A Robust Framework for Addressing Routing and Scheduling Challenges in Home Health Care

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Abstract

The nature of routing and scheduling problems for providing services to patients called home health care problems would include a remarkable level of uncertainty. These uncertainties may be due to the traffic congestion, the accessibility levels to staff members, and the service times to the patients. This paper presents a robust formulation aimed at the daily/weekly/monthly routing and scheduling of staff members under uncertainty for home health care services, which simultaneously optimize the cost factors and the service quality measures. Different requirements and preferences of patients, diverse vehicles, different skills for staff, temporal inter-dependencies between services, Continuity Of Care (COC), and blood sampling requirements are considered to construct the Robust Optimization (RO) model. The robust solutions obtained through the mixed-integer linear programming model are compared to those obtained through the deterministic and Stochastic Optimization (SO) model using some randomly small- and medium-size generated instances to evaluate the performance of the RO model. Finally, we present some efficient managerial insights to substantiate the importance of considering uncertainty in the optimization models ending up with proper routing and scheduling policies.

Keywords: Home Health Care problem, Routing and Scheduling, Robust optimization model, Mixed Integer Programming Model, Continuity of care.

1. Introduction

The supply chain of providing *home health care* (HHC) services consists of the HHC center and the patients' homes as well as the links between them. HHC services help patients, particularly the elderly, to recover from hospitalization or safely stay at home without redundant hospitalization (Goodarzian et al. [1], Larsson et al. [2]). However, performing these services encounters challenges, specifically in terms of logistical and transportation aspects (Euchi et al. [3]). Providing HHC services should decrease the transportation costs and travel distances to enhance the effectiveness of services. Typically, an HHC network usually includes caregivers, patients, and medical service requirements (Shahnejat-Bushehri et al. [4], Shiri et al. [5], Ziya-Gorabi et al. [6]).

Insufficient capacity of hospitals, increasing life expectations, high disbursements for hospitalization, and increasing elderly population are the core aspects of the growing demand for HHC services. Therefore, providing HHC services has attracted more attention of researchers and business practitioners. Figure 1 demonstrates the ratio of old-age dependency in some European countries from 2010 to 2050 (Tarricone and Tsouros [7]). For instance, in Canada, health care costs have surpassed the rate of GDP growth (Oladzad-Abbasabady and Tavakkoli-Moghaddam [8]).

[Please insert Figure 1 about here]

Vehicle Routing Problems (VRP) and Nurse Rostering Problems (NRPs) are well-known Operations Research problems. These problems form the basics of Home Health Care Routing-Scheduling Problems (HHCRSPs) (Li, Xiang and Szeto [9]). To address practical requirements, HHCRSP has been extended to cover daily/weekly/monthly planning horizons. Multi-period models have been mostly applied in short- and medium-term planning (Rivera and Zapata [10]). The COVID-19 pandemic has also highlighted the significance of HHC, as it has led to an increased demand for hospital services and redirecting medical, health and capacity resources nationwide (Almorox et al. [11], Green et al. [12], Kang et al. [13]).

Making decisions on the routing of staff members and scheduling the required services are the most important operational (i.e., short-term) decisions in managing the logistics network. Although the costs of making short-term decisions are meager compared to strategic (i.e., long-term) decisions, the incorrect decisions would result in patient dissatisfaction, reducing the demands, and increasing the costs in a long time. Recently, Grieco et al., investigated the OR approaches and resolution methods to study different decision problems in HHC. They studied the objective of the study, decisions to be made, planning horizon, modeling approach, solution method, and performance aspects, to recognize the decision hierarchies, and utilize the appropriate OR approaches (Grieco et al. [14]).

Since many HHC centers offer a wide range of services, they must hire staff members with diverse skill levels. They have to consider the skill level of staff members and the desired time window for patients and staff members. Each required service needs a special skill while patients prefer to receive services in the convinced time window within the desired period for staff members. A single staff must carry out each required service by one patient. Some temporal interdependent services should be performed at the same time or with a given sequence. While the services are often scheduled by trial and error to comply with the required interdependency between services, the planning may have poor quality. There are two types of interdependent services in our proposed problem. The first one covers all services requiring two staff members in a single time, commonly referred to as simultaneous services (e.g., bathing and lifting a disabled patient). The second one includes the services that should be performed in a specific order (e.g., giving medication to a patient at a specific time before a meal).

As another real feature in providing HHC services, *continuity of care* (CoC) is a situation for long-term perspectives, in which the favorite staff member of a patient should provide his/her required services to make more sincerity between them. It is assumed that most patients have received some of their requirements in the past days/weeks/months; therefore, they are pretty familiar with the staff members. The routing and scheduling problem with the aforementioned real features differ significantly from the well-known multiple traveling salesman problem with time-window limitations. A patient could also require multiple dependent/independent services, the patients' requirements must be well-matched with the skills of the staff members, and this is also possible that staff would visit a patient several times.

Performing a service by multiple staff might be interpreted as kind of collaboration. This collaboration could be investigated from two aspects: First, more than a single staff could implement a service because it would take less time to be finished. Although collaborating between staff members might reduce the total service time and increase the accessibility level of staff members, it might increase the traveling time due to adding the possibility of using more staff members to do a single service. Therefore, the optimization model could tradeoff between more collaboration leading to less service times and more traveling times. Adding this feature might remarkably improve the solution as the staff members can perform more services in less time with a reasonable increase in traveling time. Second, more than a single staff must perform some services.

There are many disruptions in providing health care services (e.g., increasing traffic and density of vehicles on some routes and inaccessibility to staff members at some scheduled times). There are continuously hourly/daily changes arising from dissimilar and dynamic patterns of traffic conditions, poor prediction of the traffic conditions, and the weakness of developed models. Lengthy transfer times and inaccessibility to staff members are followed by delay times to provide services. This may refer patients to other health centers, leading to disruption for providing required services. The model should consider the changes to increase obtained solutions' efficiency and be closer to reality. Transfer time between two nodes and accessibility levels of staff members, as the most important sources of uncertainty in HHC delivery networks, may not be apparent at the time of decision making. Depending on the uncertainty level of stochastic parameters, decision-making on the optimal routing pattern and scheduling of required services would also change.

There are several approaches encountering uncertain parameters in the optimization models. If the problem is addressed from the probabilistic point of view, we would like to tackle this challenge through a two-stage Stochastic Optimization (SO) model, which involves making decisions in two stages. While most papers take static information, real applications are stochastic as there is no perfect information. The main difference between stochastic and deterministic programming is the existence of uncertainty in some input parameters. As one of the efficient methods for incorporating uncertainty into the models, the scenarios has been successfully applied in various fields.

A scenario can be defined as a hypothesis about the future built based on possible states of random parameters. The decision-maker takes the first-stage decisions in the current time, while the second stage decisions are made during some future periods, after taking the first stage decisions and observing the random events. The second-stage decisions are supposed to compensate for the likely adverse effects of the first-stage decisions. It is not obligatory to make both the first- and second-stage decisions simultaneously, and it is possible to delay the second-stage decisions until random events take place. In SO, the uncertain parameters are incorporated into the optimization model using the probability distribution function. Therefore, the routing variables, independent of the type of scenario, are regarded as first-stage variables, while other variables depend on the conditions of a particular day. Since such variables could be postponed until the traffic conditions and the accessibility level of staff members are determined, they are considered second-stage variables. The constraints can be categorized into two groups of first-stage and second-stage constraints. The first-stage constraints involve only the first-stage variables, in which the values of these variables are the same in all scenarios. The latter includes the first- and second-stage decision variables.

We develop a robust MILP model considering heterogeneous staff members, possible interdependencies between services, the limitations associated with getting the blood samples, and various transfer times depending on vehicle speeds. To the best of our knowledge, the proposed method would be the first attempt to incorporate all limitations in a model under uncertainty. The transfer time is affected by the type of vehicle used by staff members. Different weather conditions or traffic situations of the service area, damaging the vehicle, as well as finding a parking lot for vehicles may cause uncertainty in the transfer times. Another important source of uncertainty is the accessibility level or efficiency of staff members. It means that staff may not be present at all their scheduled times, and the accessibility level of staff members is also different due to different efficiency on different days. The routing of staff members must be decided before determining the street traffic conditions and the accessibility level of staff members. Since we focus on Robust Optimization (RO) model, the respective model is described in more detail, as follows.

1.1 RO model

The efficient design of the staff member routes and scheduling of services are dynamic decisions whose effects might last for a few periods after providing a service, during which some situations may change. Since some important parameters may change during a time interval, the design of robust decisions would be vital as it might efficiently affect on providing services. In general, given that these fluctuations may occur abruptly, making changes in the routing of each staff and schedule of services are impossible within a short time interval. Therefore, the supply chain should be robust concerning uncertain parameters. A few recent papers have studied the variability of transfer times for routing problems in the RO framework to control the uncertainty level entailed into a problem. In RO programming, the methods have been raised for converting the planning problem with uncertainty into an equivalent deterministic problem, which are referred to as robust counterpart problems (Ben-Tal et al. [15]).

A robust supply chain is a competitive advantage for many companies, which assist them in managing inevitable increasing fluctuations. Since this logistics network might be involved with large amounts of uncertainty, it could remarkably influence the routing and scheduling decisions. With a bit of change in the nominal data, not only the deterministic solutions may not be optimal, but also there is a high possibility for being infeasible. When the values of parameters are different from their nominal data, it could violate some constraints and the optimal solution obtained for nominal data is not optimal or even feasible. Therefore, these decisions coming out of an RO model might be pretty helpful to handle increasing unavoidable fluctuations.

A robust feasible/optimal solution is also a feasible (or near feasible)/optimal (or near-optimal) solution under all scenarios. A robust (feasible) solution, especially in a problem with short-term decisions, is quite sensitive to data changes. In general, the RO overcomes some drawbacks in SO, which increases the intention for using the RO model. In the SO model, it is assumed that the values of stochastic parameters are determined through known probability distributions. The following points would be some critical reasons that decision-makers prefer the RO model instead of SO one:

1. In reality, finding enough historical data for the uncertain parameters may be problematic,
2. Since the SO solution is feasible for all corresponding scenarios in the optimization model, it could be infeasible for other realizations out of the respective scenarios when the solution is applied for decision-making in the real-time situation or online simulation.

3. Scenario-based SO is most often applied for representing uncertainty. Therefore, a large number of scenarios may result in large-sized, computationally challenging problems.

RO theory offers a framework to control the uncertainty that could immunize the optimal solution for any possible realization of the uncertainty in a given bounded uncertainty set (Ben-Tal and Nemirovski [16]). However, there is usually no information about the probability distributions in RO, and only the intervals including lower and upper bounds are available. Hence, the RO model deals with unknown real data and their probabilities in which the uncertain parameters are estimated through discrete or continuous intervals. Moreover, unlike the deterministic optimization (DO) model, whose goal is to determine the best solution for one specific scenario, the RO model looks for a feasible solution (or a set of solutions) concerning all scenarios. Therefore, we seek optimal (or near-optimal) solutions that are most likely feasible. The solution feasibility in the RO model is guaranteed in all situations with a slight disregarding of the objective function. The remainder of the paper is organized as follows: Section 2 presents a literature review on the deterministic routing and scheduling problem in providing HHC services. A comprehensive description of the parameters and decision variables is proposed in section 3. Then, section 4 is devoted to computational results by which the efficiency of the models is assessed. Finally, we conclude and recommend some possible suggestions for future researches.

2. Literature review

We classify all reviewed papers into three parts: part 1) DO models, part 2) models with considering uncertain parameters, including SO and RO models to find routing and scheduling of staff members. A comprehensive review of the routing and scheduling problems in providing HHC services was carried out by Fikar and Hirsch (2017). They classified all papers in different perspectives (e.g., different aspects of problem definition, dynamic or stochastic settings, objectives, constraints, and solution methods) (Fikar and Hirsch [17]). Furthermore, Grieco et al., explored the OR and resolution methods to review decision problems in HHC. The objective, decisions, planning horizon, modeling approach, solution approach, and performance aspects were thoroughly examined to identify the decision hierarchies, and the OR approaches (Grieco et al. [14]).

Part 1: DO

Mankowska et al. (2014) presented a mathematical formulation to optimize the daily planning of staff members. The objective function minimized the total traveling distance and total delay, and maximal delay in providing services (Mankowska et al. [18]). Decerle et al. (2018) presented a mixed-integer programming model with a hard and soft time window for patients and synchronization constraints. The objective was to minimize total transfer time and the penalties for not regarding the time windows of the patients and the synchronized visits (Decerle et al. [19]). Aliza et al. (2019) presented an exact optimization method depending on logic-based benders decomposition. Their goal is to schedule multiple visits during a given time horizon and maximize the number of served patients while considering patient requirements, travel time, and scheduling constraints (Heching et al. [20]).

Entezari ans Mahootchi (2021) [21] developed a DO model for the daily staff routing and service scheduling considering various qualifications and different vehicles for employees, different requirements, temporal interdependencies between services, CoC, and blood sampling requirements. The total transfer time, total tardiness in providing services, total overtime of the staff members, total violation of CoC, and violation of the staff's time windows were minimized. They have also presented a meta-heuristic solution scheme for finding the near-optimal solution in the deterministic version of HHCRSP published in Scientia in year 2021 (Entezari and Mahootchi [21]). Since an RO model is actually considered a deterministic model, it can be solved by all solution methodologies developed for deterministic ones and our proposed solution scheme in recently published work in Scientia would be used for solving the RO model as well. Demirbilek et.al. perform routing and scheduling for multiple nurses to maximize the number of visits. They proposed a heuristic based on several scenarios including current schedules of nurses, the new requests, and random generated future requests (Demirbilek et al. [22]).

Since the sub-problem in the HHC includes a VRP, as an NP-hard problem, the HHCRSP is also an NP-hard problem. Hence, exact methods need a significant amount of time to solve large-size problems. As a result, the majority of research studies have adopted heuristic/metaheuristic to solve large-size instances (Martin et al. [23]). Grenouilleau et.al. presented the HHCRSP for the assignment and routing of some visits during a week. The solution method is based on a set partitioning formulation and a large neighborhood search framework (Grenouilleau et al. [24]). Moreover, Cinar et.al. assigned priorities to patients so that the priorities of unvisited patients increase exponentially by day. Their goal is to maximize the overall priority of the patients while minimizing the total traveling time. They developed an adaptive large neighborhood search algorithm and a matheuristic to generate near-optimal solutions. The authors have addressed a real-world problem where the nurses are responsible for checking on patients either by visiting their homes or making phone calls. If some patients cannot be visited at home during the planning horizon, they are checked upon through phone calls (Cinar et al. [25]). In another study, one decision support framework is presented considering the real needs of HHC services under the synchronization between staff and home delivery vehicles' visits, multiple visits, multiple routes by the vehicles, and pickup/delivery visits. They developed a mixed integer programming model a hybrid genetic algorithm is proposed to solve the model (Nasir and Kuo [26]).

Furthermore, a HHCRSP modelled considering outpatient services and time windows, skill requirements, and working regulations. The problem is formulated as a mixed integer convex nonlinear programming model to minimize the total travel costs and total waiting penalties of out-patients, and maximize the overall benefit of patients’ preference satisfaction. They developed a hybrid genetic algorithm to solve the problem (Li et al. [9]). Also, Liu et.al, proposed a HHCRSP with the consideration of the time windows, qualifications of caregivers, synchronized visits, lunch breaks, and flexible departure modes of caregivers. They modeled the problem with a mixed-integer programming model and developed four hybrid metaheuristics (Liu et al. [27]). Decerle et.al. proposed a mixed integer programming model of the multi-depot HHC assignment, routing, and scheduling problem without previous assignment of caregivers to the centers. They presented a matheuristic-based approach with different assignment strategies (Decerle et al. [28]). Yadav et.al. presented a framework to address the problem of HHC routing and scheduling considering patient's preferences regarding the gender and language of the caregiver, time window constraints, and multiple visits for certain procedures. The model attempts to maximize the number of patients served without violating a predetermined maximum allowed contact limit and partially accommodating patient's requests. To tackle this, the researchers proposed two different heuristic procedures based on mixed-integer programming decomposition (Yadav and Tanksale [29]).

Frifita et al. have explored the synchronization and temporal precedence between interdependent services (Frifita and Masmoudi [30]). In another study, Mascolo et al. have focused on the scheduling and routing of health workers to provide multiple services to patients in their homes. They have developed a system that assigns patients requiring services to workers based on their skills and optimizes the routes for the health workers (Di Mascolo et al. [31]).

In addition, Decerle et al. developed an approach based on matheuristic algorithm to solve the problem of assigning visits and health professionals (Decerle et al. [28]). Khodabandeh et al. developed a model for staff routing and scheduling by reducing cost aspects and minimizing the difference between the nurses' potential skills and their actual service schedules. Another feature is considering staff members' and patients' preferences. Nurses' preferences include setting a time window for nurses' availability, limiting working hours for nurses, distributing tasks equally among nurses, rejecting patient visits, and limiting hard tasks (Khodabandeh et al. [32]). Many researchers also considered accepting new patients while conserving COC for patients (Grenouilleau et al. [33]).

Another paper presented a bi-objective model for HHCRSP, which minimized the overall service time and the total tardiness. First, a non-scalar method was provided to achieve a first solution. Then, two multi-objective evolutionary algorithms were presented to solve the problem (Belhor et al. [34]). According to the service mode of family doctor contract, a multi-period problem of combining the home health care and outpatient services was presented for multiple days. The required services with fixed frequencies and different preferences for various dates were taken into account. The authors proposed a mixed-integer nonlinear and convex model considering the patients' requirements date regulations and staff members' working regulations. The objective minimized the total operating costs and maximizes the patients' preference satisfaction. A hybrid tabu search algorithm was used to solve the problem (Xiang et al. [35]). Another paper considered skill requirements, multiple time windows, and staff members' working times. The authors proposed a new mixed integer linear programming model to minimize staff members' waiting times and balance their workloads. They presented a two-phase approach to solve the problem. The first phase consists of applying a simulated annealing based heuristic to generate the routes and assignment of staff members while the second selected a time window for each patient to ensure the synchronization of simultaneous services (Bazirha et al. [36]).

Another HHCRSP was studied with electric vehicles, for green travel and distribution, and synergistic-transport mode. The staff members may use walking, if these vehicles are recharging at a station. The authors attempted to find the synthetic-routes, including the electric vehicle and walking routes of a staff to minimize the total costs consists of the dispatching cost, the transport cost by the vehicles and walking, and the incompatibility cost of staff members and patients. They developed a tailored branch-and-price-and-cut algorithm based on a set-partitioning model to solve the pricing sub-problems (Yin et al. [37]). A multi-period with homogeneous electric vehicles and time windows was studied to minimize the total costs including the fixed cost of employing staff members, the energy charging costs, and the costs of unserved service requirements. The authors considered three charging technologies and developed an adaptive large neighborhood search metaheuristic to solve the mathematical model (Yazır et al. [38]).

Part 2: Models with Considering Uncertain Parameters

The first idea of linear programming with probability parameters was formulated by Dantzig (Dantzig [39]). The method was to solve a deterministic programming problem in which the stochastic parameter was replaced with the expected value. Stochastic service times were studied by Yuan et al. (2015), in which the objective was to minimize a combination of costs and penalties for late arrivals (Yuan et al. [40]). Hewitt et al. (2016) [41] demonstrated that a long planning horizon can save significant amount of transportation costs and staffing levels. They considered a deterministic setting, where all requests are specified in advance, and the routing cost of monthly planning is compared with the cost of weekly planning. With uncertainty in planning for a long time horizon, a presented method anticipates future unknown patient requests (Hewitt et al.[41]). Furthermore, Liu et al. (2018) developed a mathematical model for a home-caregiver routing and scheduling problem with stochastic travel and service times. A chance constraint was also incorporated to confirm the probability of promptly providing a service. The objective was to minimize the total expected operational cost of selected routes and the penalty for unvisited patients (Liu et al. [42]). Another paper studied an HHC routing problem with stochastic travel and service times. First, some approaches including hybrid genetic and simulated annealing algorithms were implemented to solve the DO model resulting from the SO model with recourse. Finally, the stochastic model was solved by the a heuristic algorithm based on simulated annealing (Shi et al. [43]).

A two-stage stochastic mixed integer programming model considered the uncertainty in travel and service times. The model considered the decisions on districting, staff dimensioning, resource assignment, scheduling, and routing. Districting and staff dimensioning were taken as the first stage, and assignment, scheduling, and routing were defined as the second stage decisions. The proposed algorithm relied on a matheuristic-based method calling on different mixed integer models (Nikzad et al. [44]). A stochastic programming model with recourse was proposed considering uncertainty in traveling and service times as well as synchronization of services. The authors aimed to minimize the travelling cost and the expected value of recourse defined as a penalty cost for delaying in patients' services and a remuneration for caregivers' overtime. While the DO model was solved by CPLEX, the genetic algorithm and the general variable neighborhood search based heuristics, the stochastic programming model was solved by Monte Carlo simulation in the GA (Bazirha et al.[45]).

A mixed integer linear model was developed for a daily cross-regional routing and scheduling problem with stochastic service times. Some real constraints such as patient time windows and skill matching were considered. A hybrid simulated annealing algorithm was used to solve the model (Du and Zhang [46]). Wang et al. [47] also proposed SO methodologies for a staffing and capacity planning problem for HHC services. Their goal was to minimize the total cost of staffing, capacity allocation, over-, and under-staffing. The authors presented two-stage SO and distributionally RO approaches considering two types of decision-makers. To enhance the applicability of the nonlinear RO model, they derived equivalent mixed-integer linear programming reformulations (Wang et al. [47]).

Furthermore, a HHCRSP considered patients' priorities and times uncertainty. A fuzzy multi-objective model was developed to maximize the total priority of patients and minimize the total service cost. To solve the model, a discrete multi-objective grey wolf optimizer was proposed (Li et al. 48]). Another work addressed a multi-center, multi-objective and stochastic problem considering the caregivers' working time, patients' requirements and resource constraints. The total operation cost and penalty cost for earliness and delay services were minimized. A chance-constrained programming model and a cooperation evolutionary algorithm using stochastic simulation were developed (Ma et al. [49]). A novel home health care routing problem was studied in high population density areas with stochastic service time. The objective function was to reduce travel and waiting times for staff members or elderly people. Considering human relationships were improved participants' satisfaction. The authors employed the Markov decision process and chance-constrained programming. To solve the problem, the Q-learning and the ant colony optimization algorithms were used (Zhang et al.[50]). Another integrated multi-period staffing, assignment, routing, and scheduling of staff members was studied under uncertainty. The aim was to construct a weekly schedule to minimize staff members' usual and overtime costs. The authors proposed a mixed-integer program for the deterministic problem. Afterwards, uncertainty was incorporated in service and travel times and develop a robust counterpart using interval and polyhedral uncertainty sets. They developed a logic-based Benders branching-decomposition algorithm to solve the DO and RO models (Naderi et al. [51]).

The first systematic methods to solve robust problems were introduced in the 1970s. In 1973, Soyster considered convex programming with the Set-inclusive constraints (Soyster [52]), in which the worst-case approach was used to solve linear programming with uncertainty in the columns of the coefficient matrix. Ben-Tal & Nemirovski (Ben-Tal and Nemirovski [53], Ben-Tal and Nemirovski [54]) and EL-Ghaoui (El Ghaoui et al. [55]) took an important step in RO theory by presenting models for uncertain linear problems with ellipsoidal uncertainties and solving the counterparts of the nominal problems as conic quadratic problems. Lanzarone and Matta (2014) consider stochastic demand in providing HHC services to generate daily robust solutions regarding overtime for staff members and continuity of care for patients (Lanzarone and Matta [56]). Also, another RO model for a HHCRSP is considered with uncertain travel and service times. They presented meta-heuristics and Monte Carlo simulation to solve the models (Shi et al. [57]).

In this paper, using the RO approach introduced by Ben-Tal and Nemirovski, we develop a robust MILP model considering stochastic transfer time and accessibility levels to staff members, considering different skills for staff members and different requirements for patients with their preferences. The model considers overtime periods and the meal-time for staff members, temporal interdependencies between services, and time windows for staff members and patients. In addition, there are no important limitations to reflect different blood, break, and depot centers for each staff. Furthermore, all services must be performed by one skilled staff member. Other medical limitations concerning returning all blood samples to the laboratory in a given time frame are also applied to the model. In general, we try to present an integrated model under uncertainty considering real features.

3. RO model

The first part of this section presents the problem definition, possible scenarios, index sets, parameters, and decision variables. Then, a mathematical formulation for the RO model is proposed, which concentrates on the density of the streets and the possibility of low service levels as uncertain parameters. Different service levels of the staff members may be caused by different efficiency on different days, holidays, and a problematic event. The transfer times and the accessibility levels to staff would be quite important in scheduling staff members. Staff do not often desire to wait until the start time of patient is started and also don't intent to stay in the patient's home after the end time of his/her time window. He would like to do the respective tasks exactly within the patient's time window because he/she can present more services in a certain period. Furthermore, patients would really like to receive their services on time without any delay.

3.1. Problem definition

The HHC logistics network is described on the completed graph, where the set of nodes includes the HHC center and patients' homes, whereas the set of edges compose all links between two nodes. Staff members are dispatched from the HHC center to the patients' homes to provide services. Each node denotes one service requirement by one patient in our model. There is only a single service in each node, which a single staff must perform. The logistic network is represented in Figure 2.

[Please insert Figure 2 about here]

We aim to determine the routing of staff members and scheduling the requirements in a specific daily/weekly/monthly time to minimize the costs for transfer times and some penalties concerning the quality of services. For example, we minimize the transfer time regarding the vehicle's speed and the distance traveled by staff. The total delay in providing services should also be reduced for on-time responsiveness to services. Each service required by one patient should be performed in a specified time window by one skilled staff (i.e., the staff members are heterogeneous). Patients are classified into two groups regarding their requirements: patients demanding single services and those demanding interdependent services (e.g., giving medication to a patient before/after a meal with a predetermined time interval). We consider some assumptions as follows:

1. All staff members must start their routes from the HHC center (this assumption can be relaxed in our models with minor changes).
2. All possible pairwise nodes are interconnected in both directions.
3. Since the staff members have different types of vehicles, we consider different speeds for staff members based on his/her vehicle.
4. Each node associates with one required service, which must be performed by only one staff member (i.e., in interdependent services mode for a patient, we consider multiple nodes with zero distances for these services.
5. Four dummy nodes, 0, n+1, *blood*, *meal*, are defined, denoting the routes' initial and end nodes, the end node for returning the blood samples, and the end node for a lunch meal, respectively.

Before explaining the problem formulation, we present a preparation step. In general, a patient may require multiple requirements of one type or several types, which may be interdependent or independent of each other. We define a requirements matrix denoting all required services such that each row of the matrix has only one identity element to integrate all situations into a model. While each matrix row represents only one service requirement, there might be different rows regarding one patient. If a required service by one patient requires more than one staff member, the service is transformed to some dummy services called simultaneous services whose numbers are equal to the number of required staff members. Therefore, each row corresponds to one node of the network representing a service requirement of one patient. Although the initial, end, blood, and meal nodes would be operationally in different locations, we assumed that all dummy nodes are located in the HHC center without loss of generality. Our proposed model could be implemented in all situations, including similar and dissimilar locations for these nodes.

3.2. Defining scenarios

In our model, a specific scenario occurs when the traffic conditions and the accessibility level of staff members on a particular day are identified. It means that such information was not available at the time of decision-making, where the sequences of visits should be determined. However, HHC centers should plan staff members before identifying the exact value of uncertain parameters. It must be decided about planning of staff members before detecting the traffic conditions and accessibility level of staff members. When the requests are daily/weekly/monthly received, there is no perfect information about the exact value of uncertain parameters, whereas the route of each staff should be set just before the real conditions are known. It should be noted that the planning of each staff has to prepare till night to coordinate staff members with the scheduled plan.

3.3. Problem formulation

The index sets, parameters, and decision variables before proposing the RO model are explained in Table 1.

[Please insert Table 1 about here]

For simplicity, we give larger indices to the nodes whose required services have to be carried out later than other services for interdependent services. If $\delta_{ij}^{\max} \geq \delta_{ij}^{\min} > 0$, the start time of performing the required service by node j should be no smaller than $\delta_{ij}^{\min}$ and no larger than $\delta_{ij}^{\max}$ after the start time of performing the required service by node i. in addition, if $\delta_{ij}^{\max} = \delta_{ij}^{\min} = 0$, both required services at nodes i and j must be started at the same time. For all simultaneous services belonging to P^{sim}, the required services can be started with the attendance of all related staff members. Furthermore, if a staff member arrives at the node i beyond the upper limit of the pertinent time window (l_i), we consider a penalty in the objective function based on the period between t_{iv}^{w} and l_i. On the contrary, if a staff arrives at node i before the lower bound of the pertinent time window (e_i), the staff must wait until e_i; otherwise, the service can be started immediately.

3.4.The RO formulation

The deterministic and the corresponding uncertain linear optimization models proposed by (Ben-Tal and Nemirovski [53]) and (Ben-Tal and Nemirovski [54]) are written as:

$$\begin{aligned} &\min cx + d \\ &s.t. Ax \le b \end{aligned} \qquad (1)$$

$$\begin{aligned} &\min cx + d \\ &s.t. Ax \le b \\ &c, d, A, b \in U \end{aligned} \qquad (2)$$

where the parameters *c*, *d*, *A*, *b* vary in a certain uncertainty set *U*. A vector x is a robust feasible solution i

An optimal solution satisfies all realizations and confirms an optimal objective value not worse than $\hat{c}x$, which is a semi-infinite linear problem and might be polynomially unsolvable. However, for a wide variety of compact, convex uncertainty sets, the RO model is a computationally tractable convex mathematical problem (Ben-Tal and Nemirovski [59]), (Ben-Tal and Nemirovski [54]), (Ben-Tal et al. [60]).

The transfer time and the accessibility to staff members are uncertain parameters, which are assumed to vary in specified closed bounded boxes (Ben-Tal et al. [15], Ben-Tal et al. [60]). The general form of this box can be denoted as follows:

$$, U_{Box} = \left\{ \mu \in R^n : \left| \mu_t - \bar{\mu}_t \right| \le \rho \gamma_t, t = 1, \ldots, n \right\} \qquad (3)$$

where $\bar{\mu}_t$ is the nominal value of the μ_t and the positive numbers γ_t signify ‘‘uncertainty scale,” and ρ >0 represents the ‘‘uncertainty level”. If $\gamma_t = \bar{\mu}_t$, the box comprises μ_t which the largest size of the relative deviation from the nominal data is ρ. Ben-Tal et al. demonstrate that in a closed bounded box, the robust counterpart problem can be effectively transformed into a tractable model considering a finite set of the extreme points of U_{Box} instead of U_{Box} in model (2). Therefore, we propose the RO model with uncertain transfer times and accessibility levels of staff members given by box sets as follows:

$\min z$	(4)
s.t. $\alpha_1 \sum_{v \in V} \sum_{i \in P^0} \sum_{j \in P^0} (t\bar{t}_{ijv} r_{ijv} + \varphi_{ijv}^{tt}) + \alpha_2 \sum_{i \in P^0} z_i + \alpha_3 \sum_{v \in V} ov_v + \alpha_4 \sum_{j \in P^0} \sum_{v \in V} CoC_{jv}.dur_j.(1 - \sum_{i \in P} r_{ijv}) + \alpha_5 \sum_{v \in V} (ltw_v + utw_v) \le z$	(5)
$\sum_{i \in P^0} r_{0,i,v} = \sum_{i \in P^0} r_{i,n+1,v} = 1, \forall v \in V$	(6)
$\sum_{j \in P^0} r_{j,i,v} = \sum_{j \in P^0} r_{i,j,v}, \forall i \in P^0, \forall v \in V$	(7)
$\sum_{v \in V} \sum_{j \in P^0} r_{jiv} = 1, \forall i \in P^0$	(8)
$r_{ijv} \le q_{vj} \forall i, j \in P^0, \forall v \in V$	(9)
$\sum_{i \in P^0} r_{i,break,v} = 1, \forall v \in V$	(10)
$r_{ijv} \in \{0,1\} \forall i, j \in P^0, \forall v \in V$	(11)
$t_{jv} - t_{iv} - dur_i + M(1 - r_{ijv}) \ge t\bar{t}_{ijv} + \rho_{tt} \gamma_{ijv}^{tt}, \forall i, j \in P^0, \forall v \in V$	(12)
$\rho_{tt} \gamma_{ijv}^{tt} r_{ijv} \le \varphi_{ijv}^{tt}, \forall i, j \in P^0, \forall v \in V$	(13)
$\rho_{tt} \gamma_{ijv}^{tt} r_{ijv} \ge -\varphi_{ijv}^{tt}, \forall i, j \in P^0, \forall v \in V$	(14)
$t_{iv} \ge e_i \sum_{j \in P} r_{jiv}, \forall i \in P^0, v \in V$	(15)
$t_{iv} \le l_i \sum_{j \in P} r_{jiv} + z_i, \forall i \in P^0, v \in V$	(16)
$t_{jv_2} - t_{iv1} \ge \delta_{ij}^{\min} - M(2 - \sum_{k \in P^0} r_{kiv_1} - \sum_{k \in P^0} r_{kjv_2}), \forall i, j \in P^0, \forall v_1, v_2 \in V$	(17)
$t_{jv_2} - t_{iv1} \le \delta_{ij}^{\max} - M(2 - \sum_{k \in P^0} r_{kiv_1} - \sum_{k \in P^0} r_{kjv_2}), \forall i, j \in P^0, \forall v_1, v_2 \in V$	(18)
$t_{n,v} - t_{0,v} \le T, \forall v \in V$	(19)
$\frac{1}{worktime_v}(ov_v - t_{n,v} + t_{0,v}) \ge -(\bar{g} - \rho_g \theta^g), \forall v \in V$	(20)
$t_{0,v} \ge estaff_v - ltw_v, \forall v \in V$	(21)
$t_{n+1,v} \le ustaff_v + utw_v, \forall v \in V$	(22)

$1+(l_{blood}-t_{i,v}) \le M\left[\sum_{j\in P} r_{j,blood,v} + 1 - \sum_{j\in P^0} r_{jiv}\right], \forall i \in P^+, \forall v \in V$	(23)
$t_{blood,v} \le l_{blood} \cdot \sum_{j\in P^0} r_{j,blood,v}, \forall v \in V$	(24)
$t_{blood,v} \ge t_{i,v}, \forall i \in P^+, v \in V$	(25)
$\rho_g \theta^g .g \le \beta^g, \forall i, j \in P^0, \forall v \in V$	(26)
$\rho_g \theta^g .g \ge -\beta^g, \forall i, j \in P^0, \forall v \in V$	(27)
$t_{iv}, z_i, ov_v, ltw_v, utw_v \ge 0, \forall i, j \in P^0, \forall v \in V$ $\varphi_{ijv}^{tt}, \beta^g \ge 0, \forall i, j \in P^0, \forall v \in V$	(28)

Constraint (5) consists of five components, which are explained as follows:

Component 1: All staff members' total time transferred between locations to provide the requirements: the transfer times enormously depend on the type and speed of the vehicles used by staff members.

Component 2: The total delay occurred in providing service requirements. If the staff member's arrival time at the patient's home is larger than the upper bound of the related time window, a penalty cost is considered in the objective.

Component 3: The total overtime of the staff members: if the conditions of a particular day led to having overtime for staff in providing services, we consider a penalty cost in the objective function.

Component 4: The total violation of CoC: if a staff member, different from the favorite staff of the patient, performs the required services, a penalty cost corresponding to the service duration is imposed into the objective; otherwise, we consider no penalty cost in the objective function.

Component 5: The violation of the staff’s time windows: it is preferred that all services should be carried out within the desired time windows for staff members.

Constraint (6) makes sure that the route of each staff member starts and terminates in the HHC center. Constraint (7) is a flow conservation constraint, ensuring that each staff member must leave the node after performing its required service. Constraints (8) specifies that each required service is performed to exactly one staff member. Constraint (9) enforces that a staff member can perform the required service if she/he has the skill corresponding to this service. Constraint (10) indicates that every staff member must visit the meal node. Integrality restrictions on the binary variables are guaranteed by constraint (11).

Constraints (12), (13), and (14) determines the start times of the services regarding the service duration and the transfer times between two nodes. Constraints (15) and (16) specify the start times of the services complying with the time windows of the patients. Although the lower bounds of the patients' time windows are considered hard constraints (i.e., they must not be violated), the upper bounds are considered soft constraints (i.e., they could be violated). Constraints (17) and (18) ensure the bounds on time distances between two interdependent services. If both the minimum and maximum time distances of two interdependent services are zero, these services must be performed simultaneously. Constraint (19) specifies a limit on the total working time for a staff member. The total overtime of each staff member considering the contract working time is also determined by constraint (20).

Constraints (21) and (22) consider the desired time windows of staff members as soft constraints. If the time window for a staff member is violated, the sum of ltw_v and utw_v are taken as penalty costs in the objective function. The last three constraints are related to blood sampling requirements. Constraint (23) enforces that whenever a staff member takes at least one blood sample, he/she has to visit the node *blood* before l_{blood}. Due to medical reasons, Constraint (24) makes sure that the staff $v \in V$ returns to the dummy destination node *blood* before the upper bound of its time window. Constraint (25) ensures that the node *blood* is visited after taking the last blood sample. Constraints (26) and (27) demonstrate the variations of the accessibility level of staff members in a specified closed bounded box. Non-negativity restrictions on second-stage decision variables are enforced by constraint (28).

Our main goal from providing SO and RO models with real-world constraints is to develop comprehensive models from stochasticity and robustness point of views. Furthermore, through finding EVPI and VSS, the deterministic and stochastic models could be compared to each other. We have not seen such comprehensive model in the literature to be comparable to our proposed models.

4. Computational experiments

In this section, after introducing the dataset and presenting the numerical results, we analyze the models' solving results.

4.1. Definition of the dataset and input parameters

Whereas there are no valid available benchmarks to compare the models, we have inevitably generated some instances with small- and medium-sizes to denote the resulting solution's efficiency. All numerical results are achieved using a 2.40 GHz Intel Core i5 CPU and 4 GB of RAM on a laptop with a 64-bit operating system. In addition, the software GAMS is used to solve the model while all results are extracted within 60000 seconds of CPU time. Two different datasets, are defined to evaluate the RO model, in which the actual number of patients is equal to 7 and 10, respectively (Table 2). Five test instances with various values for input parameters are generated for each of these sets.

[Please insert Table 2 about here]

The transfer time should be obtained based on the type of vehicle and its corresponding average speed, and the distance matrix. The distances traveled between all pairwise nodes are generated using a uniform distribution function. This function has a range between 0 and 2000 and is determined by Euclidean distance calculated from the nodes' coordinates. The distance matrix (time-traveling matrix) is first determined based on the Euclidean distance between the nodes. Then, transfer times can be calculated as formula (29):

$$tt_{ijv} = f_v . d_{ij}, \forall i, j \in P^0, \forall v \in V \tag{29}$$

where tt_{ijv} is the transfer time between nodes i and j traveled by staff member v, d_{ij} is the traveling distance between all pairwise nodes, and f_v is the conversion coefficient of distance to transfer time (i.e., inversion of speed). The exact value of f_v certainly depends on the fleet types of vehicles. It is also assumed that the transfer time matrix is symmetric.

Collaboration in our models has been considered as interdependent services. If a service requires more than a single staff (simultaneously or with a predetermined precedence) to be performed, two nodes are considered for such services, which means that there are two services that should be handled by two staff members in a same time window or two consecutive time windows. Moreover, since many HHC centers take the remarkable significance into account for the second and third performance measures, their respective weights in the objective function are considered twice the weights of other sub-goals. The coefficients have a noteworthy influence on the obtained solutions. First, we implemented different weightings to investigate trade-offs between quality and drawn the frontier surface depending on the manager of HHC center. Finally, we achieve these values for the coefficients with efficient frontier, as a Pareto optimal solution. We assume the maximum difference between lower and upper bounds for the time window of each patient is 200-time units. All input parameters are denoted in Table 3, where the random numbers are generated within the determined intervals to set the input parameters.

[Please insert Table 3 about here]

All related results, including the solutions of the SO model, analyzing the optimal number of scenarios, and calculating two performance measure, EVPI and VSS are addressed in the **c**

4.2. Comparison between the solutions of RO and DO models

While the routing and scheduling decisions are considered short-term decisions, they are of particular importance because their effects may continue after providing services. Although the cost of a problem with making short-term decisions is very low compared with strategic ones, since improper routing and scheduling may lead to low satisfaction of patients and staff members, to reduce the number of applicants, to increase costs in the long term. The importance of significant alterations in providing HHC services has incited more attention to obtain robust decisions for designing the logistics network.

To evaluate the efficiency of the RO model, we randomly generate twenty instances with two different sizes. Each of the test instances is implemented under three uncertainty levels (i.e., ρ = 0.2, 0.5, and 1). First, the DO and RO models are solved under nominal data for uncertain parameters. The uncertainty levels are considered the same to analyze the performance of two optimization models.

In the proposed model, the uncertainty set corresponding to each uncertain parameter are regarded as $[no\min alvalue - \rho_0 G_0^*, no\min alvalue + \rho_0 G_0^*]$ such that the robust model is finally feasible. There is a possibility that the decision variables regarding scheduling, delay, overtime, and the deviation of desired staff member time windows are updated in two models under each realization. However, since the routing of each staff member cannot be changed in a short time, it cannot be changed under realization.

The uncertainty is considered only for transfer times and accessibility level to staff members is assumed to be deterministic ($\rho_g = 0$) to evaluate the effects of uncertainty levels on the objective function values. The experimental results under uncertain transfer times are reported in Table 4, where the results of the models (using nominal values for transfer times) are surveyed under three different uncertainty levels for instances with 10 real patients. The results of the DO model are compared with the RO model. The *branch and bound* (B&B) algorithm is used in GAMS to solve optimization models. The following table represents the optimal values of the objective function ($W_{B\&B}$), the relative gap between the lower bound of the objective function values (LB) and $W_{B\&B}$. Since $W_{B\&B}$ and LB are the same values for all generated instances (i.e., they are solved exactly), the relative gaps are equal to 0.0% based on the equality relation $Relative-gap = \frac{(W_{B\&B} - LB)}{LB}$. As it is illustrated, the last column also denotes the gap between $W_{B\&B}$ obtained by the RO model ($W_{B\&B\text{-}RO}$) and the DO model (i.e., $W_{B\&B}$) using the equation $gap = \frac{(W_{B\&B-RO} - W_{B\&B})}{W_{B\&B}} \times 100$.

[Please insert Table 4 about here]

Three modes of uncertainty levels are considered for uncertain parameters in the robust model. If the uncertainty level of stochastic parameters is zero, their nominal values are considered, and the RO model is equivalent to the corresponding deterministic model with nominal values. On the contrary, uncertainty is wholly reflected in uncertain parameters when the uncertainty level is assumed to be one. One can distinguish between this situation with the most pessimistic situation, and it is planned based on the worst-case scenario instead of considering the robust solutions with ρ=1. This is very pessimistic and will not necessarily be occurred in reality, and as a result, the robust solutions are practically more efficient than the worst-case scenario. In the robust approach, the aim is to decide for the first-stage variables such that the objective value has to be minimized under uncertainty.

As is expected, the objective function value is increased by considering more uncertainty in the minimization problems, which is a regular outcome of adding uncertainty and increasing the variables and constraints. According to the table, if the uncertainty levels increase, the objective function values also increase in all realizations except for the test instance B1 in ρ= 0.2 and 0.5 and B2 in ρ=0.2. Increasing the uncertainty level considers a greater interval than the nominal values and may be caused to the deterioration of the objective function values. Increasing transfer times might lead to decreasing delays, overtime, and deviation from the desired time window specified by staff members. This issue makes the objective function values of the RO model lower than the DO one. The experimental results for the RO model under uncertain transfer times and accessibility to staff members and the DO model, using nominal values for two uncertain parameters, are reported in Table 5.

In the next step, uncertainty is extended to the accessibility to staff members. We obtain more efficient solutions in Table 5 compared to Table 4, considering the uncertainty in uncertain parameters. Such a model is closer to reality and is comparable with the two-stage stochastic programming model presented in the previous section.

According to the results in Table 5, the RO model has higher objective function values than DO model in all cases. A distribution function is approximately considered for uncertain parameters in the two-stage SO model, in which twenty scenarios with uniform distribution functions are considered. Obtaining perfect information on the distribution of stochastic parameters is difficult and time-consuming, and it is impossible in most cases.

[Please insert Table 5 about here]

Deterministic solutions may lead to infeasible solutions by taking uncertainty. If only the transfer times are uncertain, this event does not happen. Since fluctuating in the transfer times only leads to changing the objective function's optimal value and no constraints are violated, the deterministic solutions also remain feasible. However, if there is uncertainty about the accessibility level to staff members, then the deterministic solutions may not be not only optimal but also feasible. Lack of full accessibility to staff members may disrupt providing services, and as a result, all patients may not be received their requirement services in given time intervals and may lead to problem infeasibility with this number of staff members. For this purpose, outsourcing requirements to other centers (i.e., using the staff members outside the health center) or employing additional staff could be used with higher costs to get patients' satisfaction. As a result, the robust approach is justified when the accessibility to staff members and the transfer times is assumed to be uncertain. The health centers prepare themselves to cope with a possible worst-case scenario.

The RO model determines the routes of each staff member such that they can also be used in a possible worst-case of a critical parameter. Therefore, further staff members would be employed compared to the DO and RO models. The DO model gives rise to more efficient solutions for nominal data, but it may lead to infeasible solutions for many other realizations, and the amount of uncertainty is further increased as the uncertainty levels increase. With the increasing uncertainty level for the accessibility rate to staff members, the infeasibility rate of the DO model also increases. Hence, the application of the RO model is entirely justified with increasing uncertainty levels. Numerical experiments demonstrate the efficacy of the RO model in controlling uncertainty. However, considering the most fluctuation in uncertain parameters, the RO approach would be followed with higher costs.

5. Main achievements and Managerial insights

The following achievements for our proposed mathematical model would be summarized:

1. The HHC managers would plan more accurate and comprehensive schedules via the proposed models, in which the preferences of the patients and staff members are taken into consideration.
2. Since a single staff can provide two interdependent services with serial orders, the staff members are efficiently employed, and the total costs might be remarkably reduced.
3. We assure a feasible solution for all problems, in which all service requirements are performed on time or with limited delays based on the given time windows for patients.
4. Incorporating the measure of CoC into the model makes the relationship between patients and staff members more friendly. Therefore, the service duration might be relatively decreased.

The following managerial insights can be concluded for our RO model:

1. Optimum planning: Since we develop a RO model, the HHC centers are prepared for the pessimistic event. Although the number of service requirements performed by skilled staff members might be decreased, the quality of providing services is improved in terms of satisfying patients' time windows and reducing overtime. Furthermore, some extra cost might be imposed into the objective because of increasing total waiting time for staff. However, it might cause an increased satisfaction for patients leading to more benefit in long-term perspectives.
2. Improvement: HHC centers do not usually have a robust solution once inevitable events triggers the guarantee of providing services. To improve the quality of providing services, all policy-makers need to be engaged for developing a substitute policy to reduce risks particularly for rerouting and rescheduling of providing services.
3. Cost-effectiveness with regard to risk reduction: Since it is not cost effective to employ extra human resources for occasional events, the existing resources must be efficiently used to avoid spending unnecessary budget. Robust planning performance measures are cost-effective, prevent unsatisfactory of patients and ensure effective response to service requirements. Although re-planning staff members is a time-consuming process, decision- makers can find the most proper plan by the model within the shortest timeframe.

6. Conclusions

Due to an increasing average population aging and life expectancies, the necessity to provide services at homes is felt more than ever. The accidental and unpredictable nature of traffic conditions and the accessibility level to staff members on a certain day/week/month necessitates finding efficient policies for providing services, called home health care (HHC). The main goals of providing HHC services are to control and manage some important criteria such as the total traveling times, the number of service delays, and the overtime of staff members. Considering uncertainty in the HHC network would increase the effectiveness of the policy by properly handling the traffic conditions and accessibility level of staff members and subsequently increases the responsiveness of HHC firms. Providing HHC services deals with specific skills, different types, and speeds of vehicles, consideration of overtime and waiting time, the meal-time period for each staff, predetermined time windows for patients and staff members, Continuity Of Care (COC), interdependent required services, and blood sampling requirements.

We attempted to incorporate all the above-mentioned features into an optimization model with consideration of uncertainty. Two optimization models have been extended to cope with uncertainty: two- stage Stochastic Optimization (SO) and Robust Optimization (RO) models. The first one is designed based on the different scenarios. The traffic situations and the availability level to qualified staff members are represented as various scenarios for real cases. In the second, the uncertainty is handled using the defined bounds in closed boxes, specified by expert or some limited historical data. Moreover, as the RO model is used to obtain the policy for the pessimistic scenario, this policy would be feasible for all scenarios happening in the future when it is utilized for decision-making process in real-time situations.

We have also demonstrated that taking decision based on the expected value of uncertain parameters through a deterministic model does not usually end up with an efficient policy. Using some generated instances, we have demonstrated that the decisions with consideration of future scenarios and their probabilities or solely a pessimistic scenario might be better than the solution obtained using the expected value of uncertain parameters. Therefore, a scenario- or pessimistic-based decision-making might be usually suitable in the case of providing HHC services.

To make the presented models reliable for real situations, one can propose stochastic and robust formulations when the service time parameter is uncertain. The stochastic model can also be studied as a multistage stochastic programming model, which might improve the respective policy in real cases. Furthermore, other general forms of uncertainty sets, such as ellipsoidal uncertainty sets, may be taken into account in development of RO models. A deterministic or metaheuristic solution approach need to be developed to cope with the large-scale RO or SO models. In addition, a good forecasting model could be extended to predict the traffic congestion or travelling times using the historical data. In this situation, the respective scenarios would be generated for the residual terms of the forecasting model. These are left for future studies.

7. Declarations

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The paper has not been published in other journals. It is an original work performed jointly and solely by the two authors as a part of research in the Master thesis of the first author under the supervision of the second author. Both two authors are consent to submitting and publishing the paper in the Journal. There are no conflicts of interest/competing interests for this paper. The data and the respective codes are not included in the submission files; however, they are available if requested by the Editor-in-chief. The authors have written all the text, and the respective sentences in the literature review and the introduction sections have been suitably cited and restructured such that they became our own words with sufficient dissimilarity.

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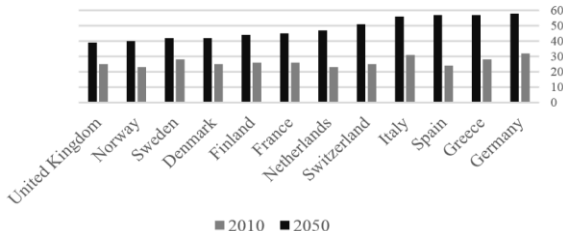

Figure 1 The ratio of old-age dependency in some European countries (The ratio refers to the estimated proportion of individuals aged 65 and above expressed as a percentage of the estimated proportion of individuals aged between 15 and 64 (Eurostat, last updated: 17–06–2011))

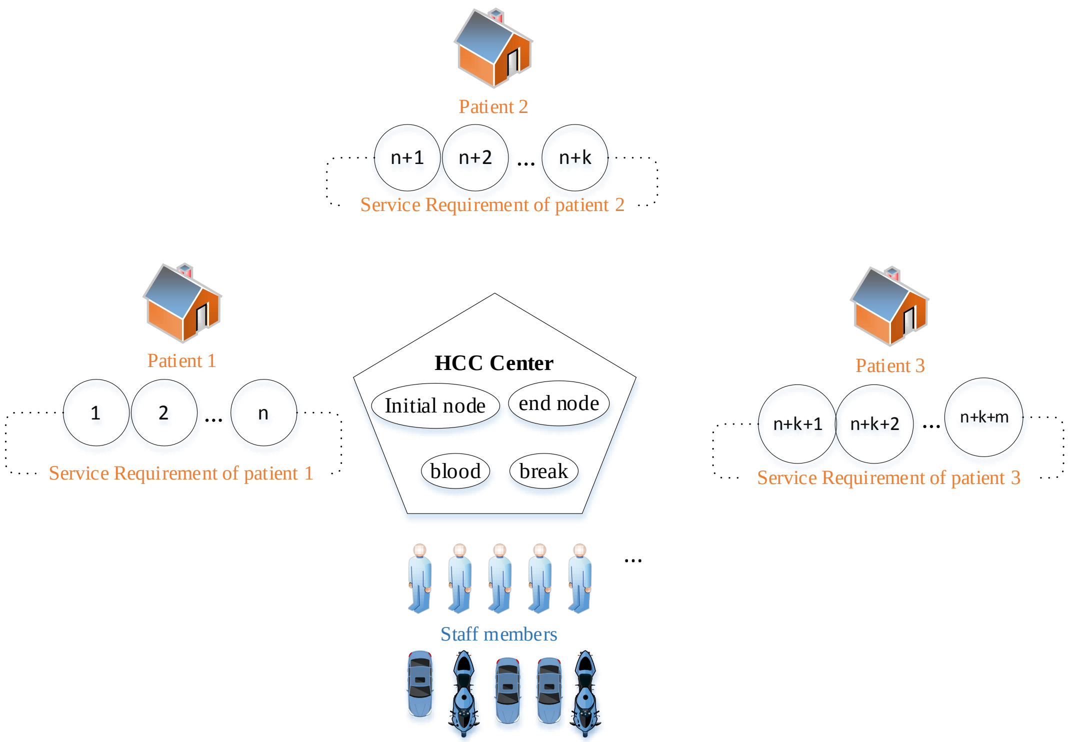

Figure 2 The logistic network of providing HHC services

Table 1 Index sets, Parameters, and decision variables of the model

Index sets	**Explanations**
P	Index set of all services required by the patients
V	Index set of all staff members
P^0	Index set of all nodes on the logistics network; i.e., $P^0 = P \cup \{0, n+1, blood, meal\}$
P^d	Set of pair of nodes requiring timely interdependent services
P^{sim}	Set of pair of nodes requiring simultaneous services
P^{prec}	Set of pair of nodes requiring services with precedence
P^+	Index set of all patients (nodes) requiring a blood sampling
Deterministic Parameters	**Explanations**
q_{vi}	The staff members' skills are equal to 1 if staff member $v \in V$ is qualified to perform the required service at node $i \in P^0$ and 0 otherwise.
$\delta_{ij}^{\min}$	The minimal time interval between service start times at nodes i and j, i.e., $(i,j) \in P^d$
$\delta_{ij}^{\max}$	The maximal time interval between service start times at nodes i and j, i.e., $(i, j) \in P^d$
$[e_i, l_i]$	The time window for the start of the required service at node $i \in P^0$
dur_i	The time duration of performing the service required by node $i \in P^0$
d_{ij}	Traveling distance between nodes $i \in P^0$ and $j \in P^0$
COC_{jv}	The preferences of patients, which is equal to 1 iff node $j \in P^0$ prefers staff member $v \in V$ for providing its required service and 0 otherwise
$worktime_v$	The contract working time of staff member $v \in V$
$[estaff_v, lstaff_v]$	Time window of staff member $v \in V$
T	Time frame
M	A very large number, which is equal to the maximum working time allowed to staff
$\alpha_1, \alpha_2, \alpha_3, \alpha_4,$ and α_5	The coefficients of the respective terms in the objective function
$\overline{tt}_{ijv}$	The nominal value of the transfer time from node $\boldsymbol{i} \in \boldsymbol{P^0}$ to node $\boldsymbol{j} \in \boldsymbol{P^0}$ in the route of staff member $\boldsymbol{v} \in \boldsymbol{V}$
$\overline{g}$	The nominal value of the availability level to staff members
ρ_{tt}	The uncertainty level of the uncertain parameter tt_{ijv}
ρ_g	The uncertainty level of the uncertain parameter g
γ_{ijv}^{tt}	The uncertainty scale of the uncertain parameter tt_{ijv}
θ^g	The uncertainty scale of the uncertain parameter g
Uncertain parameters	**Explanations**
tt_{ijv}	The transfer time from node $\boldsymbol{i} \in \boldsymbol{P^0}$ to node $\boldsymbol{j} \in \boldsymbol{P^0}$ in the route of staff member $\boldsymbol{v} \in \boldsymbol{V}$ (proportional to the distance $\boldsymbol{d_{ij}}$)
g	The availability level to staff members
Decision variables	**Explanations**
r_{ijv}	The routing variables of the staff members If staff $v \in V$ moves from $i \in P^0$ to $j \in P^0$ to provide the service required by the node j, $r_{ijv} = 1$; otherwise, $r_{ijv} = 0$
t_{iv}	The start time of providing service at node $i \in P^0$ by staff member v
z_i	Delay time of service required by node $i \in P^0$
ov_v	The overtime of staff member $v \in V$
ltw_v, utw_v	The deviations from the desired time window of staff member $v \in V$
φ_{ijv}^{tt}	The variations of the uncertain parameter tt_{ijv} in a closed bounded box
β^g	The variations of the uncertain parameter g in a closed bounded box

Table 2 Characteristics of datasets

Dataset	Number of patients	Number of patients requiring interdependent services	Number of nodes along with the dummy nodes ($\|P^0\|$)	Number of staff members ($\|V\|$)
A	7	2	13	3
B	10	3	17	3

Table 3 The values of input parameters

Parameter	Value	Description
α_1	1	The coefficient associated with the first performance measure in (1) and (2)
α_2	2	The coefficient associated with the second performance measure in (1) and (2)
α_3	2	The coefficient associated with the third performance measure in (1) and (2)
α_4	1	The coefficient associated with the fourth performance measure in (1) and (2)
α_5	1	The coefficient associated with the fifth performance measure in (1) and (2)
f_v	0.80, 0.80 and 0.68	The conversion coefficient of traveled distance to transfer time
$\delta_{ij}^{\min}$	[0, 70]	The minimal time interval between service start times at each of two interdependent nodes i and j (in minutes)
$\delta_{ij}^{\max}$	$\delta_{ij}{}^{min}$ + [5, 50]	The maximal time interval between service start times at each of two interdependent nodes i and j (in minutes)
dur_i	[5, 40]	The time duration to carry out the service required by patient-related node$i$ (in minutes)
T	600, 1000	Time frame (in minutes)

Table 4 Numerical results of DO and RO models for uncertain transfer times with different sizes

Problem size ($(\|\boldsymbol{P0}\|*\|\boldsymbol{P0}\|-\mathbf{1}*\|\boldsymbol{V}\|)$)	**Instance**	**Uncertainty level** ($(\boldsymbol{\rho})$)	**DO model**		**RO model**	
			$W_{B\&B}$	relative gap%	$W_{B\&B-RO}$	relative gap%
$13\times12\times3$	B1	0.2	180.579	0.00	145.762	-19.28
		0.5			175.208	-2.97
		1			226.810	25.60
	B2	0.2	246.181	0.00	229.178	-6.91
		0.5			303.337	23.22
		1			444.618	80.61
	B3	0.2	345.537	0.00	363.644	5.24
		0.5			390.805	13.10
		1			441.798	27.86
	B4	0.2	405.980	0.00	433.376	6.75
		0.5			488.224	20.26
		1			563.585	38.82
	B5	0.2	248.395	0.00	259.712	4.56
		0.5			275.890	11.07
		1			302.853	21.92
	B6	0.2	676.990	0.00	705.388	4.19
		0.5			747.985	10.49
		1			833.572	23.13
	B7	0.2	632.497	0.00	673.996	6.56
		0.5			740.386	17.06
		1			915.216	44.70
$17\times16\times3$	C1	0.2	493.777	0.00	Infeasible	—
		0.5			Infeasible	—
		1			Infeasible	—
	C2	0.2	620.683	0.00	Infeasible	—
		0.5			Infeasible	—
		1			Infeasible	—
	C3	0.2	376.333	0.00	409.397	8.79
		0.5			442.496	17.58
		1			504.361	34.02
	C4	0.2	255.111	0.00	309.025	21.13
		0.5			372.443	45.99
		1			481.090	88.58
	C5	0.2	729.383	0.00	762.859	4.59
		0.5			829.868	13.78
		1			976.641	33.90
	C6	0.2	257.923	0.00	280.107	8.60
		0.5			313.384	21.50
		1			391.268	51.70
	C7	0.2	648.969	0.00	710.363	9.46
		0.5			803.214	23.77
		1			978.264	50.74

Table 5 Numerical results of the DO and RO models for two uncertain parameters with different sizes

Problem size ($\|P0\|*\|P0\|-1*\|V\|$)	Instance	Uncertainty level (ρ)	DO model $W_{B\&B}$	DO model relative gap%	RO model $W_{B\&B-RO}$	RO model relative gap%
$13 \times 12 \times 3$	B1	0.2	497.321	0.00	642.177	29.13
		0.5			854.817	71.88
		1			1215.422	144.39
	B2	0.2	856.161	0.00	964.795	12.69
		0.5			1128.922	31.86
		1			1435.882	67.71
	B3	0.2	300.490	0.00	407.552	35.63
		0.5			597.691	98.91
		1			912.550	203.69
	B4	0.2	1084.281	0.00	1193.587	10.08
		0.5			1350.484	24.55
		1			1613.650	48.82
	B5	0.2	1077.873	0.00	1224.368	13.59
		0.5			1442.869	33.86
		1			1835.359	70.28
$17 \times 16 \times 3$	C1	0.2	594.012	0.00	679.415	14.38
		0.5			819.859	38.02
		1			1122.929	89.04
	C2	0.2	525.036	0.00	623.920	18.83
		0.5			825.493	57.23
		1			1174.658	123.73
	C3	0.2	1171.970	0.00	1285.644	9.70
		0.5			1472.949	25.68
		1			1892.057	61.44
	C4	0.2	344.793	0.00	479.481	39.06
		0.5			718.277	108.32
		1			1185.276	243.76
	C5	0.2	1321.667	0.00	1468.601	11.12
		0.5			1725.028	30.52
		1			2231.231	68.82