# Corrections to "On the global practical stabilization of discrete-time switched affine systems: application to switching power converters" [Scientia Iranica (2021) 28(3), 1621-1642] and "Global Practical Stabilization of Discrete-time Switched Affine Systems via Switched Lyapunov Functions and State-dependent Switching Functions" [Scientia Iranica (2021) 28(3), 1606-1620]

Mohammad Hejri*

*Department of Electrical Engineering, Sahand University of Technology, P.O. Box: 51335-1996, Sahand New Town, Tabriz, Iran*

**Abstract**

This note corrects the proof of Theorem 1 in [1], the statement and proof of Lemma 1, and a part of proofs in Theorems 1 and 2 of [2].

*Keywords:* Discrete-time Switched Affine Systems, Stabilization, Bilinear Matrix Inequalities (BMIs), DC-DC Switching Power Converters, Practical Stability

## 1. Corrections to [1]

A step is incorrect in the proof of Theorem 1 of [1] when showing a positive constant $\gamma$ exists to satisfy item (b) of Lemma 1. The error lies in the definition of function $\phi(s)$ in (35) where the norm function $||e(k)||$ must be replaced by the distance function $||e(k)||_{\mathcal{E}(P)} = \inf_{y\in\mathcal{E}} ||e(k) - y||$ denoting the distance from point $e(k)$ to the ellipsoid $\mathcal{E}(P)$. Therefore, using (32) in [1] and Lyapunov set asymptotic stability arguments [3], we can only conclude that $\phi(||e(k)||_{\mathcal{E}(P)}) \to 0$ as $k \to \infty$, and the existence of a positive constant $\gamma$ is not proved.

### *1.1. Correction to the proof of Theorem 1*

In [1], pages 1627-1628, a new relation (25) in this note is added, minor modifications are made in Eqs. (25)-(32) of [1] by changing " $< 0$" to " $\leq -\gamma$" in these relations, and arguments between (32) and (37) of [1] are omitted.

**Remark 1.** *According to the preceding modifications in the proof of Theorem 1 in [1], the relation numbers (25)-(32) in [1] must be increased by 1. For instance, the equation number (32) in [1] must be changed to (33). Furthermore, the equation numbers (38)-(62) in [1] must be reduced by 4. For example, the relation (62) must be changed to (58) in the modified version.*

In what follows, all equation numbers are set based on the comment in Remark 1 of this note. The rest of correct proof of Theorem 1 in the paragraph after (24), page 1627 of [1], is given in below.

In the sequel, we plan to prove the attractiveness of the set $\mathcal{E}(P)$ via Relations (7) and (8) and $v(e(k)) = e(k)^T Pe(k)$. From (7), one can conclude $\exists\gamma > 0$ such that

$$\begin{bmatrix} \tau_2 P + \tilde{M}_1 - \sum_{i\in\mathbb{K}} \lambda_{2i} P & * \\ \tilde{M}_2 & \tilde{M}_3 - \tau_2 \end{bmatrix} \leq -\gamma I_{n+1} \tag{25}$$

*Corresponding author, Tel: +984133459363, Mob: +989127969411, Fax: +984123444322
*Email address:* hejri@sut.ac.ir (Mohammad Hejri*)

This comes from the fact that for any real symmetric matrix $M = M^T \in \mathbb{R}^{n\times n}$ one can write $\lambda_{min}(M)I_n \leq M \leq \lambda_{max}(M)I_n$. This inequality can be derived via Rayleigh's inequality in the context of the Matrix Analysis. Now, if $M$ is a negative definite matrix as well, i.e. $M = M^T \prec 0$, then its all eigenvalues are negative, including $\lambda_{max}(M)$. Therefore, one can choose $-\gamma = \lambda_{max}(M)$ which always exists. Pre-multiplying Relation (25) by $[e(k)^T 1]$ and post-multiplying by $[e(k)^T 1]^T$ one can obtain

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tau_2 P + \tilde{M}_1 - \sum_{i\in\mathbb{K}} \lambda_{2i} P & * \\ \tilde{M}_2 & \tilde{M}_3 - \tau_2 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma, \forall e(k) \in \mathbb{R}^n \tag{26}$$

Relation (26) can be rewritten as Relation (27).

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_1 - \sum_{i\in\mathbb{K}} \lambda_{2i} P & * \\ \tilde{M}_2 & \tilde{M}_3 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} - \tau_2 \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P & * \\ 0_{1\times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma, \forall e(k) \in \mathbb{R}^n \tag{27}$$

Using S-procedure, Relation (27) implies Relation (28).

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P & * \\ 0_{1\times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} < 0 \Rightarrow \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_1 - \sum_{i\in\mathbb{K}} \lambda_{2i} P & * \\ \tilde{M}_2 & \tilde{M}_3 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma, \tag{28}$$

By substituting $\tilde{M}_1$, $\tilde{M}_2$ and $\tilde{M}_3$ from Eqs. (13)-(15) into Relation (28) and after some algebra, one can reach

$$e(k)^T Pe(k) > 1 \Rightarrow \sum_{i\in\mathbb{K}} \lambda_{2i}[(A_i e(k) + l_i)^T P(A_i e(k) + l_i) - e(k)^T Pe(k)] \leq -\gamma \tag{29}$$

According to Relation (9), since $\sum_{i\in\mathbb{K}} \lambda_{2i} > 0$, $\lambda_{2i} \geq 0, i \in \mathbb{K}$, Lemma 2 implies that Relation (29) can be rewritten as

$$e(k)^T Pe(k) > 1 \Rightarrow \exists i \in \mathbb{K} \text{ such that } (A_i e(k) + l_i)^T P(A_i e(k) + l_i) - e(k)^T Pe(k) \leq -\gamma \tag{30}$$

Relation (30) means for each $e(k)$ satisfying $e(k)^T Pe(k) > 1$ there exists an index $i$ such that $(A_i e(k) + l_i)^T P(A_i e(k) + l_i) - e(k)^T Pe(k) \leq -\gamma$. This is because outside the set $\mathcal{E}(P) = \{e(k) \in \mathbb{R}^n | e(k)^T Pe(k) \leq 1\}$ the switching function is not constant. According to the switching rule (5), we have

$$(A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P(A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))}) \leq (A_i e(k) + l_i)^T P(A_i e(k) + l_i) \tag{31}$$

From Relations (30) and (31) one can reach

$$\begin{aligned} e(k)^T Pe(k) > 1 \Rightarrow & (A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P(A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))}) - e(k)^T Pe(k) \\ & \leq (A_i e(k) + l_i)^T P(A_i e(k) + l_i) - e(k)^T Pe(k) \leq -\gamma \end{aligned} \tag{32}$$

or equivalently

$$\begin{aligned} e(k)^T Pe(k) > 1 \Rightarrow & v(A_{\sigma(e(k))} e + l_{\sigma(e(k))}) - v(e(k)) = v(e(k+1)) - v(e(k)) = \Delta v(e(k)) \\ & \leq (A_i e(k) + l_i)^T P(A_i e(k) + l_i) - e(k)^T Pe(k) \leq -\gamma \end{aligned} \tag{33}$$

which implies condition (b) in Lemma 1. Since $P = P^T > 0$, $v(e(k)) = e(k)^T Pe(k)$ is positive in the whole state space unless $e(k) = 0_{n\times 1}$ which satisfies condition (c) of Lemma 1. Therefore, all conditions of Lemma 1 are satisfied. Since during proof of Theorem 1 no restriction is imposed on the selection of $e(k)$, i.e., $e(k) \in \mathbb{R}^n$, as a result $D = \mathbb{R}^n$ and according to Remark 3 the switched affine system (2) is globally practically stable under switching rule (5) and the proof is concluded.

## 2. Corrections to [2]

In Lemma 1 of [2], the proposed conditions in items (c) and (d) do not imply the finite-time convergence of the trajectories according to item (c) of Def. 1. In fact, in the proof of Lemma 1, when we reach $\lim_{k\to\infty} v(e(k)) = h \geq 0$, the proposed condition in item (c) of Lemma 1, implies that

$$\lim_{k\to\infty} [v(e(k+1)) - v(e(k))] = h - h = 0 \leq \lim_{k\to\infty} -\phi(\|e(k)\|) < 0. \tag{1}$$

By this, one can conclude that $\lim_{k\to\infty} \phi(\|e(k)\|) = 0$. Therefore, from $\|e(k)\| \geq \gamma$, for some $\gamma > 0$, one cannot conclude that $\phi(\|e(k)\|) \geq \phi(\gamma)$, $\forall k \in \mathbb{Z}_{\geq 0}$. As a result, the existence of a positive constant $\phi(\gamma)$ is not proved and Relation (4) in [2] is not verified.

*2.1. Correction to the statement and proof of Lemma 1:*

The correct statement and proof for Lemma 1 of [2] are given in below.

**Lemma 1.** *System (2) is practically stable in the large in a given domain $D \subset \mathbb{R}^n$ containing the origin in the sense of Definition 3 if there exist a bounded set $\mathcal{V} \subset D$ and a scalar function $v(e(k)) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that*

*(a) $0_{n\times 1} \in \mathcal{V}$*

*(b) if $e(k) \in \mathcal{V}$ then $e(k+1) = A_{\sigma(e(k))}e(k) + l_{\sigma(e(k))} \in \mathcal{V}$*

*(c) if $e(k) \in D - \mathcal{V}$ then $v(e(k+1)) - v(e(k)) = \Delta v(e(k)) \leq -\gamma < 0$ where $\gamma$ is a positive constant.*

*(d) $v(e(k)) > 0$ where $e(k) \in D - \mathcal{V}$.*

*Proof.* Conditions (a) and (b) are the same as conditions (a) and (b) of Def. 1 and therefore, fulfill the invariant property of the bounded set $\mathcal{V}$. To prove the attractiveness property of the set $\mathcal{V}$, according to condition (c) of Definition 1 it is necessary to show that starting any initial state $e(0) \in D-\mathcal{V}$, there exists a finite time $T = T(e(0)) > 0$ such that for $k \geq T$ the state $e(k)$ eventually enters within the set $\mathcal{V}$, i.e., $\exists T > 0$ such that $e(k) \in \mathcal{V}$ for $k \geq T$. We show this by contradiction. Using condition (c), one can write

$$v(e(k)) = v(e(0)) + \sum_{n=0}^{k-1}(v(e(n+1) - v(e(n))) \leq v(e(0)) - k\gamma \tag{4}$$

The right side of Eq. (4) will be eventually negative when $k$ takes large values. This leads to contradiction to the condition (d) where it is assumed $v(e(k))$ is positive definite on $D-\mathcal{V}$. Therefore, there exists a finite time $T = T(\epsilon) > 0$ such that for $k \geq T$ the state $e(k)$ eventually enters within the set $\mathcal{V}$. As a result, $\mathcal{V}$ is an invariant set of attraction according to Definition 1, and therefore, according to Definition 3, system (2) is practically stable in the large on the domain $D$ and under switching function $\sigma(e(k))$. Thus, the proof is completed. □

**Remark 2.** *During foregoing modifications in Lemma 1 proof, the relation (5) in [2] is omitted, and relation numbers in the range (6)-(27), must be reduced by 1. For instance, the relation number (27) must be changed to (26).*

In what follows, a part of Theorems 1 and 2 proofs established based on items (b) and (c) of Lemma 1 in [2] are modified as follows.

*2.2. Correction to a part of the proof of Theorem 1:*

The correct proof of Theorem 1 for the implication of the attractiveness property of the set $\mathcal{V}$, in the paragraph after (27), page 1612 of [2], is given in below. In summary, a new equation (27) in this note is added, minor modifications are made in Eqs. (28)-(38) of [2] by changing " $<0$" to " $\leq -\gamma$" in these relations, and arguments between (38) and (42) of [2] are omitted. In the following, a detailed explanation is given where all equation numbers are adjusted based on the Remark 1 of this note.

In the sequel, we plan to prove the attractiveness of the set $\mathcal{V}$ via matrix inequalities (12). From (12), one can conclude $\exists\gamma > 0$ such that

$$\begin{bmatrix} \tau_{2h}P_h + \tilde{M}_{1h} & * \\ \tilde{M}_{2h} & \tilde{M}_{3h} - \tau_{2h} \end{bmatrix} \leq -\gamma I_{n+1}, \forall h \in \mathbb{K} \tag{27}$$

Pre-multiplying matrix inequalities (27) by $[e(k)^T 1]$ and post-multiplying by $[e(k)^T 1]^T$ one can obtain

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tau_{2h}P_h + \tilde{M}_{1h} & * \\ \tilde{M}_{2h} & \tilde{M}_{3h} - \tau_{2h} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma, \forall h \in \mathbb{K} \tag{28}$$

Relation (28) can be rewritten as Relation (29).

$$-\tau_{2h} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P_h & * \\ 0_{1\times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} + \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_{1h} & * \\ \tilde{M}_{2h} & \tilde{M}_{3h} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma, \forall h \in \mathbb{K} \tag{29}$$

Using S-procedure, from Relation (29) one can conclude (30), $\forall e(k) \in \mathbb{R}^n$:

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P_h & * \\ 0_{1\times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} < 0 \Rightarrow \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_{1h} & * \\ \tilde{M}_{2h} & \tilde{M}_{3h} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma, \quad \forall h \in \mathbb{K} \tag{30}$$

By substituting $\tilde{M}_{1h}$, $\tilde{M}_{2h}$ and $\tilde{M}_{3h}$ from Eqs. (16)-(18) into Relation (30) and after some algebra, one can reach

$$e(k)^T P_h e(k) > 1 \Rightarrow \sum_{i\in\mathbb{K}} \beta_{hi} (\sum_{j\in\mathbb{K}} \lambda_{hj} [(A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k)]) \leq -\gamma, \forall h \in \mathbb{K} \tag{31}$$

Since $\beta_{hi} \geq 0$, $i, h \in \mathbb{K}$, and $\sum_{i\in\mathbb{K}} \beta_{hi} > 0$ according to Lemma 2 and (31) one can conclude

$$e(k)^T P_h e(k) > 1 \Rightarrow \exists i \in \mathbb{K} \text{ such that} \sum_{j\in\mathbb{K}} \lambda_{hj} [(A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k)] \leq -\gamma, \forall h \in \mathbb{K} \tag{32}$$

Again according to Lemma 2, since $\lambda_{hj} \geq 0$, $j, h \in \mathbb{K}$ and $\sum_{j\in\mathbb{K}} \lambda_{hj} > 0$, Relation (32) implies

$$e(k)^T P_h e(k) > 1 \Rightarrow \exists i, j \in \mathbb{K} \text{ such that } (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma, \forall h \in \mathbb{K} \tag{33}$$

Now, according to the definition of the set $\mathcal{V}$ in Eq. (3), we have

$$e(k) \notin \mathcal{V} \Rightarrow \exists h \in \mathbb{K} \text{ such that } e(k)^T P_h e(k) > 1 \tag{34}$$

From Relations (33) and (34), one can infer

$$e(k) \notin \mathcal{V} \Rightarrow \exists i, j \in \mathbb{K} \text{ such that } (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma \tag{35}$$

since $e(k) \notin \mathcal{V}$, according to Relation (5) and item 2) in the switching Algorithm 1, one can conclude there exist $\sigma(e(k)), i, j \in \mathbb{K}$ satisfying the following expression

$$(A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P_i (A_{\sigma(e(k))} e + l_{\sigma(e(k))}) - e(k)^T P_{\sigma(e(k))} e(k) \leq (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \tag{36}$$

From Relations (35) and (36) one can conclude that there exist $\sigma(e(k)), i \in \mathbb{K}$ such that

$$e(k) \notin \mathcal{V} \Rightarrow \exists i, j, \sigma(e(k)) \in \mathbb{K} \text{ such that } (A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P_i (A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))}) - e(k)^T P_{\sigma(e(k))} e(k) \leq (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma \tag{37}$$

or equivalently

$$e(k) \notin \mathcal{V} \Rightarrow \exists i, j \in \mathbb{K} \text{ such that } v(e(k+1)) - v(e(k)) = \Delta v(e(k)) \leq (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma \tag{38}$$

which implies condition (c) in Lemma 1. Since $P_i = P_i^T > 0$, $v(e(k)) = e(k)^T P_{\sigma(e(k))} e(k)$ is positive in the whole state space unless $e(k) = 0_{n\times 1}$ which satisfies condition (d) of Lemma 1.

Therefore, according to Relations (38) and (26), all conditions of Lemma 1 are fulfilled. Since during the proof of Theorem 1 no restriction is imposed on the selection of $e(k)$, i.e., $e(k) \in \mathbb{R}^n$, as a result $D = \mathbb{R}^n$ and according to Rem. 2 the switched affine system (2) is globally practically stable under switching Algorithm 1 and the proof is concluded.

**Remark 3.** *During the preceding modifications in a part of Theorem 1 proof, the relations (39)-(42) of [2] are omitted. Thus, the relation numbers in the range (43)-(50) of [2] must be reduced by 4. A new equation (47) is added through this note. Finally, all equation numbers in the range (51)-(68) of [2] are reduced by 3.*

*2.3. Correction to a part of the proof of Theorem 2:*

The correct proof of Theorem 2 for the implication of the attractive property of the set $\mathcal{V}$, in the paragraph after (50) of [2], is given in below. In summary, a new relation (47) is added by this note, and minor modifications are made to the relations (51)-(60) of [2] by changing " $<0$" to $\leq -\gamma$ in these relations. In the following, a detailed explanation is given where all equation numbers are adjusted based on the Remark 3 of this note.

In the sequel, the attractive property of the set $\mathcal{V}$ is proved via conditions in (40). From (40), one can conclude $\exists\gamma > 0$ such that

$$\begin{bmatrix} \tau_{2hi}P_h + \tilde{M}_{1hi} & * \\ \tilde{M}_{2hi} & \tilde{M}_{3hi} - \tau_{2hi} \end{bmatrix} \leq -\gamma I_{n+1}, \forall i, h \in \mathbb{K} \tag{47}$$

Pre-multiplying matrix inequality (47) by $[e(k)^T 1]$ and post-multiplying by $[e(k)^T 1]^T$ one can obtain

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tau_{2hi}P_h + \tilde{M}_{1hi} & * \\ \tilde{M}_{2hi} & \tilde{M}_{3hi} - \tau_{2hi} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma, \forall i, h \in \mathbb{K} \tag{48}$$

Relation (48) can be rewritten as:

$$-\tau_{2hi} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P_h & * \\ 0_{1\times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} + \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_{1hi} & * \\ \tilde{M}_{2hi} & \tilde{M}_{3hi} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma, \forall i, h \in \mathbb{K} \tag{49}$$

Using S-procedure, from Relation (49) one can reach to (50), $\forall e(k) \in \mathbb{R}^n$.

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P_h & * \\ 0_{1\times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} < 0 \Rightarrow \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_{1hi} & * \\ \tilde{M}_{2hi} & \tilde{M}_{3hi} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \leq -\gamma, \quad \forall i, h \in \mathbb{K} \tag{50}$$

By substituting $\tilde{M}_{1hi}$, $\tilde{M}_{2hi}$ and $\tilde{M}_{3hi}$ from Eqs. (44)-(46) into Relation (50) and after some algebra, one can reach

$$e(k)^T P_h e(k) > 1 \Rightarrow \sum_{j\in\mathbb{K}} \lambda_{hj}[(A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k)] \leq -\gamma, \forall i, h \in \mathbb{K} \tag{51}$$

Since $\lambda_{hj} \geq 0$, $j, h \in \mathbb{K}$, and $\sum_{j\in\mathbb{K}} \lambda_{hj} > 0$, according to Lemma 2, Relation (51) implies Relation (52).

$$e(k)^T P_h e(k) > 1 \Rightarrow \exists j \in \mathbb{K} \text{ such that } (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma, \forall i, h \in \mathbb{K} \tag{52}$$

According to the definition of the set $\mathcal{V}$ in (3), we have

$$e(k) \notin \mathcal{V} \Rightarrow \exists h \in \mathbb{K} \text{ such that } e(k)^T P_h e(k) > 1 \tag{53}$$

Now from Relations (52) and (53), one can reach

$$e(k) \notin \mathcal{V} \Rightarrow \exists j \in \mathbb{K} \text{ such that } (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma, \forall i \in \mathbb{K} \tag{54}$$

since $e(k) \notin \mathcal{V}$, according to Relation (54) and Item (2) in the switching Algorithm 1, one can conclude there exists a $\sigma(e(k))$ satisfying the following expression:

$$\begin{aligned}(A_{\sigma(e(k))}e(k) + l_{\sigma(e(k))})^T P_i (A_{\sigma(e(k))}e + l_{\sigma(e(k))}) - e(k)^T P_{\sigma(e(k))}e(k) \leq \\ (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k), \forall i \in \mathbb{K}\end{aligned} \tag{55}$$

From Relations (54) and (55) one can reach

$$\begin{aligned}e(k) \notin \mathcal{V} \Rightarrow \exists j \in \mathbb{K} \text{ such that } (A_{\sigma(e(k))}e(k) + l_{\sigma(e(k))})^T P_i (A_{\sigma(e(k))}e(k) + l_{\sigma(e(k))}) - e(k)^T P_{\sigma(e(k))}e(k) \leq \\ (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma, \forall i \in \mathbb{K}\end{aligned} \tag{56}$$

or equivalently

$$e(k) \notin \mathcal{V} \Rightarrow \exists j \in \mathbb{K} \text{ such that } v(e(k+1)) - v(e(k)) = \Delta v(e(k)) \leq (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma, \forall i \in \mathbb{K} \quad (57)$$

Similar to our argument in Theorem 1, the attractive property of the set $\mathcal{V}$ is inferred according to items (a), (c), and (d) of Lemma 1. Therefore, all conditions of Lemma 1 are fulfilled. Similar to Theorem 1, since no restriction is imposed on the selection of $e(k)$, namely, $e(k) \in \mathbb{R}^n$, therefore $D = \mathbb{R}^n$ and according to Rem. 2 the switched affine system (2) is globally practically stable under switching Algorithm 1 and the proof is completed.

## References


[1] Hejri, M. "On the global practical stabilization of discrete-time switched affine systems: application to switching power converters", *Scientia Iranica, Transaction D, Computer Science & Electrical Engineering*, **28**(3), pp. 1621-1642 (2021).

[2] Hejri, M. "Global Practical Stabilization of Discrete-time Switched Affine Systems via Switched Lyapunov Functions and State-dependent Switching Functions", *Scientia Iranica, Transaction D, Computer Science & Electrical Engineering*, **28**(3), pp. 1606-1620.

[3] Goebel, R., Sanfelice, R. G., and Teel, A. R. "*Hybrid Dynamical Systems: Modeling, Stability and Robustness*" Princeton University Press (2012).


## Biography

Mohammad Hejri, for biography see p. 1642, or pp. 1619–1620 of 2021, vol. 28, issue 3 of this Transactions.