences the results.

In the present study, the non-linear effect of empty two-dimensional valleys on the amplification or attenuation of seismic waves is investigated in time domain using PLAXIS software.

It is noteworthy that in the time domain analysis, equation of motion is solved using step by step time integration method, which is integration of small time intervals. In such a method, time intervals should be small in order to present accurate simulation of loading and mechanical properties of materials.

The results are presented in the form of non-dimensional diagrams for displacement amplitudes. A comparison is also made between the results of the current study and those obtained by the FE-IFE method. In this investigation, realistic simulation of the model was conducted to avoid reflection of seismic waves at boundaries using special boundary conditions known as “absorbing boundaries”. In fact, they are coupled to the area modeled by FEM. As mentioned before, the aforementioned boundaries can absorb all body and surface waves at all angles of incidence and frequencies. Application of such boundaries proposed in this research provides a powerful FE tool which diminishes difficulties caused by FE-IFE method and also helps achieving a more rational approach, and hence more acceptable results.

## 2. Theoretical principles

### 2.1. FEM

Among the methods employed for the analysis of ground response, FEM is one of the most powerful one. The reason is that the method is capable of simulating complicated geological and geotechnical conditions, which are the matter of concern in this paper. Nevertheless, accurate simulation of boundary conditions is of great importance for this concept. In this method, soil profile is divided into a finite (limited) number of elements. The dynamic response of soil can be calculated by applying the concepts of soil dynamical behavior, described in detail in some references (e.g., [25]).

Formulations of FEM were excluded from this paper as they are widely known and can be found elsewhere. In order to understand the principles and formulations associated with this method, readers are referred to [26]. The boundary conditions as well as the discretization of the model used in FEM analysis are discussed in the following sections, since they are of great importance.

### 2.2. FE-IFE method

In this method, the near and far fields of the topography are simulated using FE and IFE, respectively. A summary of the basics of the FE-IFE method, with regard to simulation of infinite environments, gives a general understanding of this method. More information is also available in [8,24].

The mapping relationships between the global and local coordinate systems for the six-node elements shown in Figure 1 are as follows [24]:

$$x = \sum_{q=1}^6 M_q x_q, \quad (1)$$

$$y = \sum_{q=1}^6 M_q y_q, \quad (2)$$

where  $M_q$  ( $q = 1, 2, \dots, 6$ ) are the mapping functions of the IFE, which are further expressed as follows:

$$M_1 = 1/2(\xi - 1)(\eta - 1), \quad (3)$$

$$M_2 = M_5 = 0, \quad (4)$$

$$M_3 = -1/2(\xi - 1)(\eta + 1), \quad (5)$$

$$M_4 = 1/2\xi(\eta + 1), \quad (6)$$

$$M_6 = -1/2\xi(\eta - 1). \quad (7)$$

$M_2$  and  $M_5$  are assumed to be equal to zero in order to simplify the IFE coordinates conversion relations.

In order to consider displacement compatibility at the interface of the FEs and IFEs, the nodal displacements of dynamic two-dimensional IFEs are defined as follows:

$$u = \sum_{q=1}^6 N_q u_q, \quad (8)$$

$$v = \sum_{q=1}^6 N_q v_q, \quad (9)$$

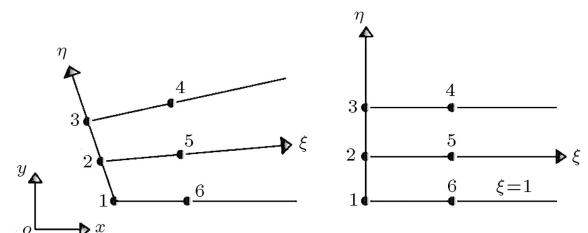


Figure 1. Six-node IFE used in FE-IFE method [24].

where  $N_q$  ( $q = 1, 2, \dots, 6$ ) are the displacement shape functions defined by the following relations:

$$N_q = P_q(\xi) \frac{\eta(\eta - 1)}{2} \quad (q = 1, 6), \quad (10)$$

$$N_q = -P_q(\xi)(\eta + 1)(\eta - 1) \quad (q = 2, 5), \quad (11)$$

$$N_q = P_q(\xi) \frac{\eta(\eta + 1)}{2} \quad (q = 3, 4), \quad (12)$$

where  $P_q(\xi)$  are the wave propagation functions. The general form of the wave propagation function for infinite two-dimensional dynamic elements is formulated as:

$$P_q(\xi) = e^{-\alpha^* \xi} (c_1 e^{-i\beta_1 \xi} + c_2 e^{-i\beta_2 \xi}) \quad (q = 1, 2, \dots, 6), \quad (13)$$

where  $\alpha^*$  is the nominal decay coefficient, which represents the reduction in wave amplitude due to the dissipation of wave energy in the material and the geometric divergence of the medium.

$\beta_q$  ( $q = 1, 2$ ) are also the nominal wave numbers of the wave associated with  $S$  and  $P$  waves in two-dimensional environments. These parameters are used to express the phase characteristics of the wave during propagation in the medium. In addition,  $C_q$  ( $q = 1, 2$ ) are constants for adjusting the displacement of IFEs and surface of the bedrock. These constants are obtained through equality of two different nodal displacements. For example, the following expression is used to obtain such constants based on Figure 1:

$$\begin{Bmatrix} u_1 \\ u_6 \end{Bmatrix} = \begin{bmatrix} 1 \\ e^{(\alpha^* + i\beta_1)} & e^{(\alpha^* + i\beta_2)} \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix} = [C] \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix}. \quad (14)$$

Further, the following relation can be obtained using Eq. (14):

$$\begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix} = [E] \begin{Bmatrix} u_1 \\ u_6 \end{Bmatrix}, \quad (15)$$

where  $[E]$  is the inversion matrix for  $[C]$ . According to Eqs. (10) to (15), one obtains:

$$P_q(\xi) = \frac{1}{\Delta} \left[ e^{-(\alpha^* + i\beta_2)} e^{-(\alpha^* + i\beta_1)\xi} - e^{-(\alpha^* + i\beta_1)} e^{-(\alpha^* + i\beta_2)\xi} \right] \quad (q = 1, 2, 3), \quad (16)$$

$$P_q(\xi) = \frac{1}{\Delta} \left[ e^{-(\alpha^* + i\beta_1)\xi} + e^{-(\alpha^* + i\beta_2)\xi} \right] \quad (q = 4, 5, 6), \quad (17)$$

in which:

$$\Delta = P_q(\xi) = \left[ e^{-(\alpha^* + i\beta_2)} - e^{-(\alpha^* + i\beta_1)} \right]. \quad (18)$$

Also, according to Eqs. (10) to (12), it can be inferred that:

$$P_q(\xi_r) = \delta_{qr}, \quad (19)$$

where  $\delta_{qr}$  is the Kronecker Delta. Hence, for every shape function  $N_q$  ( $q = 1, 2, \dots, 6$ ) and  $q \neq r$ , one may reach:

$$N_q = 1 \text{ if } \eta = \eta_q \quad \text{and} \quad \xi = \xi_q,$$

and:

$$N_q = 0 \text{ if } \eta = \eta_r \quad \text{and} \quad \xi = \xi_r.$$

According to the above relations, the mass and stiffness matrices for dynamic two-dimensional IFEs are written as follows:

$$[M]^e = \int_0^\infty \int_{-1}^1 [N]^T \rho[N] |J| d\eta d\xi, \quad (20)$$

$$[K]^e = \int_0^\infty \int_{-1}^1 [B]^T [D^*] [B] |J| d\eta d\xi, \quad (21)$$

where  $|J|$  is the Jacobian determinant, which is developed using Eqs. (1) to (7).

Substituting Eqs. (3) to (7) and (10) to (12) into Eqs. (20) and (21), the following extended integral equation can be used for assessing the mass matrix and stiffness of IFEs:

$$I = \int_0^\infty F(\xi) e^{-(2\alpha^* + i\beta_q + i\beta_r)\xi} d\xi \quad (q = 1, 2, r = 1, 2). \quad (22)$$

The above integral can be solved through any numerical integration scheme described in many references (e.g., [27]).

### 3. Boundary conditions

Minimizing the number of elements in FE analysis leads to a reduction in calculation time and required memory. As the size of the divisions decreases, the influence of boundary conditions becomes more significantly. Simulation of the radial attenuation of wave energy is of particular importance to FE dynamic analysis. The most commonly used boundaries for FE analysis are classified as follows [1].

#### 3.1. Elementary boundaries

Zero-displacement conditions or zero-stress conditions are defined at elementary boundaries. Elementary boundaries can be used as stress-free boundaries in order to model ground surface. However, with lateral boundaries, full reflection of elementary boundaries leads to the confinement of energy in the model. The resulting box effect leads to serious errors and spurious results in the analysis of ground response. In real conditions, these waves radially pass through the medium and vanish gradually.

### 3.2. Local boundaries

It can be shown that the damping coefficient required for full absorption of energy depends on the angle of incidence of the incident wave. Since waves with different angles may hit the boundary, a local boundary with a certain damping coefficient always reflects a portion of the energy of the incident wave. Additional problems arise when divergent surface waves reach the local boundary. Since the phase velocity of these waves depends on frequency, a frequency-independent damper is required to fully absorb the energy. The effects of reflections of local boundaries can be reduced by increasing the distance between the boundary and the range of concern. However, it should be noted that, depending on the dimensions of the model and software capabilities as well as the extent of stress zone, an increase in such distance is not always feasible. Even with probable and feasible options, the process is always associated with excessive time for analysis and requires large core storage.

### 3.3. Consistent boundaries

Consistent boundaries are boundaries that absorb all body and surface waves at all angles of incidence and frequencies. Consistent boundaries can be obtained through the frequency-independent boundary stiffness matrix resulting from integral equations.

## 4. Selection of appropriate software

In previous studies related to ground response analysis, various softwares have been introduced to simulate infinite or semi-infinite soil and rock medium. One of the most powerful and applicable one is PLAXIS that has been developed based on FEM. Using this software, one may apply proper boundaries to the FE models to simulate the infinite extent of a soil medium and prevent reflecting of artificial waves to the model.

### 4.1. Application of suitable boundary conditions in PLAXIS

In static deformation analysis, the vertical boundaries of the mesh are often synthetic boundaries; thus, they do not affect the deformation behavior of the environment to be modeled [28]. In other words, these boundaries are distant. However, for dynamic analysis, boundaries have to be placed far enough and farther than the boundaries in static analysis. Otherwise, stress waves are reflected and calculations usually lead to spurious results. Furthermore, introduction of boundaries at far distance virtually means large mesh required in FE model with excessive elements which entails also extra calculation time. The absorbing boundaries used in this research behave similarly to the aforementioned consistent boundaries.

### 4.2. Absorbing boundaries

When absorbing boundaries are applied, equivalent dampers are used instead of commonly used boundary constraints. Such equivalent dampers absorb stresses induced to the boundary. It further means that such dampers act as if stress waves are travelling the region outwards.

Components of absorbed normal and shear stress are defined as follows when equivalent dampers are introduced in  $x$  direction:

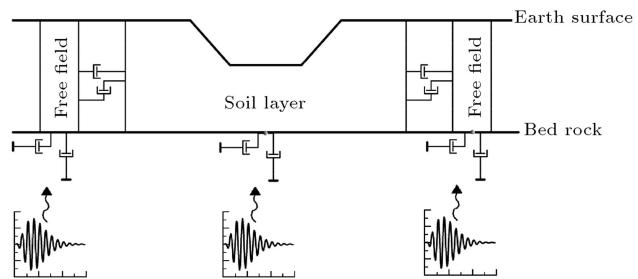
$$\sigma_n = -c_1 \rho v_p \dot{u}_x, \quad (23)$$

$$\tau = -c_2 \rho v_s \dot{u}_y. \quad (24)$$

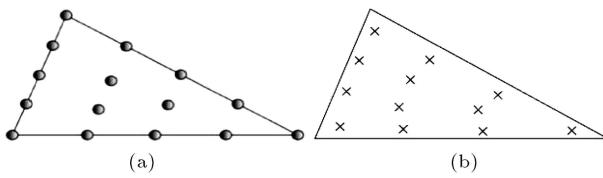
In the above equations,  $\rho$  is the density of materials,  $v_p$  and  $v_s$  are the velocities of the compressional and shear waves, respectively, while  $c_1$  and  $c_2$  are relaxation coefficients that are applied to the model to enhance the performance of the absorbing boundaries. The interesting point is that if incident compressional waves reach the vertical boundaries of the model,  $c_1$  and  $c_2$  coefficients are reduced to unity ( $c_1 = c_2 = 1$ ). However, in presence of shear waves, the damping effect of absorbing boundaries would not be adequate if coefficients  $c_1$  and  $c_2$  are neglected. In fact, the effect of these boundaries is increased directly with the increase in  $c_2$  value. Recent studies have shown that application of  $c_1 = 1$  and  $c_2 = 0.25$  would optimize the absorbing effect of these boundaries [28]. Fundamental formulation of absorbing boundaries is based on the procedure described in [29].

## 5. Problem under study

In this study, to investigate the effect of topographic irregularities on seismic ground response and also application of proposed boundaries in FE model, the valley environment was considered as shown in Figure 2. Seismic excitation was applied to the base of the model where bedrock exists at bottom of a valley. Response was obtained at the ground level at various points in order to investigate the efficiency of the absorbing boundaries coupled with FEM as



**Figure 2.** Schematic image of the model used in this research.



**Figure 3.** Fifteen-node element used in PLAXIS analysis: (a) Fifteen-node element, and (b) Gaussian points.

well as the effect of soil layer and topographic irregularities on amplification or attenuation of seismic waves.

Determination of the extent of the FE model and the mesh size is of importance, since PLAXIS is also based on FEM. Adoption of large elements for a FE model filters high-frequency components whose short wave length cannot be simulated with nodal points with long intervals. Considering the mechanism governing the propagation of seismic waves, it has been found that size of elements should be limited to 1/10 to 1/8 of the wave length corresponding to the highest frequency content of the input motion [28].

Fifteen-node elements were used to model the soil medium because they provide more accurate results, since they benefit a better interpolation scheme. These elements have two degrees of freedom defined at every node and have 12 Gaussian points (Figure 3).

### 5.1. Input motion

Since seismic waves can be decomposed into a set of harmonic waves with different frequencies and amplitudes, they are, therefore, used for analysis of ground motion characteristics [24,30].

The seismic excitation adopted in this study is a planar vertically propagating SV wave which is induced too deep in soil layer and at surface of the bedrock. Response at the ground level and different points are obtained. Comparison is made between the results obtained here and those of the FE-IFE method to verify the applicability of different proposed boundaries.

### 5.2. Frequency content of seismic excitation

Effects of two-dimensional site are important where the dimension of topography is approximately equal to the wavelength of the seismic wave [31]. From the earthquake engineering point of view, different researchers present the frequency content of a strong earthquake which almost ranges from 0.1 to 20 Hz. On the other hand, since the velocity of seismic waves near the ground surface lies between 0.1 to 3 km/s, topographies with dimensions larger than tens of meters to several kilometers usually behave as in two-dimensional site response models (e.g., [1,21,32]).

In order to facilitate the study on the effect of the frequency content of the induced harmonic wave, a non-dimensional parameter, known as the dimensionless

frequency,  $a_o$ , is defined based on the following relation:

$$a_o = \frac{\omega H}{\pi v_s}, \quad (25)$$

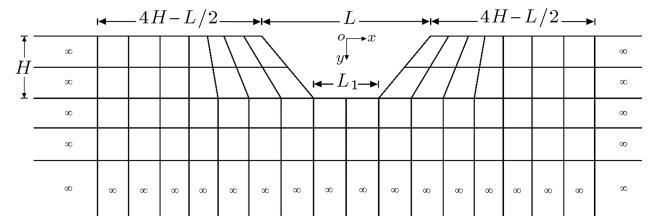
where  $\omega$  is the angular frequency of the incident wave at the bedrock,  $H$  is the maximum valley depth, and  $v_s$  is the velocity of shear wave travelling through the soil medium. In this study, constant values as  $a_o=0.5$  and  $a_o=1$  were adopted for this non-dimensional frequencies of seismic excitation.

Because of frequency-dependent formulations and in order to apply reasonable frequency contents, the acceleration time history of an S25W component of Parkfield, California, Earthquake which occurred on June 27, 1966, was used. The maximum circular frequency of the harmonic component with considerable amplitude is found to be 60 rad/s. It reveals that although the unit earthquake wave has been decomposed into a set of harmonic waves, the harmonic wave with a circular frequency above 60 rad/s might be roughly considered or even dropped, without loss of accuracy in numerical results [24]. Since the circular frequency in this sample problem adopted here is 60.91 rad/s in the case of  $a_o = 1$ , it was considered that the present model is valid for studying the effect of valley topography on ground motion under this earthquake wave incidence.

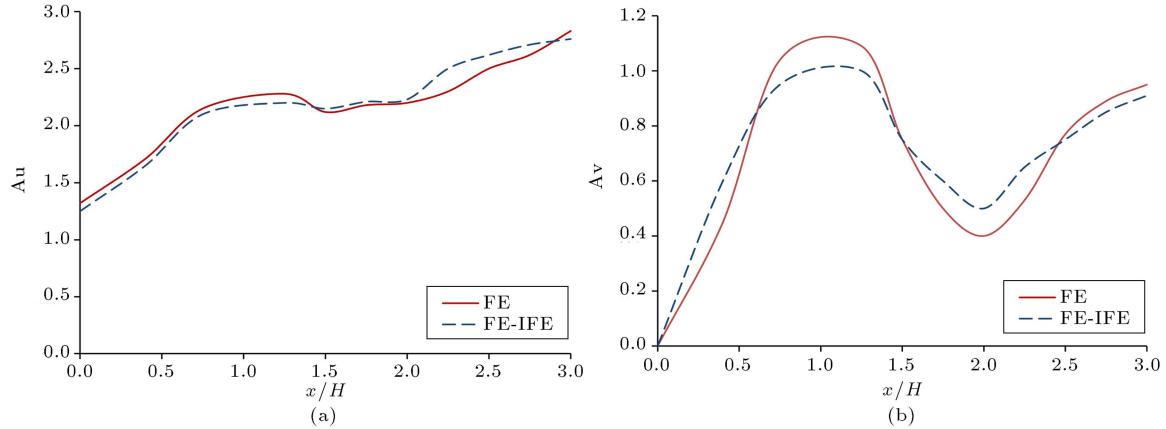
### 5.3. Discussion on the results obtained and verification

As seen in Figure 4,  $H$  is the maximum valley depth,  $L$  is the valley span, and  $L_1$  is the width of the valley bottom. In this study,  $L/H = 3$  and  $H = 100$  m were adopted. In order to examine the impact of boundaries at different conditions,  $L_1/L = 0, 1/3$ , and 1 were used for V-shaped, trapezoidal, and rectangular valleys, respectively. In addition, the Poisson's ratio was assumed to be constant and equal to 0.33, the soil modulus of elasticity was adopted as  $2.4 \times 10^7$  KN/m<sup>2</sup>, while the unit weight of soil was considered to be 23.54 KN/m<sup>3</sup> [24].

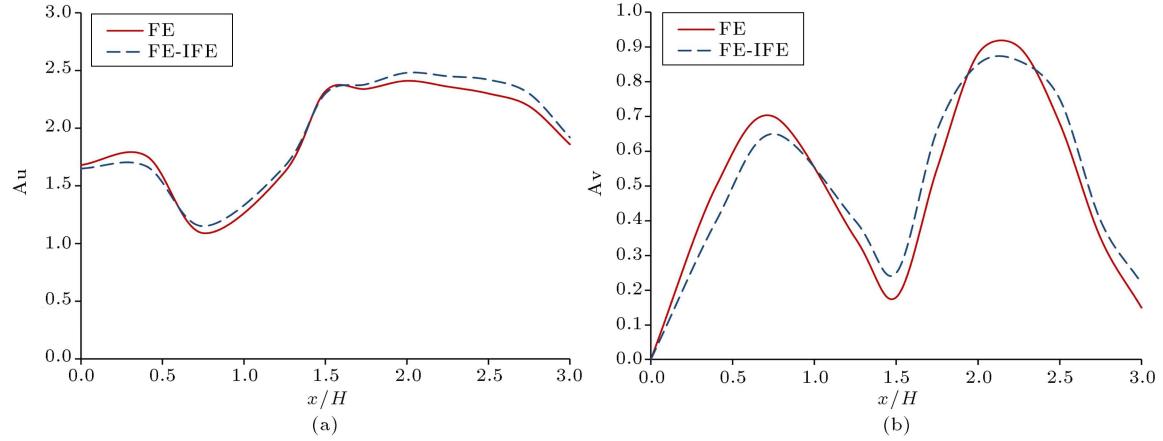
The normalized response resulting from two-dimensional and non-linear modelings developed in PLAXIS software is compared to those obtained in FE-IFE method through graphical scheme shown in



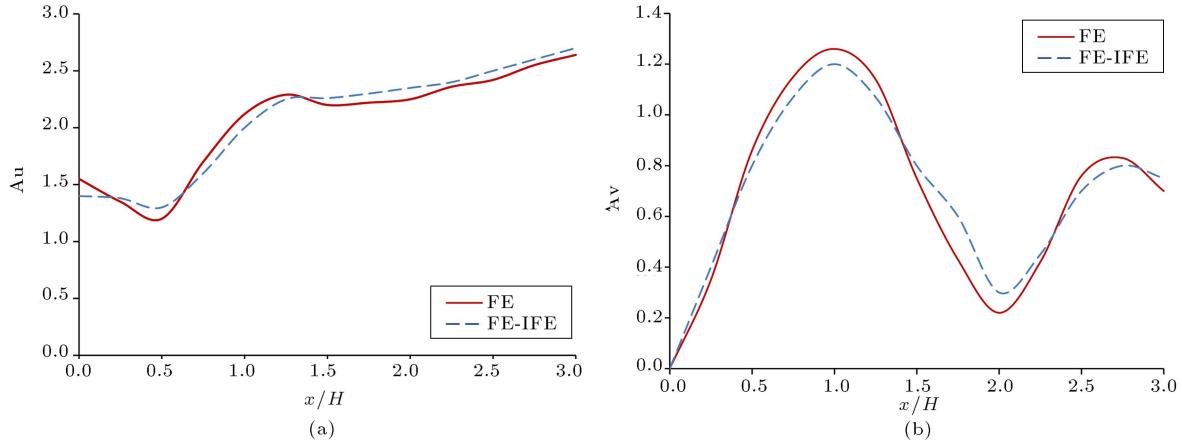
**Figure 4.** Schematic view of the valley model adopted and FE-IFE discretization [24].



**Figure 5.** (a) Horizontal displacement amplitude ( $a_o = 0.5$ ,  $L_1/L = 0$ ). (b) Vertical displacement amplitude ( $a_o = 0.5$ ,  $L_1/L = 0$ ).



**Figure 6.** (a) Horizontal displacement amplitude ( $a_o = 1$ ,  $L_1/L = 0$ ). (b) Vertical displacement amplitude ( $a_o = 1$ ,  $L_1/L = 0$ ).



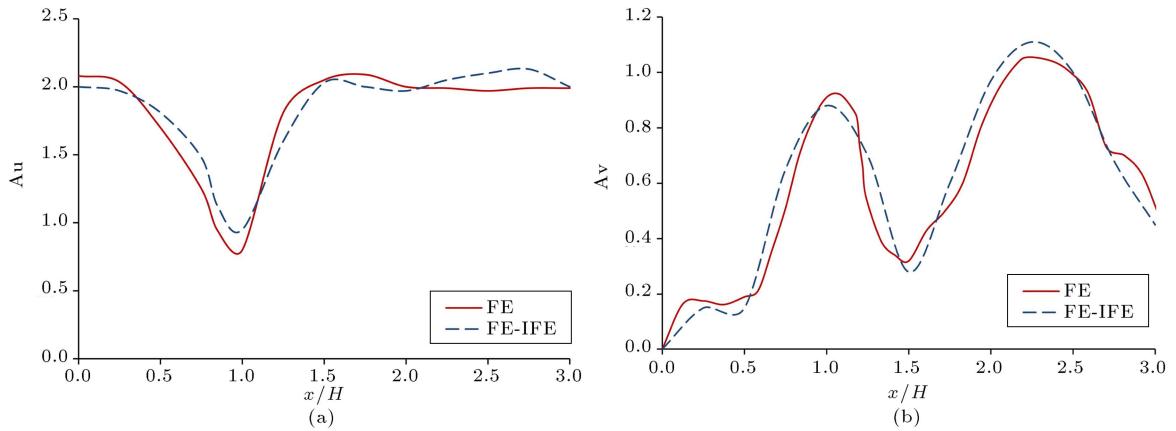
**Figure 7.** (a) Horizontal displacement amplitude ( $a_o = 0.5$ ,  $L_1/L = 1/3$ ). (b) Vertical displacement amplitude ( $a_o = 0.5$ ,  $L_1/L = 1/3$ ).

Figures 5 to 10. Results are analyzed considering displacement amplitude defined through the following equations:

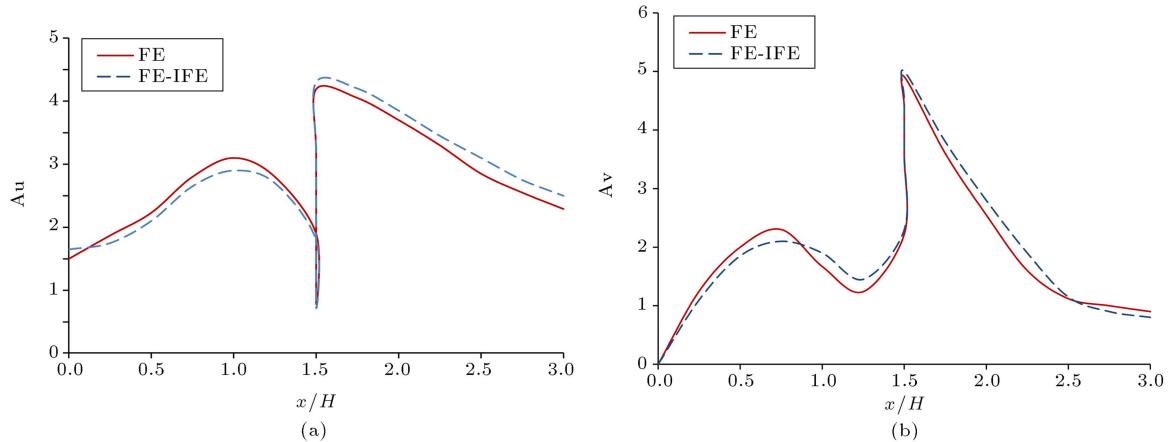
$$A_u = \sqrt{(R(u))^2 + (\text{Im}(u))^2}, \quad (26)$$

$$A_v = \sqrt{(R(v))^2 + (\text{Im}(v))^2}, \quad (27)$$

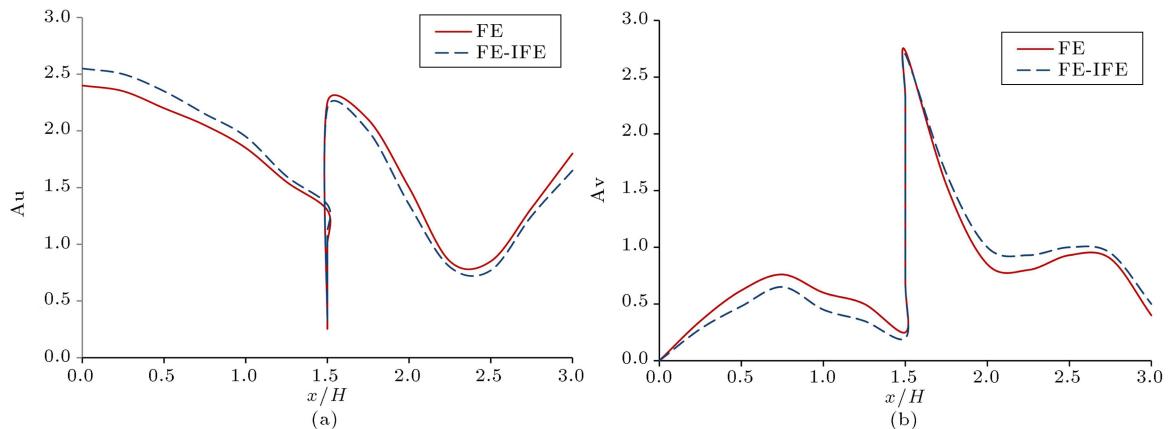
where  $A_u$  and  $A_v$  are non-dimensional displacement amplitudes along  $x$  and  $y$  directions, respectively.  $u$  and  $v$  denote corresponding displacements.  $\text{Re}$  and  $\text{Im}$  are also the real and imaginary parts of displacements



**Figure 8.** (a) Horizontal displacement amplitude ( $a_o = 1, L_1/L = 1/3$ ). (b) Vertical displacement amplitude ( $a_o = 1, L_1/L = 1/3$ ).



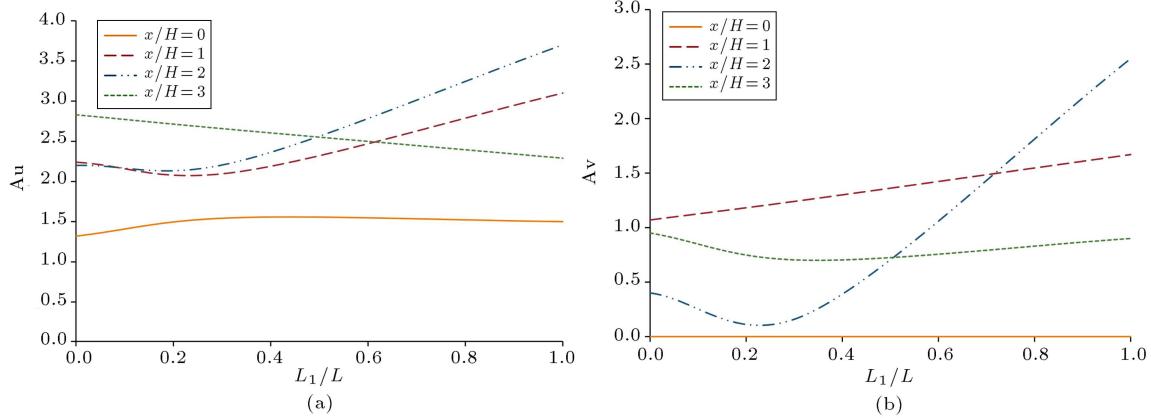
**Figure 9.** (a) Horizontal displacement amplitude ( $a_o = 0.5, L_1/L = 1$ ). (b) Vertical displacement amplitude ( $a_o = 0.5, L_1/L = 1$ ).



**Figure 10.** (a) Horizontal displacement amplitude ( $a_o = 1, L_1/L = 1$ ). (b) Vertical displacement amplitude ( $a_o = 1, L_1/L = 1$ ).

expressed in complex notation. The diagrams of displacement amplitude facilitate understanding the pattern of response along the valley span and its surrounding environment, and also help visualization of the displacement amplification factor using the amplitude of the incident harmonic waves. In this study, investigations showed that  $x/H = 3$  is, in

fact, the upper limit at which free field condition is observed. Complementary investigations also revealed that in large distances (relative to valley centre), the same phenomenon is observed. Therefore, it can be concluded that topographic irregularities, especially those investigated in this study, have little effect on the results when  $x/H$  exceeds limiting value of 3.



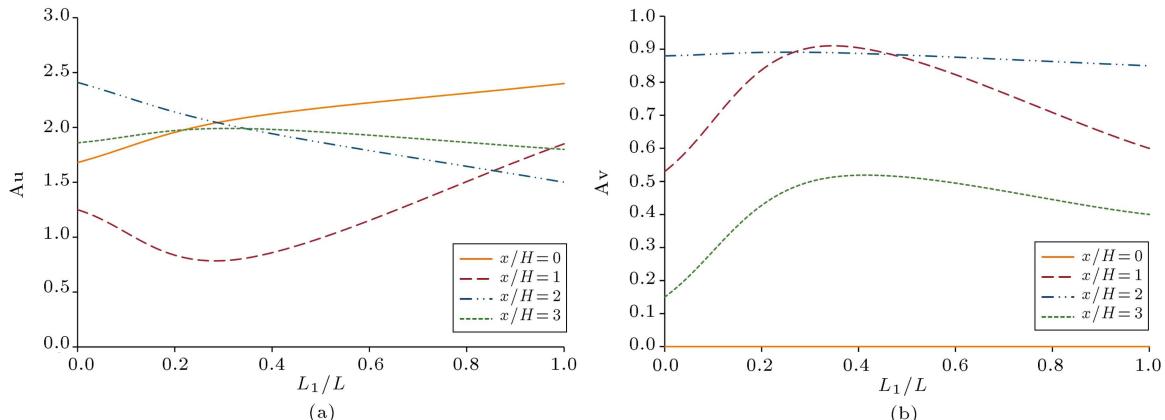
**Figure 11.** (a) Horizontal displacement amplitude ( $a_o = 0.5$ ). (b) Vertical displacement amplitude ( $a_o = 0.5$ ).

Comparison of Figures 5 to 10 reveals that using coupled 2D FEM and absorbing boundaries in seismic response analysis is efficient. In addition, obtained results suggest that the proposed boundaries, in general, prevent the inward reflection of waves to the model. These boundaries can be used, associated with the general FEM, for accurate simulation of semi-infinite and infinite soil and rock mediums. Some advantages of this method over the FE-IFE method are as follows: In FE-IFE method, accuracy of the results are strongly dependent on the location of the interfaces between FE-IFE's. If these interfaces are not properly placed on the model, results may be regarded as inaccurate or even suspicious. Whereas in current approach such dependency is substantially removed. Moreover, calculation time and required memory in the current approach are considerably reduced due to efficient mechanism of absorbing boundaries.

On the other hand, Figures 5 to 10 show that topography influences the amplification or attenuation of seismic waves to a great extent. As the distance from the valley centre increases and free field conditions are met, the amplification of the horizontal component nearly doubles (Figures 5(a) to 10(a)), which reflects

the extent of outcrop motion to the corresponding bedrock. Therefore, ground surface motions are always amplified compared with other points at depths. Although the incident wave was considered to be an SV wave propagating in vertical direction, the vertical component of the output wave at the ground surface did not vanish. This phenomenon can be attributed to the interference of incident waves and their consecutive reflections in soil medium. At the valley centre, due to the assumed symmetry of the valley and the vertical propagation of incident wave, the vertical component is virtually reduced to zero. Further investigation into Figures 5 to 12 reveals that with an increase in the ratio of the valley area to its maximum depth, amplification increases. For example, in the case of rectangular valleys, ground response is larger than that of trapezoidal or V-shaped valleys.

As can also be inferred from Figures 11(a) and 12(a) in  $x/H = 0$  (center of the valley), there is no significant difference on horizontal component in different valleys. Although, with decreasing wavelength of the incident wave, the effect of valley slope becomes more. In  $x/H = 1, 2$ , the horizontal component increases with the increase in valley slope. However, in the V-



**Figure 12.** (a) Horizontal displacement amplitude ( $a_o = 1$ ). (b) Vertical displacement amplitude ( $a_o = 1$ ).

shaped valley, because of sharp angle at the bottom of topography, the waves are trapped and multiple reflections occur. With decreasing the wavelength of incident wave, the effect of V-shape valleys increases drastically. In  $x/H = 3$ , almost free-field condition is obtained, although slight differences can be seen in different valleys. Figures 11(b) and 12(b) show that the locus of maximum and minimum of vertical components depends on wavelength of incident wave. The rate of this component varies irregularly through the valley span and its surrounding area. complementary investigations are underway which prove the effect of different factors such as thickness of soil layer, depth of valley, slope of valley, characteristics of the incident wave, and etc. in variation of this component. These will be presented in future publications.

However, it should be noted that in a two-dimensional modeling, the realistic simulation of ground surface curvature causes some incident body waves to repropagate as surface waves. These waves, which decay in a slower rate than does body waves, can develop stronger seismic vibrations even at longer distances. However, in a one-dimensional modeling, soil layers are assumed to be horizontal, while propagation of  $S$  waves is considered vertical; therefore, the effects of topographic irregularities are neglected.

## 6. Conclusion

A realistic model based on FEM coupled with absorbing boundaries is presented in this study to evaluate the effect of wave scatter in valleys with different forms and to investigate the amplification and attenuation of SV waves. The precision and accuracy of the proposed model was assessed through comparison between the non-dimensional diagrams of  $Au$  and  $Av$  obtained here with those of the FE-IFE method. The satisfactory compliance between the results of the two methods proved the acceptable performance of absorbing boundaries in simulation of semi-infinite and infinite environments. Hence, the existing model is capable of simulating similar conditions.

On the other hand, variations of displacement were seen inside the valley and around it. The displacement variations were caused by surface waves and their interference with incident and reflected waves. The variations could have a significant effect on the seismic response of structures constructed in and around the valley. Therefore, considering the effect of these topographies on the seismic design of structures constructed on them is a necessity.

## References

- Kramer, S.L., *Geotechnical Earthquake Engineering*, In Prentice-Hall International Series in Civil Engineering and Engineering Mechanics, Prentice-Hall, New Jersey (1996).
- Idriss, I. "Finite element analysis for the seismic response of earth banks", *Journal of Soil Mechanics & Foundations Div.*, **94**, pp. 617-636 (1968).
- Celebi, M. "Topographical and geological amplifications determined from strong-motion and aftershock records of the 3 March 1985 Chile earthquake", *Bull. Seismol. Soc. Am.*, **77**, pp. 1147-1167 (1987).
- Celebi, M. "Topographical and geological amplification: case studies and engineering implications", *Struct. Saf.*, **10**, pp. 199-217 (1991).
- Bouchon, M. and Barker, J.S. "Seismic response of a hill: the example of Tarzana, California", *Bull. Seismol. Soc. Am.*, **86**, pp. 66-72 (1996).
- Celebi, M. "Northridge (California) earthquake: unique ground motions and resulting spectral and site effects", *International Conference on Seismic Zonation*, pp. 988-995 (1996).
- Gazetas, G., Kallou, P. and Psarropoulos, P. "Topography and soil effects in the MS 5.9 Parnitha (Athens) earthquake: the case of Adames", *Nat. Hazards.*, **27**, pp. 133-169 (2002).
- Bagheripour, M.H. and Marandi, S.M. "A Numerical model for unbounded soil domain in earthquake SSI analysis using periodic infinite elements", *Int. J. Civ. Eng.*, **3**, pp. 96-111 (2005).
- Gatmiri, B., Arson, C. and Nguyen, K. "Seismic site effects by an optimized 2D BE/FE method I. Theory, numerical optimization and application to topographical irregularities", *Soil Dyn. Earthquake Eng.*, **28**, pp. 632-645 (2008).
- Bagheripour, M.H., Rahgozar, R. and Malekinejad, M. "Efficient analysis of SSI problems using infinite elements and wavelet theory", *Geomech. Eng.*, **2(4)**, pp. 229-252 (2010).
- Nimtaj, A. and Bagheripour, M.H. "Non-linear seismic response analysis of the layered soil deposit using hybrid frequency-time domain (HFTD) approach", *European Journal of Environmental and Civil Engineering*, **17**, pp. 1039-1056 (2013).
- Kara, H.F. and Trifunac, M.D. "Two-dimensional earthquake vibrations in sedimentary basins-SH waves", *Soil Dyn. Earthquake Eng.*, **63**, pp. 69-82 (2014).
- Ghaemian, M. and Sohrabi-Gilani, M. "Seismic responses of arch dams due to non-uniform ground motions", *Scientia Iranica*, **19**, pp. 1431-1436 (2012).
- Khanbabazadeh, H. and Iyisan, R. "A numerical study on the 2D behavior of the single and layered clayey basins", *Bull. Earthquake Eng.*, **12**, pp. 1515-1536 (2014).
- Lermo, J. and Chávez-García, F.J. "Site effect evaluation using spectral ratios with only one station", *Bull. Seismol. Soc. Am.*, **83**, pp. 1574-1594 (1993).

16. LeBrun, B., Hatzfeld, D., Bard, P. and Bouchon, M. "Experimental study of the ground motion on a large scale topographic hill at Kitherion (Greece)", *J. Seismolog.*, **3**, pp. 1-15 (1999).
17. Fu, L.Y. "Rough surface scattering: comparison of various approximation theories for 2D SH waves", *Bull. Seismol. Soc. Am.*, **95**, pp. 646-663 (2005).
18. Bouckovalas, G.D. and Papadimitriou, A.G. "Numerical evaluation of slope topography effects on seismic ground motion", *Soil Dyn. Earthquake Eng.*, **25**, pp. 547-558 (2005).
19. Kamalian, M., Jafari, M.K., Sohrabi-Bidar, A., Razmkhah, A. and Gatmiri, B. "Time-domain two-dimensional site response analysis of non-homogeneous topographic structures by a hybrid BE/FE method", *Soil Dyn. Earthquake Eng.*, **26**, pp. 753-765 (2006).
20. Asgari, A. and Bagheripour, M.H. "Earthquake response analysis of soil layers using HFTD approach", *The GeoShanghai 2010 International Conference*, Shanghai, China (2010).
21. Di Fiore, V. "Seismic site amplification induced by topographic irregularity: Results of a numerical analysis on 2D synthetic models", *Eng. Geol.*, **114**, pp. 109-115 (2010).
22. Bazrafshan Moghaddam, A. and Bagheripour, M.H. "Ground response analysis using non-recursive matrix implementation of hybrid frequency-time domain (HFTD) approach", *Scientia Iranica*, **18**, pp. 1188-1197 (2011).
23. Tripe, R., Kontoe, S. and Wong, T. "Slope topography effects on ground motion in the presence of deep soil layers", *Soil Dyn. Earthquake Eng.*, **50**, pp. 72-84 (2013).
24. Zhao, C. and Valliappan, S. "Incident P and SV wave scattering effects under different canyon topographic and geological conditions", *Int. J. Numer. Anal. Methods Geomech.*, **17**, pp. 73-94 (1993).
25. Yoshida, N., *Seismic Ground Response Analysis*, Springer (2015).
26. Desai, C.S. and Kundu, T. "Introductory finite element method", CRC Press (2001).
27. Zhao, C., Zhang, C. and Zhang, G. "Analysis of 3-D foundation wave problems by mapped dynamic infinite elements", *Science In China Series A-Mathematics Physics Astronomy*, **32**, pp. 479-491 (1989).
28. Brinkgreve, R., *Plaxis: Finite Element Code for Soil and Rock Analyses: 2D-Version 8:[user's guide]*, Balkema (2002).
29. Lysmer, J. and Kuhlemeyer, R.L. "Finite dynamic model for infinite media", *Journal of Engineering Mechanics Division*, **95**, pp. 859-878 (1969).
30. Weihua, L. and Chenggang, Z. "Scattering of plane SV waves by cylindrical canyons in saturated porous medium", *Soil Dyn. Earthquake Eng.*, **25**, pp. 981-995 (2005).
31. Bouchon, M. "Effect of topography on surface motion", *Bull. Seismol. Soc. Am.*, **63**, pp. 615-632 (1973).
32. Pagliaroli, A., Lanzo, G. and D'Elia, B. "Numerical evaluation of topographic effects at the Nicastro ridge in Southern Italy", *J. Earthquake Eng.*, **15**, pp. 404-432 (2011).

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