Evaluation of the Efficiency of Mother Wavelet Functions for Simulating Endurance Time Excitations

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Abstract

The Endurance Time (ET) method is employed as a dynamic time history technique to analyze structures under artificially intensified acceleration time histories, known as Endurance Time Excitation Functions (ETEFs). Prior studies have shown that discrete wavelet transform (DWT) is an effective approach for generating ETEFs by representing signals as transform coefficients determined through optimization procedures. However, the impact of the chosen mother wavelet function on simulated ETEF accuracy remains unexplored. This study introduces a methodology to investigate the influence of mother wavelet functions on simulated ETEFs. Specifically, 31 mother wavelet function candidates from four families (Daubechies, Coiflet, Symlet, and Bio-Orthogonal) are examined. Results reveal that the choice of the mother wavelet function can lead to approximately 15% variation in simulated ETEFs' accuracy. The Daubechies wavelet family stands out as the preferred choice, exhibiting a diminished impact compared to alternative families. Remarkably, this wavelet family is associated with an importance factor of 5.5%, significantly lower than the 13% observed for the other families. Within the Daubechies family, db12 demonstrates optimal efficiency in generating linear response-based ETEFs. The research highlights the superiority of the Daubechies wavelet family, offering valuable insights to enhance ETEF simulation accuracy and reliability for effective ET method implementation.

Key Words: Endurance Time method; discrete wavelet transform; mother wavelet functions; Daubechies, Coiflet, Symlet, Bio-Orthogonal

1 Introduction

Several researchers have tried to offer new alternative frameworks to overcome difficulties associated with the conventional nonlinear time history analyses [1-3]. One potential solution to address this problem is employing source-based or site-based ground motion simulation, which finds application in nonlinear time history analysis [4-7]. Additionally, researchers have examined the effectiveness of this method by comparing it with actual ground motion data. As another solution, Endurance Time (ET) method was devised to be as a simple yet accurate alternative for an nonlinear dynamic analysis [8]. The ET method is a dynamic time history procedure in which the structure is subjected to a set of predefined acceleration functions. These endurance time excitation functions (ETEFs) are produced to be in an increasing form, which are replaced by the real ground motions (GMs) being used in the conventional time history analysis. The conceptual viewpoint behind the ET method works in a way that seismic behavior of a specific structure or building can be readily assessed by how long it can endure the seismic vibration of an ETEF [9].

The ET method can be also compared to a more recent dynamic analysis that is in an increasing form as well—the Incremental Dynamic Analysis (IDA). To perform an IDA, selected recorded ground motions should be scaled from a lower intensity level up to an upper seismic intensity level that is typically decided to become related to the collapse mechanism of the considered structural system [10]. In

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earthquake engineering, the IDA is widely used for many applications such as risk assessment [11], collapse assessment [12] and fragility model developments [13]. The major advantage of the ET method over the IDA procedure is the limited and reduced number of required dynamic analyses that are essential for an ET procedure [14]. Mashayekhi and Mostafaei [15] utilized the ET method to determine the intensity measure at which cracks initiate in the concrete dam.

As can be recognized from the concept of the ET method, ETEFs are the core part of the ET analysis [16]. This means that the accuracy of the ET method can heavily depend on the accuracy and exactness of the ET records, the ETEFs. Therefore, simulating efficient and more reliable ETEFs is an important step toward improving the results obtained from the ET framework. ETEFs are generated and optimized in a way that they maintain their compatibility with the responses computed using recorded GMs. To reach this ultimate goal, dynamic characteristics of the generated ETEFs are set to be compatible with the ones found to be significant in those of GMs selected for the ET record generation procedure. In this case, there are a number of investigations in which the significance of different dynamic characteristics for the generation of ETEFs has been examined [17, 18]. In a tradition for this case, discrepancies between dynamic characteristics of ETEFs and those corresponding factors associated with selected GMs are defined as target equations or objective functions which are normally solved by unconstrained nonlinear optimization algorithms. For more information on this issue, readers are referred to available literatures in this regard [14, 18].

There are several ways to define decision variables in simulating ETEFs problem. In the conventional practice of simulating ETEF, acceleration values of the ET signals are selected as the decision variables in the optimization procedure. It is well known that acceleration values are not able to solely express the dynamic characteristics of signals, i.e., the frequency properties and content within the signal. In fact, the dynamic characteristics of the signals can be detected and extracted by methods frequently used in signal processing—the Fourier and Wavelet transformations. Fourier transform is able to decompose a signal record into its frequency components. However, it should be mentioned that it cannot detect and extract frequency variations along a signal, so the Fourier transform seems to be appropriate for stationary signals only. Conversely, the wavelet transform expresses the time history of a signal in a time-frequency domain [19]. Therefore, contrary to the Fourier transformation technique, the wavelet transform is capable of identifying the time variation of frequency in a signal. The temporal variations of signals can be identified and extracted by the wavelet transformation procedure, so it is applicable to the non-stationary signals such as real GMs. Wavelet functions are produced based on a basic wavelet function called *mother wavelet function*. Wavelet functions are basis functions for wavelet analysis. There are several mother wavelet functions developed for wavelet analysis. Recent generations of ETEFs have been efficiently produced by an algorithm that is based on a wavelet transformation or discrete form of it—the discrete wavelet transform (DWT) [20]. DWT finds extensive application in earthquake engineering, particularly in seismic response evaluation [21-24]. Simulating ETEFs in DWT space is employed in fourth and fifth ETEFs generation [25, 26]. Although the recent generations of ETEFs—which are based on a wavelet transform—turned to be highly efficient in terms of matching and exactness quality, no study has been performed to investigate the influence of the mother wavelet functions used for the generation procedure of ET records. It is worth to add that there are several mother wavelet functions for this purpose and finding an appropriate one is a significant and important step for having more accurate and exact ETEFs. Presently, there is a lack of dedicated research on identifying the optimal mother wavelet function for generating ETEFs. The selection of the mother wavelet is often done randomly or by using multiple mother wavelets to produce ETEFs, and then choosing the most suitable one. However, both approaches fall short of yielding an ideal solution, and the latter option is particularly time-consuming. Therefore, it is crucial to conduct comprehensive research focused on the selection of mother wavelets for ETEF production. Such research efforts are essential to enhance the accuracy and efficiency of the process. Mashayekhi et al. [27] employed increasing sine functions to generate long-time ETEFs.

In this study, a methodology to perform a parametric study for finding the best mother wavelet functions that can be incorporated in simulating ETEFs via DWT is proposed. So, the appropriate mother wavelet functions found from this study can be utilized to simulate ETEFs in the DWT space. The primary novelty of this study lies in exploring the impact of different mother wavelet functions on the production of ETEFs, as well as introducing the most effective mother wavelets for generating ETEFs. In this paper, first, the generation procedure of ETEFs is concisely described. Then matters related to the wavelet theory are explained shortly. Afterwards, explanations pertinent to the mother wavelet functions as well as the issues related to the methodology framework are put forth. Finally, the results associated with this parametric study on the mother wavelet functions and their influence on the quality of resulted ETEFs are presented and thoroughly discussed.

2 Generation of Endurance Time Excitations

In this section, a brief review on generating Endurance time excitation functions (ETEF) is presented. As previously stated, ETEFs are intensifying acceleration time histories that their intensities increase with time. Each time in ETEFs corresponds to a specific intensity level. ETEFs are generated in a way that the response of structures subjected to them at each analysis time would be representative of the response of structures under GMs with the intensity level that is identical to the level of intensity found from the ET record at a given time. In order to reach this condition, dynamic characteristics of ETEFs at each time should be compatible with those associated with recorded GMs with the intensity level associated with that time. Acceleration time history of ETA20kd01 at four time windows $[0, 5\,\mathrm{sec}]$, $[0, 10\,\mathrm{sec}]$, $[0, 15\,\mathrm{sec}]$, and $[0, 20\,\mathrm{sec}]$ are shown in Figure 1. It should be mentioned that these windows are arbitrarily chosen and the sole purpose is to show the intensifying feature of ETEFs. Acceleration spectra of these time windows are also shown in this figure. The intensifying trend of the acceleration spectrum for this ETEF is apparent for longer time windows. There are numerous dynamic characteristics that can be considered in simulating ETEFs. With different considered dynamic characteristics of the recorded GMs, diverse ETEFs generations can be and have been created so far. Some examples of the dynamic characteristics that can be considered are linear response spectrum, Cumulative Absolute Velocity (CAV) and hysteresis energy. In this study, in order to avoid extra complexity, only acceleration spectra are adopted. But it should be noted that acceleration spectra are one of the most prominent dynamic characteristics of the earthquake records.

In the ET method, time is the measure of intensity. Normally, the spectrum is the function of the period Single Degree of Freedom (SDOF) structures, hereafter represented as only period. According to these two facts, ETEFs are functions of time (t) and period (T). In order to simulate ETEFs, acceleration spectra of ETEFs at each time and each period should be specified. This matter contradicts with generating artificial motions in which only acceleration spectra at each period must be determined. This difference originates from the different natures of ETEFs and recorded GMs. Acceleration spectra of ETEFs gradually increases with a specified pattern as time increases. In order to simulate ETEFs, acceleration spectra of ETEFs must increase with time and must be compatible with considered GMs. These two required conditions are the targets of simulating ETEFs. Since the number of targets considerably exceeds the number of variables, optimization process must be used to simulate ETEFs. In optimization context, equations are defined in the objective functions form. The objective function of Equation (1) calculates the discrepancy between ETEFs acceleration spectra and targets:

$$F_{\mathrm{ETEF}}\left(a_g\right) = \int_{T_{\min}}^{T_{\max}} \int_{0}^{t_{\max}} \left\{ \left[S_a\left(t,T\right) - S_{aC}\left(t,T\right) \right]^2 \right\} \mathrm{d}t\mathrm{d}T \tag{1}$$

Where $S_a\left(t,\tau\right)$ denotes acceleration spectra produced by time window $[0,t]$ of ETEFs at period of T. S_{aC} is target acceleration spectra of ETEFs. $t_{\max}$ is duration of ETEFs. Also, $T_{\min}$ and $T_{\max}$ are the minimum and maximum of the periods considered in the generating process, respectively. In this case, $S_a\left(t,T\right)$ is calculated by Equation (2).

$$S_a(t,T) = \max\left(\left|\ddot{x}(\tau) + a_g(\tau)\right|\right) \; 0 \le \tau \le t \tag{2}$$

Where $\ddot{x}(\tau)$ is the relative acceleration response of an SDOF with a period of T and damping ratio of 5% under the ETEFs, and $a_g(\tau)$ is the acceleration time history of ETEFs.

$S_{aC}(t,T)$ is calculated by Equation (3) :

$$S_{ac}(t,T) = \frac{t}{t_{\text{target}}} * S_a^{\text{Target}}(T) \tag{3}$$

Where t_{target} is the time at which ETEFs are compatible with normalized GMs. S_a^{target} is the median acceleration spectra of normalized GMs. In this study, the GMs suite recommended by [28] is used as target motions. These ground motions are recorded from large magnitude events ($M > 6.5$) at sites located greater than or equal to 10 km from fault rupture. This set includes records from soft rock, stiff sites, and shallow crustal sources. Acceleration spectra of GMs in logarithmic scale are depicted in Figure 2. It should be mentioned that normalizing procedure proposed by FEMAP695 is used in this study [29].

To minimize the objective function Equation (1) for simulating ETEFs, unconstrained nonlinear optimization procedure is employed. It should be noted that discretization is required in solving such objective functions; not to mention that the type of discretization to be used might affect the results. For discretization purpose, times are sampled at *n* points $t_j\,(j=1:n)$, and periods are sampled at *m* points $T_i\,(i=1:m)$. When discretization is applied, objective function of Equation (1) converts double integrals to double summations, as stated in Equation (4).

$$F_{\text{ETEF}}(a_g) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\left[S_a(T_i,t_j) - S_{aC}(T_i,t_j)\right]^2\right\} \tag{4}$$

In this study, 120 periods with a logarithmic distribution between 0.02 seconds and 5 seconds are employed. The logarithmic distribution produces more data in the low period region where fluctuation of acceleration spectra is considerably higher than the high period region. In this way, *t* is also sampled at 2048 points with equal intervals of 0.01seconds.

There are several optimization algorithms that can be employed for simulating ETEFs. In this study, *trust region reflective* algorithm, a simple yet powerful concept in optimization, is employed as the optimization method [30]. The basic idea is to approximate function *f* with a simpler function *q*, which reasonably reflects the behavior of function *f* in the neighborhood *N* around the point of *x*. This neighborhood is called the trust region [31]. In the standard trust region, *q* is defined by the first two terms of the Taylor expansion of *f* around *x*, and *N* is hyper-spherical with radius of Δ, as stated in Equation (5). As can be found, this is a constrained optimization problem, where *g* and *H* are the gradient and Hessian matrix of *f* at point *x* and *s* is the step size to be determined at each iteration.

$$\min\left\{\frac{1}{2}s^T Hs + s^T g\right\} \text{such that} \left\|s\right\| \le \Delta \tag{5}$$

In order to compare ETEFs produced with different objective functions, a quantity called *Normalized Relative Residual* (NRR) is defined [32]. Although this quantity can be defined for different dynamic characteristics, the one associated with acceleration spectra as given in (6) is utilized in this study. The main property of this quantity is its independency to the target acceleration spectra. With this quantity, comparison of different ETEFs produced by different objective functions would be possible. This quantity expresses the incorrectness of ETEFs in percent. In fact, the accuracy of ETEFs can be defined as one hundred minus NRR in percent. The discretized form of Equation (6) is given in Equation (7).

$$\mathrm{NRR}_{\mathrm{S_a}} = \frac{1}{t_{\max}} \int_0^{t_{\max}} \left(\frac{\int_{T_{\min}}^{T_{\max}} \left| \left(S_a(t,T) - S_{ac}(t,T) \right) \right| dT}{\int_{T_{\min}}^{T_{\max}} S_{ac}(t,T)\, dT} \right) dt \tag{6}$$

$$\mathrm{NRR}_{\mathrm{S_a}} = \frac{1}{t_{\max}} \sum_{i=1}^{m} \frac{\sum_{j=1}^{n} \left| S_a(t_j, T_i) - S_{aC}(t_j, T_i) \right|}{\sum_{j=1}^{n} S_{aC}(t_j, T_i)} \tag{7}$$

3 Proposed Method for Selecting Mother Wavelet Functions

In this section, it is investigated whether or not the used mother wavelet function can play an important role in the accuracy of the generated ETEFs. A methodology to explore the influence of the mother wavelet function on the accuracy of simulated ETEFs is presented. The steps of the proposed methodology are presented here:

1- Select a suite of mother wavelet families (e.g., Daubechies, Symlet, Coiflet, and Bio-Orthogonal). Given that *n* wavelet families are selected, these wavelet families are consecutively numbered as $i = 1, 2, \ldots, n$.

2- For each mother wavelet family, a set of mother wavelet functions is chosen. For example, the set $\{2, 4, 6, \ldots, 20\}$ is chosen for Daubechies family. This selection is based on engineering judgment. The members of the selected set are consecutively numbered as $j = 1, 2, \ldots, m$, where *m* is the set size. The *j*-th member of this set is ψ_{ij}.

3- For each mother wavelet functions ψ_{ij}, *k* number of ETEFs are generated. The process of generating ETEFs is briefly described in Section 2. The generation procedure is completely explained by [20]. The objective functions of the generated excitations are denoted as $F_{ijl}\ \ l = 1, 2, \ldots, k$.

4- Find the average objective function value of the simulated ETEFs produced by the mother wavelet function ψ_{ij} as follow:

$$F_{ij} = \frac{\sum_{l=1}^{k} F_{ijl}}{k} \tag{8}$$

5- Find the best wavelet function for each wavelet family

$$j^*(i) = \arg\min_j F_{ij} \tag{9}$$

6- Find the best wavelet family

$$i^* = \arg\min_i F_{ij^*(i)} \tag{10}$$

With this approach, the wavelet family associated with the number i^* and the mother wavelet function associated with $j^*(i^*)$ leads to the more accurate ETEFs. This methodology is illustrated in Figure 3.

In order to investigate the influence of the used mother wavelet function on the accuracy of simulated ETEFs the *importance factor* denoted by α is defined. The equation of determining the *importance factor* is given in Equation (11). This parameter is expressed in percent. The high value of this parameter shows the sensitivity of the simulated ETEFs to the selected mother wavelet function. The *importance factor* can be obtained either for each separate mother wavelet family or for all considered mother wavelet families. In the earlier one, the sensitivity of the ETEFs accuracy to the selected mother wavelet function that belongs to a mother wavelet family is of concern, while, in the latter one, the mother wavelet function can be selected from all mother wavelet families.

$$\alpha = \frac{\sqrt{\frac{1}{mk}\left(F_{ijl} - \frac{\sum F_{ijl}}{mk}\right)}}{\frac{\sum F_{ijl}}{mk}} \times 100 \tag{11}$$

4 Application of the Proposed Method

In this section, the influence of the mother wavelet on the accuracy of the produced ETEFs is investigated through applying the methodology presented in previous section. Four wavelet families are considered for this purpose, i.e., the wavelet families of Daubechies, Coiflet, Symlet and Bio-Orthogonal. On the other words, *n* is taken four in the methodology presented in previous section. The first three wavelet families belong to the class three (orthogonal) and the other one belongs to the class four (bio-orthogonal). No wavelet families are selected from two first classes because FWT cannot be performed by these two classes. For each wavelet family, several members of the wavelet set belonging to that family will be considered as mother wavelets, and the corresponding wavelet coefficients are taken as optimization variables in the production of ETEFs. The procedure explained in the methodology section is followed.

The first wavelet family considered in this study is Daubechies wavelet family. This wavelet family is an orthogonal wavelet class. For the Daubechies wavelet family, ten mother wavelets—db2, db4, db6, db8, db10, db12, db14, db16, db18, db20—are considered, *m*=10. The function support of db2, db4, db6, db8, db10, db12, db14, db16, db18, and db20 are 1, 3, 5, 7, 9, 11, 13, 15, 17, and 19, respectively. According to the methodology presented in previous section, six ETEFs are produced for each mother wavelet function. On the other words, *k* is taken six. Ten scenarios with the names of ETEF-db2, ETEF-db4, ETEF-db6, ETEF-db8, ETEF-db10, ETEF-db12, ETEF-db14, ETEF-db16, ETEF-db18, and ETEF-db20 are defined. The second part of each scenario name shows the used wavelet function. Number of sample points of each scenario is 2048. Number of optimization variables for each scenario is taken 512.

According to the proposed methodology, 60 ETEFs are generated. The average objective function value of ETEFs produced by db2 is 85.9, while this value for other generated ETEFs is 35.3; this shows that the wavelet function db2 does not provide sufficient accuracy, so this wavelet function is not considered for further investigation. The accuracy of the produced ETEFs in the boxplot format is shown in Figure 4. By the methodology presented in previous section, $j^*(1) = 6$. On the other words, db12 mother wavelet function results in a more accurate ET excitation function than the other mother wavelet functions applied in this scenario. However, it can be seen the accuracy of the production functions in the other wavelets has approximately the same error values if we exclude the wavelet db2. Figure 4 shows that the cost function of db12 is less than other considered wavelet functions; This is consistent with the results of the methodology. The average CPU time of production of ETEFs in the Daubechies family was 7348 seconds. It should be mentioned that Intel® Xeon® Processor E5-2698 v4 system is used to simulate ETEFs.

The second mother wavelet family considered in this paper is Coiflet. Coiflet wavelet family corresponds to $i = 2$ in the methodology. Coiflet is an orthogonal wavelet family. For Coiflet wavelet family, five mother wavelet functions are considered, i.e., $m = 5$. The function support of Coif1, Coif2, Coif3, Coif4, and Coif5 are 5, 11, 17, 23, and 29, respectively. Nine levels of decomposition have been used to generate the ETEFs of this scenario using the upper-pass and lower-pass filters. The reason for choosing nine levels of decomposition is that it can be possible to cover different significant frequencies of the signal. To reduce the optimization variables and eliminate inefficient decision variables in this case, level 1 and 2 detail coefficients are not considered in the optimization procedure. The values of these coefficients are set to zero. Five optimization scenarios are introduced to investigate the generated ETEFs in Coiflet wavelet family. The details of these optimization scenarios are given in Table 1. In this table, number of sample points is the total number of wavelet coefficients and number of decision variables is the number of wavelet coefficients that are considered as variables in the optimization procedure. As said before, wavelet coefficients of levels 1 and 2 are not considered as variable in this case.

The cost functions of the produced ETEFs in the Coiflet wavelet family are given in Figure 5. It is seen that the definition of wavelet coefficients using the coif4 mother wavelet results in more accurate ETEFs than the rest of the Coiflet family functions. It should be mentioned that all of the considered Coiflet-family mother wavelet functions lead to acceptable ETEFs. It is contrast with Daubechies wavelet family that db2 cannot be used to simulate ETEFs with acceptable accuracy. The average CPU time of production of ETEFs in the Coiflet family was 5721 seconds.

The third mother wavelet family considered in this study is Symlet. Symlet wavelet family corresponds to i=3 in the methodology. Symlet belongs to Orthogonal wavelet families. For Symlet, eight wavelet functions are considered, m=8. The function support of Sym1, Sym2, Sym3, Sym4, Sym5, Sym6, Sym7, and Sym8 are respectively 1, 3, 5, 7, 9, 11, 13, and 15. To investigate the wavelet functions of the Symlet family, the optimization scenarios defined in

are presented in this section. For each defined optimization scenario, three ET acceleration functions are generated. In other words, k is assigned to three for Symlet mother wavelet family in the methodology. To reduce the optimization variables and eliminate inefficient decision variables in this case, level 1 and 2 detail coefficients are not considered in the optimization procedure. The values of these coefficients are set to zero.

The average cost function of ETEFs simulated by sym1 is 106.1, while the average cost function of ETEFs generated by other wavelet functions of Symlet family is 38.3; this shows the inappropriateness of sym1 in simulating ETEFs. So, sym1 is removed from mother wavelet function candidates for generating ETEFs.The results of the generated ETEFs in the Symlet family, except for sym1, are presented in Figure 6. It can be seen that the produced ETEFs using the sym5 mother wavelet are more accurate than the rest of the functions computed from other wavelet functions in the Symlet family. The criteria for choosing sym5 is the multiplication sign in Figure 6 that shows the average value of the simulated ETEFs. The average CPU time of production of ETEFs in the Symlet family was 5721 seconds.

The fourth mother wavelet family considered in this study is Bio-Orthogonal. Bio-Orthogonal wavelet family corresponds to i=4 in the methodology. In this wavelet family, eight mother wavelets—including bior1.3, bior2.4, bior2.6, bior3.1, bior3.3, bior3.7, bior4.4 and bior6.8—are considered for the optimization scenarios aimed to be included for the production of ETEFs in the Bio-orthogonal wavelet family. As it can be seen, the function support of bior1.3, bior2.4, bior2.6, bior3.1, bior3.3, bior3.7, bior4.4, and bior6.8 are 4.99, 8.99, 12.99, 2.99, 6.99, 14.99, 8.99, and 16.99, respectively. These optimization scenarios are accessible from Table 3. For each optimization scenario shown, three ET acceleration functions have been simulated for each optimization scenario, k=3. To reduce the optimization variables and eliminate inefficient decision variables in this case, level 1 and 2 detail coefficients are not considered in the optimization procedure. The values of these coefficients are set to zero.

bior1.3 leads to ETEFs that their cost function values are considerably larger than the ETEFs produced by other wavelet functions of Bio-orthogonal wavelet family. The results of the ETEFs within the Bio-orthogonal wavelet family, except for bior1.3, are revealed in Figure 7. It is seen that the use of the bior3.3 mother wavelet in the Bio-orthogonal family results in more accurate ET acceleration functions. The average CPU time of production of ETEFs in the Bio-orthogonal family was 5638 seconds.

Top ten mother wavelet functions in simulating ETEFs and their corresponding cost function values are shown in Figure 8. As it can be seen, db12 is the most efficient mother wavelet function in generating ETEFs. From these ten mother wavelet functions, six of them belongs to Daubechies wavelet family. This matter highlights the effectiveness of Daubechies wavelet family in producing ETEFs. Two of these ten mother wavelet functions belong to Symlet wavelet family. Bio-orthogonal and Coiflet have only one share in top ten mother wavelet functions. As can be seen from this figure, discrepancies between the cost function of these ten mother wavelet functions are not significant. The difference between the cost function of db12 and coif4 is about seven percent. The Daubechies wavelet family demonstrates superior performance in producing ETEFs.

The average cost function value of ETEFs simulated by using each mother wavelet family is shown in Figure 9. This figure provides a tool to compare different mother wavelet families. It should be mentioned that outliers are removed from the averaging process. The outliers are associated with db2, sym1, and bior1.3 wavelet functions. Figure 9 demonstrates that Daubechies wavelet family outperform other considered wavelet families in producing ETEFs. Bio-Orthogonal, Symlet, and Coiflet wavelet families have respectively second, third, and fourth rank among considered wavelet families. The average cost function of ETEFs simulated by Daubechies wavelet function is 35.28 while this value for Bio-Orthogonal, Symlet, and Coiflet wavelet functions are respectively are 37.86, 38.28, and 39.37. This means that Bio-Orthogonal, Symlet, and Coiflet wavelet functions have the efficiency about 7.3%, 8.5%, and 11.6% less than Daubechies wavelet family. The average cost function index serves as a criterion for selecting a mother wavelet function from one of the wavelet families randomly, determining which one performs better. In this context, it is observed that Daubechies outperforms the others.

Importance factor, which shows the extent of impact of selecting the appropriate mother wavelet function in generating more accurate ETEFs according to Equation (11), are calculated either for each considered wavelet family separately or for all wavelet families together. The calculated values are depicted in Figure 10. Since the desired accuracy is not achieved in ETEFs produced by db2, sym1, and bior1.3, these mother wavelet functions are not included in calculating *Importance factor*. It can be seen that selecting appropriate mother wavelet function from all considered mother wavelet families has 13.8% impact on the accuracy of produced ETEFs. Moreover, selecting appropriate mother wavelet function from Daubechies wavelet family, has 5.5% impact on the accuracy of produced ETEFs. The *Importance factor* of Daubechies wavelet family is the lowest among all considered wavelet families, and this shows that the selection of the best wavelet function in this family is less significant in generating ETEFs. As evident from this figure, the Importance factor associated with the Bio-Orthogonal family is comparatively higher when compared to the other considered mother wavelet families.

Summary results of comparing mother wavelets in generating ETEFs are presented in Table 4. This table shows the order of the wavelet family that results in more accurate ETEFs and also shows the first five best mother wavelet in each family. Moreover, *importance factor* for each wavelet family is provided. The same information for all considered wavelet families together is given in this table. It is seen that the Daubechies family with the db12 mother wavelet yields more accurate results all together.

In order to analyze the results, the mother wavelet functions that are unsuitable for simulating ETEFs is shown in Figure 11. This figure shows that all these wavelet functions have discontinuity and have sudden jumps at several points. On the other hand, top ranked wavelet functions are shown in Figure 12.

In this section, the effectiveness of the proposed mother wavelet is evaluated by utilizing the Goodness-of-Fit (GOF) index to assess the quality of the generated ETEFs. GOF is proposed as a metric to quantify the misfit of broadband synthetic seismograms [33-35]. The formula for calculating this index is given below. It's important to acknowledge that the GOF criterion is a numerical measure falling within the range of 0 to 100, where a value of 100 signifies the perfect fit. While the GOF index can be applied to various intensity measures, this study focuses on using it specifically for Peak Ground Acceleration (PGA) since only acceleration spectra were utilized in the production process. For ETEFs produced with db12, the Goodness-of-fit for PGA is 94.50%, indicating an excellent level of fit. The GOF index for spectral acceleration is computed by averaging across various periods ranging from 0.02 seconds to 5 seconds, resulting in a value of 91.66%.

$$GOF = 100 * \mathrm{erfc}\left[\mathrm{NR}\right] \text{where NR=} \frac{2|x-y|}{x+y} \quad (12)$$

where x and y represent the observed and simulated values, respectively, while the complementary error function erfc(.) is employed to measure the discrepancy between the two datasets.

5 Simulating new ETEFs by the Selected Mother Wavelet Functions

In order to show the validity of the results presented in the previous section, four ETEFs by using the first rank mother wavelet functions associated with four considered wavelet families, i.e., db12, bior3.3, sym5, and coif4, are produced. Target acceleration spectra of these ETEFs differ from that considered for determining the influence of the mother wavelet function in producing ETEFs in previous sections. The reason that the considered spectrum in this section is different from the target spectrum of the previous section is to investigate whether the results are dependent on the target spectrum or not. Iranian national building code [36] spectrum for soil type C and high seismic zone is considered as target spectrum. Both the target spectrum for finding the appropriate mother wavelet functions and the target spectrum defined in this section are depicted in Figure 13. The main difference between these two spectra is that the target spectrum of this section has sharp corner. ETEFs simulated by using db12, sym5, bior3.3, and coif4 are respectively named ETA20NBCdb12, ETA20NBCsym5, ETA20NBCbior3.3, and ETA20NBCcoif4. Acceleration time histories of these ETEFs are shown in Figure 14. It can be seen that all these four ETEFs have the intensifying trend as expected. The comparison between target spectrum and ETEFs spectrum at four times, i.e., 5sec, 10sec, 15sec, and 20sec, are demonstrated in Figure 15. Acceptable correspondence between target spectrum and ETEFs spectrum is evident. Acceleration time history response of ETEFs is compared with the corresponding target at four periods, i.e., 0.1sec, 0.8sec, 2sec, and 4.5sec. This comparison is shown in Figure 16. Figure 15 and Figure **16** show that although all these wavelet functions lead to acceptable ETEFs, ETA20NBCdb12 is more accurate than other ETEFs. In order to quantify the accuracy of the generated ETEFs, *normalized relative residual* (NRR) which was defined by [32] is used. The NRR (in percent) of these four ETEFs are provided in Table 5. This table approves that db12 leads to more accurate ETEFs than other considered mother wavelet functions. ETA20NBCdb12 had 8.6% error or 91.4% accuracy. In Section 5, it was seen that the order of the wavelet functions in terms of efficiency in simulating ETEFs was db12, sym5, bior3.3 and coif4. In this section, this order becomes db12, bior3.3, sym5 and coif4 respectively. The first and the last wavelet functions in these two orders are same. But the second and the third ones are changed. Nevertheless, it can be seen that this difference is very small. Table 5 shows that bior3.3 is only about 0.2% better than sym5. As can be seen, this difference can be ignored. In addition, the best mother wavelet function is important and the best mother wavelet function is same for two cases.

6 Discussion

The production of ETEFs is a crucial step in the development of the ET method, achieved through unconstrained nonlinear optimization by selecting appropriate decision variables, often using the Discrete Wavelet Transform (DWT). An algorithm is proposed to identify the optimal mother wavelet for ETEFs generation, evaluating various mother wavelets' accuracy. Due to computational complexity, it's preferable to identify the optimal mother wavelets for fundamental problems and extend the results. This study focuses on generating ETEFs suitable for linear analysis, assuming that mother wavelets obtained for linear response ETEFs can also apply to non-linear responses. Estekanchi et al. [37] demonstrated that incorporating long periods into the process of generating ETEFs simulated based on linear response can lead to an acceptable level of consistency in capturing nonlinear responses. However, the efficiency of these mother wavelets in producing non-linear ETEFs remains an avenue for future research.

The ETEFs presented in this article serve a specific purpose: identifying the optimal mother wavelet function for generating ETEFs based on the linear responses of structures. This particular class of ETEFs finds extensive utility in the linear analysis of structures. Moreover, their viability extends to non-linear analysis if long periods are incorporated during the generation process. However, it's worth noting that ETEFs intended for practical applications must undergo a rigorous qualification process for compatibility with both linear and non-linear spectra before they can be deemed fit for use.

7 Summary and Conclusion

The Endurance Time (ET) method involves subjecting structures to intensifying excitations called Endurance Time Excitation Functions (ETEFs) in a time history dynamic analysis. The accuracy of these ETEFs is vital for the successful implementation of the method, often simulated through an unconstrained optimization procedure. Previous studies have shown that defining decision variables in the discrete wavelet transform (DWT) space results in more precise ETEFs. This study presents a methodology to assess the impact of the mother wavelet function on the accuracy of simulated ETEFs, particularly for acceleration spectra-consistent ETEFs. The results are applied to ETEFs simulated based on linear response, emphasizing their compatibility with far-field ground motions recommended by FEMAP695. The ranking of mother wavelets is based on the examined wavelet families.

1- It is demonstrated that the accuracy of ETEFs simulated by db2 (Haar), sym1 and bior1.3 wavelet functions is meaningfully less than those ETEFs simulated by other considered wavelet functions. So, it is not recommended to utilize these wavelet functions in simulating ETEFs. The common property of these mother wavelet functions is the presence of rectangular part in their functions and the existence of some sharp jumps.
2- It is observed that selecting the best mother wavelet function, db12 as shown in this paper, for simulating ETEFs has about 15% improvement on the accuracy of produced ETEFs. This matter shows the importance of selecting the mother wavelet function in producing ETEFs.
3- Among four considered mother wavelet families, it is shown that Daubechies wavelet family leads to more accurate ETEFs. After that, the Bio-Orthogonal, Symlet, and Coiflet are consecutively next positions. Bio-Orthogonal, Symlet, and Coiflet wavelet functions have the more residual, the difference between targets and ETEFs, about 7.3%, 8.5%, and 11.6% than Daubechies, respectively.
4- Among the different mother wavelet families examined, the Daubechies wavelet stands out as the least influenced by the choice of the mother wavelet function, as revealed by the Importance Factor index. Specifically, the Importance Factor index for Daubechies is 5.5%. In comparison, the Coiflet, Symlet, and Bio-Orthogonal families exhibit Importance Factor indices of 12.4%, 13.5%, and 14.6%, respectively.
5- Among 31 considered mother wavelet functions, it is demonstrated that db12 is the best mother wavelet function to be chosen for simulating ETEFs. After that, db14, db18, db6, sym5, sym7, db16, bior3.3, and db10 are the next positions. This ranking is based on the observations of this study.
6- For each wavelet family, the best mother wavelet function for simulating ETEFs is identified. The best mother wavelet function for Daubechies, Bio-Orthogonal, Symlet, and Coiflet are respectively db12, bior3.3, sym5, and coif4.
7- The best mother wavelet functions for each considered wavelet family are employed to simulate new ETEFs. Among db12, bior3.3, sym5, and coif4, db12 leads to the more accurate ETEFs as

shown by the methodology. The residuals of ETEFs generated by db12, bior3.3, sym5, and coif4 are 8.6%, 8.9%, 8.9%, and 9.5%, respectively.

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Biographies

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Figures and Tables captions:

Figure 1. (a) ETA20kd01 acceleration time history at four times windows [0-5sec], [0-10sec], [0-15sec], and [0 20sec], (b) ETA20kd01 acceleration spectra at four times windows [0-5sec], [0-10sec], [0-15sec], and [0-20sec]

Figure 2. Acceleration spectra of the recorded GMs used as the basis for generating ETEFs (logarithmic scale)

Figure 3. Flowchart of finding the optimum mother wavelet function for generating ETEFs

Figure 4. Cost function of ETEFs produced by Daubechies wavelet family

Figure 5. Cost function of ETEFs produced by Coiflet wavelet family

Figure 6. Cost function of ETEFs produced by Symlet wavelet family

Figure 7. Cost function of ETEFs produced by Bio-orthogonal wavelet family

Figure 8. Top ten mother wavelet functions in generating ETEFs.

Figure 9. Comparison between the efficiency of different wavelet families in simulating ETEFs.

Figure 10. Importance factor of selecting the appropriate mother wavelet in simulating ETEFs

Figure 11. Unsuitable mother wavelet functions for simulating ETEFs, (a) db2, (b) sym1, and (c) bior1.3.

Figure 12. Top ranked mother wavelet functions for simulating ETEFs, (a)db12, (b)sym5, (c)bior3.3, (d)coif4.

Figure 13. Target spectrum for finding the best mother wavelet function in simulating ETEFs vs the target spectrum for generating new ETEFs

Figure 14. Acceleration time histories of the produced ETEFs: (a) ETA20NBCdb12, (b) ETA20NBCbior3.3, (c) ETA20NBCsym5, and (d) ETA20NBCcoif4.

Figure 15. Comparison of target spectrum and ETEFs spectrum: (a) ETA20NBCdb12, (b) ETA20NBCbior3.3, (c) ETA20NBCsym5, and (d) ETA20NBCcoif4.

Figure 16. Comparison between acceleration time history of ETEFs with target acceleration time history at four periods: (a) 0.1sec, (b) 0.8sec, (c) 2sec, and (d) 4.5sec.

Table 1. Optimization scenarios for generating ETEFs using the Coiflet wavelet family

Table 2. Optimization scenarios for the generation of ETEFs using Symlet 's wavelet

Table 3. Optimization scenarios for the generation of ETEFs by the Bio-orthogonal wavelet family

Table 4. Comparison of different mother wavelets in simulation of ET excitation functions

Table 5. Normalized Relative residuals (NRR) of the generated ETEFs

Fig. 1

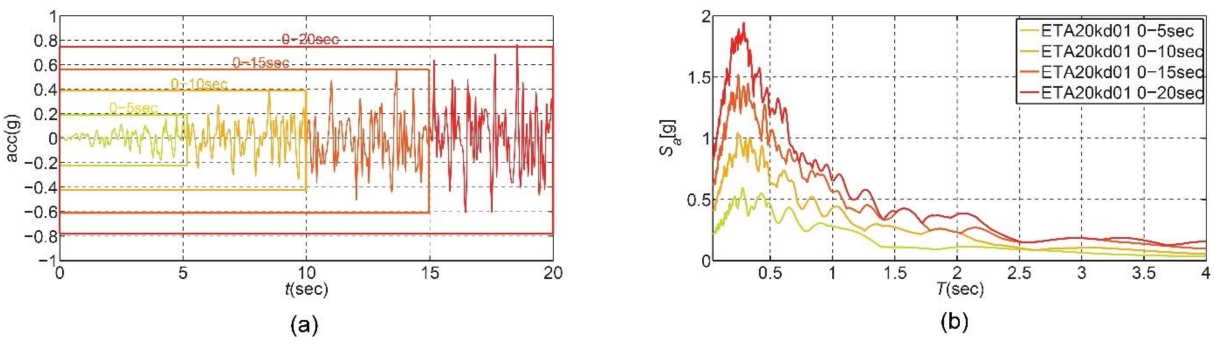

0-20sec
0-15sec
0-10sec
0-5sec
acc(g)
t(sec)
(a)
ETA20kd01 0-5sec
ETA20kd01 0-10sec
ETA20kd01 0-15sec
ETA20kd01 0-20sec
S_a[g]
T(sec)
(b)

Fig. 2

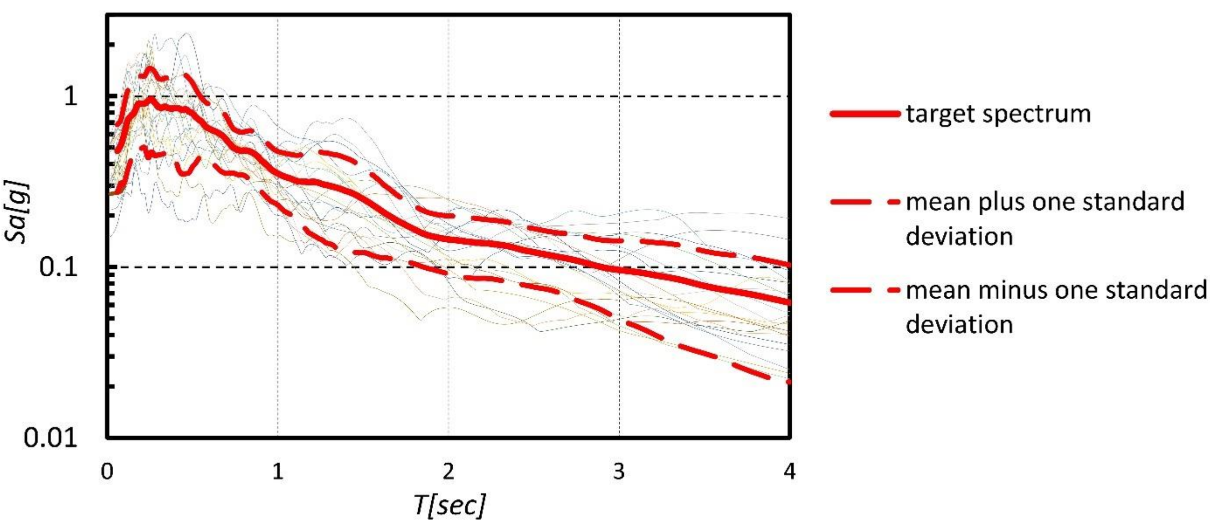

target spectrum
mean plus one standard deviation
mean minus one standard deviation
Sa[g]
T[sec]

Fig.3

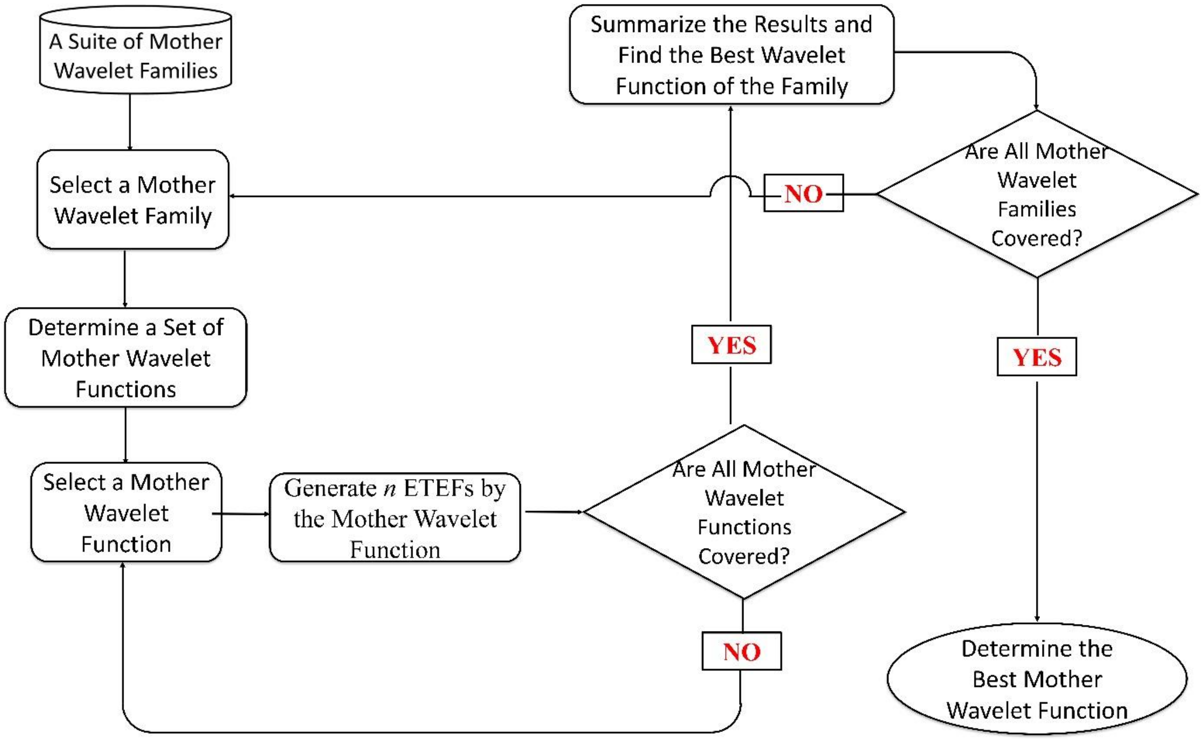

A Suite of Mother Wavelet Families
Select a Mother Wavelet Family
Determine a Set of Mother Wavelet Functions
Select a Mother Wavelet Function
Generate *n* ETEFs by the Mother Wavelet Function
Are All Mother Wavelet Functions Covered?
NO
YES
Summarize the Results and Find the Best Wavelet Function of the Family
Are All Mother Wavelet Families Covered?
NO
YES
Determine the Best Mother Wavelet Function

Fig. 4

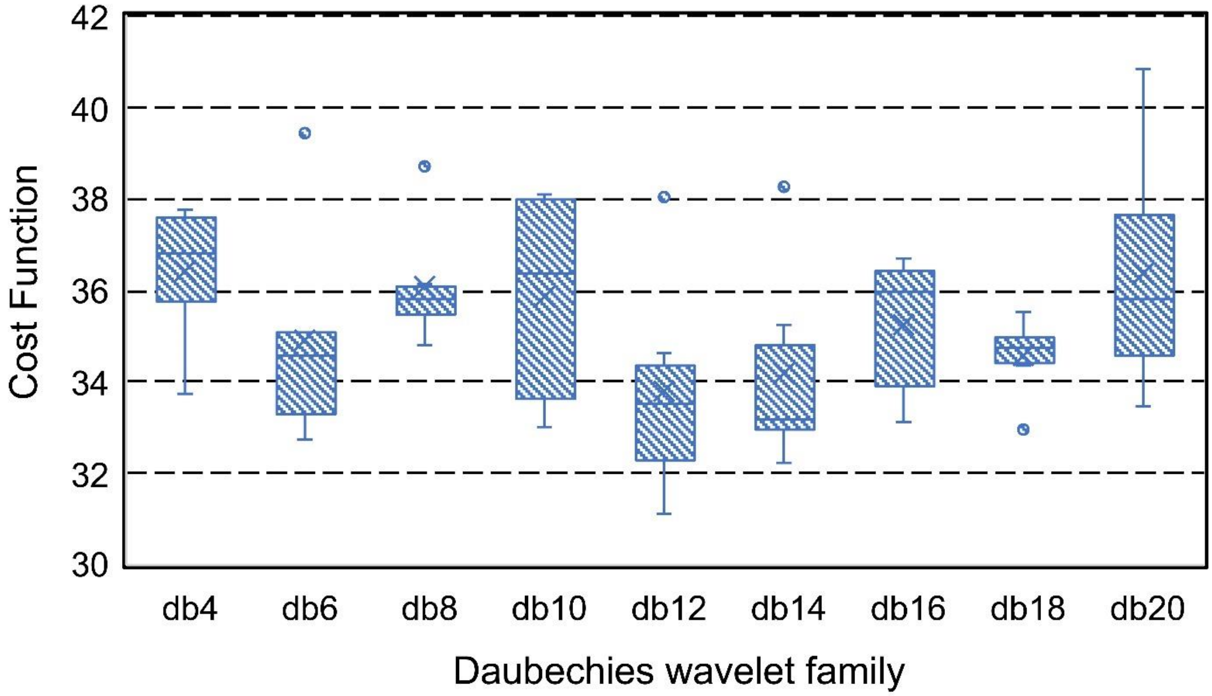

42
40
38
36
34
32
30
Cost Function
db4
db6
db8
db10
db12
db14
db16
db18
db20
Daubechies wavelet family

Fig. 5

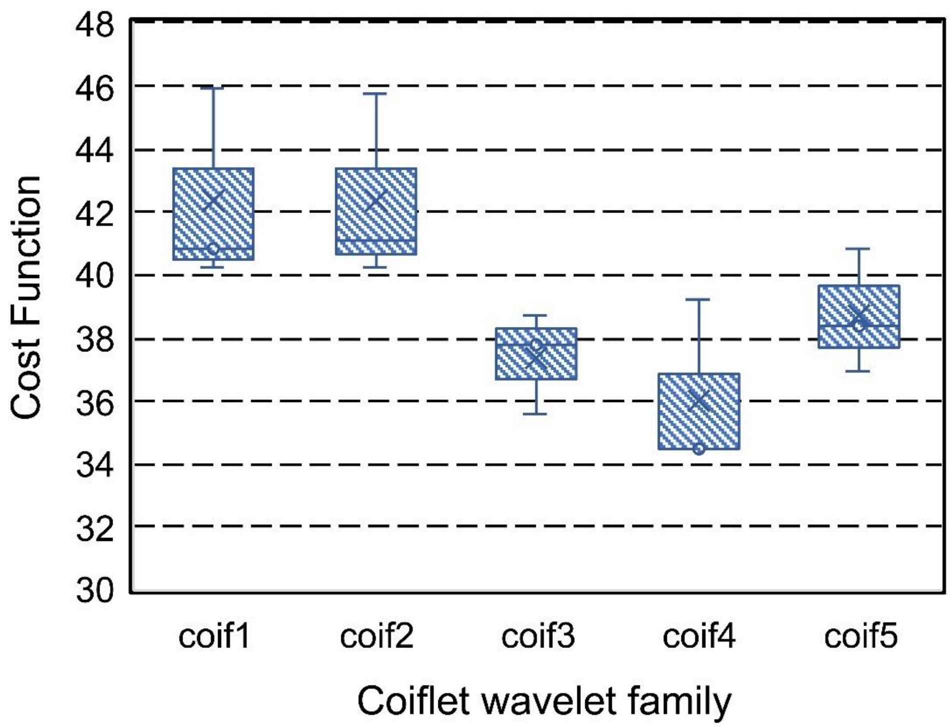

48
46
44
42
40
38
36
34
32
30
Cost Function
coif1
coif2
coif3
coif4
coif5
Coiflet wavelet family

Fig. 6

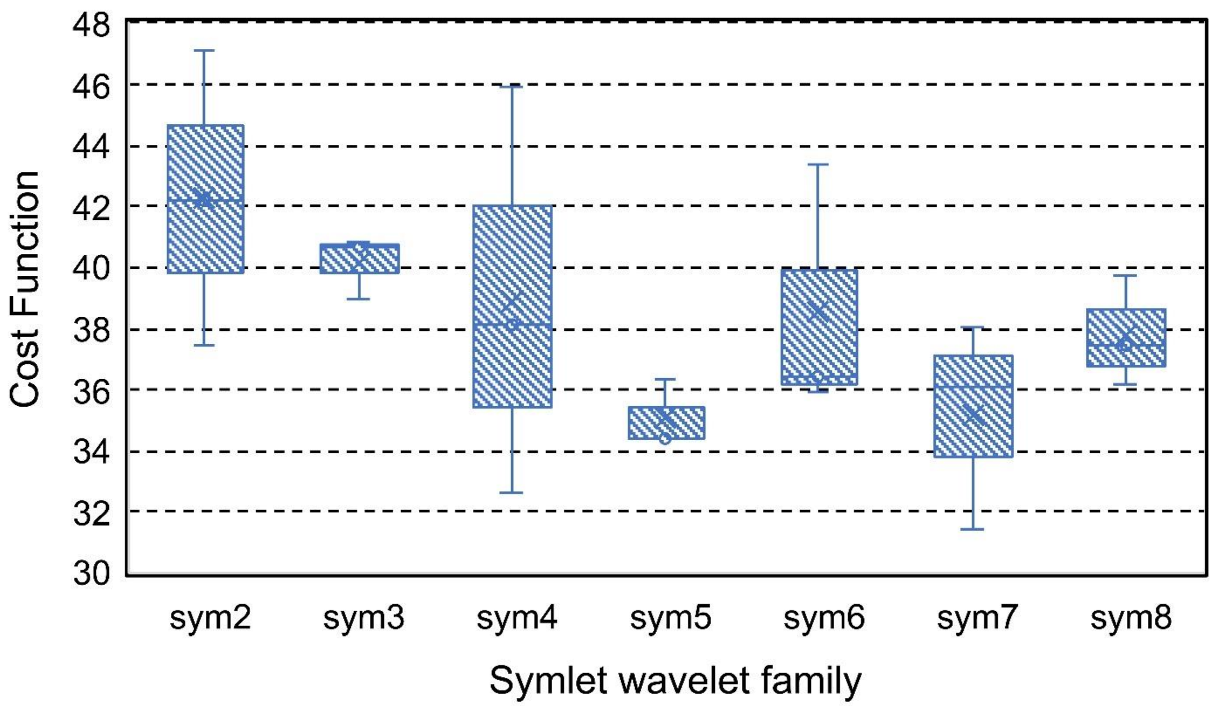

48
46
44
42
40
38
36
34
32
30
Cost Function
sym2
sym3
sym4
sym5
sym6
sym7
sym8
Symlet wavelet family

Fig. 7

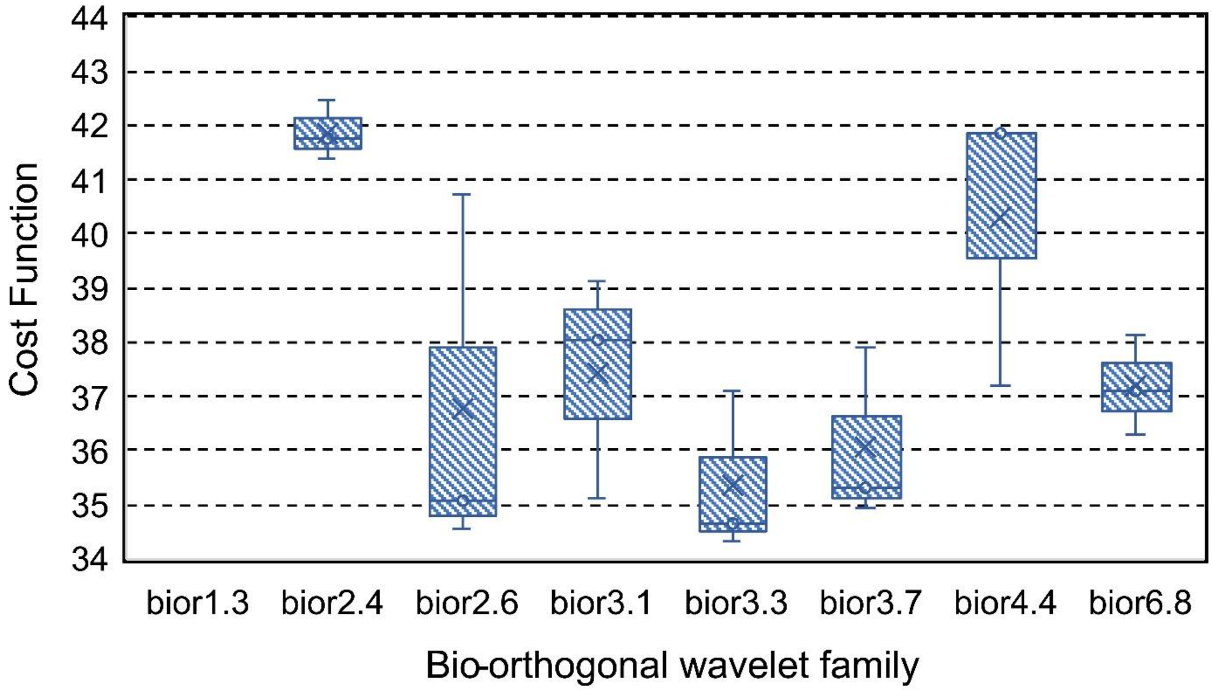

Cost Function
44
43
42
41
40
39
38
37
36
35
34
bior1.3
bior2.4
bior2.6
bior3.1
bior3.3
bior3.7
bior4.4
bior6.8
Bio-orthogonal wavelet family

Fig. 8

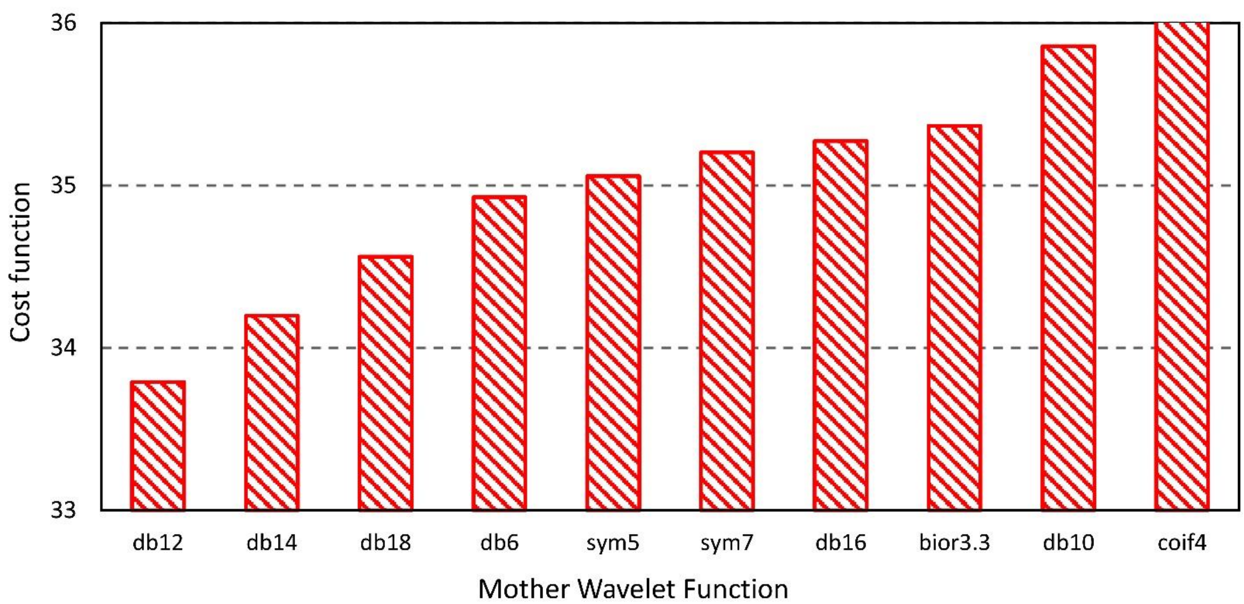

36
35
34
33
Cost function
db12
db14
db18
db6
sym5
sym7
db16
bior3.3
db10
coif4
Mother Wavelet Function

Fig. 9

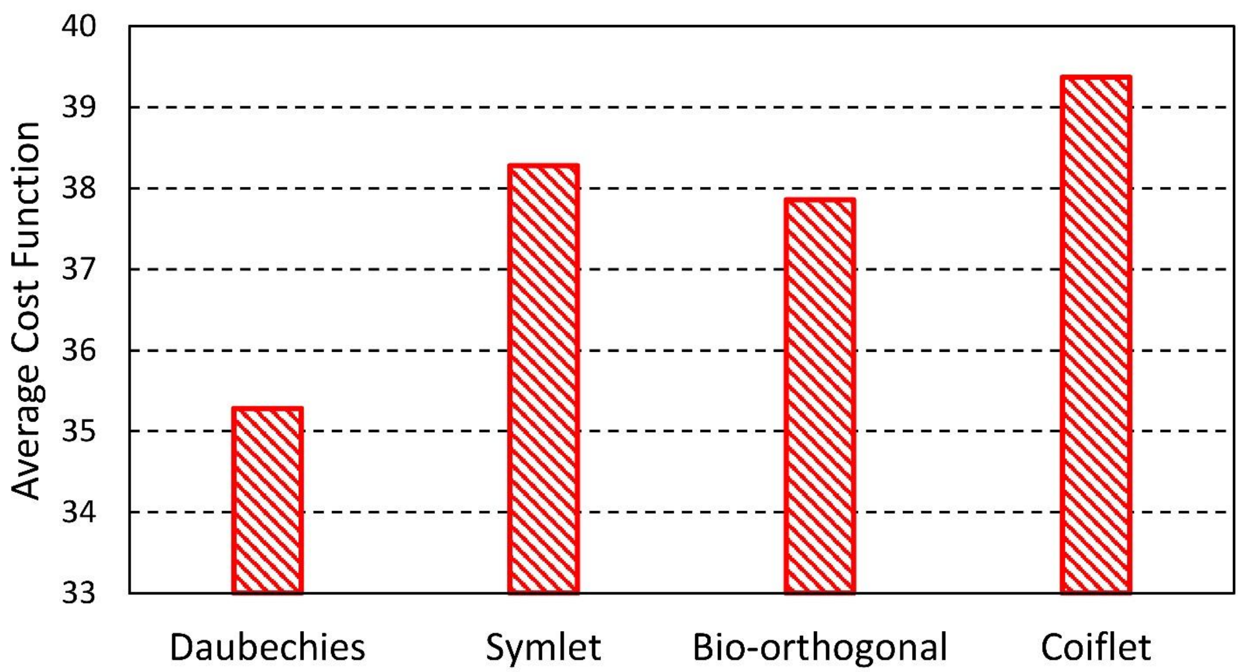
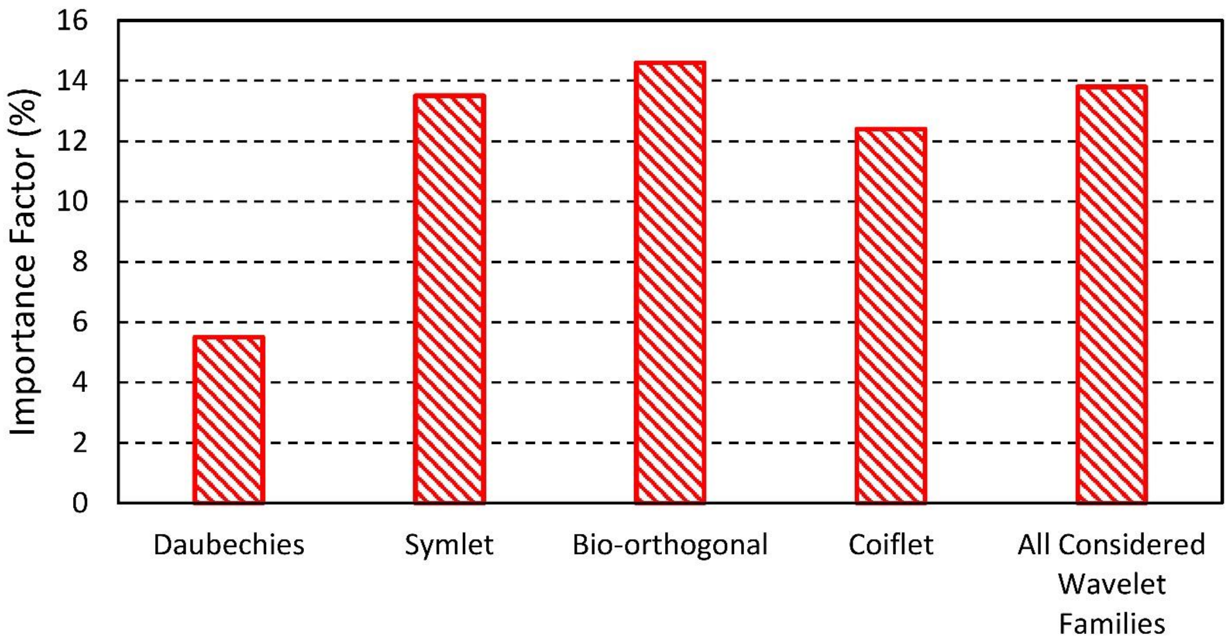
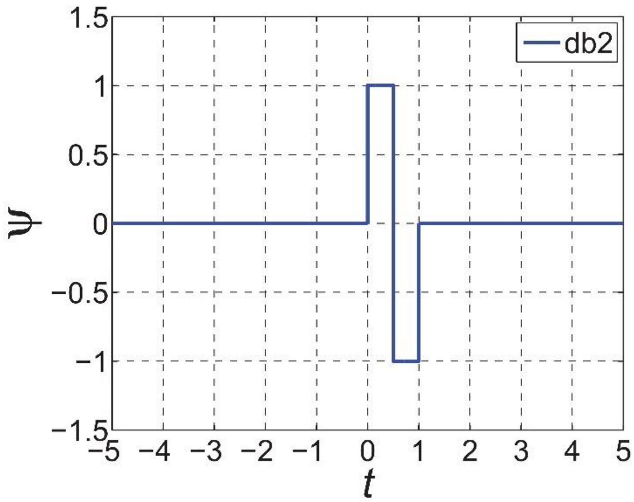

40
39
38
37
36
35
34
33
Average Cost Function
Daubechies
Symlet
Bio-orthogonal
Coiflet

Fig. 10

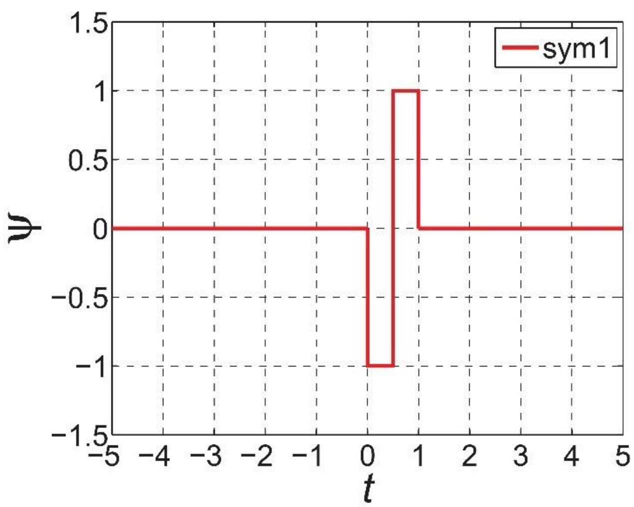


16
14
12
10
8
6
4
2
0
Importance Factor (%)
Daubechies
Symlet
Bio-orthogonal
Coiflet
All Considered Wavelet Families

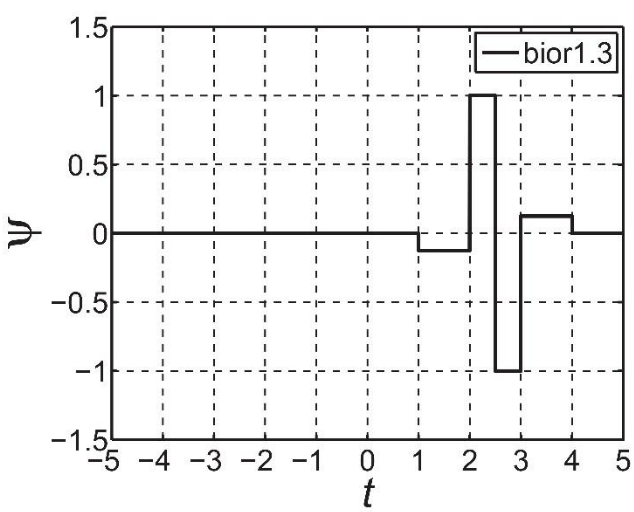
Fig. 11


db2
ψ
t
1.5
1
0.5
0
−0.5
−1
−1.5
−5 −4 −3 −2 −1 0 1 2 3 4 5

(a)


sym1
ψ
t
1.5
1
0.5
0
−0.5
−1
−1.5
−5 −4 −3 −2 −1 0 1 2 3 4 5

(b)


bior1.3
ψ
t
1.5
1
0.5
0
−0.5
−1
−1.5
−5 −4 −3 −2 −1 0 1 2 3 4 5

(c)

Fig. 12

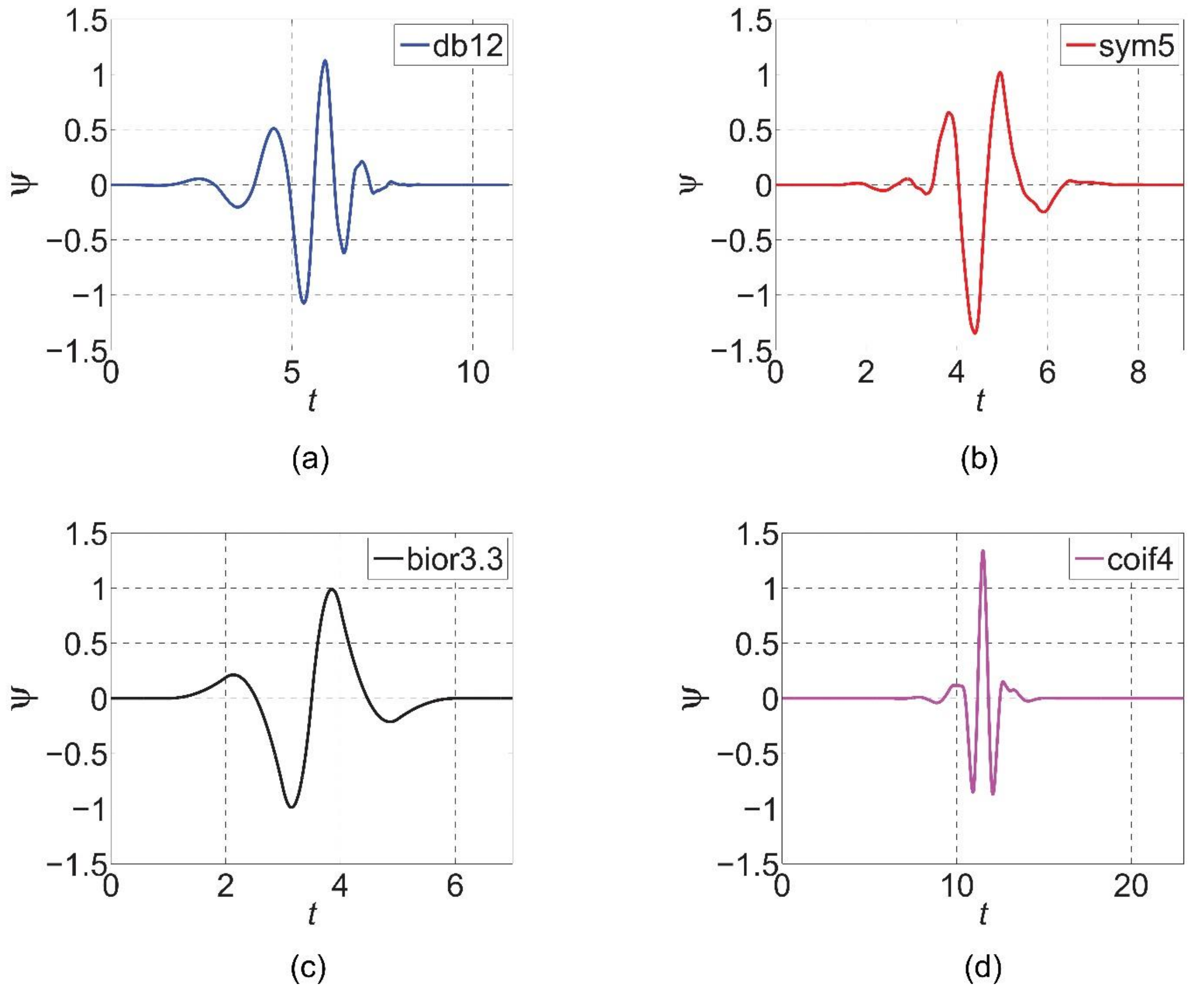


Fig. 13

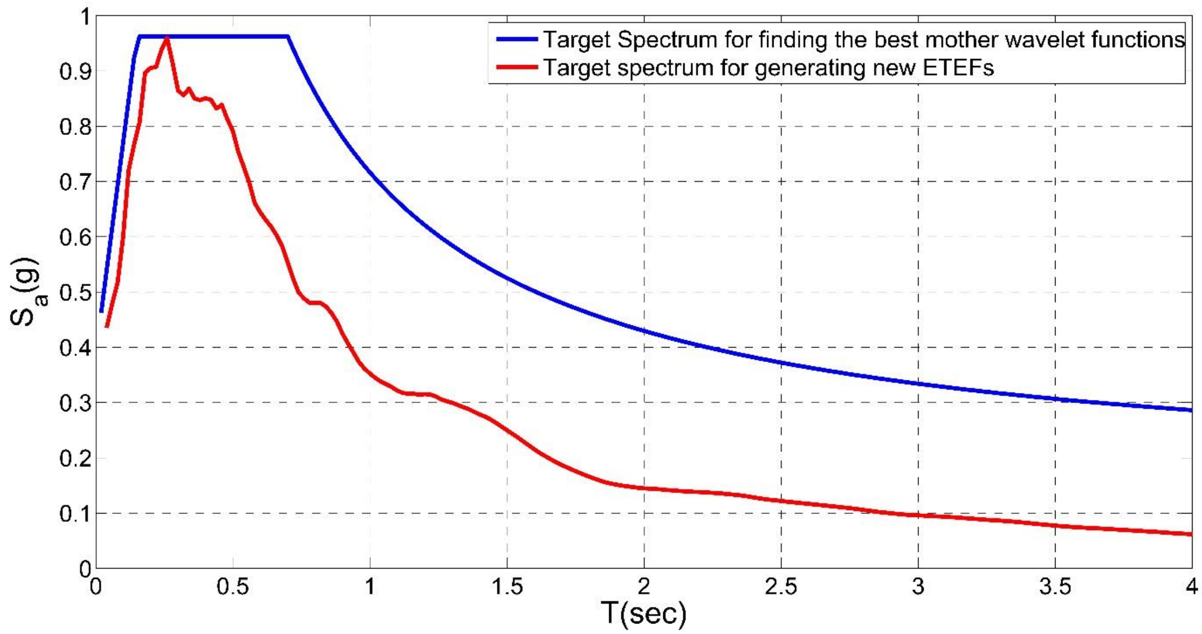
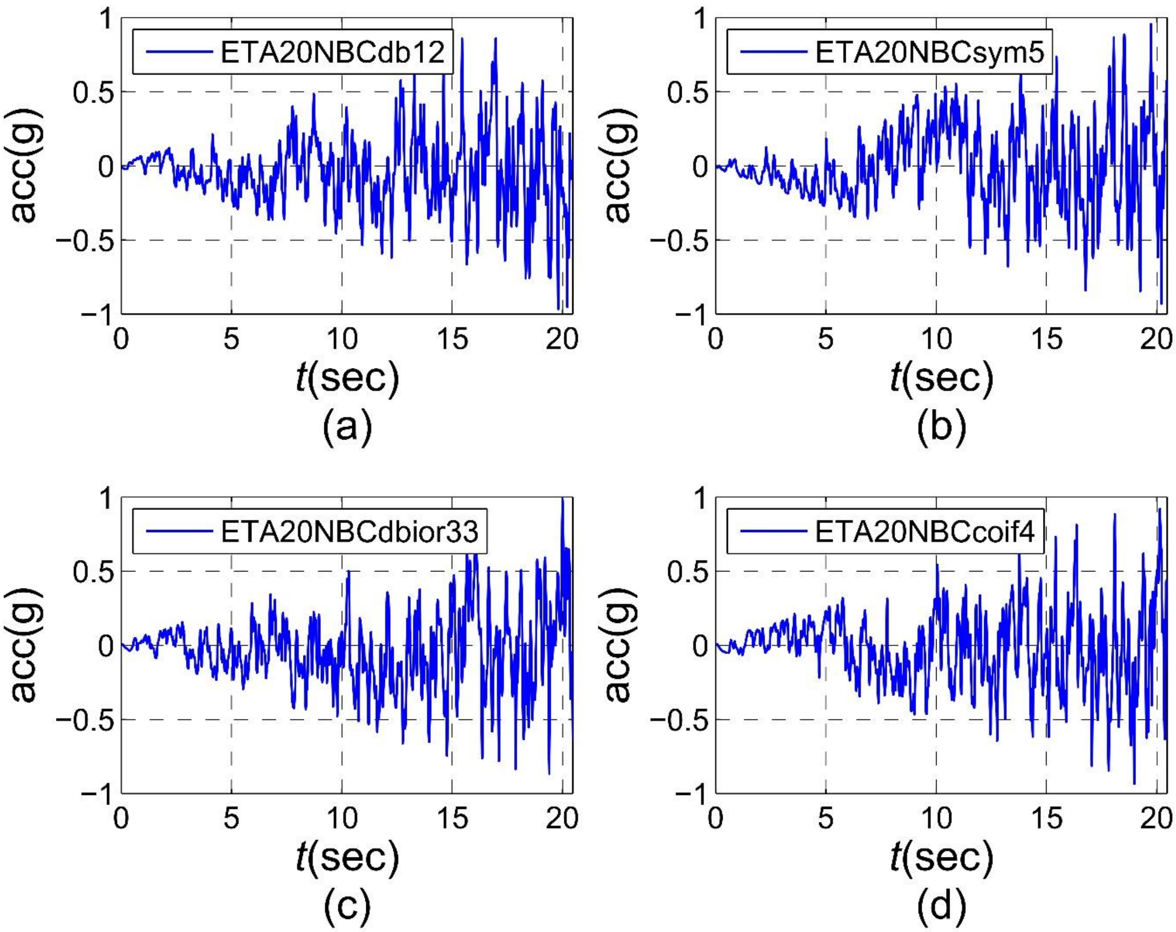

Target Spectrum for finding the best mother wavelet functions
Target spectrum for generating new ETEFs
S_a(g)
T(sec)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1
1.5
2
2.5
3
3.5
4

Fig. 14



ETA20NBCdb12
ETA20NBCsym5
ETA20NBCdbior33
ETA20NBCcoif4
acc(g)
t(sec)
(a)
(b)
(c)
(d)

Fig. 15

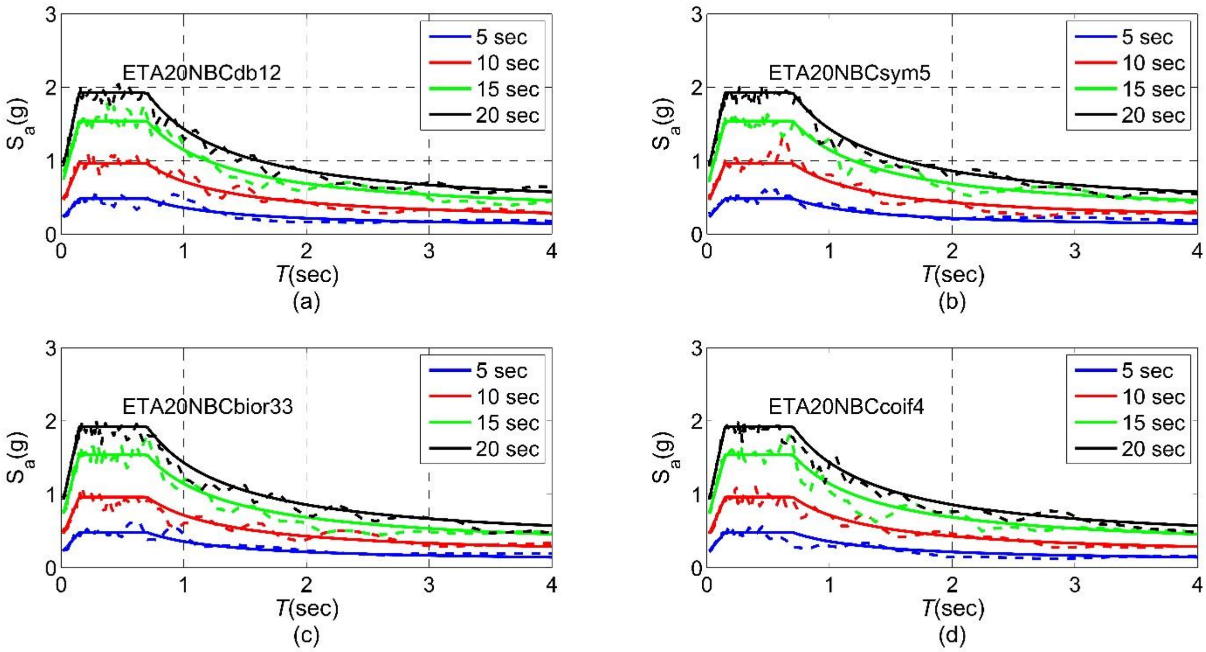

ETA20NBCdb12
ETA20NBCsym5
ETA20NBCbior33
ETA20NBCcoif4
5 sec
10 sec
15 sec
20 sec
S_a(g)
T(sec)
(a)
(b)
(c)
(d)

Fig. 16

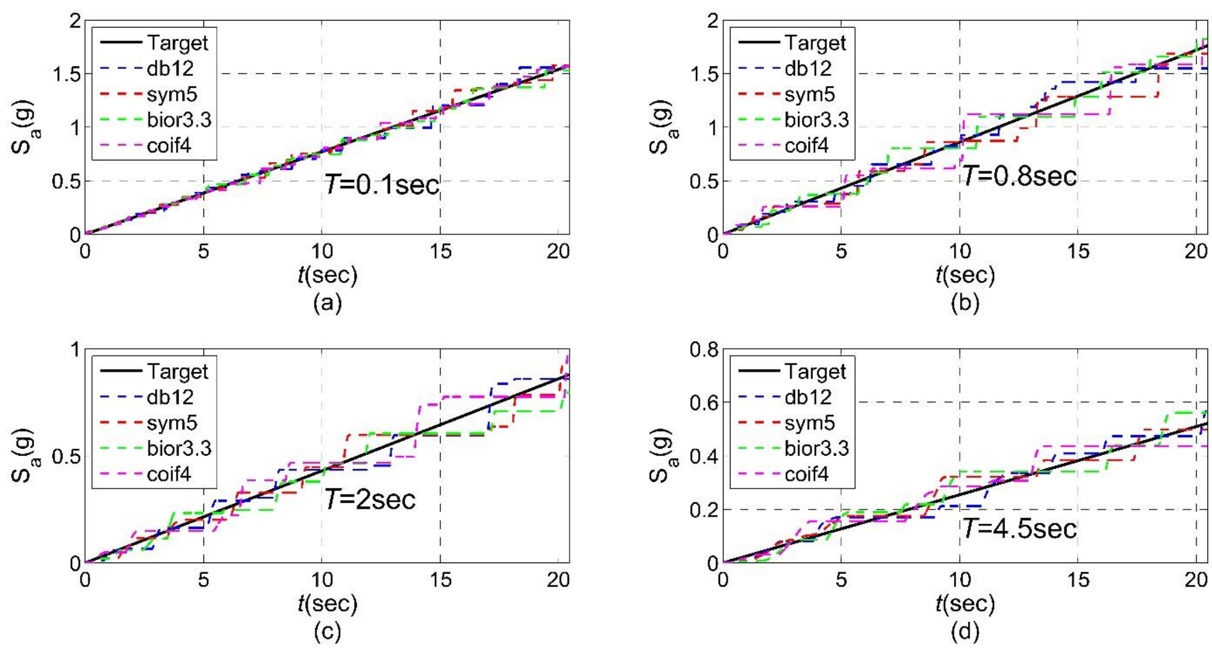

Target
db12
sym5
bior3.3
coif4
T=0.1sec
T=0.8sec
T=2sec
T=4.5sec
S_a(g)
t(sec)
(a)
(b)
(c)
(d)

Table 1

Scenario	**Mother wavelet**	**Number of sample points**	**Number of decision variables**
ETEF-cf1	Coif1	2085	544
ETEF-cf2	Coif2	2140	591
ETEF-cf3	Coif3	2193	637
ETEF-cf4	Coif4	2294	685
ETEF-cf5	Coif5	2303	732

Table 2

Scenario	**Mother wavelet**	**Number of sample points**	**Number of decision variables**
ETEF-sym1	Sym1	2048	512
ETEF-sym2	Sym2	2067	528
ETEF-sym3	Sym3	2085	544
ETEF-sym4	Sym4	2104	560
ETEF-sym5	Sym5	2121	575
ETEF-sym6	Sym6	2140	591
ETEF-sym7	Sym7	2158	607
ETEF-sym8	Sym8	2177	623

Table 3

Scenario	**Mother wavelet**	**Number of sample points**	**Number of decision variables**
ETEF-bior1.3	bior1.3	2085	544
ETEF-bior2.4	bior2.4	2121	575
ETEF-bior2.6	bior2.6	2158	607
ETEF-bior3.1	bior3.1	2067	528
ETEF-bior3.3	bior3.3	2104	560
ETEF-bior3.7	bior3.7	2177	623
ETEF-bior4.4	bior4.4	2121	575
ETEF-bior6.8	bior6.8	2193	637

Table 4

Wavelet family	Rank in simulating ETEFs	Importance Factor	First rank	Second rank	Third rank	Fourth rank	Fifth rank
Daubechies	1	5.5%	db12	db14	db18	db6	db16
Bio-Orthogonal	2	14.6%	sym5	sym7	sym8	sym6	sym4
Symlet	3	13.5%	bior3.3	bior3.7	bior2.6	bior6.8	bior3.1

Coiflet	4	12.4%	coif4	coif3	coif5	coif1	coif2
All Considered Families	---	13.8%	db12	db14	db18	db6	sym5

Table 5

ETEF	Normalized Relative Residual (%)
ETA20NBCdb12	8.63
ETA20NBCsym5	8.91
ETA20NBCbior3.3	8.86
ETA20NBCcoif4	9.46