Biogas Reverse Supply Chain Network Design based on Biomass Quality Levels using Robust Programming and Benders decomposition approach

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Abstract

Today, renewable energy generation infrastructures are increasingly developed due to reduced fossil fuel resources and increased energy consumption. In this respect, a biogas supply chain has a high potential to generate energy. This paper aims to design a bi-objective biogas supply chain network for power and fertilizer generation. A mixed-integer linear programming (MILP) model was developed for the multi-level biogas supply chain with biomass input under different parameter uncertainties. A stochastic-robust programming approach was adopted to cope with the intrinsic uncertainties of such value chains. Realistic uncertainty modeling allowed for adjusting the conservatism level for a trade-off between performance and robustness. The adopted stochastic-robust programming pathway not only diminished the optimality fluctuations and provided a reasonable allocation space for uncertainties but also enhanced network flexibility and alleviated decision-making risks. Finally, the model was solved using the Benders decomposition (BD) algorithm. This research obtained more efficient and effective solutions by enhancing the Benders cuts based on previously generated solutions and Pareto optimal cuts. The implemented algorithm converged to the optimal solution at a reasonable rate.

Keywords: *Biogas, Network design, Robust programming, Bender's decomposition, Biomass*

1. Introduction

The increased demand for fossil fuels has imposed substantial environmental impacts. In addition, wars, sanctions, and terrorism challenge access to oil and gas resources. In this regard, governments have sought to generate renewable and sustainable energy due to the increased energy consumption in public, industrial, and domestic sectors [1]. The lack of primary energy supply, energy security, environmental protection, and impact on climate change are among the most critical challenges facing the future of energy. Therefore, renewable energies will play an essential role in meeting the energy demand, diversifying the

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energy portfolio, and reducing the environmental effects caused by the increase in energy consumption [2]. In this respect, in 2019, the world's new renewable capacity grew by 10.3% compared to the previous year. In the same year, the share of renewable energy in electricity production reached 29%, of which 16.8% was related to hydroelectric power. In 2019, about 11.2% of the world's energy consumption for heating, energy production, and transportation was provided through renewable energies, including biomass, hydroelectricity, wind, and biofuel [3]. According to the New Energy Association of the European Community report, in the most optimistic case, half of the world's energy in 2040 can be supplied by new energies. In the most pessimistic case, this ratio will not be less than 27%. In this regard, it is predicted that in the United States of America, electricity production from biomass will double every ten years and mature from 4% of the electricity market and industry in 2010 to 5% in 2030. Hence, The European Union (EU) required countries to supply over 30% of the energy demand using renewable energy resources by 2023 [4].

Biomass energy can be obtained from various sources, such as agricultural and forest residues, energy products, animal fats, urban waste, and animal waste. Low environmental pollutants, dispersion, and easy access are key factors considered when choosing renewables. Biomass energy sources can provide domestic and industrial needs in the main form of electricity or energy carriers such as gaseous and liquid fuels. In this regard, biogas is one of the leading carriers of energy from biomass resource processing. The product of anaerobic digestion is a gas with a medium calorific value called "biogas". In converting biogas into electricity, 1.5 to 2.2 kilowatt hours of electricity can be produced from each cubic meter (m^3) when using existing biogas engines [5].

The major advantages of biomass are as follows [6-8]:

- Biofuel generation across the world to enhance energy security
- Eco-friendly energy with lower negative impacts on the ecosystem
- Biodegradability, renewability, and contribution to sustainability
- Development of poultry and associated industries
- Contribution to regional growth and job opportunities
- Abundance and availability

These advantages diminish energy generation costs and enhance social responsibility performance, contributing to sustainable social growth [9]. Hence, designing a biomass supply chain network would strongly contribute to economic development and have positive environmental outcomes. Such a design may also enhance sustainability aspects in supply chain management [10, 11]. The coordination of material flows and raw material uncertainty remain significant challenges of a biomass supply chain, particularly in the commercial-scale implementation of biofuel projects. A noteworthy issue to handle in this field is the coordination of resource provision for energy generation and storage [12, 13].

Biomass raw materials for energy generation are classified into four groups [14-16]:

- First-generation biofuels: They are obtained from food industries, such as starch, sugar, and herbal oils.
- Second-generation biofuels: These biofuels are obtained from inedible materials, e.g., agricultural waste, forestry, energy products, and urban and rural municipal waste.
- Third-generation biofuels: These biofuels (e.g., bio-oil) are produced from algae.
- Fourth-generation biofuels: They include engineering plants or biomass.

Uncertainties are also a significant challenge in managing biomass supply chain networks. Climatic and geographical parameters impose substantial uncertainties on the supply of biomass resources. Biomass logistics, e.g., price, storage, and transportation costs, also have uncertainties. These uncertainties are significant in bio-refineries and combined biomass power plants, including technology, production rate, and operational cost uncertainties [17]. In addition, economic fluctuations strongly impact the demand, generation, distribution capacity, and product quality, disturbing investments in some cases [18]. The uncertainties even worsen in the event of floods, earthquakes, sanctions, and droughts, thereby challenging network management and leading to complete network failure [19].

The present study developed a mathematical bi-objective supply chain to produce biogas (primary product) and a bio fertilizer (secondary product). The objective function would guarantee network performance by maximizing profitability. This function included selling the primary and secondary products, investment, storage, excess demand, and penalizing unmet customer demand. Moreover, a mixed-integer linear programming (MILP) model was developed for a multi-product reverse biogas supply chain network with production, distribution, and recycling quality levels. The model would maximize profitability under the pessimistic scenario. The strategic decisions included location, quality maximization at high-pressure levels of the network, ensuring the inventory, network sustainability improvement through anaerobic digestion enhancement, and enhancing the power generation capacity. A hybrid stochastic-robust programming approach with flexibility and adjustable conservatism was also proposed to establish a trade-off between model performance and robustness to cope with uncertainties. The model would remain robust while generating a solution that would remain reasonable for the entire uncertain dataset. The Benders decomposition was used to solve the model. Meanwhile, the upper and lower bounds were improved by generating optimal cuts.

The remainder of the paper is organized as follows: Section 2 provides a literature review on the research topic. Section 3 provides a problem statement and describes the assumptions and mathematical model. Section 4 describes the hybrid stochastic-robust programming approach. Section 5 implements the Benders decomposition algorithm. Section 6 analyzes the results. Section 7 provides the sensitivity analysis of the two-stage approach. Finally, Section 8 concludes the paper.

2. Literature review

Sustainable industrial development requires sustainable resources. In this regard, raw materials and novel manufacturing approaches are required to set a sustainable industrial future. The unsustainability of fossil fuels and population growth have raised energy consumption [20], leading to global warming. Researchers have proposed exploiting renewable resources to reduce Greenhouse Gas (GHG) emissions. Biomass is a significant and eco-friendly energy resource [21, 22]. Since converting biomass into clean energy alleviates the dependence on fossil fuels, it is a sustainable resource with environmental advantages [23]. However, producing bio-products with complex conversion processes, biomass provision, and resource uncertainties remain commercial-scale challenges [24]. Furthermore, increased environmental concerns and annual demand have motivated the recycling of waste/products. Hence, the reverse direction of material flow helps the development of the value chain [25]. Firms seek to increase their values and focus on controlling value drivers. In this regard, developing return routes absorbs many sustainability benefits in the biogas supply chain [26].

Network flows are covered by four primary levels of the biomass supply chain network. The first one is the level of suppliers, which creates the main flow of biomass [27]. The second level is the storage and separation of biomass. At this level, three biomass flows are created. The first flow is the undesirable outputs. Converting them into fuel provides no economic or environmental justification [28,29]. The second category includes desirable outputs and is for biomass that can be recycled and used. These types of outputs are often obtained from dry urban waste. The third flow is process outputs that are transferred to the power plant to be converted into fuel. The third level is the product refinement and production field. This level leads to the bioenergy flow production, which is transferred to the final level (i.e., the customer level) [30]. In the real world, network flows are affected by several uncertainties, which create disturbances in the supply chain network's upstream, middle, and downstream facilities [31]. The research in this field has covered the uncertainty to be closer to reality and increase the reliability level of the studied axes. In previous studies, the parameters of supply and demand, biomass and biofuel price, various costs (e.g., transportation and purchase of resources), and environmental effects were considered uncertain [32,33].

On the other, a biomass supply chain is exposed to different sources of uncertainty. In the upstream supply chain, production is quantitatively and qualitatively affected by weather conditions. The biomass supply from one season to another or in diverse climates is subject to high uncertainty [34]. Hence, biomass prices and transportation and storage costs face high tolerance [2, 4]. According to Santos et al., biological refineries also have uncertain parameters such as operating cost and production rate [35]. Also, biofuel demand points are surrounded by price and demand uncertainty [36]. However, the main challenge is the quality of biomass, which has been neglected by previous research despite its direct impact on bioenergy production.

Parker et al. [37] modeled a forward biofuel supply chain network consisting of biofuel supply routes and commercially viable technologies. Next, they developed a robust model to use spatial distributions based on the GIS of biomass resources to optimize bio-refinery locations. The results showed that biofuel production from agricultural biomass streams, forestry, and municipal has significant energy potential. Dal-Mas et al. [38] investigated the multi-layer direct supply chain network of ethanol biofuel from corn biomass. Next, they extended a dynamic, spatial, and multi-level mathematical programming model incorporating the uncertainty of biomass production cost and product sales price. This model reduced the investment risk and improved the economic performance of the chain simultaneously. Čučeka et al. [39] developed a multi-criteria optimization model for a multi-echelon direct biofuel supply chain network with sustainability approaches. This model minimized the environmental and social footprint while improving the economic performance of the network. The material flow of the above network converts agricultural biomass into biofuel. Yilmaz and Selim [40] proposed a comprehensive multi-phase mathematical programming model under intrinsic uncertainties for bio-energy supply chain network design. The government and private investors could exploit the model to design a region's most profitable biomass supply chain based on anaerobic digestion and estimate the costs and profit. Poudel et al. [41] developed a two-stage stochastic MILP model to design and manage a simultaneous supply chain problem of biomass fuel with coal under the uncertainty of raw material supply. The generated results described the seasonal use of multimodal facilities, the number of containers transported between multimodal facilities, and the amount of biomass processed, stored, and transported from several feedstock supply sites to coal-fired power plants under biomass supply uncertainty. Abdul Quddus et al. [42] presented an optimization method to improve the design and planning of waste biomass-based supply chains for producing different types of bio-products. This model works based on various biomass pre-processing technologies and energy production with mathematical modeling and fuzzy multi-objective decision-making. The extended supply chain model determined the location of the optimal size and routing plan of multiple warehouse facilities for raw material storage and processing plants. Woo et al. [43] proposed a mathematical model to cope with uncertainty in the selection of biomass raw materials and the operation programming problem in the biomass supply chain to help decision-makers in the supply sector. The model aimed to minimize the total cost of a biomass supply chain system. An advanced and regular L-shaped algorithm was employed to solve the two-stage stochastic programming model. Rahemi et al. [44] proposed a mixed integer linear programming model (MILP) for the optimal design and planning of a bioethanol supply chain network to reduce supply chain costs and maximize the appropriateness of the allocated lands with the crops planted. The extended model followed strategic decision-making (i.e., location and capacity of facilities, source, and allocation of biomass raw materials to bio-refineries) and tactical decisions (i.e., land planning, inventory, and production of biomass raw materials and bio-ethanol). Saghaei et al. [45] presented a non-linear mathematical programming model for formulating a wood biomass forward supply chain network for electricity generation under material quality and demand uncertainties. This model provided optimal solutions for decisions such as location and layout of facilities, nodes stream, supply of materials, and inventory policy.

Yahya et al. [46] investigated the need for various energy characteristics and a movement toward renewable energy resources. Increasing challenges unexpectedly appear in the form of uncertainties in this respect. They introduced a Monte Carlo simulation (MCS) model for the techno-economic feasibility evaluation of biomass gasification under five uncertainties, namely 1) biomass quality, 2) biomass supply, 3) biomass price, 4) synthetic gas price, and 5) transportation fuel cost. Guo et al. [47] analyzed spatiotemporal uncertainties in collecting residues as significant challenges in developing a supply chain network to convert biomass into biofuel. The model was solved through a stochastic programming approach. Aranguren et al. [48] introduced a modeling approach to design a large-scale biomass supply chain to minimize investment. The model was solved using a metaheuristic algorithm. Previous studies have developed biogas production with different network structures and flows and various modeling approaches. Umakanth et al. [49] introduced the sustainable availability of biomass flow as the critical criterion for choosing the location of a biofuel plant. Syahira Mohd et al. [50] investigated the optimization of the biomass supply chain for refineries based on carbon reduction goals. To this end, they considered the flow of uncertainties, such as changes in the biomass supply, caused by the seasonal dependence of biomass.

Examining the above studies reveals that most biogas supply chain research has investigated the forward network, emphasizing single-product and single-source models. Another point considered in these studies is social and environmental concerns without providing an alternative to separate the biomass streams and clean up a huge amount of residual waste from the biomass stream, which causes lasting environmental damage. Notably, most biomass and biofuel supply chain research has been conducted in conditions of parametric uncertainty, such as the uncertainty of demand, supply, and weather conditions. Meanwhile, biomass quality as a primary factor of uncertainty that is caused by changing weather conditions and different economic and environmental policies has been neglected in many studies. However, the production capacity parameter caused by investment incentives covers an essential part of the uncertainty of the production rate. Another point neglected in previous studies is the high volume of residue flow from the direct flow of biomass, which is one of the main obstacles to developing bioenergy power plants. Table 1 reveals the research gaps by providing a more detailed classification and reviewing recent reverse supply chain studies.

$$Please\ insert\ Table 1$$

Reviewing the literature on the design of biogas supply chain networks reveals the novelty of the present work as follows:

- Integrated strategic and tactical decisions based on flexibility to ensure met customer demand and increased power generation capacity;
- Incorporating the three typical uncertainties in the operation of biogas networks, i.e., demand, quality, and generation capacity uncertainties;
- Separating biomass flows in successive “separation and storage” centers and bioenergy power plants;
- Producing bioenergy as the main product and bio fertilizer as a side product
- Enhanced network sustainability through anaerobic digestion maximization and increased power generation capacity and second-type bio-fertilizer production;

- Developing a hybrid stochastic-robust programming approach by generating several events in the form of future scenarios under frequent operational uncertainties; and
- Utilizing the accelerated Benders decomposition (BD) algorithm in the biogas supply chain network with complex variables to shorten the computational time and increase the convergence rate.

3. Problem statement

The present study focused on the supply chain network of biogas (primary product) and bio-fertilizers (secondary product) for energy generation. Biomass would be used as a fertilizer for power generation using anaerobic processes [54]. This material is the biodegradable resource of products, sewage, agricultural and domestic wastes of forest industries and other related industries, urban and industrial wastes, and waste [35]. Anaerobic digestion is the process of bacterial decomposition of biomass resources in the absence of air. This process produces methane and byproducts with moderate calorific value (biogas) through hydrolysis, acidification, and methanation. The materials that can be used in this process are animal husbandry and poultry waste, livestock waste, agricultural products, urban and rural sewage solid waste, urban and rural waste, etc. Taking the above materials allows for biogas production to be used as an energy source. For this purpose, it is necessary to establish suitable environmental and temperature conditions, pH, carbon-to-nitrogen ratio of materials, material concentration, material stopping time, stirring, and the amount of daily loading of materials inside the reactor [55]. This study developed a two-stage stochastic MILP model of the reverse supply chain network. In this model, biomass would be collected, inspected, and organized in storage reservoirs and then transported to the bio-refinery to produce biogas. High-quality biomass would lead to a high biogas yield in the bio-refinery, raising the energy generation capacity. As depicted in Fig. 1, biomass conversion into biogas includes poultry litter, pre-processing, fermentation, biogas production, residue separation, and impurity discharge. Biogases are classified into pure and impure biogases [56]. A bio-refinery does not convert 100% of the biogas into power as input as conversion processes have high biomass losses, which are introduced to recycling centers. Research has shown that biogas has an energy generation rate of 1.7 kW/m^3 [57]. Biogas quality has a direct relationship with energy demand and capacity. Due to the uncertain quality of the produced biogas, experts separate biomass-derived products based on impurity content, and the biogas with lower impurities is transferred to power plants. In the reverse direction, the impure fractions are collected from the biogas facilities in the collection centers. The collected fractions are inspected and separated to enhance the performance of the supply chain. This operation is carried out in the second stage, through which the impure fractions of biomass are transferred to recycling centers. In this stage, the quality of the impure fractions is inspected, and the usable fractions are utilized as fertilizers. In this process, the unusable fractions disposed of biogas have a volumetric fertilizer yield rate of about 8% [58]. Furthermore, since demand and recycling cannot be predicted, it is crucial to incorporate the uncertainties of parameters into biomass supply chains due to possible changes in climatic parameters that often challenge the extraction of biomass resources. Therefore, the demand (for power and fertilizers), production capacity, and returned product quality are assumed to be uncertain. The demand may be a function of unexpected incidents such as weather or the production of other products, depending on the competitors and energy consumption. Hence, the probability distribution cannot be predicted, particularly in a time horizon.

$Please\ insert\ figure\ 1$

The assumptions applied in our extended model are presented as follows:

- The multi-level model produces a unique product in each forward and reverse path.
- Biogas production capacity, demand, and quality level are considered uncertain parameters.
- The location of biorefinery facilities, the number of customers, the transportation cost of the supply chain network, and biomass suppliers are fixed and predetermined.
- A penalty will be charged for unsatisfied customer demands.
- The quality level of biogas is checked at three levels (i.e., production, recycling, and distribution).
- The potential locations of poultry farms, biological refineries, recycling, distribution, and destruction have been determined.
- The forward path of the network follows the location decisions, capacity of facilities, and distribution management of the first type of product. In the reverse path, the recycled product and the amount of production of the second product (bio-fertilizer) are considered.
- The raw material (biomass) for biogas production enters the reverse biogas supply chain network cycle from the anaerobic digestion mechanism.
- The processing capacity of biomass production and biomass storage centers is limited.

3.1. Mathematical model

Indices

p	*Biogas facilities* $(p = 1, \ \ldots, 6)$
s	*Suppliers* $\left(poultry\ centers\right)$ $\left(s = 1.2, 3\right)$
u	*Demand* $\left(customers\right)$ $(u = 1, \ \ldots, \ n)$
d	*Disposal centers* $\left(d = 1, 2, 3\right)$
se	*Scenario* $\left(se = 1, \ \ldots, 6\right)$
i	*Separation locations* $\left(i = 1, 2\right)$
c	*Biogas production capacity* $\left(c = 1, \ \ldots, \ n\right)$
j	*Distribution and collection centers* $\left(j = 1, 2, 3\right)$
r	*Recycling centers* $\left(r = 1, 2, 3\right)$
m	*Material* $\left(biogas\right)$ $\left(m = 1, \ \ldots, \ n\right)$

Parameters

pw_S	*Daily biomass production capacity of poultry centers*
mwu_c	*Maximum usable waste in biogas facilities for biogas production capacity c*
mc_s	*Maximum capacity of supplier* **s**
mp_j	*Maximum capacity of distribution and collection center* **j**
ms_i	*Maximum capacity of separation center* **i**
coc_d	*Capacity of disposal center* **d**
$C\,\mathrm{Re}_r$	*Capacity of recycling center* **r**
MCp_i	*Distribution capacity allocated to distribution and collection center* **j**
HC_j	*Collection capacity allocated to distribution collection center* **j**
RE_c	*Minimum power payback of power plants over a given period based on the biogas production capacity c*
CE_c	*Energy generation constraint for biogas facilities in scenario se based on the biogas production capacity c*
Eg_m	*Maximum storable biogas*
Mc_c	*Maximum anaerobic digestion – based biogas production that can be stored*
GE	*Biogas – energy conversion rate*
BE	*Biogas – energy conversion rate*
COF_c	*Investment cost* $\left(per\ kW\right)$ *in biogas facilities with capacity c*
COB	*Unit price of biomass*
UD	*Unit price of biogas produced using anaerobic digestion*
COE	*Unit price of electrical energy*
op_s	*Fixed opening cost of poultry centers*
os_i	*Fixed opening cost of separation center i*
Cow_d	*Fixed opening cost of disposal center d*
$\mathrm{Re}\,p_r$	*Fixed opening cost of recycling center r*
cp_S	*Reproduction / reassembly cost in s*
cs_i	*Separation cost in separation center i*
CR_r	*Recycling cost in recycling center* **r**
cp_d	*Disposal cost in disposal center* **d**
oE_{pd}	*Unit disposal cost of biogas of facility* **p** *in disposal center* **d**
CT_{uj}	*Transportation cost between customer* **u** *and distribution and collection center* **j**
TC_{ju}	*Transportation cost between distribution and collection center* **j** *and customer* **u**
CTE_{jp}	*Transportation cost between distribution and collection center* **j** *and biogas facility* **p**
GT_{ji}	*Transportation cost between distribution and collection center* **j** *and separation center* **i**
Kr_{ir}	*Transportation cost between separation center* **i** *and recycling center* **r**
csp_{id}	*Transportation cost between separation center* **i** *and disposal center* **d**
CNe	*Fertilizer price*
RSe_{pse}	*Successful recycling ratio of biogas residues in biogas facility* **p** *in scenario* **se**
mub_{mpse}	*Use ratio of biogas* **m** *in biogas facility* **p**

bp_{mse}	The ratio of usable biogas **m** in separation under scenario **se**
MCT_{ms}	Capacity coefficient of biogas **m** in supplers **s**
Cbp_{p}	Capacity coefficient of biogas facility **p** in disposal centers
car_{p}	Capacity coefficient of biogas facility **p** in recycling centers
Tm_{p}	Transpiration coefficient of biogas facility **p** compared to one product
prs_{se}	Probability of scenario **se**
Py_{se}	Penalty in scenario se
wp_{s}	Waste transportation from suppliers **s**
pu_{j}	Collection processing in distribution and collection center j
lq_{q}	Quality level **p** of biogas
den_{use}	Fertilizer demand of customer **u** in scenario **se**
der_{use}	The power demand of customer **u** in scenario **se**

Decision variables

qt_{juse}	Transported product from distribution and collection center **j** to customer **u** in scenario **se**
Hom_{jsse}	Transported product from distribution and collection center **j** to suppliers **s**
tq_{uj}	Transported product from customer **u** to distribution and collection center **j**
td_{jd}	Transported product from distribution and collection center **j** to disposal center **d**
ts_{jise}	Transported product from distribution and collection center **j** to separation **i**
Hmo_{sjse}	Recycled product transferred from suppliers **s** to distribution and collection center **j** in scenario **se**
$\sec_{juse}$	Recycled product transferred from distribution and collection center **j** to customer **u** in scenario **se**
puf_{spse}	Poultry litter transferred from suppliers **s** to biogas facility **p** in scenario **se**
HBc_{pse}	Biomass consumed in biogas facility **p** in scenario **se**
qpd_{mpse}	Biogas **m** produced using anaerobic digestion in biogas facility **p** in scenario **se**
CTe_{pse}	Biogas residue in biogas facility **p** in scenario **se**
Egf_{p}	Power generation in biogas facility **p** in scenario **se**
Bi_{p}	Maximum biogas production in biogas facility **p** in scenario **se**
TP_{pisse}	Biogas facility **p** transferred from separation center **i** to suppliers **s**
Tse_{pir}	Biogas facility **p** transferred from separation center **i** to recycling center **r**
sep_{pirse}	Biogas facility **p** transferred from separation center **i** to disposal center **d**

Binary variables

Lu_{q}	1 *if q is higher than disposal; otherwise, it is* 0.
h_{pc}	1 *if biogas facility p with a capacity of c is selected; otherwise, it is* 0.
N_{d}	1 *if disposal center d is selected; otherwise, it is* 0.
G_{s}	1 *if supplier s is selected; otherwise, it is* 0.

cc_j	1 *if distribution center i is selected; otherwise, it is* 0.
ch_r	1 *if recycling center r is selected; otherwise, it is* 0.
sp_i	1 *if separation center i is selected; otherwise, it is* 0.

3.2. Objective function

$$\max\ profit = FIC + SIC - (TCC) \tag{1}$$

$$FIC = \sum_{se} prs_{se}\left(\sum_{p} COE.Egf_{p} + \sum_{m}\sum_{p}\sum_{se} ud.qpd_{mpse}\right)$$

$$SIC = \sum_{d}\sum_{se}\sum_{j}\sum_{u}\sum_{q} cp_{d}\cdot prs_{se}\cdot qt_{juse}\cdot Lq_{q}\left(1 - Lq_{q}\right) + \sum_{se}\sum_{j}\sum_{u} prs_{se}.CNe.sec_{juse}$$

$$TCC = \sum_{se} prs_{se}\ (\sum_{s} op_{s}\cdot G_{s} + os.sp_{i} + \sum_{d} CoW_{d}\cdot N_{d} + \sum_{r} Rrp_{r}\cdot ch_{r} + \sum_{c}\sum_{p} CoF_{c}\cdot h_{pc}$$

$$\sum_{pc} Cbp_{p}.h_{pc} + \sum_{s}\sum_{p} CoE.CoB.Puf_{spse} + \sum_{j}\sum_{u}\sum_{q}\sum_{p} qp_{juse}\cdot Tc_{ju}\cdot lq_{q}\cdot Hbc_{pse} + \sum_{s}\sum_{p} Cp_{s}\cdot Hbc_{pse} +$$

$$\sum_{j}\sum_{s}\sum_{u}\left(Hom_{jsse} + Hmo_{sjse}\right)\cdot\left(wp_{s} + Tc_{ju}\right) + \sum_{j}\sum_{u}\left(qt_{juse} + sec_{juse}\right)\cdot TC_{ju} + \sum_{u}\sum_{j} tq_{uj}\cdot\left(CT_{uj} + Pu_{j}\right) +$$

$$\sum_{j}\sum_{i} ts_{jise}\cdot\left(gT_{ji} + Cs_{i}\right) + \sum_{j}\sum_{d} td_{jd}\cdot\left(CTE_{jd} + CP_{d}\right) + \sum_{p}\sum_{i}\sum_{s} TP_{pisse}\cdot Tm_{p}$$

$$+\sum_{p}\sum_{i}\sum_{r} Tse_{pir}\cdot\left(Kr_{ir}.Tm_{p} + CR_{d}\right) + \sum_{p}\sum_{i}\sum_{r}\sum_{d} sep_{pirse}\cdot\left(Csp_{id}\cdot Tm_{p} + OE_{pd}\right) + \sum_{j}\sum_{s}\left(Hom_{jsse} + Hmo_{sjse} - \sum_{p}\sum_{i} TP_{pisse}\right)\cdot CP_{s})$$

The objective function ensures the expected profit maximization of the biomass supply chain network (Eq. 1). The profit of the network includes biogas-generated power income, fertilizer income, quality maximization at three levels (i.e., production, recycling, and distribution), storage costs, transportation costs, excess product demand costs, excess warehouse capacity costs, penalty minimization, and total investment cost minimization in all the scenarios.

s.t.

$$\sum_{c} h_{pc} \le 1 \forall p \tag{2}$$

Based on Eq. (2), the production capacity level of each biogas facility is 1

$$\sum_{p}\sum_{c} h_{pc}.mwu_{c} \le \sum_{s} pw_{s} \tag{3}$$

Eq. (3) represents the poultry litter quantity that can be transferred to biogas facilities

$$\sum_{m}\left[\sum_{s}\sum_{j}\left(Hom_{sjse} + Hmo_{jsse}\right).bp_{mse} - \sum_{j} ts_{jise}\right] = \sum_{p}\sum_{i}\sum_{r} Tse_{pir} \forall se, i, p \tag{4}$$

Eq. (4) formulates the recycled product equal to the poultry biomass quantity.

$$\sum_{u}\sum_{p} \sec_{juse}\cdot Rse_{pse} \ge \sum_{i} ts_{jise} \forall j, se \tag{5}$$

Eq. (5) constrains the product quantity that can be transferred to recycling centers.

$$\sum_{j}\sum_{d}\left(1 - Hbc_{pse} - Rse_{pse}\cdot td_{jd}\right) \le \sum_{i}\sum_{r} sep_{pirse}\cdot lq_{qse} \forall p, se \tag{6}$$

Eq. (6) indicates that the usable products are delivered to either recycling centers or customers.

$$pw_s \geq \sum_j \sum_{se} \left(Hom_{jsse} + Hmo_{sjse} \right) + \sum_p \sum_i pu_p \cdot Tp_{pisse} \forall s \tag{7}$$

Eq. (7) represents the constraint of poultry litter production capacity.

$$G_s \cdot ms_i \geq \sum_m MCT_{ms} \forall s \tag{8}$$

Eq. (8) formulates the supplier constraint.

$$sp_i \cdot ms_i \geq Hp_i \forall i \tag{9}$$

Eq. (9) determines the maximum recycled product separation capacity.

$$chr_r \cdot C\,\mathrm{Re}_r \geq \sum_p \sum_i car_p \cdot Tse_{pir} \forall r \tag{10}$$

Eq. (10) constrains the capacities of disposal centers and recycling centers.

$$cc_j \cdot bp_j \geq mcp_j + Hc_j \forall j \tag{11}$$

Eq. (11) constrains the capacities of distribution and collection centers.

$$N_d \cdot coc_d \geq \sum_j td_{jd} + \sum_p \sum_i \sum_r \sum_{se} sep_{pirse} \cdot cbp_p \forall d \tag{12}$$

Eq. (12) constrains the capacities of separation centers.

$$\sum_j qt_{juse} \leq den_{use} \forall u, se \tag{13}$$

Eq. (13) formulates the power demand (primary product).

$$\sum_j \sec_{juse} \leq der_{use} \forall u, se \tag{14}$$

Eq. (14) specifies the fertilizer demand (secondary product).

$$\sum_s \sum_{se} puf_{spse} - Hbc_{pse} + \sum_s \sum_{se} puf_{spse} - \sum_{se} Hbc_{pse-1} \leq \sum_c Eg_c \cdot h_{pc} \forall p \tag{15}$$

Eq. (15) maximizes the poultry litter storage.

$$\sum_m \sum_{se} \left(qpd_{mpse} - CTe_{pse} + \left(qpd_{(mpse-1)} - CTe_{(pse-1)} \right) \right) \leq \sum_c mc_c \cdot h_{pc} \forall p \tag{16}$$

Eq. (16) determines the maximum biogas production from anaerobic digestion.

$$\sum_{se} Hbc_{pse} \cdot GE.BE \leq \sum_c CE_c \cdot h_{pc} \forall p \tag{17}$$

Eq. (17) formulates the minimum power generation capacity.

$$\sum_{se} Hbc_{pse} \cdot GE.BE \leq \sum_c RE_c \cdot h_{pc} \forall p \tag{18}$$

Eq. (18) minimizes the biogas production capacity.

$$\sum_{se} CTe_{pse} \geq \sum_i \sum_c cs_i \cdot h_{pc} \forall p \tag{19}$$

Eq. (19) specifies the minimum separation capacity of recycled products.

$$Egf_p = \sum_{se} Hbc_{pse} \cdot GE.BE \forall p \tag{20}$$

Eq. (20) shows the maximum power generation capacity.

$$Bi_p = \sum_{se} Hbc_{pse} . GE \forall p \tag{21}$$

Finally, Eq. (21) represents the maximum fertilizer production capacity.

$$qt_{iuse}, Hom_{isse}, tq_{ui}, td_{jd}, ts_{jise}, TP_{pisse}, Tse_{pir}, sep_{pirse}, Hmo_{sjse}, \sec_{juse}, puf_{spse}, \tag{22}$$

$$HBc_{pse}, Egf_p, Bi_p, CTe_{pse}, qpd_{mpse} \geq 0 \forall j, u, se, s, d, p, i, r, m$$

$$qpd_{mpse}, h_{pc}, N_d, G_s, cc_j, ch_r, sp_i \in \{0,1\} \forall m, p, se, c, d, j, r \tag{23}$$

Constraints (22) and (23) impose binary and non-negative restrictions on the decision variables.

4. Hybrid stochastic-robust programming approach

In the real decision-making process, parameters face a mix of random and cognitive uncertainties. These parameters have cognitive uncertainty under different scenarios. This uncertainty occurs when there is a cognitive uncertainty in estimating the exact value of random parameters under different scenarios [59]. Generally, there is not sufficient data available to find the probability distribution of each uncertainty parameter in the estimation of the parameters of each scenario. This shortcoming is due to the unrepeatability and other specific characteristics of each scenario. Hence, vague values in the form of fuzzy numbers are used to define the parameters under each scenario. Therefore, fuzzy stochastic optimization problems have emerged to deal with such uncertainty [60]. In continuation of this two-step path, the robust programming approach (as a risk-averse method) covers the aspects of optimality and feasibility robustness under conditions of combined uncertainty. Accordingly, we seek near-optimal solutions and validate them with a high probability, guaranteeing the robustness of the problem decisions. The optimality solution would be kept secure and robust in the event of uncertainties in a given bounded uncertainty set [61]. Also, feasibility robustness is ensured when the objective function value has the minimum undesired deviation from the optimal value for each scenario [62].

The uncertainty set was developed by defining the positive and then negative deviations from the nominal scenario as follows:

$$\eta^{+}_{den_{use}} = \frac{den_{use} - den_{use}}{\Delta den^{+}_{use}} \; if \; den_{use} > den_{use}, \; \eta^{-}_{den_{use}} = \frac{den_{use} - den_{use}}{\Delta den^{-}_{use}} \; if \; den_{use} < den_{use} \tag{24}$$

$$\eta der^{+}_{use} = \frac{der_{use} - der_{use}}{\Delta der^{+}_{use}} \; if \; der_{use} > der_{use}, \; \eta der^{-}_{use} = \frac{der_{use} - der_{use}}{\Delta der^{-}_{use}} \; if \; der_{use} < der_{use} \tag{25}$$

$$\eta pw^{+}_{s} = \frac{pw_s - pw_s}{\Delta pw^{+}_{s}} \; if \; pw_s > pw_s, \; \eta pw^{-}_{s} = \frac{pw_s - pw_s}{\Delta pw^{-}_{s}} \; if \; pw_s < pw_s \tag{26}$$

$$\eta lq^{+}_{q} = \frac{\hat{lq}_q - lq_q}{\Delta lq^{+}_{q}} \; if \; lq_q > \hat{lq}_q, \; \eta lq^{-}_{q} = \frac{\hat{lq}_q - lq_q}{\Delta lq^{-}_{q}} \; if \; lq_q < \hat{lq}_q \tag{27}$$

Then, the uncertainty set is formulated as:

$$j_{se}^{den}=\left\{den_{use}\middle|den_{use}=\hat{den}_{use}+\Delta den^{+}_{use}\times\eta den^{+}_{use}-\Delta den^{-}_{use}\times\eta den^{-}_{use},\forall u,\forall\Delta den_{use},\eta den_{use}\in u^{den}\right\}\tag{28}$$

Then,

$$u^{den}=\left\{\eta^{+}_{den_{use}},\eta^{-}_{den_{use}}\middle|0\le\eta^{+}_{den_{use}}\le1,0\le\eta^{-}_{den_{use}}\le1,\sum_{u}\left(\eta den^{+}_{use}+\eta den^{-}_{use}\right)\le\mu_{se}^{den}\right\}\tag{29}$$

$$j_{se}^{der}=\left\{der_{use}\middle|der_{use}=\hat{der}_{use}+\Delta der^{+}_{use}\times\eta^{+}_{der_{use}}-\Delta der^{-}_{use}\times\eta^{-}_{der_{use}},\forall u,\forall\Delta der_{use},\eta_{der_{use}}\in u^{der}\right\}\tag{30}$$

$$u^{der}=\left\{\eta^{+}_{der_{use}},\eta^{-}_{der_{use}}\middle|0\le\eta^{+}_{der_{use}}\le1,0\le\eta^{-}_{der_{use}}\le1,\sum_{u}\left(\eta der^{+}_{use}+\eta der^{-}_{use}\right)\le\mu_{se}^{der}\right\}\tag{31}$$

$$c_{se}^{pw}=\left\{pw_{sse}\middle|pw_{sse}=\hat{pw}_{sse}+\Delta pw^{+}_{sse}\times\eta^{+}_{pw_{sse}}-\Delta pw^{-}_{sse}\times\eta^{-}_{pw_{sse}},\forall s,\forall\Delta pw_{sse},\eta_{pw_{sse}}\in s^{pw}\right\}\tag{32}$$

$$s^{pw}=\left\{\eta^{+}_{pw_{sse}},\eta^{-}_{pw_{sse}}\middle|0\le\eta^{+}_{pw_{sse}}\le1,0\le\eta^{-}_{pw_{sse}}\le1,\sum_{s}\left(\eta pw^{+}_{sse}+\eta pw^{-}_{sse}\right)\le\mu_{sse}^{pw}\right\}\tag{33}$$

$$i_{se}^{lq}=\left\{lq_{qse}\middle|lq_{qse}=\hat{lq}_{qse}+\Delta lq^{+}_{qse}\times\eta lq^{+}_{qse}-\Delta lq^{-}_{qse}\times\eta lq^{-}_{qse},\forall q,\forall\Delta lq_{qse},\eta lq_{qse}\in q^{lq}\right\}\tag{34}$$

$$q^{lq}=\left\{\eta lq^{+}_{qse},\eta lq^{-}_{qse}\middle|0\le\eta lq^{+}_{qse}\le1,0\le\eta lq^{-}_{qse}\le1,\sum_{q}\left(\eta lq^{+}_{qse}+\eta lq^{-}_{qse}\right)\le\mu_{se}^{lq}\right\}\tag{35}$$

The constraints are reduced based on the assumption that uncertain constraints have been violated. The decision-maker is allowed to implement constraint violations by penalizing the objective function for the violated constraints. The objective is to minimize the worst costs in the violated constraints. Inserting Constraints (30-35) in Eq. (1) gives:

$$\begin{aligned}py_{s}(td,mwu,qt,\sec)=&\max_{den_{use}\in j_{se}^{den}}\left[\sum_{u}\left(den_{use}-\sum_{j}qt_{juse}\right)\times py^{den},\sum_{u}\left(\sum_{j}qt_{juse}-den_{use}\right)\times CoB^{den}\right]\\&+\max_{der_{use}\in j_{se}^{der}}\left[\sum_{u}\left(der_{use}-\sum_{j}\sec_{juse}\right)\times py^{der},\sum_{u}\left(\sum_{j}\sec_{juse}-der_{use}\right)\times CoB^{der}\right]\\&+\max_{lq_{qse}\in q_{se}^{lq}}\left[\sum_{p}\sum_{i}\sum_{r}\sum_{q}sep_{pirse}.lq_{qse}-\sum_{p}\left(1-Hbc_{pse}-\sum_{n}\sum_{i}Rs_{pse}.td_{jd}\right)\times py^{lq},\sum_{p}\left(1-Hbc_{pse}-\sum_{j}\sum_{d}Rs_{pse}.td_{jd}\right)-\sum_{p}\sum_{i}\sum_{r}\sum_{q}sep_{pirse}.lq_{qse}\times CoB^{lq}\right]\\&+\max_{der_{use}\in j_{se}^{der}}\left[\sum_{s}pw_{sse}-\sum_{p}\sum_{c}h_{pc}.mwu_{c}\times py^{pw},\sum_{p}\sum_{c}h_{pc}.mwu_{c}\times py^{pw}-\sum_{s}pw_{se}\times CoB^{pw}\right]\end{aligned}\tag{36}$$

Constraint (36) is linearly formulated using auxiliary variables $z1_{se}, z2_{se}, z3_{se}$ and $z4_{se}$:

$$\min_{z1_{se},z2_{se},z3_{se},z4_{se}}Py_{s}(td,mwu,qt,\sec)=z1_{se}+z2_{se}+z3_{se}+z4_{se}\tag{37}$$

$s.t.$

$$\sum_{u}\left(den_{use}-\sum_{j}qt_{juse}\right)\times py^{den}\le z1_{se},\forall den_{use}\in j_{se}^{den}\tag{38}$$

$$\sum_{u}\left(den_{use}-\sum_{j}qt_{juse}\right)\times CoB^{den}\le z1_{se},\forall den_{use}\in j_{se}^{den}\tag{39}$$

$$\sum_{u}\left(der_{use}-\sum_{j}\sec_{juse}\right)\times py^{der}\le z2_{se},\forall der_{use}\in j_{se}^{der}\tag{40}$$

$$\sum_u \left(der_{use} - \sum_j \sec_{juse} \right) \times CoB^{der} \le z2_{se}, \forall der_{use} \in j^{der}_{se} \tag{41}$$

$$\sum_p \sum_i \sum_r \sum_q sep_{pirse} \cdot lq_{qse} - \sum_p \left(1 - Hbc_{pse} - \sum_j \sum_d Rs_{pse} \cdot td_{jd} \right) \times py^{lq} \le z3_{se}, \forall lq_{qse} \in i^{lq}_{qse} \tag{42}$$

$$\sum_p \sum_i \sum_r \sum_q sep_{pirse} \cdot lq_{qse} - \sum_p \left(1 - Hbc_{pse} - \sum_j \sum_d Rs_{pse} \cdot td_{jd} \right) \times CoB^{lq} \le z3_{se}, \forall lq_{qse} \in i^{lq}_{qse} \tag{43}$$

$$\sum_s pw_s - \sum_p \sum_c h_{pc} \cdot mwu_c \cdot py^{pw} \le z4_{se}, \forall pw_{sse} \in s^{pw}_{se} \tag{44}$$

$$\sum_s pw_s - \sum_p \sum_c h_{pc} \cdot mwu_c \cdot CoB^{pw} \le z4_{se}, \forall pw_{sse} \in s^{pw}_{se} \tag{45}$$

$$z1_{se}, z2_{se}, z3_{se}, z4_{se} \ge 0$$

$$\max_{den_{use} \in j^{den}_{se}} \left[\sum_u \left(den_{use} - \sum_j qt_{juse} \right) \times py^{den} \right] \le z1_{se} \tag{46}$$

It can be converted into Constraint (46):

$$\sum_u \left(den_{use} - \sum_j qt_{juse} \right) \times py^{den} + \max_{\eta den^{+}_{use}, \eta den^{-}_{use}} \times \left[\sum_u \left(\Delta den^{+}_{use} \times \eta den^{+}_{use} - \Delta den^{-}_{use} \times \eta den^{-}_{use} \right) \times py^{den} \right] \le z1_{se}$$

$$- \min_{\eta den^{+}_{use}, \eta den^{-}_{use}} \times \left[\sum_u \left(-\Delta den^{+}_{use} \times \eta den^{+}_{use} + \Delta den^{-}_{use} \times \eta den^{-}_{use} \right) \times py^{den} \right] \tag{47}$$

$$s.t$$

$$-\eta den^{+}_{use} \ge -1, \forall u : \alpha 1_{use}$$

$$-\eta den^{-}_{use} \ge -1, \forall u : \alpha 2_{use} \tag{48}$$

$$\sum_u \left(-\eta den^{+}_{use} - \eta den^{-}_{use} \right) \ge -\mu^{den}_{se} : \beta 1^{den}$$

$$-\eta den^{+}_{use} - \eta den^{-}_{use} \ge 0$$

The dual of the problem is modeled as:

$$\max_{\alpha 1_{use}, \alpha 2_{use}, \beta 1^{den}} \left[-\mu^{den}_{se} \times \beta 1^{den} - \sum_u \left(\alpha 1_{use} + \alpha 2_{use} \right) \right]$$

$$s.t$$

$$-\alpha 1_{use} - \beta 1^{den} \le \Delta den^{+}_{use}, \forall u \tag{49}$$

$$-\alpha 2_{use} - \beta 1^{den} \le \Delta den^{-}_{use}, \forall u$$

$$\beta 1^{den}, \alpha 1_{use}, \alpha 2_{use} \ge 0, \forall u$$

In Model (49), since the second constraint is excess, $\alpha 2_{ds}$ is excluded based on robust duality theory. Then, the objective function (49) is inserted into Constraint (46) without the excluded constraint. The robust counterpart of Constraint (38) is as follows:

$$\left(\sum_u \left(den_{use} + \alpha 1_{use} \right) + \beta 1^{den} \times \Gamma^{den}_{se} - \sum_j \sum_u qt_{juse} \right) \times py^{den} \le z1_{se}$$

$$\alpha 1_{use} + \beta 1^{den} \geq \Delta den^{+}_{use}, \forall u \tag{50}$$

$$\alpha 1_{use}, \beta 1^{den} \geq 0, \forall u$$

The hybrid stochastic-robust supply chain network is expressed as:

$$Max z = (\sum_{se} prs_{se} \left(\sum_{p} COE \cdot Egf_{p} + \sum_{m} \sum_{p} \sum_{se} ud \cdot qpd_{mpse} \right) \tag{51}$$

$$-\sum_{se} (\sum_{u} \left(\Delta den^{+}_{use} t7_{use} + \Delta den^{-}_{use} t8_{use} \right) + \sum_{u} \left(\Delta der^{+}_{use} t9_{use} + \Delta der^{-}_{use} t10_{use} \right)$$

$$+\sum_{q} \left(\Delta lq^{+}_{qse} t11_{qse} + \Delta lq^{-}_{qse} t12_{qse} \right) + \sum_{s} \left(\Delta pw^{+}_{sse} t13_{sse} + \Delta pw^{-}_{sse} t14_{sse} \right))$$

$$\left(\sum_{u} \left(der_{use} + \alpha 1_{use} \right) + \beta 2^{den} \times \mu^{den}_{se} - \sum_{j} \sum_{u} sec_{juse} \right) \times py^{der} \leq z2_{se}, \forall se$$

$$-\sum_{d} \sum_{se} \sum_{j} \sum_{u} \sum_{q} cp_{d} \cdot prs_{se} \cdot qt_{juse} \cdot Lq_{q} \left(1 - Lu_{q} \right) + \sum_{se} \sum_{j} \sum_{u} prs_{se} \cdot CNe \cdot sec_{juse} - \sum_{se} prs_{se} (\sum_{s} op_{s} \cdot G_{s} + os \cdot sp_{i} +$$

$$\sum_{d} CoW_{d} \cdot N_{d} + \sum_{r} Rrp_{r} \cdot ch_{r} + \sum_{c} \sum_{p} CoF_{c} \cdot h_{pc} + \sum_{p} \sum_{c} Cbp_{p} \cdot h_{pc} + \sum_{s} \sum_{p} CoE.CoB.Puf_{spse} +$$

$$\sum_{j} \sum_{u} \sum_{q} \sum_{p} qp_{juse} \cdot Tc_{ju} \cdot lq_{q} \cdot Hbc_{pse} + \sum_{s} \sum_{p} Cp_{s} \cdot Hbc_{pse} + \sum_{j} \sum_{s} \sum_{u} \left(Hom_{jsse} + Hmo_{sjse} \right) \cdot \left(wp_{s} + Tc_{ju} \right) +$$

$$\sum_{j} \sum_{u} \left(qt_{juse} + sec_{juse} \right) \cdot TC_{ju} + \sum_{u} \sum_{j} tq_{uj} \cdot \left(CT_{uj} + Pu_{j} \right) +$$

$$\sum_{j} \sum_{i} ts_{jise} \cdot \left(gT_{ji} + Cs_{i} \right) + \sum_{j} \sum_{d} td_{jd} \cdot \left(CTE_{jd} + CP_{d} \right) + \sum_{p} \sum_{i} \sum_{s} TP_{pisse} \cdot Tm_{p}$$

$$+\sum_{p} \sum_{i} \sum_{r} Tse_{pir} \cdot \left(Kr_{ir} \cdot Tm_{p} + CR_{r} \right) + \sum_{p} \sum_{i} \sum_{r} \sum_{d} sep_{pirse} \cdot \left(Csp_{id} \cdot Tm_{p} + OE_{pd} \right) + \sum_{j} \sum_{s} \left(Hom_{jsse} + Hmo_{sjse} - \sum_{p} \sum_{i} TP_{pisse} \right) \cdot CP_{s})$$

$$-Z1_{se} - Z2_{se} - Z3_{se} - Z4_{se})$$

Constraints (2-21) are rewritten as:

$$\left(\sum_{u} \left(den_{use} + \alpha 1_{use} \right) + \beta 1^{den} \times \mu^{den}_{se} - \sum_{j} \sum_{u} qt_{juse} \right) \times py^{den} \leq z1_{se}, \forall se \tag{52}$$

$$\left(\sum_{u} \left(den_{use} + \alpha 2_{use} \right) + \beta 2^{den} \times \mu^{den}_{se} - \sum_{j} \sum_{u} qt_{juse} \right) \times CoB^{den} \geq -z1_{se}, \forall se \tag{53}$$

$$\left(\sum_{u} \left(der_{use} + \tau 1_{use} \right) + \varphi 1^{der} \times \mu^{der}_{se} - \sum_{j} \sum_{u} sec_{juse} \right) \times py^{der} \leq z2_{se}, \forall se \tag{54}$$

$$\left(\sum_{u} \left(der_{use} + \tau 2_{use} \right) + \varphi 2^{der} \times \mu^{der}_{se} - \sum_{j} \sum_{u} sec_{juse} \right) \times CoB^{der} \geq -z2_{se}, \forall se \tag{55}$$

$$\left(\sum_{p} \sum_{i} \sum_{r} \sum_{q} \left(sep_{pirse} \cdot lq_{se} + \lambda 1_{qse} \right) + \gamma 1^{lq} \mu^{lq}_{se} - \sum_{p} \left(1 - Hbc_{pse} - \sum_{j} \sum_{d} Rs_{pse} \cdot td_{jd} \right) \right) \times py^{lq} \leq z3_{se}, \forall se \tag{56}$$

$$\left(\sum_{p} \sum_{i} \sum_{r} \sum_{q} \left(sep_{pirse} \cdot lq_{se} + \lambda 2_{qse} \right) + \gamma 2^{lq} \mu^{lq}_{se} - \sum_{p} \left(1 - Hbc_{pse} - \sum_{j} \sum_{d} Rs_{pse} \cdot td_{jd} \right) \right) \times CoB^{lq} \geq -z3_{se}, \forall se \tag{57}$$

$$\left(\sum_{s} \left(pw_{sse} + \delta 1_{sse} \right) + \zeta 1^{pw} \mu^{pw}_{se} - \sum_{p} \sum_{c} h_{pc} \cdot mwu_{c} \right) \times py^{pw} \leq z4_{se}, \forall se \tag{58}$$

$$\left(\sum_{s} \left(pw_{sse} + \delta 2_{sse} \right) + \zeta 2^{pw} \mu^{pw}_{se} - \sum_{p} \sum_{c} h_{pc} \cdot mwu_{c} \right) \times py^{pw} \geq -z4_{se}, \forall se \tag{59}$$

$$\alpha 1_{use} + \beta 1^{den} \geq \Delta den^{+}_{use} \forall u, se \tag{60}$$

$$\alpha 2_{use} + \beta 2^{den} \geq \Delta den^{-}_{use} \quad \forall u, se \tag{61}$$

$$\tau 1_{use} + \varphi 1^{der} \geq \Delta der^{+}_{use} \quad \forall u, se \tag{62}$$

$$\tau 2_{use} + \varphi 2^{der} \geq \Delta der^{-}_{use} \quad \forall u, se \tag{63}$$

$$\lambda 1_{qse} + \gamma 1^{lq} \geq \Delta lq^{+}_{qse} \quad \forall q, se \tag{64}$$

$$\lambda 2_{qse} + \gamma 2^{lq} \geq \Delta lq^{-}_{qse} \quad \forall q, se \tag{65}$$

$$\delta 1_{pse} + \zeta 1^{pw} \geq \Delta pw^{+}_{pse} \quad \forall p, se \tag{66}$$

$$\delta 2_{pse} + \zeta 2^{pw} \geq \Delta pw^{-}_{pse} \quad \forall p, se \tag{67}$$

$$z1_{se}, z2_{se}, z3_{se}, z4_{se}, \alpha 1_{use}, \beta 1^{den}, \alpha 2_{use}, \beta 2^{den}, \tau 1_{use}, \varphi 1^{der}, \tau 2_{use}, \varphi 2^{der}, \lambda 1_{qse}, \gamma 1^{lq}, \lambda 2_{qse}, \gamma 2^{lq}, \delta 1_{pse}, \zeta 1^{pw}, \delta 2_{pse}, \zeta 2^{pw} \geq 0 \quad \forall u, q, s, se$$

5. Benders Decomposition Algorithm

Heuristic, meta-heuristic, and exact methods are usually used to solve mathematical models in large dimensions. Typically, heuristic and meta-heuristic methods generate reasonable and close-to-optimal results. Besides, there are exact methods that bring the mathematical model to the exact solution. In the case of a gap between the exact solution and the obtained answer and the presence of a huge penalty after that, the exact method of the Benders analysis algorithm is developed. This approach decomposes the main problem into smaller sub-problems to reduce the complexity of the problem and converge to the optimal solution during fewer iterations. In the first stage, desirable cuts are added to the main problem (MP) to increase the speed of convergence of Bander's decomposition algorithm. To this end, suitable initial solutions are estimated, and suitable values are obtained for the variables of the dual problem [61]. The next workaround is to modify the original problem in each step, select appropriate cuts, and add it to the original problem in each iteration of the algorithm. However, the main challenge is the low quality of the answers obtained from the MP section. This inefficiency can be avoided by limiting the solution space of the problem by defining valid limits in the MP part, thereby generating high-quality solutions [62].

The BD algorithm is a two-stage technique for stochastic linear programming problems. The linear programming of the main problem (MP) includes complex variables and is decomposed into a sub-problem to optimize dual variables and the primal relations (DSP). Let vectors Y and t denote the dual variables of Constraints (2-5), (7), (9-12), (15-21), and (47-67). The dual of the DSP provides a lower bound for the objective function. The main biomass reverse supply chain problem is formulated in each iteration as:

$$DSP: \min z = \sum_{se} \Big(\sum_{p}\sum_{i}\sum_{r} Tse_{pir}\, y2_{pirse} + \Big(\sum_{j}\sum_{s} \big(Hom_{sjse} + Hmo_{sjse} \big) + \sum_{p}\sum_{i}\sum_{s} pu_{p} \cdot TR_{pisse} \Big) y3_{psise} \tag{68}$$

$$+ \sum_{i} HP_{i}\, y4_{ise} + \sum_{p}\sum_{i}\sum_{r} car_{p} \cdot Tse_{pir}\, y5_{pirse} + \Big(\sum_{j} mcp_{j} + \sum_{j} Hc_{j} \Big) y6_{jse} +$$

$$\sum_{u} der_{use}\, y8_{use} + \sum_{c}\sum_{p} Eg\,ch_{pc}\, y9_{pcse} \times \sum_{p}\sum_{c} mc\,ch_{pc}\, y10_{pcse} + \sum_{c}\sum_{p} CE\,ch_{pc}\, y11_{pcse} +$$

$$\sum_{c}\sum_{p} RE\,ch_{pc}\,y12_{pcse} + \sum_{i}\sum_{p}\sum_{c} cs_i\,h_{pc}\,y13_{pcs} + \sum_{p} Hbc_{pse}.GE.BE.\,y14_{pse} + \sum_{p} Hbc_{pse}.GE.\,y15_{pse})$$

$$-\sum_{se}\left(\sum_{u}\left(\Delta \overset{+}{den}_{use}t7_{use} + \Delta \overset{-}{den}_{use}t8_{use}\right) + \sum_{u}\left(\Delta \overset{+}{der}_{use}t9_{use} + \Delta \overset{-}{der}_{use}t10_{use}\right) + \sum_{q}\left(\Delta \overset{+}{lq}_{qse}t11_{qse} + \Delta \overset{-}{lq}_{qse}t12_{qse}\right) + \sum_{s}\left(\Delta \overset{+}{pw}_{sse}t13_{sse} + \Delta \overset{-}{pw}_{sse}t14_{sse}\right)\right)$$

s.t

$$h_{pc}y16_{pcs} - y2_{pirse} + y3_{psise} - py^{den}\,t1_{se} + CoB^{den}\,t2_{se} \geq sep_{pirse} - Tse_{pir} + Cp_s \quad \forall p, c, i, r, s \tag{69}$$

$$-h_{pc}y16_{pcs} + y2_{pirse} - y10_{pcse} + y7_{jrpise} + y9_{cpse} \geq -ts_{jise} - Cp_s \quad \forall p, c, j, i, r, se \tag{70}$$

$$y3_{psise} - y4_{ise} + y6_{jse} + y12_{pcse} \geq -TP_{pisse} - CoF_c \quad \forall p, s, i, se, j, c \tag{71}$$

$$mwu_c y1_{pcsse} + y5_{pirse} + y2_{pirse} + y10_{pcse} + y13_{pcse} - py^{lq}\,t3_{se} + CoB^{lq}\,t4_{se} \geq -pw_{pse} - sep_{pirse} - h_{jc} \quad \forall p, c, s, i, r, se \tag{72}$$

$$-y5_{pirse} + y14_{pse} + y15_{pse} - py^{pw}\,t5_{se} - CoB^{pw}\,t6_{se} \geq -kr_{ir} - bp_{pse} \quad \forall p, i, r, se \tag{73}$$

$$py^{den}\,\mu^{den}_{se}\,t1_{se} - t7_{use} \geq 0 \quad \forall u, se \tag{74}$$

$$CoB^{den}\,\mu^{den}_{se}\,t2_{se} - t8_{use} \geq 0 \quad \forall u, se \tag{75}$$

$$py^{der}\,\mu^{der}_{se}\,t3_{se} - t9_{use} \geq 0 \quad \forall u, se \tag{76}$$

$$CoB^{der}\,\mu^{der}_{se}\,t4_{se} - t10_{use} \geq 0 \quad \forall u, se \tag{77}$$

$$py^{lq}\,\mu^{lq}_{se}t5_{se} - t11_{qse} \geq 0 \quad \forall q, se \tag{78}$$

$$CoB^{lq}\,\mu^{lq}_{se}t6_{se} - t12_{qse} \geq 0 \quad \forall q, se \tag{79}$$

$$py^{pw}\,\mu^{pw}_{se}t7_{se} - t13_{sse} \geq 0 \quad \forall s, se \tag{80}$$

$$CoB^{pw}\,\mu^{pw}_{se}t8_{se} - t14_{sse} \geq 0 \quad \forall s, se \tag{81}$$

$$t1_{se} + t2_{se} \leq 1 \tag{82}$$

$$t3_{se} + t4_{se} \leq 1 \tag{83}$$

$$t5_{se} + t6_{se} \leq 1 \tag{84}$$

$$t7_{se} + t8_{se} \leq 1 \tag{85}$$

$$y1_{pcsse}, y6_{jse}, y7_{jrpise}, y8_{use}, y9_{cpse}, y10_{pcse}, y11_{pcse}, y14_{pse}, y15_{pse}, y16_{pcse} \tag{86}$$

$$, t1_{se}, t2_{se}, t3_{se}, t4_{se}, t5_{se}, t6_{se}, t7_{se}, t8_{se}, t9_{use}, t10_{use}, t11_{qse}, t12_{qse}, t13_{sse}, t14_{sse} \geq 0$$

Based on DSP, an upper-bound MP is written in each iteration for the objective function of the biomass reverse supply chain as:

$$\mathrm{MP}: \mathrm{Max}^{\mathrm{MP}} = \sum_{se} \mathrm{prs}_{se}\left(\sum_{p} \mathrm{COE}.\mathrm{Egf}_{p} + \sum_{m}\sum_{p}\sum_{se} \mathrm{ud}.\mathrm{qpd}_{mpse}\right) \tag{87}$$

$$-\sum_{d}\sum_{se}\sum_{j}\sum_{u}\sum_{q} \mathrm{cp}_{d}\cdot\mathrm{prs}_{se}\cdot\mathrm{qt}_{juse}\cdot\mathrm{Lq}_{q}\left(1 - \mathrm{Lu}_{q}\right) + \sum_{se}\sum_{j}\sum_{u} \mathrm{prs}_{se}.\mathrm{CNe}.\mathrm{sec}_{juse} - \sum_{se} \mathrm{prs}_{se}$$

$$
\begin{aligned}
&(\sum_{s} op_s \cdot G_s + os \cdot sp_i + \sum_{d} CoW_d \cdot N_d + \sum_{r} Rrp_r \cdot ch_r + \sum_{c}\sum_{p} CoF_c \cdot h_{pc} + \sum_{p}\sum_{c} Cbp_p \cdot h_{pc} + \sum_{s}\sum_{p} CoE.CoB.Puf_{spse} + \\
&\sum_{j}\sum_{u}\sum_{q}\sum_{p} qp_{juse} \cdot Tc_{ju} \cdot lq_q \cdot Hbc_{pse} + \sum_{s}\sum_{p} Cp_s \cdot Hbc_{pse} + \sum_{j}\sum_{s}\sum_{u} \left(Hom_{jsse} + Hmo_{sjse}\right) \cdot \left(wp_s + Tc_{ju}\right) + \\
&\sum_{j}\sum_{u} \left(qt_{juse} + sec_{juse}\right) \cdot TC_{ju} + \sum_{u}\sum_{j} tq_{uj} \cdot \left(CT_{uj} + Pu_j\right) + \\
&\sum_{j}\sum_{i} ts_{jise} \cdot \left(gT_{ji} + Cs_i\right) + \sum_{j}\sum_{d} td_{jd} \cdot \left(CTE_{jd} + CP_d\right) + \sum_{p}\sum_{i}\sum_{s} TP_{pisse} \cdot Tm_p \\
&s.t \\
&\theta \le \sum_{se} (\sum_{p}\sum_{i}\sum_{r} Tse_{pir}\, y2_{pirse} + \left(\sum_{j}\sum_{s} \left(Hom_{sjse} + Hmo_{sjse}\right) + \sum_{p}\sum_{i}\sum_{s} pu_p \cdot TR_{pisse}\right) y3_{psise} \\
&+ \sum_{i} HP_i\, y4_{ise} + \sum_{p}\sum_{i}\sum_{r} car_p \cdot Tse_{pir}\, y5_{pirse} + \left(\sum_{j} mcp_j + \sum_{j} Hc_j\right) y6_{jse} + \\
&\sum_{u} der_{use}\, y8_{use} + \sum_{c}\sum_{p} Eg_c h_{pc}\, y9_{pcse} \times \sum_{p}\sum_{c} mc\, ch_{pc}\, y10_{pcse} + \sum_{c}\sum_{p} CE\, ch_{pc}\, y11_{pcse} + \\
&\sum_{c}\sum_{p} RE\, ch_{pc}\, y12_{pcse} + \sum_{i}\sum_{p}\sum_{c} cs_i h_{pc}\, y13_{pcs} + \sum_{p} Hbc_{pse} \cdot GE.BE. \cdot y14_{pse} + \sum_{p} Hbc_{pse} \cdot GE. \cdot y15_{pse}) \\
&-\sum_{se}\left(\sum_{u}\left(\Delta \overset{+}{den}_{use} t7_{use} + \Delta \overset{-}{den}_{use} t8_{use}\right) + \sum_{u}\left(\Delta \overset{+}{der}_{use} t9_{use} + \Delta \overset{-}{der}_{use} t10_{use}\right) + \sum_{q}\left(\Delta \overset{+}{lq}_{qse} t11_{qse} + \Delta \overset{-}{lq}_{qse} t12_{qse}\right) + \sum_{s}\left(\Delta \overset{+}{pw}_{sse} t13_{sse} + \Delta \overset{-}{pw}_{sse} t14_{sse}\right)\right)
\end{aligned}
\tag{88}
$$

Acceleration of the algorithm can be enhanced using an approach based on enhancing Pareto optimal cuts [63]. A cut is Pareto optimal until the new cut does not make it redundant such that the dual optimal solution of the above cut is Pareto optimal. If a DSP (main problem) has several optimal solutions, in BSP (Bender's sub-problem), the strongest Pareto optimal cut can replace all production cuts. Accordingly, the convergence rate is improved, and the acceleration of reaching the final cuts increases.

6. Sensitivity analysis

Generally, real-life problems are complex and have uncertainties. This study assumed quality, demand, and capacity to be uncertain parameters. Quality was treated at three levels, i.e., input, supply, and recycled products. As mentioned, a robust optimization approach was adopted to cope with supply chain uncertainties. Then, the BD algorithm was employed to solve complex variables. The proposed model was formulated and solved in GAMS 24.7.3 and evaluated in ten iterations. Next, the deterministic and robust models were compared. Eventually, the robust and BD models were compared and analyzed. Effective planning of the biomass supply chain requires identifying biomass production centers. Fig. 2 depicts the poultry biomass frequency in the potential Iranian provinces.

$Please\ insert\ figure\ 2$

6.1. Proposed model under deterministic and robust scenarios

The effects of various cost components on the supply chain network under different scenarios were explored to evaluate the proposed model. Table 2 represents the profit of residue recycling for different supply chain costs. An increase in the coefficient of variation of costs reduces the demand and profit. The profit reduces as the operating costs increase in collection and distribution. As a result, producing new products would no longer be cost-efficient, with recycling costs having no contribution to the supply chain profit.

$Please\ insert\ table\ 2$

According to Table 3, an increase in the production cost reduces the demand, profit, and capacity of biomass production and raises the warehouse storage capacity. Upon the reduced demand, the products are stored in the biogas facilities, thereby increasing the storage capacity. Based on reverse logistics, the residues of biogas facilities are collected and transferred to recycling centers to produce new products and save production costs.

$please\ insert\ table\ 3$

In the next step, the effects of the unmet demand and unproduced product penalization costs on the expected demand and output were studied. Overproduction and penalty have an inverse relationship; the network receives a lower penalty in the case of overproduction. The proposed network seeks to bring a trade-off between overproduction and penalty. Network optimization enables a trade-off between overproduction and the penalty cost to minimize them. As a result, it provides a minimal trade-off in the system.

According to Table 4, an increase in the unmet demand penalty lowers the cost and expected demand coverage and raises the quality level. Quality is directly related to the total profit and customer demand. The high quality of raw materials raises production and avoids product storage. Fig. 3 illustrates product storage versus quality and profit. As can be seen, the quality of products directly influences the profit of the logistic network. The increased quality of products prevents additional storage costs and lowers storage quality (an inverse relationship).

$Please\ insert\ table\ 4$

$Please\ insert\ figure\ 3$

According to Table 5, the return of products to the production cycle saves on purchasing raw materials. It prevents excess costs for purchasing raw materials and increases the profit of the entire supply chain. Also, the increase in recycled products increases raw materials, which is directly related to the output of products.

$Please\ insert\ table\ 5$

According to Fig. 4, an increase in overproduction costs reduces the quality and total profit. In other words, storage (increased capacity) has inverse relationships with demand, quality, and total profit.

$Please\ insert\ figure\ 4$

In the robust counterpart model, the risk level of the problem becomes controllable according to the decision-maker's approach. The number of uncertain parameters in the objective function and constraints should be calculated to determine conservatism in the objective function and constraints. In this process, the total number of uncertain data points for the objective function is maximized; i.e., conservatism reduces as uncertain data is reduced. The conservatism level is 100% when the maximum uncertain dataset is the case. On the other hand, the objective function has no conservatism when the uncertain dataset is zero. The level of conservatism for the constraints was assumed in the range of $\mu = [0.1, 0.25, 0.4, 0.65, 0.8, 0.9]$, considering their uncertainties. The model was executed once to define the level of conservatism. The constraints are most likely to be feasible for a high level of conservatism.

According to Bertsimas and Sim approach [64], the lower-bound robustness of uncertain parameters based on mathematical logic contradicts the objective; hence, the lower-bound parameters are assumed to be certain, whereas upper-bound parameters are treated as uncertain parameters. It is required to solve the model independently six times for the objective function levels of conservatism and six times for the constraints. Scenarios were defined for the supply chain network. A set of demand, capacity, and quality uncertainties was developed for each scenario. For this purpose, a sensitivity analysis was carried out on the objective function to understand the main effects of the parameters on the solutions (i.e., the costs, penalty, and uncertainties). The uncertainty radius could be changed using this parameter while $\mu_{se}^{den} = 0$ is evaluated to investigate the conservatism of the demand uncertainty. For the violation of one constraint, demand is assumed to have a symmetric distribution based on Eq. (24). Table 6 exhibits the changes in the total objective function under different protection levels. Values can vary from 0.1 to 0.65. The increase in credibility levels causes the risk aversion level of the model to increase and the value of the objective function to go up. Table 7 shows that the increase in conservatism intensifies the volume of calculations and increases the computational time of the model.

$Please\ insert\ table\ 6$

$Please\ insert\ table\ 7$

According to the above Tables, the profit increases for the variability level while the uncertainty probability declines. Hence, only the violation probability of robust constraints is calculated. Table 8 describes the changes in the total objective function at different protection levels. It focuses on the risk associated with the model, objective function values, and computational time. Thus, the violation probability is 1 when $\mu_{se}^{den}, \mu_{se}^{lq}$, and $\mu_{se}^{pw} \leq 3$; i.e., when the solution has the maximum value and zero robustness to changes. An increase in $\mu_{se}^{den}, \mu_{se}^{lq}$, and μ_{se}^{pw} from 3 to 12 reduces the constraint violation. Thus, a value below 12 should be considered as it would be closer to 0 and has no significant difference. The objective function value may be unacceptable. The selection of values in the range of 3-12 provides a good trade-off in the supply chain.

$Please\ insert\ table\ 8$

6.2. Evaluation of the proposed model using Bender’s decomposition Algorithm

The model was evaluated regarding the computational time, the number of BD cuts, and lower-bound convergence through Pareto optimal cuts. Table 9 represents the effects of valid inequalities on the number of iterations, computational time, lower bound, and optimal gap. According to Table 9, the CPLEX computational time was longer for fewer scenarios. However, the BD algorithm showed higher performance at larger numbers of scenarios. In addition, the Pareto cuts had a longer computational time than the BD algorithm since it is difficult to obtain a convergent cut. The maximum gap occurred at iteration 7. Hence, the Pareto-optimality cut of the reduction generation scheme had the highest performance.

$Please\ insert\ table\ 9$

Fig. 5 depicts the convergence of the BD algorithm for the six scenarios at all iterations. The lower bound is distant from the optimal solution at iterations 1, 2, and 3; i.e., the BD optimality cut constraint is not limited at iterations 1-3. The upper and lower bounds became closer after two iterations, converging toward each other at iteration 7 (optimal solution). This convergence is a major advantage of the BD algorithm.

$Please\ insert\ figure\ 5$

$Please\ insert\ figure\ 6$

Fig. 6 compares problem-solving methods and demand uncertainty concerning the set of biomass supply chain network costs. The problem has been investigated with three different quality levels. CPLEX cannot solve the problem in the third experiment and does not have a suitable answer for the effect of demand uncertainty on the cost. At the same time, the robust approach is used to deal with the uncertainty of the parameters. Based on the obtained results, the supply chain cost increased with the increase in the biomass quality. This process is fully compatible with the model robustification approach. In other words, with increasing uncertainty, more conservative solutions are produced. However, this method is not efficient in large and complex dimensions. Therefore, the Benders algorithm is used to solve this problem. The classic Banders algorithm at a higher level requires more time and cuts to solve the problem. The accelerated Benders algorithm was used to improve the model’s efficiency and solution time. The results of the accelerated Benders method show that it will get a better solution in complex dimensions with more conservatism.

7.Managerial insights

The main goal of the current research is to maximize profit based on biogas production based on different levels of biomass quality. The final product has a strategic value for business areas. Based on this value, the mathematical model minimizes the problem of fines caused by not meeting the customer’s demand. The studied network model simultaneously follows strategic and tactical decisions. In strategic decision-making, critical axes, including locating the main facilities, determining the capacity of biogas facilities and the quality level of biogas, and the circular use of biomass, were considered. Also, the tactical decisions of the present problem have covered the estimation of biomass flows, the amount of biogas produced and chemical fertilizers, the explanation of network costs, and the coverage of the demand for final and secondary products.

In the following, the optimal policies were analyzed under uncertain conditions, and the key determining uncertainties were applied in the flows related to the studied network. Overall, increasing the coefficient of variability of operating and production costs in 6 scenarios declined the production flow of biogas. As a result, a significant loss of profit is created. In this respect, a significant drop of 44.45% in profit was achieved for up to six initial operating scenarios. On the other hand, with the increase in the uncertainty level of biogas's quality, the lost demand rate for the main and secondary products increases, and the resulting penalty on the network intensifies. The increase in fines has caused a 52% decrease in profit and a drop in the storage level of manufactured products. In this case, the level faced with uncertainty is estimated by employing the robust approach. By increasing the robustness confidence level in each scenario, risk aversion increases by about 20% in the logistics network. Although the network's overall cost increases, the constraint's violation has decreased significantly. Therefore, the ability to control the conservatism level of the network is improved, and less profit is lost. In addition, as the conservatism level increases, we reduce the number of violations of the customer's demand limit and achieve fewer lost sales. Hence, with an integrated management approach, the appropriate level of protection is set by balancing unsatisfied demand violations, quality, capacity, and cost. Furthermore, in the extended network, from every 1 m^3 of biogas, 1.7 kilowatts per cubic meter are directly used as electricity. Meanwhile, of every 1 m^3 of biogas, about 8% is supplied to customers as bio-fertilizers from recycled channels. Therefore, the existence of production channels in direct and reverse routes improves the sustainability of the chain.

By increasing the level and finding the convergent cut, the Benders algorithm needs a longer computation time. However, from Scenario 3 onward, the accelerated Benders and the Benders algorithms outperform the other algorithms. This issue indicates the optimality of Pareto cuts iteration. More specifically, the optimal gap in Benders and accelerated Benders algorithms has increased from 0.22 and 0.25 to 0.76 and 0.81, respectively. Optimized Pareto cuts have better performance in the plan's production cost reduction. In this respect, by increasing the quality level and reducing waste, this algorithm lowers production costs and declines the storage capacity of products. Waste minimization will ensure delivery of the orders and prevent the re-production of raw materials. Subsequently, this minimization lowers costs and increases the network profits significantly.

8. Conclusion

Resilience and sustainability play vital roles in supply chains. This paper introduced a two-stage stochastic mathematical programming model for reverse supply chain networks. A robustness approach was adopted to evaluate the proposed model based on the network optimization results. Due to uncertainties in logistic networks, conservatism and maintenance levels are crucial for managers. In this study, the coefficient of conservatism of the robust programming model was employed to maintain conservatism. Conservatism is maintained through changes in the coefficient of conservatism, thereby raising the level of maintenance. The proposed model indicated that a rise in the operating and reconstruction costs would increase the capacity and reduce demand and profitability in the biomass reverse supply chain network. The customer demand increased as the biomass quality level increased. Consequently, the total profitability and saving warehouse (storage) costs increased. This study exploited the BD to improve supply chain performance. The BD algorithm uses various methods, including Pareto-optimal cuts. In this research, the computational time, the number of BD cuts, and lower-bound convergence were evaluated for Pareto-optimal cuts. The reduction generation scheme is an advantage of the BD algorithm. The Pareto-optimal cuts had a longer computational time than the BD algorithm since it is complex to find a convergent cut. Restrictions in the storage of poultry biomass due to the existence of diverse weather conditions and the increase in the degree of perishability of biomass lead to an increase in uncertainty in the supply and transfer of materials. On the other hand, limitations in the location of biogas facilities and production centers. Biomass poses a serious challenge to the planners of this field due to the environmental effects and limitations of the transmission route and transportation systems. Accordingly, the following suggestions are presented for developing various aspects of the biogas supply chain research.

- Development of location/allocation axes according to regional economic indicators;
- Covering the uncertainty of vital parameters such as weather information and supply of raw materials; and
- Focusing on the reliability approach in the production and distribution centers of the biogas network concerning partial and general disturbances

References:

1. Simionescu, M., Albu, L. L., Szeles, M. R., et al. “The impact of biofuels utilization in transport on the sustainable development in the European Union”, Technological and Economic Development of Economy, 23(4), pp. 667–686 (2017).
2. Khatun, M.L., Nime, J., Nandi, R., Alam, M.M., et al. “Co-digestion of poultry droppings and banana waste for maximizing biogas production in Bangladesh”, Fuel, 346, pp. 128346-128346(2023).
3. Morato, T., Vaezi, M., Kumar, A. “Techno-economic assessment of biomass combustion technologies to generate electricity in South America: A case study for Bolivia”, Renewable and Sustainable Energy Reviews, 134, p. 110154(2020).
4. Wakeel, M., Hayat, T., Shah, N.S., et al. “Biogas Energy Resources in Pakistan Status, Potential, and Barriers”, Utilities Policy, 84, p. 101643(2023).
5. Zhang, F., Wang, J., Liu, S., et al. “Integrating GIS with optimization method for a biofuel feedstock supply chain”, Biomass Bioenergy, 98, pp. 194-205(2017).
6. Arshad, M., Bano, I., Khan, N., et al. “Electricity generation from biogas of poultry waste: an assessment of potential and feasibility in Pakistan”, Renewable and Sustainable Energy Review, 81, pp. 1241-1246(2018).
7. Sharma, B., Ingalls, R.G., Jones, C.L., et al. “Biomass supply chain design and analysis: Basis, overview, modeling, challenges, and future”, Renewable and Sustainable Energy Review, 24, pp. 608-627(2013).
8. Yue, D., and You, F. “Optimal supply chain design and operations under multi-scale uncertainties: Nested stochastic robust optimization modeling framework and solution algorithm”, AIChE Journal, 62, pp. 3041–3055(2016).

9. Badri, H., Fatemi Ghomi, S. M.T., and Hejazi, T.H. “A two-stage stochastic programming approach for value-based closed-loop supply chain network design”, Transportation Research Part E: Logistics and Transportation Review, 105, pp. 1–17(2017).
10. Chen, Y., Guo, T., Kainz, J., et al. “Design of a biomass-heating network with an integrated heat pump: A simulation-based multi-objective optimization framework”, Applied Energy, 326, p. 119922(2022).
11. Bice, K., Batun, S. “Closed-loop supply chain network design under demand, return and quality uncertainty”, Computer and Industrial Engineering, 155, p. 107081(2020).
12. Ika Rhofita, E., Rachmat, R., Meyer, M., et al. “Mapping analysis of biomass residue valorization as the future green energy generation in Indonesia”, Journal of Cleaner Production, 354, p. 131667(2022).
13. Bruce da Silva, T., Baptista, P., Santos Silva, C., et al. “The use of alternative fuels to mitigate climate change impacts in the transportation sector in Rio de Janeiro, Brazil”, Transportation Research Procedia, 62, pp. 752-759(2022).
14. Hao Ngo, H., Nguyen, T.K.L., Guo, W., et al. “Chapter 8 - Life-cycle assessment on sequestration of greenhouse gases for the production of biofuels and biomaterials”, Climate Change Mitigation: Sequestration of Green House Gases, pp. 179-202(2022).
15. An, H. “The closed-open truck and loader routing problem for biomass transportation from satellite storage locations to a bioenergy plant, Computers and Electronics in Agriculture, 194, p. 106764(2022).
16. Zhao, B., Wang, H., Huang, Zh., et al. “Location mapping for constructing biomass power plant using multi-criteria decision-making method”, 49, p. 101707(2022).
17. Tumusiime, E., Kirabira, J.B., Musinguzi, W.B. “Optimization of substrate mixing ratios for wet anaerobic digestion of selected organic waste streams for productive biogas systems”, Energy Reports, 8, pp. 10409-10417(2022).
18. pishvaee, M.S., Mohseni, SH., Bairamzadeh, S. Chapter7- Strategic planning in biofuel supply chain under uncertainty”, Biomass to Biofuel Supply Chain Design and Planning under Uncertainty, pp. 183-212(2021).
19. Zeballos, L.J., Mendez, C.A., Barbosa-Pavoa, A.P., et al. “Multi-period design and planning of closed-loop supply chains with uncertain supply and demand”, Computers and Chemical Engineering, 66, pp. 151–164(2014).
20. Sahoo, K., Mani, S. “GIS based discrete event modeling and simulation of biomass supply chain”, 2015 Winter Simulation Conference(WSC), (2015).
21. Ghaderi, H., Pishvaee, M.S., Moini A. “Biomass supply chain network design: an optimization-oriented review and analysis”, Industrial Crop and Products, 94, pp. 972–1000(2016).
22. Jeihoonian, M., Zanjani, M.K., and Gendreau, M. “Closed-loop supply chain network design under uncertain quality status: Case of durable products”, International Journal of Production Economics, 183, pp. 470–486(2017).
23. Kamyabi, E., Moazzez, H., Husseinzadeh Kashan, A. “A hybrid system dynamics and two-stage mixed integer stochastic programming approach for closed-loop battery supply chain optimization”, Applied Mathematical Modelling, 106, pp. 770-798(2022).
24. Fathollahi-Fard, A.M., Hajiaghaei-Keshteli, M. “A stochastic multi-objective model for a closed-loop supply chain with environmental considerations”, Applied Soft Computing, 69, pp. 232–249(2018).
25. Pazhani, S., Mendoza, A., Nambirajan, R., et al. “Multi-period multi-product closed loop supply chain network design: A relaxation approach”, Computer and Industrial Engineering, 155, pp. 107191(2021).
26. Ahmadi, S., Amin, S. H. “An integrated chance-constrained stochastic model fora mobile phone closed-loop supply chain network with supplier selection”, Journal of Cleaner Production, 226, pp. 988–1003(2019).
27. Lemire, P.O., Delcroix, B., Audy, J.F., et al. “GIS method to design and assess the transportation performance of a decentralized biorefinery supply system and comparison with a centralized system: case study in southern Quebec, Canada”, Biofuels, Bioproducts Biorefining, 13, pp. 552–67(2019).
28. Murele, O.C., Zulkafli, N.I., Kopanos, G., et al. “Integrating biomass into energy supply chain networks”, Journal of Cleaner Production, 248, pp. 119246(2020).
29. Vazifeh, Z., Mafakheri, F., An, Ch. “Biomass supply chain coordination for remote communities: A game-theoretic modeling and analysis approach”, Sustainable Cities and Society, 69, pp. 102819-102819(2021).
30. Lee Yuen Lo, Sh., Shen How, B., Yong Teng, S., et al. “Stochastic techno-economic evaluation model for biomass supply chain: A biomass gasification case study with supply chain uncertainties”, Renewable and Sustainable Energy Reviews, 152, p.
31. Tittmann, P.W., Parker, N.C., Hart, Q.J., et al. “A spatially explicit techno-economic model of bioenergy and biofuels production in California”, Journal of Transport Geography, 18, pp. 715-728(2010).

32. Shaverdi, M., Yaghoubi, S., Ensafian, H. “A multi-objective robust possibilistic model for technology portfolio optimization considering social impact and different types of financing”, Applied Soft Computing, 86, pp. 105892(2020).
33. Fattahi, M. and Govindan, K. “Integrated forward/reverse logistics network design under uncertainty with pricing for collection of used products”, Annals of Operations Research, 253, pp. 193–225(2016).
34. Ramezani, M., Kimiagari, A.M., Karimi, B., et al. “Closed-loop supply chain network design under a fuzzy environment”, Knowledge- Based Systems, pp. 108-120(2014).
35. Santos, V.O., Guimaraes, M.N., and Colpani, “Biomass Conversion by Pyrolysis Process”, Reference Module in Earth Systems and Environmental Sciences, (2023).
36. Li, D., Ge, A. “New energy technology innovation and sustainable economic development in the complex scientific environment”, Energy Reports, 9, pp. 4214-4223(2023).
37. Parker, N., Tittmann, P., Hart, Q., et al. “Development of a. biorefinery optimized biofuel supply curve for the Western United States”, Biomass and Bioenergy, 34, pp. 1597–1607(2010).
38. Dal-Mas, M., Giarola, S., and Zamboni, A. “Strategic design and investment capacity planning of the ethanol supply chain underprice uncertainty”, Biomass and Bioenergy, 35, pp. 2059-2071(2011).
39. Čučeka, L., Sabev, P., Jiří, V., et al. “Total footprints-based multi-criteria optimisation of regional biomass energy supply chains”, Energy, 44, pp. 135-145(2012).
40. Yılmaz, S., Selim, H. “A decision model for cost effective design of biomass based green energy supply chains, Bioresource Technology, 191, pp. 97-109(2015).
41. Poudel, S.R., Marufuzzaman, M., Bian, L., “A hybrid decomposition algorithm for designing a multi-modal transportation network under biomass supply uncertainty”, Transportation Research Part E: Logistics and Transportation, 94, pp. 1–25(2016).
42. Abdul Quddus, M., Hossain, N.U.I., Raed, M., Mohammad, M., et al. “Sustainable network design for multi-purpose pellet processing depots under biomass supply uncertainty”, Computers and Industrial Engineering, 110, pp. 462-483(2017).
43. Woo, H., Acuna, M., Moroni, M., et al. “Optimizing the location of biomass energy facilities by integrating Multi-Criteria Analysis (MCA) and Geographical Information Systems (GIS)”, Forests, 9, p. 585(2018).
44. Rahemi, H., Torabi, A., Avami, A., et al. “Bioethanol supply chain network design considering land characteristics”, Renewable and Sustainable Energy Reviews, 119, p. 109517(2020).
45. Saghaei, M., Ghaderi, H., Soleimani, H. “Design and optimization of biomass electricity supply chain with uncertainty in material quality, availability and market demand”, Energy, 197, 117165(2020).
46. Yahya, N.S.M., Yin Ng, L., Andiappan, V. “Optimisation and planning of biomass supply chain for new and existing power plants based on carbon reduction targets”, Energy, 237, p. 121488(2021).
47. Guo, Ch., Hu, H., Wang, Sh., et al. “Multiperiod stochastic programming for biomass supply chain design under spatiotemporal variability of feedstock supply”, Renewable Energy, 186, pp. 378-393(2022).
48. Aranguren, M., F., Castillo-Villar, K.K. “Bi-objective stochastic model for the design of large-scale carbon footprint conscious co-firing biomass supply chains”, Computers and Industrial Engineering, 171, p. 108352(2022).
49. Umakanth, A.V., Ashok Kumar, A., Vermerris, W., et al. “ Sweet Sorghum for Biofuel Industry”, Breeding Sorghum for Diverse End Uses, p.p. 255-270(2019).
50. Syahira Mod Yahya, N., Yin Ng, L., Andiappan, V. “Optimisation and planning of biomass supply chain for new and existing power plants based on carbon reduction targets” , 237, p.121488(2021).
51. Chen, A., Liu, Y. “Designing globalized robust supply chain network for sustainable biomass-based power generation problem”, Journal of Cleaner Production, 413, p. 137403(2023).
52. Masel Ullah, K., Hossain Masum, F., Field, J.L., et al. “Designing a GIS-based supply chain for producing carinata-based sustainable aviation fuel in Georgia”, Biofuels, Bioproducts and Biorefining, 17, pp. 786-802(2023).
53. Zarei, M., Cherif, A., Yoon, H.J., et al. “Optimal design of a biofuel supply chain using an augmented multi-objective and TOPSIS method”, Green Chemistry, 25, pp. 4067-4075(2023).
54. Saidur, R., Abdelaziz, E.A., Demirbas, A., et al. “A review on biomass as a fuel for boilers”, Renewable and Sustainable Energy Review, 15, 2262-2289(2011).
55. Tosarkani, B.M., Amin, S.H. “A robust optimization model for designing a wastewater treatment network under uncertainty: Multi-objective approach”, Computers and Industrial Engineering, 146, p. 10661(2020).
56. McGauran, Th., Dunne, N., Smyth, B.M., et al. “Feasibility of the use of poultry waste as polymer additives and implications for energy, cost and carbon”, Journal of Cleaner Production, 291, p. 125948(2021).

57. Daneshmandi, M., Sahebi, H., Ashayeri, J. “The incorporated environmental policies and regulations into bioenergy supply chain management: A literature review”, Science of The Total Environment, 820, p. 153202(2022).
58. Hu, J., Wang, Y., Pang, Y., et al. “Optimal maintenance scheduling under uncertainties using Linear Programming-enhanced Reinforcement Learning”, Engineering Applications of Artificial Intelligence, 109, p. 104655(2022).
59. Ebrahim Sharifnia, S.M., Amrollahi Biyouki, S., Sawhney, R., et al. “Robust simulation optimization for supply chain problem under uncertainty via neural network metamodeling”, Computers and Industrial Engineering, 162, p. 107693(2021).
60. Sa, E.M., Morabito, R., Camargo, R.S. “Benders decomposition applied to a robust multiple allocation incomplete hub location problem”, Computers and Operations Research, 89, pp. 31-50(2018).
61. Hamidieh, A., Babaei, S. “Flexible and Robust Optimization Combination for Reliable Forward Reverse Logistic Network Design using Benders’ Decomposition Method”, Journal of Quality Engineering and Production Optimization, 7, pp. 107-134(2022).
62. Abbasi, Y., Momeni, H.R., Ramezani, A. “Robust tube-based MPC with enlarging the region of attraction for tracking of switched systems”, Journal of the Franklin Institute, 358, pp. 8886-8913(2021).
63. Keyvanshokooh, E., Ryan, S.M., Kabir, E. “Hybrid robust and stochastic optimization for closed-loop supply chain network design using accelerated Benders decomposition”, European Journal of Operational Research, 249, pp. 76-92(2016).
64. Bertsimas, D., and Sim, M. “The price of robustness”, Operations Research, 52, pp. 35–53(2004).

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Table 1. Summary of the literature

Ref.	**Biomass**		**Uncertainty**	**Recycling**	**Objective Function**		**Constraints**					**Model**	**Network flows**		**Method**
	Poultry	Livestock			Single-stage	Other (cost and benefit)	Location	Quality	Capacity	Demand	Transportation		Forward	Reverse	
Parker et al. [37]		*				*				*	*	MILP	*		GAMS
Dal-mas [38]		*	*			*	*			*	*	MILP	*		GAMS
Čučeka et al. [39]		*				*			*				*		GAMS
Sharma et al. [7]		*				*			*	*				*	GAMS
Yilmaz and Selim [40]		*				*			*	*	*	MILP	*		Simplex
Podel et al. [41]		*	*			*			*	*	*	MILP	*		GAMS
Alghodos et al. [42]		*	*			*			*	*		SMINLP	*		Combined decomposition
woo et al. [43]		*				*			*	*	*	LP	*		L-shaped decomposition
Rahemi et al. [44]		*				*			*	*	*	MILP	*		GAMS
Saghaei et al. [45]	*		*			*	*		*	*		MINLP	*		GAMS and multi-objective method
Yahya et al. [46]		*	*			*			*	*	*		*		Multi-stakeholder analysis
Lee Yuen Lo et al. [30]		*	*			*		*			*		*		Monte Carlo
Guo et al. [47]		*	*			*					*		*		Stochastic programming
Arangorn et al. [48]		*	*			*					*		*		Metaheuristic
Chen, Liu[51]		*	*			*				*		MILP	*		Robust Programming
Masel Ullah et al.[52]		*				*	*				*	MILP	*		
Zarei et al.[53]		*				*					*	MILP	*		Limitation of ε
Present Work (2023)	*		*	*	*	*		*	*	*		MILP	*	*	Benders decomposition

Table 2. Effects of operating costs on the reverse supply chain profit

Scenario	Coefficient of Variability	Profit	Demand (%)	Recycling (%)
1	0	547012	97.23	99.47
2	0.4	412368	95.14	96.35
3	0.9	359673	86.78	89.23
4	2	234567	74.23	76.35
5	4	132587	60.86	67.41
6	6	-724876	52.78	59.14

Table 3. Effects of production costs on the profit, demand, and capacity

Scenario	Coefficient of Variability	Profit	Demand (%)	Recycling (%)
1	0.2	521347	96.54	67.28
2	0.6	396542	94.28	70.89
3	1	302564	81.18	79.28
4	3	195472	75.35	84.25
5	5	-125476	69.87	89.47
6	7	-824876	49.25	98.54

Table 4. Expected demand coverage, unmet demand penalty, and quality

Scenario	Coefficient of Variability	Profit	Demand (%)	Recycling (%)
1	0	77412	38.54	54.78
2	15	64538	47.24	60.34
3	30	55612	67.89	67.21
4	45	48735	71.54	78.59
5	80	35478	84.92	89.65
6	100	24578	98.78	99.58

Table 5. Returned product cycle

Scenario	Recycled Products (m3)	Total Profit	Output Products (%)
1	10264	35791	67.54
2	30254	65874	79.68
3	50546	75469	81.57
4	64644	76589	82.46
5	54650	87691	87.57
6	10054	95547	98.87

Table 6. Overall objective function changes under levels of conservatism $(\mu_{se}^{den}, \mu_{se}^{lq}, \mu_{se}^{pw})$

$\boldsymbol{\mu_{se}^{den}, \mu_{se}^{lq}, \mu_{se}^{Pw}}$	Objective Function	Cost Objective	Overall Objective Function

	Rise	Function	
0.10	0	724563	215489
0.25	14.56	892354	254625.98
0.40	18.45	914523	263541.87
0.65	19.27	102456.67	278569.24
0.80	20	115698.36	286574.48
0.90	20	145268.78	293546.98

Table 7. Overall objective function change and computational time

$\mu_{se}^{den}, \mu_{se}^{lq}, \mu_{se}^{Pw}$	**Objective Function Rise**	**Computational Time (s)**	**Overall Objective Function**
0.10	0	326.00	256489.57
0.25	4	338.20	257625.14
0.40	8	349.10	258541.97
0.65	12	352.80	258669.31
0.80	16	376.20	258974.54
0.90	20	402.25	259546.61

Table 8. Overall objective function at different levels of conservatism

$\mu_{se}^{den}, \mu_{se}^{lq}, \mu_{se}^{pw}$	**Objective Function Rise**	**Computational Time (s)**	**Overall Objective Function**	**Cost Objective Function**
0.10	0	326.00	256489.57	724563
0.25	14.56	338.20	257625.14	892354
0.40	18.45	349.10	258541.97	914523
0.65	19.27	352.80	258669.31	102456.67
0.80	20	376.20	258974.54	115698.36
0.90	20	402.25	259546.61	145268.78

Table 9. Lower bounds, optimal gap, number of iterations, and computational time (BD)

No.	**BD**				**Accelerated BD**				**CPLEX**	
	Time (s)	**Iteration**	**Optimal Gap**	**Objective Function**	**Time (s)**	**Iteration**	**Optimal Gap**	**Objective Function**	**Time**	**Objective Function**
1	10	5	0.25	8785	12	4	0.22	9902	8	9785
2	24	10	0.35	9254	34	14	0.32	10356	20	10154
3	38	30	0.99	11254	45	30	0.95	12354	30	11054
4	59	18	0.86	12354	69	16	0.81	12456	-	12132
5	68	10	0.68	13547	78	9	0.61	13698	-	13547
6	88	20	0.81	13869	90	19	0.76	13965	-	13869
7	100	30	1.35	14256	111.53	30	1.32	14256	-	14256

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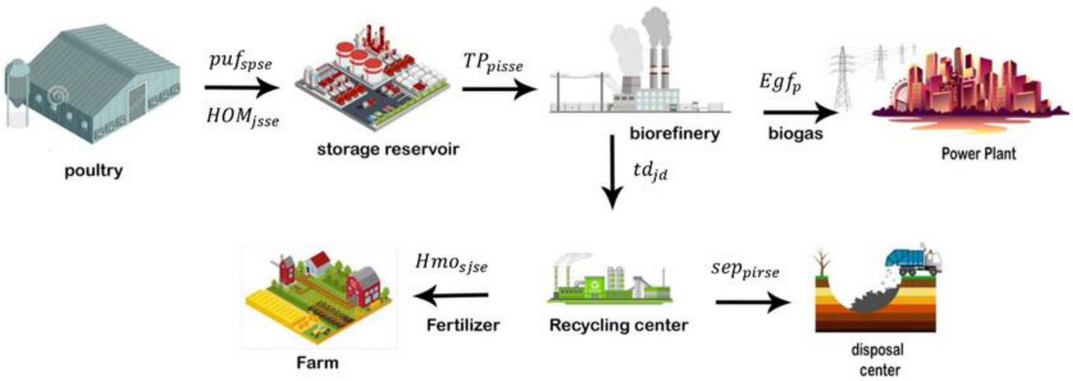


Figure. 1. Schematic of the proposed biomass supply chain network

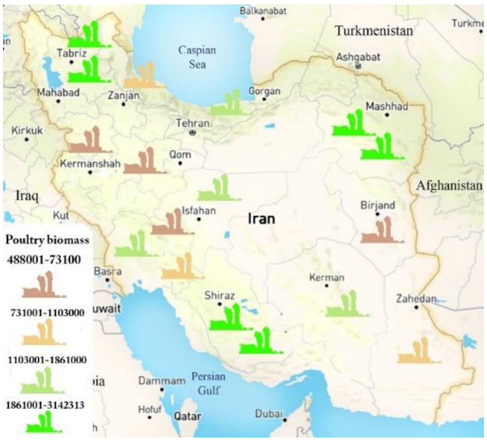


Figure. 2. Poultry biomass map of Iran

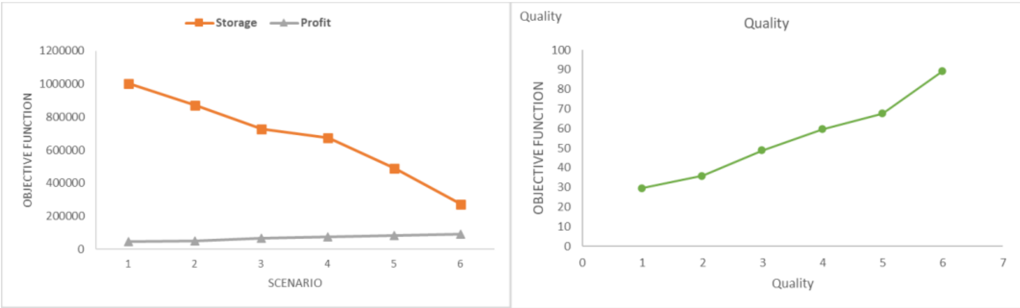


Figure. 3. Quality, profit, and storage

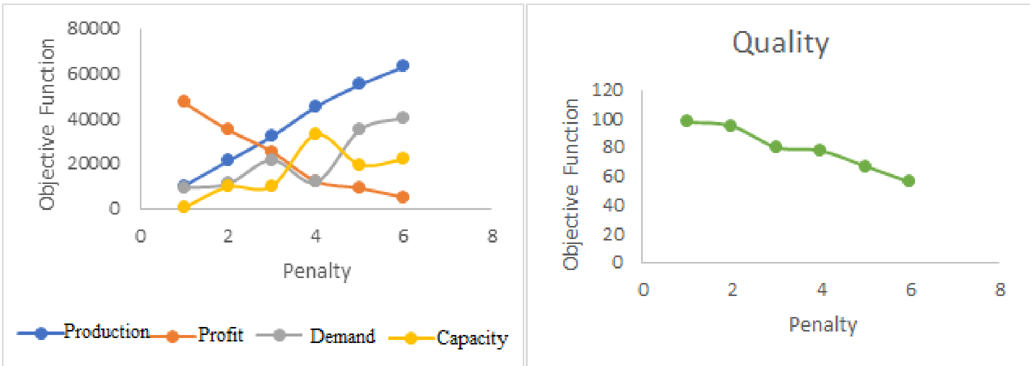


Figure. 4. Overproduction effects on supply chain network parameters

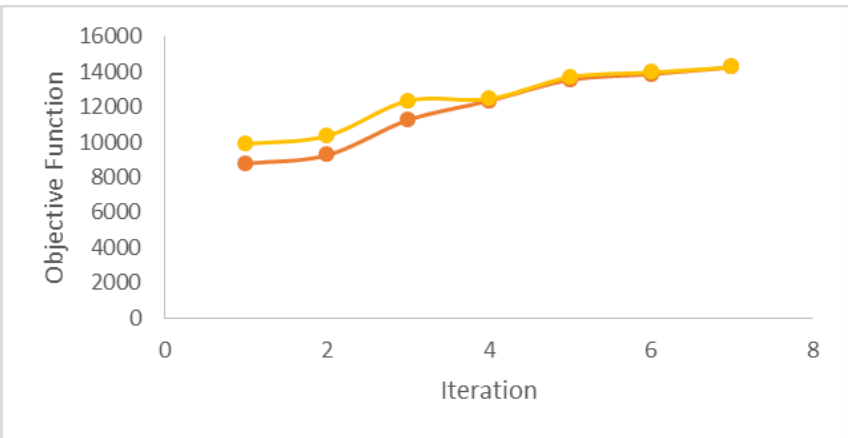
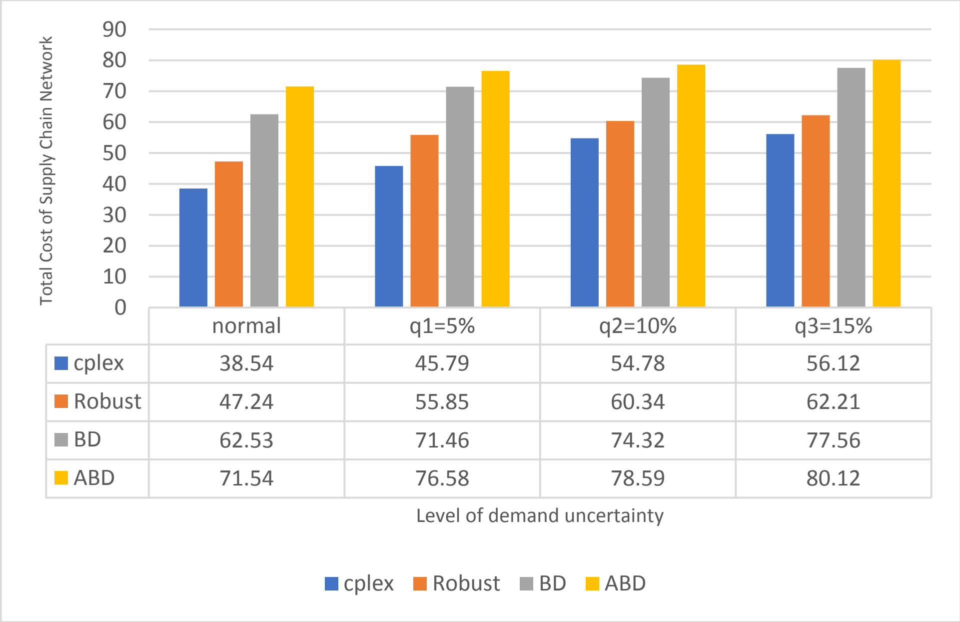


Figure. 5. Convergence of the BD algorithm



	normal	q1=5%	q2=10%	q3=15%
cplex	38.54	45.79	54.78	56.12
Robust	47.24	55.85	60.34	62.21
BD	62.53	71.46	74.32	77.56
ABD	71.54	76.58	78.59	80.12

Figure. 6. Impact of demand uncertainty and biomass quality level on supply chain costs

Biograghy

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