Physical Insights on Bio-Convection in Prandtl Nanofluid over an Inclined Stretching Sheet in Non-Darcy Medium: Numerical Simulation

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Abstract

The innovative aspect of this work is to understand how the intricate interplay between bio-convection, heat transfer, and other behaviours of nanoparticles in a porous zone is affected by Prandtl nanofluid flow across an inclined stretched sheet. The stated equations are transformed into dimensionless form using appropriate similarity transformations, and the resultant set of equations is then numerically solved using MATLAB bvp4c. The acquired results are additionally verified against existing data. The incorporation of special parameters, including the Forchheimer drag (F_r), bio-convection Rayleigh number (R_b), density ratio of motile microorganism (Ω), stretching parameter (ε), Prandtl fluid parameter (δ), and elastic parameter (β), adds novelty and complexity to the analysis. The density ratio of motile microorganism plays a crucial role in determining the impact of microorganisms on bio-convection. Depending on whether this parameter is higher or lower than the surrounding fluid, the behaviour of velocity can vary, leading to different fluid flow patterns and dynamics within the system. The higher concentration causes the density of mobile microorganisms to increase, which has a stronger effect on the dynamics of bio-convection. The motile microorganisms considerably contribute to convective heat transmission, and the bacteria's density is extremely excessive compared to the fluid around them.

Keywords: Prandtl fluid, bio-convection Rayleigh number, Forchheimer drag, density ratio, stretching parameter, and elastic parameter.

1. Introduction –

Bio-convection, the collective motion of microorganisms induced by gradients in their surrounding fluid medium, has been a subject of significant exploration interest owing to its vital role in several natural processes, such as nutrient transport, ecological dynamics, and harmful algal blooms. Moreover, bio-convection has attracted attention for its potential applications in biotechnology, bioengineering, and wastewater treatment. This literature intends to provide an overview of the existing studies on the behaviour of microorganisms in bio-convection over a Prandtl-nanofluid flowing into an inclined, stretched flat surface in a non-Darcy background. By examining the current state of research in this field, this review aims to identify knowledge gaps and potential avenues for further investigation.

The inclusion of nanofluids in bio-convection studies introduces additional complexities due to the presence of nanoparticles. Prandtl-nanofluids, characterised by enhanced thermal properties, have gained prominence in recent years. The Forchheimer drag term captures the additional drag forces arising from non-Darcy flow. Several studies have inspected the outcome of the Forchheimer drag on flow patterns and bio-convection performance. For instance, researchers have investigated the impact of different drag coefficients on the motion and distribution of microorganisms in the fluid. Several studies have investigated the impact of the Forchheimer drag on microorganisms in bio-convection with different parameters and conduits. For instance, Li et al. [1] examined the effects of the Forchheimer drag coefficient on the stability and patterns of bio-convection. They detected an upsurge in the drag coefficient directed towards enhanced mixing and a more uniform distribution of microorganisms. A numerical investigation on mixed bioconvection in porous media saturated with nanofluid, including oxytactic microorganisms, was carried out by Bég et al. [2]. They looked at how different parameters affected the flow and heat transmission properties, highlighting the significance of microbial behaviour in porous media filled with nanofluid. In a computer examination of bioconvection in Prandtl nanofluid Darcy-Forchheimer flow across different conduits, Waqas et al. [3], Ahmad et al. [4], and Wang et al. [5] presented their findings. They investigated how inclination affected the bioconvection process, which has repercussions for a number of engineering applications. Yaseen et al.'s [6] investigation of the Cattaneo-Christov heat flux model in the MoS2-SiO2/kerosene oil Darcy-Forchheimer radiative flow.

The microorganisms may cause flow instabilities that result in the formation of bioconvection cells, which are patterns and structures. The microorganisms can move around and disperse throughout the fluid thanks to the stretched sheet's dynamic environment. The features of the system's heat and mass transmission can be considerably impacted by this interaction between the microorganisms and the flow. A numerical analysis of the movement of gyrotactic microorganisms in nanofluids via a porous medium over a stretched surface was done by Shahid et al. [7], Alharbi et al. [8], and Waqas et al.'s [9]. They investigated the behaviour of the random motion of microorganisms via bio-convection in nanofluids and develop some significant findings and resolve then numerically.

Khan et al. [10] examined the combined effects of bioconvection and velocity slip in the three-dimensional flow of the Eyring-Powell nanofluid with Arrhenius activation energy and binary chemical reactions, this work leads to the incorporation of the study of chemical reaction by microorganisms in bio-fluid which is quite significant they mentioned. Muhammad et al.'s [11] investigation of the bioconvection flow of magnetised Carreau nanofluid under the influence of slip over a wedge with motile microorganisms; they were stretched the study on the impact of slip flow under the action Lorentz force in bio-fluid and resolve the model by numerical scheme.

Majeed et al. [12] examined thermal radiation in a flow of magneto-hydrodynamic motile gyrotactic microorganisms that included minute nanoparticles moving at a slipping velocity towards a nonlinear surface, they claimed their work is novel under the applications of slipping velocity of the nanoparticles in bio-fluid and also it has feasibility due to the movement of microorganisms. The computational modelling of bioconvection and heat transfer studies of Prandtl nanofluid in an inclined stretched sheet was presented by Das and Ahmed [13] using a finite difference approach. The study [13] asserts that microorganisms can move thermophoretically in response to temperature gradients, but the density ratio can influence this motion. A computational solution for chemically reactive and thermally radiative MHD Prandtl nanofluid over a curved surface with convective boundary conditions was presented by Rasheed et al. [14]. Babu and Sandeep [15] looked into how nonlinear thermal radiation affected the flow of a magnetic nanofluid across a stretching sheet when it reached a non-aligned bio-convective stagnation point. In a water-based nanofluid, Waqas et al.'s study [16] examined how heat radiation and convective circumstances affected bio-convection. The bio-convection flow of a Casson nanofluid caused by a revolving and stretched disc was studied by Siddiqui et al. [17], while Wang et al. [18] explored a numerical modelling of a hybrid Casson nanofluid flow taking into account the impact of a magnetic dipole and gyrotactic microorganism. The findings contribute to a better understanding of the complex behaviour of nanofluids with magnetic and biological influences. Wang et al.'s study [19] concentrated on the MHD Williamson nanofluid flow through a thin elastic sheet with an erratic thickness. Additionally, Wang et al. [20] used the modified Mittag-Leffler kernel of Prabhakar's kind to analyse the time-dependent thermal transport flow of Casson nanofluids. In 2003, Shampine [21] presented the crucial numerical solution by utilizing the finite difference scheme to resolve the ODEs via the bvp4c MATLAB algorithm.

The authors [22–28] purposefully brought attention to the examination of the characteristic's nanoparticles using hybrid-nanofluid and CNTS under specific boundary conditions for the relevant different surfaces with non-axisymmetric flow, and they discovered appropriate recommendations. The mathematical modelling of blood flow was investigated by the authors [29 – 31] via bio-convection characteristics with suitable configurations. The shapes of various surfaces, like heated wavy-walled lid-driven enclosures, open-sided cubical enclosures, and heated flexible-walled cavities in a rotating cylinder, are crucial to investigating the motion of nanoparticles in different base fluids under the action of magnetic drag force, CNTs, and free convections that have been studied by the authors [32–34]. The authors [35 – 41] presented various flow models through the Darcy-Forchheimer model for ferromagnetic nanoparticles and bioconvection Casson nanofluid with gyrotactic microorganisms and activation energy aspects; they also analysed the behaviour of generalised Eyring-Powell liquid subject to Cattaneo–Christov double diffusion aspects for magnetised Carreau and Maxwell viscoelastic nanofluids in different boundary conditions. Finally, the behaviour of power law fluids using a variety of numerical schemes in relation to the bioconvection flow of nanofluids of microorganisms inside a wavy wall of porous materials, an omega-shaped porous enclosure, or a lid-driven cavity have also been presented by [42–46]. The idea of microorganisms has attracted a lot of attention from contemporary researchers [47–57] due to its application in commercial and industrial items, such as fertilisers, biofuel, and medicine delivery. Multi-physical factors and different geometries are taken into account in these investigations [47–57], and the results show that gyrotactic bacteria and nano-liquid consideration stabilise the adjourned nanoparticles.

This inquiry is distinctive and novel in several ways. The originality of this study is to explore the behaviour of microorganisms in bio-convection over a Prandtl-nanofluid, which is a relatively new and emerging research area. Combining bio-convection with nanofluid flow introduces novel complexities and interactions between microorganisms and nanoparticles, affecting the overall fluid dynamics and heat transfer processes. The study considers an inclined, stretching flat surface in a non-Darcy background, which is less explored in the context of bio-convection. The inclusion of the Forchheimer drag term allows for the investigation of deviations from Darcy flow, providing a more realistic representation of fluid flow in practical scenarios. The study incorporates special parameters, including the Forchheimer drag, bio-convection Rayleigh number, density ratio, stretching parameter, and elastic parameter, to analyse their effects on the bio-convection phenomenon. The study employs a comprehensive numerical analysis by solving the coupled nonlinear partial differential equations using a finite difference scheme. The study provides insights for optimising bio-convection-based systems and can guide the design and operation of such applications. The study expands the understanding of bio-convection phenomena and provides a foundation for further research in this exciting and evolving field.

2. Mathematical Formulation –

The study of how microorganisms move through bioconvection in Prandtl nanofluids provides the novel background for this investigation and provides an insight into new techniques for improving heat transfer and fluid mixing, with applications ranging from advanced cooling systems to more effective chemical processes. The interaction of microorganisms and nanoparticles in nanofluids is a fascinating confluence of biology, nanotechnology, and fluid dynamics, giving intriguing potential for scientific research and technological advancement.

Figure 1 depicts the flow configuration of the model, which simulates the flow of microorganism bio-convection in a Prandtl nanofluid over an inclined stretched surface with an inclination of γ and a stretching velocity of $u_w = ax$ along the x-axis. The surface normal to it is subject to a magnetic drag force of strength B_0. The thermal and molar species as well as the motile microorganisms at the wall are designated as $T_f = T_0 + a_1 x, C_f = C_0 + a_2 x$ and $N_f = N_0 + a_3 x,$ respectively.

The Prandtl fluid equation is (Wang et al. [5])

$$\tau^* = \frac{A \sin^{-1}\left(\frac{1}{C}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]^{0.5}\right)}{\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]} \tag{1}$$

where, the material constants for the Prandtl fluid are A and $\overline{C}$.

The vectorial equations for fluid flow are given by (Wang et al. [5] and Shahid et al. [7]):

Continuity *(*mass conservation*)* equation:

$$\nabla \cdot \vec{q} = 0 \tag{2}$$

Momentum equation:

$$\left(\vec{q} \cdot \nabla\right)\vec{q} = \begin{Bmatrix} \frac{A\nu}{C}\left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{A\nu}{2C^3}\left(\frac{\partial u}{\partial y}\right)^2 \left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{\rho}\left(\vec{J} \times \vec{B}\right) - \frac{\nu}{k}u - Fu^2 \\ +g\begin{bmatrix} \beta_1 \rho_f \left(1 - C_\infty\right)\left(T - T_\infty\right) - \left(\rho_p - \rho_f\right)\left(C - C_\infty\right) \\ -\gamma\left(\rho_m - \rho_f\right)\left(N - N_\infty\right) \end{bmatrix}\cos\alpha \end{Bmatrix} \tag{3}$$

Energy equation:

$$\vec{q} \cdot \nabla T = \frac{\kappa}{\rho C_P}\nabla^2 T + \tau\left[D_B \nabla C \cdot \nabla T + \frac{D_T}{T_\infty}\nabla T \cdot \nabla T\right]\frac{\sigma}{\rho C_P}B^2(x)\left(\sin^2\gamma\right)u^2 \tag{4}$$

Oxygen conservation equation:

$$\vec{q} \cdot \nabla C = D_B \nabla^2 C + \frac{D_T}{T_\infty}\nabla^2 T, \tag{5}$$

Conservation equation for microorganisms:

$$\nabla \cdot \vec{J}_N = 0, \tag{6}$$

where, $\vec{q}$ is the velocity vector, $\vec{J}$ is the electric current density in the fluid, $\vec{B}$ is the magnetic field, $\vec{J}_N$ is the flux of gyrotactic micro-organisms.

The flux of microorganisms, $\vec{J}_N$, is defined as

$$\vec{J}_N = N\bar{\vartheta} - D_N \nabla^2 N, \tag{7}$$

where,

$$\bar{\vartheta} = \frac{dw_c}{C_f - C_0}\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right) \tag{8}$$

$$\vec{J}\times\vec{B} = \sigma B^2(x)\left(\sin^2\gamma\right)\cdot u \tag{9}$$

The Eqs. (2) – (6) become with the aid of (7) – (9) in Cartesian coordinates:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left\{\begin{array}{l} \dfrac{A\nu}{C}\left(\dfrac{\partial^2 u}{\partial y^2}\right) + \dfrac{A\nu}{2C^3}\left(\dfrac{\partial u}{\partial y}\right)^2\left(\dfrac{\partial^2 u}{\partial y^2}\right) - \dfrac{\sigma B^2(x)}{\rho}\left(\sin^2\gamma\right)\cdot u - \dfrac{\nu}{k}u - Fu^2 \\ +g\left[\begin{array}{l}\beta_1\rho_f\left(1-C_\infty\right)\left(T-T_\infty\right) - \left(\rho_p - \rho_f\right)\left(C - C_\infty\right) \\ -\gamma\left(\rho_m - \rho_f\right)\left(N - N_\infty\right)\end{array}\right]\cos\alpha \end{array}\right\} \tag{11}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_P}\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]\frac{\sigma}{\rho C_P}B^2(x)\left(\sin^2\gamma\right)u^2 \tag{12}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{13}$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{dw_c}{C_f - C_0}\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right) + D_N\frac{\partial^2 N}{\partial y^2} \tag{14}$$

The formulated boundary conditions are (Wang et al. [5], Das et al. [13]):

$$\left\{\begin{array}{l} y = 0 : \left[\begin{array}{l} u = \varepsilon u_w = ax,\ v = v_0,\ T = T_f = T_0 + a_1 x, \\ C = C_f = C_0 + a_2 x,\ N = N_f = N_0 + a_3 x, \end{array}\right] \\ y \to \infty : \left[\begin{array}{l} u \to 0,\ T \to T_\infty = T_0 + d_1 x,\ C \to C_\infty = C_0 + d_2 x, \\ N \to N_\infty = N_0 + d_3 x, \end{array}\right] \end{array}\right\} \tag{15}$$

Physical explanations of the boundary conditions (15) –

The boundary conditions (15) describe the behaviour of a bio-convective nanofluid near a boundary $(y = 0)$ and how it behaves as it moves towards the far field $(y \to 0)$. The specific values of the constants occurred in Eq. (15) will determine the exact behaviour of the fluid, but this set of conditions is often used in mathematical modelling and simulations for bio-convective nanofluid dynamics, heat, and mass transfer models.

At y=0:

Velocity $\left(u=\varepsilon u_w=ax\right)$: The velocity component u, in the x-direction, is defined as a function of x and has two components:

The term εu_w represents the velocity at $y=0$, scaled by a factor ε and it suggests that the velocity near the wall is influenced by the wall's motion or properties.

The term ax represents an additional velocity component that varies linearly with x. This means that the velocity increases linearly away from the wall in the positive x-direction.

Velocity $\left(v=v_0\right)$: It indicates that there is no change in velocity in the x-direction, and it remains equal to v_0 at all points along the $y=0$ boundary.

Temperature $\left(T=T_f=T_0+a_1x\right)$: The temperature T is constant at $y=0$ and the temperature increases linearly with x away from the wall.

Concentration $\left(C=C_f=C_0+a_1x\right)$: The concentration C is constant at $y=0$ and the concentration also increases linearly with x away from the wall.

Number Density $\left(N=N_f=N_0+a_3x\right)$: The number density N is constant at $y=0$ and the number density increases linearly with x.

At the free stream $\left(y\rightarrow\infty\right)$:

Velocity $\left(u\rightarrow 0\right)$: The velocity becomes negligible, and the fluid comes to rest in the x−direction.

Temperature $\left(T\rightarrow T_\infty=T_0+d_1x\right)$: The temperature T increases without bound as $y\rightarrow\infty$ and it continues to rise linearly with x in the free stream.

Similar to temperature, the concentration C and the number density N also increases without bound as $y\rightarrow\infty$ and they continue to increase linearly with x in the free stream.

To alter the above system of equations, subsequent similarity transformations [Wang et al. [5] and Das and Ahmed [13]] are defined as:

$$\left\{\begin{array}{l}\psi=\sqrt{a\nu x}\,f\left(\eta\right),\ \eta=\left(\dfrac{a}{\nu}\right)^{\frac{1}{2}}y,\ u=\dfrac{\partial\psi}{\partial y}=axf'\left(\eta\right),\\ \mathrm{v}=-\dfrac{\partial\psi}{\partial x}=-\sqrt{a\nu}x\ f\left(\eta\right),\ \theta\left(\eta\right)=\dfrac{T-T_\infty}{T_f-T_0},\\ \phi\left(\eta\right)=\dfrac{C-C_\infty}{C_f-C_0},\ \chi\left(\eta\right)=\dfrac{N-N_\infty}{N_f-N_0}\end{array}\right\} \tag{16}$$

here, ψ is the stream function.

Using (16), the converted system of Eqs. are:

$$\left\{\begin{array}{l} f'''\delta\left(1+\beta f''^2\right)+ff''-M\sin^2(\gamma)f'-K_p f' \\ +\left(F_r+1\right)f'^2+w\left(\theta-N_r\phi-R_b\chi\right)\cos(\alpha) \end{array}\right\}=0 \tag{17}$$

$$\frac{1}{Pr}\theta''-f'\theta+f\theta'+N_b\theta\phi'+N_t\theta'^2+M\,Ec\,f'^2=0 \tag{18}$$

$$\phi''-Sc\left(f'\phi+f\phi'\right)+\frac{N_t}{N_b}\theta''=0 \tag{19}$$

$$\chi''-L_b\chi f'+L_b\chi' f-P_e\left(\phi'\chi'+\chi\phi''+\Omega\phi''\right)=0 \tag{20}$$

The converted boundary conditions are:

$$\left[\begin{array}{l} \eta=0:\left\{f(\eta)=S,\ f'(\eta)=\varepsilon,\ \theta(\eta)=1-S_1,\ \phi(\eta)=1-S_2,\ \chi(\eta)=1-S_3\right\} \\ \eta\to 0:\ f'(\eta)\to 0,\ \theta(\eta)\to 0,\ \phi(\eta)\to 0 \end{array}\right] \tag{21}$$

From the perspective of Eqs. (17) to (20), non-dimensional variables are

$$\left\{\begin{array}{l} \delta=\dfrac{A}{C},\ \beta=\dfrac{a^3x^2}{2C^2\nu},\ M=\dfrac{\sigma B^2(x)}{\rho a},\ K_p=\dfrac{\nu}{Ka},\ F_r=Fx=\dfrac{C_b x}{\sqrt{K}}, \\ w=\dfrac{g\beta_1\rho_f}{U_w^2}\left(T_f-T_0\right)\left(1-C_\infty\right),\ N_r=\dfrac{\left(\rho_p-\rho_f\right)\left(C_f-C_0\right)}{\beta_1\rho_f\left(T_f-T_0\right)\left(1-C_\infty\right)},\ Pr=\dfrac{\rho C_P\nu}{\kappa}, \\ R_b=\dfrac{\left(\rho_m-\rho_f\right)\left(N_f-N_0\right)}{\beta_1\rho_f\left(T_f-T_0\right)\left(1-C_\infty\right)},\ N_b=\dfrac{\tau D_B\left(C_f-C_0\right)}{\nu},\ N_t=\dfrac{\tau D_T\left(T_f-T_0\right)}{\nu T_\infty}, \\ L_b=\dfrac{\nu}{D_B},\ P_e=\dfrac{dw_c}{D_N},\ \Omega=\dfrac{n_\infty}{N_f-N_0},\ S_1=\dfrac{d_1}{a_1},\ S_2=\dfrac{d_2}{a_2},\ S_3=\dfrac{d_3}{a_3}, \end{array}\right\} \tag{22}$$

3. Physical quantities of interests:

The features of a few physical quantities are defined in this section. The following list includes the density number of motile microorganisms, Nusselt number, Sherwood number, and coefficient of skin friction:

$$\left\{\begin{array}{l} Nn_x=\dfrac{xq_n}{D_N\left(N-N_\infty\right)},\ Sh_x=\dfrac{xq_m}{D_B\left(C-C_\infty\right)}, \\ Nu_x=\dfrac{xq_w}{\kappa\left(T_w-T_\infty\right)},\ C_{fx}=\dfrac{\tau_w}{\frac{1}{2}\rho_f U_w^2(\mathrm{x})} \end{array}\right\} \tag{23}$$

$$\begin{Bmatrix} q_n = -D_N\left(\dfrac{\partial N}{\partial y}\right)_{y=0}, \; q_m = -D_B\left(\dfrac{\partial C}{\partial y}\right)_{y=0}, \\ q_w = -\kappa\left(\dfrac{\partial T}{\partial y}\right)_{y=0}, \; \tau_w = -\left(\dfrac{A}{C}\dfrac{\partial u}{\partial x} + \dfrac{A}{6C^3}\left(\dfrac{\partial u}{\partial y}\right)^2\right)_{y=0}, \end{Bmatrix} \tag{24}$$

Here q_n, q_m, q_w and τ_w are the density of motile microorganisms' flux, mass flux, heat flux and shear stress respectively.

By using of (16) and (24), the Eq. (23) is converted to:

$$Re_x^{-\frac{1}{2}} Nn_x = -(\chi'(0)), \tag{25}$$

$$Re_x^{-\frac{1}{2}} Sh_x = -(\phi'(0)), \tag{26}$$

$$Re_x^{-\frac{1}{2}} Nu_x = -(\theta'(0)), \tag{27}$$

$$Re_x^{\frac{1}{2}} C_{fx} = -\left(\frac{\delta\beta}{3} f''^3(0) + \delta f''(0)\right), \tag{28}$$

$Re_x^{\frac{1}{2}} = \sqrt{\dfrac{U_w x}{\nu}}$, is the local Reynolds number.

4. Solution Methodology –

Using similarity variables, the simulated non-linear PDEs were transformed into a two-point boundary value problem. There is no closed-form analytical solution for the non-linear boundary value problem given by Eqs. (17) to (20) and boundary conditions given by Eq. (21), and so numerically solved the systems of ODEs using bvp4c via MATLAB algorithm by **Shampine [21]**. The behaviour of the solutions is precisely captured by this solver, which applies the ***finite difference approach*** that was widely analysed by **Das and Ahmed [13]** in conjunction with adaptive mesh refinement techniques. To utilize bvp4c, we need to specify the system of ODEs, the boundary conditions, and an initial guess for the solution. It then iteratively refines the solution by adjusting the mesh until the desired accuracy is achieved. For purposes of computation, a step length of $\Delta\eta = 0.0001$ is used, and a relative tolerance of 10^{-6} is taken into account.

The Eqs. (17 – 20) have to be transformed into the order-one ODE's with help of the substitutions:

$$\begin{Bmatrix} f = y_1, \; f' = y_2, \; f'' = y_3, \\ \theta = y_4, \; \theta' = y_5, \\ \phi = y_6, \; \phi' = y_7, \\ \chi = y_8, \; \chi' = y_9 \end{Bmatrix} \tag{29}$$

Now, the transformed system of equations is:

$$y_3' = \left(\frac{1}{\delta(1+\beta y_3^2)}\right)\begin{pmatrix} -y_1 y_3 + M \sin^2(\gamma) y_2 + K_P y_2 \\ -\left(F_r + 1\right) y_2^2 - w\left(y_4 - N_r y_6 - R_b y_8\right)\cos(\alpha) \end{pmatrix} \tag{30}$$

$$y_5' = Pr\left(y_4 - y_1 y_5 - N_b y_4 y_7 - N_t y_5^2\right) - M\, Ec\, y_2^2 \tag{31}$$

$$y_7' = Sc\left(y_2 y_6 + y_1 y_7\right) - \frac{N_t}{N_b} Pr\left(y_4 - y_1 y_5 - N_b y_4 y_7 - N_t y_5^2\right), \tag{32}$$

$$y_9' = L_b y_2 y_8 - L_b y_1 y_9 + Pe\left(y_7 y_9 + \begin{pmatrix} Sc\left(y_2 y_6 + y_1 y_7\right) \\ -\frac{N_t}{N_b} Pr\begin{pmatrix} y_4 - y_1 y_5 \\ -N_b y_4 y_7 - N_t y_5^2 \end{pmatrix} \end{pmatrix}\left(\chi + \Omega\right) \right) \tag{33}$$

The Boundary conditions are:

$$\begin{Bmatrix} y_1(0) = S,\ y_2(0) = \varepsilon,\ y_4(0) = 1 - S_1, \\ y_6(0) = 1 - S_2,\ y_8(0) = 1 - S_3, \\ y_2(\infty) \to 0,\ y_4(\infty) \to 0,\ y_8(\infty) \to 0, \end{Bmatrix} \tag{34}$$

5. Validity and Stability Analysis –

To demonstrate the validity of the present inquiry, a comparison of the velocity distribution has been conducted and found excellent agreement with the available results attained by Wang et al. [5] and Das and Ahmed [13]. Here, the numerical values for **Table 1** are taken to be $\delta = 0.5$, $Pr = 0.71,\ M = 0.5,\ \gamma = \frac{\pi}{4},\ \varepsilon = -1.5$, and the other parameters are assumed as zero.

According to **Table 2**, it can inferred that taking $h = 0.001$ and $h = 0.0001$, the solutions of velocity, temperature, concentration, and bio-convection's density are stable and convergent, as the boundary conditions are also satisfied. That is from **Table 1** and **Table 2,** it is concluded that our proposed model is validated and the chosen numerical scheme is stable and convergent.

6. Results and Discussion –

It has been discussed in this section how several significant parameters effect on $f'(\eta)$, $\theta(\eta), \phi(\eta)$ and $\chi(\eta)$, taking the numerical values of the prime parameters as $\delta = 0.5,\ Pr = 0.71,$ $M = 0.5,\ \gamma = \frac{\pi}{4},\ \varepsilon = 0.2,\ \alpha = \frac{\pi}{4},\ R_b = 0.5,\ \Omega = 0.5,\ F_r = 1.5$.

In this study, the flow is more likely to remain laminar when taking into account lower nanoparticle concentrations, where the nanoparticles have a limited impact on fluid viscosity and flow behaviour. Several factors, including the concentration of nanoparticles, flow velocity, and the existence of the microorganisms driving the bioconvection, determine whether the flow of bioconvection in a nanofluid maintains laminar or transitions to turbulent behaviour. Low nanoparticle concentrations and diluted nanofluids are more likely to have laminar flow than high concentrations or severe bioconvection effects.

The novel physical variables of this study are summarized as –

***Pr*-nanofluid (δ):** The Prandtl number nanofluid, also known as *Pr*-nanofluid or the Prandtl number nanofluid, is a particular kind of nanofluid with exceptional thermal and heat transmission characteristics. Its importance comes from its superiority over conventional fluids in terms of improving heat transfer in a variety of applications.

Forchheimer drag (F_r): It is known as Forchheimer inertial resistance, is the additional resistance that fluid flow experiences when it passes through a porous media, as a result of the presence of solid particles or barriers inside the medium. It is a non-Darcy flow effect and is often defined by a quadratic relationship between the velocity of the fluid and the pressure drop across the porous media. It comprises both viscous resistance (Darcy's law) and inertial resistance (Forchheimer's law).

Bio-convection Rayleigh number (R_b): In the study of biological convection, where fluid velocity is influenced by the presence of microorganisms, it is a dimensionless parameter that is used. It is referred to as the Grashof-Péclet product and is used to assess the relative importance of buoyancy forces brought on by density differences as well as the impacts of biological activity on fluid motion within a system.

Density ratio of motile microorganism (Ω): It characterizes the relative density of the microorganisms in relation to the density of the fluid in which they are suspended. By calculating the buoyant forces that these microorganisms exert on the fluid, this parameter is essential for comprehending how they affect fluid flow and buoyancy-driven phenomena, such bio-convection.

Figures 2(a-d) depict the impact of Ω and R_b on $f'(\eta)$, $\theta(\eta)$, $\phi(\eta)$ and $\chi(\eta)$. When considering the bio-convection Rayleigh number (R_b) and the density ratio of motile microorganism parameter (Ω), the behaviour of velocity, temperature, concentration, and microorganism's density can differ depending on the specific parameter values. When $\Omega = 5$, and the bio-convection Rayleigh number is high, it suggests that motile microorganisms' density is relatively higher compared to the surrounding fluid. In this case, the motile microorganisms contribute significantly to the convective motion and fluid flow. As a result, the velocity is heightened, indicating a more vigorous and pronounced fluid motion due to the active movement and behaviour of the microorganisms. On the contrary, motile microorganisms have less influence on convective heat transfer. The enhanced temperature indicates that the convective heat transfer due to fluid motion dominates over the heat generated by the microorganisms. This can lead to an increase in temperature in the system.

Furthermore, the profiles of $\chi(\eta)$ is elevated, which means that there is a higher concentration of microorganisms present in the system. The density of motile microorganisms rises due to the greater concentration, resulting in a more significant impact on the bio-convection dynamics. On the other hand, when $\Omega = 0.5$ and the bio-convection Rayleigh number is high, it implies that the density of the motile microorganisms is relatively lower compared to the surrounding fluid. In this scenario, the motile microorganisms have less influence on the convective motion and fluid flow. Consequently, the velocity may exhibit an opposite behaviour, potentially showing a decrease or a different pattern of fluid motion compared to the case with a higher density ratio. In addition, microorganisms' density is relatively extreme compared to the surrounding fluid, and the motile microorganisms contribute significantly to convective heat transfer. The opposite behaviour of temperature recommends that the heat generated by the microorganisms dominate over the convective heat transfer due to fluid motion. This can lead to a drop in temperature or a different pattern of temperature distribution compared to the case with a lower density ratio. Furthermore, the concentration of microorganisms in this system is reduced, resulting in a lower density of motile microorganisms. These observations endorse that Ω influences the concentration and density of motile microorganisms within the bio-convection system. Depending on whether the density of the microorganisms is higher or lower than the surrounding fluid, the impact of the density and concentration can vary. These variations in the density and concentration of motile microorganisms can have implications for the bio-convection dynamics, nutrient transport, and overall behaviour of the system. Bio-convection Rayleigh number $(R_b < 1)$ suggests that buoyancy-driven convection is not a dominant factor in the system of microorganisms of nanofluid that depends on the higher and lower and higher motile microorganism parameter (Ω). The fluid velocity $f'(\eta)$ enhanced by the action of $R_b < 1$ when $\Omega = 5$, which indicates that the microorganisms' buoyancy forces are becoming more pronounced, strengthening bio-convection and raising the system's fluid velocity. This behaviour of $f'(\eta)$ is reversed when $\Omega = 0.5$ and it means that a decrease in fluid velocity can result from the suppression of buoyancy-driven convection caused by the density contrast between the microorganisms and the surrounding fluid. A reversed trend has been observed for the profiles of $\theta(\eta), \phi(\eta)$ and $\chi(\eta)$ in comparison to $f'(\eta)$ for the effects of $R_b < 1$ when $\Omega = 0.5$ and $\Omega = 5$.

The behaviour of velocity, temperature, concentration, and density of microorganism with respect to the Prandtl fluid parameter (δ) and the Forchheimer drag parameter (F_r) are displayed in Figures 3 (a – d) respectively. In Figure 3 (a), when the Forchheimer drag parameter (F_r) is taken as 0.1, the velocity is boosted. This means that the fluid exhibits a higher velocity under these conditions. On the other hand, when $F_r = 1.5$, the velocity declines. The interaction between momentum and thermal diffusion is the cause of these observations. It is implied by a smaller

Forchheimer drag parameter (0.1) that the fluid has greater momentum diffusivity than heat diffusivity. Due to the dominance of momentum transport, augmented velocity is noticed. In contrast, a higher Forchheimer drag parameter $(F_r = 1.5)$ results in greater thermal diffusivity, which causes a reduction in velocity. Furthermore, the Forchheimer drag parameter (F_r) reduces the velocity. This shows that the Forchheimer drag causes an increase in flow resistance, which causes a drop in velocity. The existence of impediments or porous structures that obstruct the flow and raise drag forces is often related to the Forchheimer drag. These drag forces act against the motion of the fluid, leading to a drop in velocity.

The temperature, concentration, and microorganism's density are enhanced when the Forchheimer drag parameter is taken as 1.5, but decreased when it is taken as 0.1. When $F_r = 1.5$, the momentum diffusivity and thermal diffusivity of the fluid behave alike, which asserts that an improvement has occurred in the temperature, concentration, and microorganisms' density. This implies that heat and mass are transmitted more effectively, leading to enhanced profiles of temperature, concentration, and microorganism density. On the contrary, when $F_r = 0.1$, it infers that momentum diffusivity surpasses heat diffusivity. As a result, the efficiency of the movement of heat, mass, and microorganisms' density declines, which diminishes temperature, concentration, and the microorganism's density. The presence of F_r indicates increase in flow resistance due to obstacles or porous structures. This enhances flow resistance and causes the fluid to experience more convective mixing, which leads to improved transmission of heat and mass. Consequently, temperature, concentration, and microorganism density are enhanced. Additionally, the augmented concentration leads to improved mass transfer, which raises the concentration of the transported species. In addition, the amplified density of the microorganisms suggests stronger convective mixing, enabling the microorganisms to be more evenly dispersed and densely packed throughout the fluid. When a fluid's Prandtl fluid parameter (δ) is less than one, it means that it resists flow more effectively than it conducts heat (high viscosity). This results in slower heat transfer and is a property of some substances, such as excessively viscous oils or polymers. Significantly, the Prandtl fluid parameter, declines all the profiles $\theta(\eta), \phi(\eta)$ and $\chi(\eta)$ in bio-convection motion.

In **Table 3,** we have portrayed the effects of some of the significant parameters on Effect of various governing parameters on $Re_x^{\frac{1}{2}}C_{fx}$, $Re_x^{-\frac{1}{2}}Nu_x$, $Re_x^{-\frac{1}{2}}Sh_x$ and $Re_x^{-\frac{1}{2}}Nn_x$.

Impact of δ on $Re_x^{\frac{1}{2}}C_{fx}$, $Re_x^{-\frac{1}{2}}Nu_x$, $Re_x^{-\frac{1}{2}}Sh_x$ and $Re_x^{-\frac{1}{2}}Nn_x$:

A lower skin friction due to δ indicates that the fluid is undergoing less drag or resistance as it flows over the stretching surface. A non-dimensional parameter known as the Nusselt number $(Re_x^{-\frac{1}{2}}Nu_x)$ connects the convective rate of change of heat to the conductive rate of change of heat at a boundary surface. An increased Nusselt number designates that the convective heat transmute is more efficient, leading to enhanced heat dissipation or transfer from the surface to the fluid. The enhancement of the Sherwood number suggests an improvement in the mass transfer characteristics of the system. The Sherwood number $(Re_x^{-\frac{1}{2}} Sh_x)$ is a non-dimensional parameter that narrates the convective rate of mass transfer to the diffusive rate of mass transfer at a boundary surface. An increased Sherwood number indicates that the mass transfer from the surface to the fluid is more efficient, i.e., there is an amplified rate of mass transfer, which could be important in applications where the transport of chemical species or nutrients is crucial, such as in biofilm growth or biological reactors. The rise in rate of change of microorganism's density intends that the concentration of motile microorganisms in the fluid is higher. This higher density could be influenced by the Prandtl fluid parameter, which may provide more favourable conditions for the growth and movement of microorganisms.

Impact of F_r on $Re_x^{\frac{1}{2}} C_{fx}$, $Re_x^{-\frac{1}{2}} Nu_x$, $Re_x^{-\frac{1}{2}} Sh_x$ and $Re_x^{-\frac{1}{2}} Nn_x$:

The increase in skin friction acclaims the resistance to the flow of the fluid over the inclined, stretching flat surface is heightened due to the Forchheimer drag parameter. The reduction in the Nusselt number signifies a reduction in convective heat transfer efficiency. A lower Nusselt number indicates less efficient convective heat transfer, which may be accredited to the increased skin friction and altered flow patterns caused by the Forchheimer drag parameter. The decrease in the Sherwood number indicates a decline in mass transfer efficiency. A lower Sherwood number suggests reduced mass transfer from the surface to the fluid. The increased skin friction and altered flow patterns caused by the Forchheimer drag parameter could potentially hinder the transport of species or nutrients, leading to a decrease in mass transfer efficiency. The decrease in the rate of change of the microorganism's density suggests a lower concentration of motile microorganisms in the fluid. This reduction could be influenced by the altered flow patterns and reduced mass transfer efficiency associated with the Forchheimer drag parameter.

Impact of R_b on $Re_x^{\frac{1}{2}} C_{fx}$, $Re_x^{-\frac{1}{2}} Nu_x$, $Re_x^{-\frac{1}{2}} Sh_x$ and $Re_x^{-\frac{1}{2}} Nn_x$:

The reduced $Re_x^{\frac{1}{2}} C_{fx}$ and enhanced $Re_x^{-\frac{1}{2}} Nu_x$, $Re_x^{-\frac{1}{2}} Sh_x$ and $Re_x^{-\frac{1}{2}} Nn_x$ due to the Bio-convection Rayleigh number suggest favourable outcomes in terms of reduced flow resistance, improved convective heat transfer, and enhanced mass transfer efficiency. These findings highlight the significant role of bio-convection and motile microorganisms in influencing the system behaviour and optimizing heat and mass transfer processes. Understanding these effects is crucial in various applications, such as in biological systems, environmental engineering, where convective transport and microorganism dynamics play essential roles.

Impact of Ω on $Re_x^{\frac{1}{2}}C_{fx}$, $Re_x^{-\frac{1}{2}}Nu_x$, $Re_x^{-\frac{1}{2}}Sh_x$ and $Re_x^{-\frac{1}{2}}Nn_x$:

The presence of motile microorganisms and their influence on fluid flow can alter the flow behaviour and reduce the skin friction experienced by the fluid. This declination indicates a more efficient flow with reduced drag or resistance. The increase in the Nusselt number due to the density ratio of motile microorganisms signifies improved convective heat transfer efficiency, i.e., enhanced convective heat transfer. The presence of motile microorganisms can enhance convection, leading to improved heat transmutation or dissipation from the surface to the fluid. An augmented Sherwood number designates improved transmission of mass from the surface to the fluid, which can be influenced by the presence of motile microorganisms and their role in promoting fluid mixing and enhancing mass transport. The increase in rate of change of density directs a higher concentration of motile microorganisms in the fluid, which can lead to more intense bio-convection, alter flow patterns, and promote convective heat and mass transfer.

7. Conclusions –

The physical situation being modelled involves understanding how microorganisms interact with nanoparticles under the implications of Prandtl fluid parameter, Forchheimer drag force, density ratio of motile microorganism, and bio-convection Rayleigh number in the nanofluid and how these interactions influence the flow patterns. In these contexts, the major findings of this investigation on bio-convection heat diffusion along a stretching flat surface are summarised as –

- In this study, the behaviour of moving microorganisms that are added in a nanofluid via bio convection has been analysed. Bio-convection is a natural process that occurs as microorganisms move randomly in single-celled or colony-like formation. The directional motion of various forms of microorganisms is the basis for various bio-convection systems.
- This investigation emphasises the significant role played by mobile microorganisms in shaping the behaviour of the system and enhancing mass and heat transfer operations.
- The incorporation of the Prandtl fluid parameter and elastic parameter in this numerical investigation of bio-convection and heat transfer analysis significantly affects the system behaviour. These parameters have notable impacts on momentum, transmission of heat and mass, and microorganism's density.
- The presence of Forchheimer drag increases flow resistance and hinders convective heat transfer, mass transfer efficiency, and the concentration of motile microorganisms.
- The elastic parameter has led to enhance $Re_x^{-\frac{1}{2}}Nu_x$, $Re_x^{-\frac{1}{2}}Sh_x$ and $Re_x^{-\frac{1}{2}}Nn_x$, and reduced $Re_x^{\frac{1}{2}}C_{fx}$. The elasticity in the system improves convective heat transfer, mass transfer efficiency, and promotes a higher concentration of motile microorganisms.

- A higher Nusselt number indicates a more effective convective heat transfer process, which improves heat transmission from the surface to the fluid. The improvement in the Sherwood number shows that the system's mass transport properties have improved.
- The Prandtl fluid parameter may have an impact on this greater density, which might result in more hospitable conditions for the growth and movement of microorganisms.
- Since the mobility of nanoparticles has no effect on the movement of microorganisms, the interface between bio convection and nanofluids arises for microfluid appliances.
- Some of the limitations of magnetic nanoparticles in drug delivery is that they cannot be concentrated into a three-dimensional space, since the application of an external magnetic field organizes the magnetic nanoparticles into a two-dimensional area.
- Tiny microorganisms that float in a fluid's upper layer cause irregular development and instability through a process known as bioconvection. Because they swim so quickly, gyrotactic microorganisms like algae are likely to collect in the fluid's upper layer, creating an unstable peak that leads to heavy density stabilisation.
- Convective transport and microorganism's dynamics play crucial roles in many applications, including biological systems, environmental engineering, and bioreactors, therefore understanding these impacts is important in future projects.

It is expected that a combination of scientific, technological, and interdisciplinary efforts will be necessary to overcome the restrictions in the research of magnetohydrodynamic bioconvection in nanofluids. These restrictions might be overcome in the near future by appropriate computer simulations, nanoparticle engineering, real-world applications, and interdisciplinary research.

Nomenclature –

M	Magnetic Parameter	u, v	Velocity component along x and y directions respectively, (m/s)
Pr	Prandtl Number	x, y	Chosen co-ordinate system
w	Mixed Convection	A, C	Material constants for the Prandtl fluid
F_r	Forchheimer drag	T, C, N	Dimensional Temperature (K), Concentration (mol/ m^3), Microorganisms density (kg/m^3).
K_P	Permeability	$T_\infty, N_\infty, C_\infty$	Dimensional Temperature (K), Concentration (mol/ m^3), Microorganisms density (kg/m^3) at free stream.
N_r	Buoyancy parameter	f, θ, ϕ, χ	Non-dimensional velocity, Temperature, Concentration, Microorganisms density.
Sc	Schimdt number	D_B	mass diffusivity, (m^2s^{-1})

R_b	Bio-convection Rayleigh Number	D_T	co-efficient of mass flux, through temperature gradient, ($Kgm^{-2}s^{-1}$)
$\overline{\vartheta}$	mean swimming velocity vector of the gyrotactic micro-organisms	d	chemotaxis constant
w_C	maximum cell swimming speed	D_N	diffusivity of micro-organisms
dw_c	constant	$C_f - C_0 = \Delta C$	Characteristic nanoparticle volume fraction
N_b	Brownian Parameter	δ	Prandtl fluid parameter
N_t	Thermophoresis parameter	β	Elastic parameter
L_b	Lewis number	γ	Angle of inclination of magnetic field
P_e	Peclet number	α	Angle of inclination of the sheet
Ec	Eckert number	Ω	Density ratio of motile microorganism
S	Suction/ Injection	ε	Stretching parameter
S_1, S_2, S_3	Thermal stratification parameters	η	Similarity variable

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Figure captions –

Figure 1. Flow configuration of the model.

Figures 2(a-d). Distribution of $f'(\eta)$, $\theta(\eta)$, $\phi(\eta)$ and $\chi(\eta)$ **for** Ω and R_b**.**

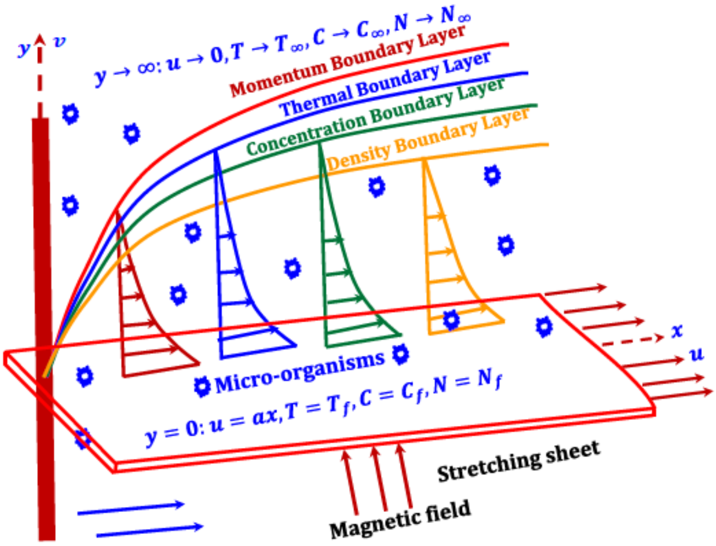
Figures 3 (a-d). Distribution of $f'(\eta)$**,** $\theta(\eta)$**,**$\phi(\eta)$ **and** $\chi(\eta)$ **for** δ **and** F_r**.**

Tables –

Table 1. Velocity distribution for S .

Table 2. Grid Point Stability and Convergence Analysis**.**

Table 3. Influence of some noteworthy parameters on $Re_x^{\frac{1}{2}}C_{fx}$, $Re_x^{-\frac{1}{2}}Nu_x$, $Re_x^{-\frac{1}{2}}Sh_x$ and $Re_x^{-\frac{1}{2}}Nn_x$.



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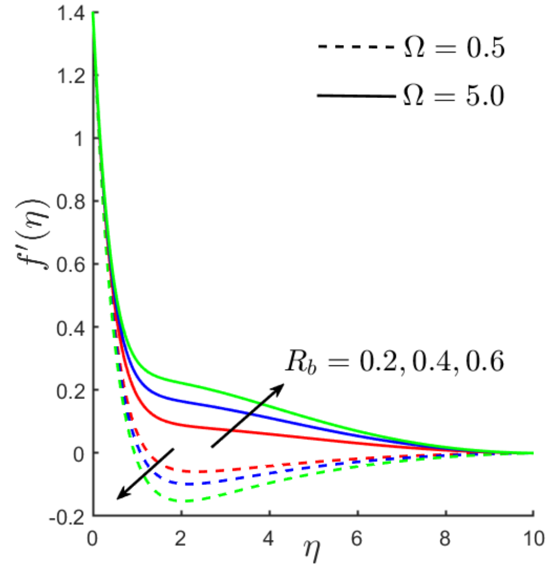


Figure 2(a)

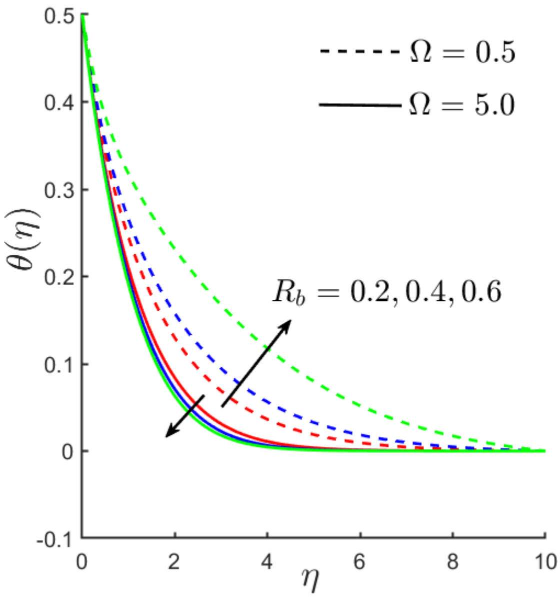


Figure 2(b)

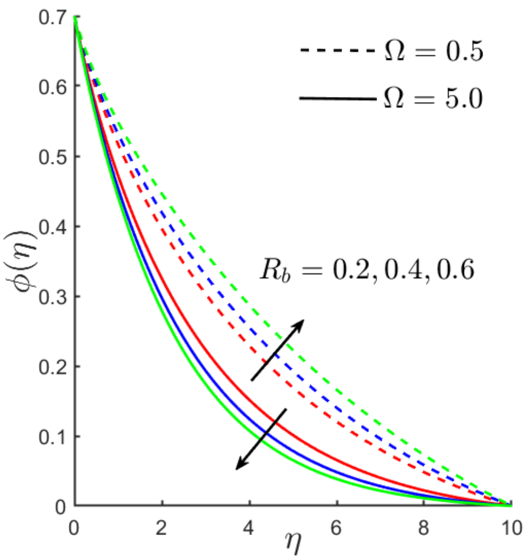


Figure 2(c)

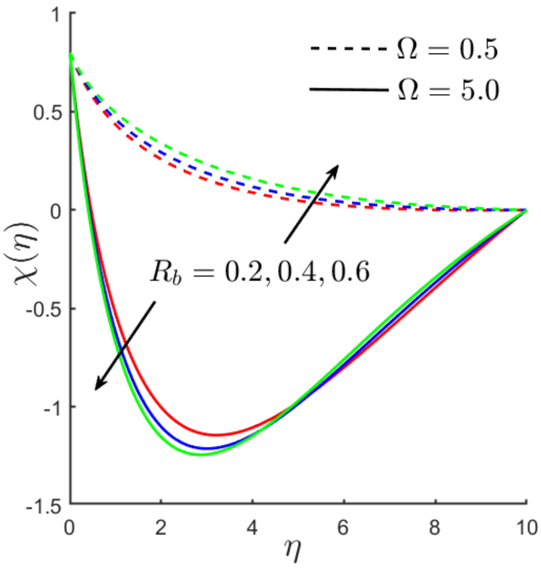


Figure 2(d)

Figures 2(a-d). Distribution of $f'(\eta)$, $\theta(\eta)$, $\phi(\eta)$ and $\chi(\eta)$ **for** Ω and R_b**.**

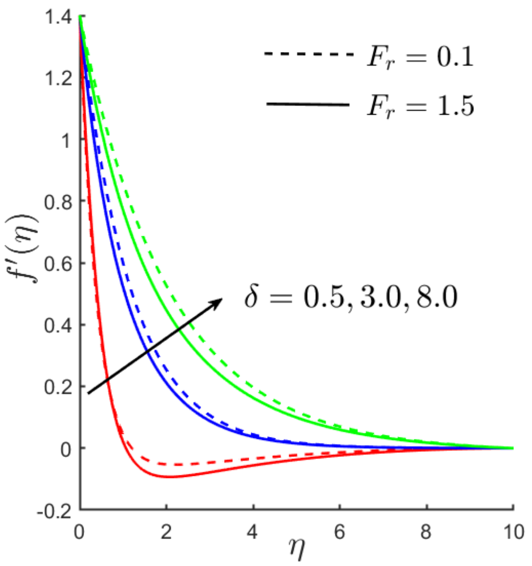
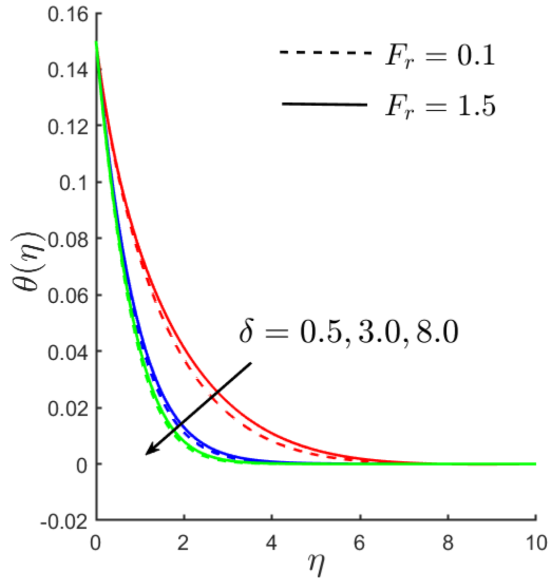


Figure 3(a)



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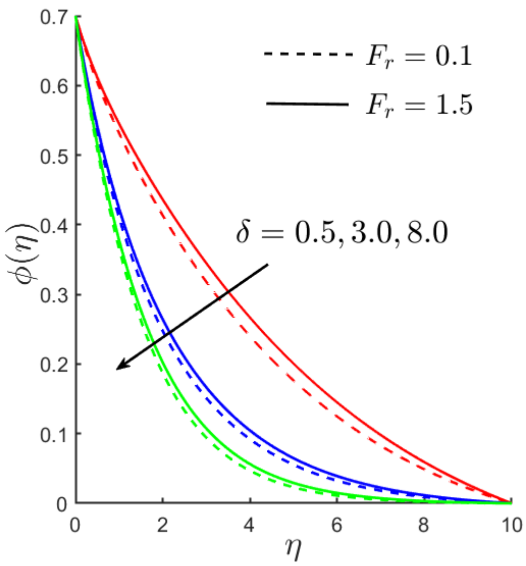

Figure 3(c)

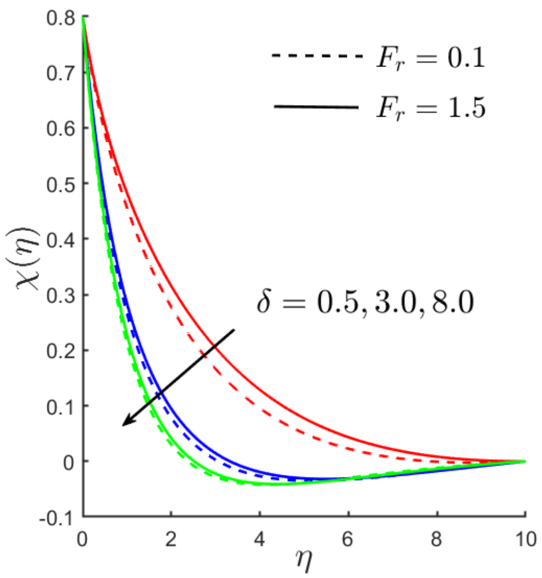

Figure 3 (d)

Figures 3 (a-d). Distribution of $f'(\eta)$, $\theta(\eta)$, $\phi(\eta)$ and $\chi(\eta)$ for δ and F_r.

Table 1. Velocity distribution for S.

S	**Present study**	**Wang et al. [5]**	**Das and Ahmed [13]**
2	3.3143	3.3142	3.3141
3	4.2028	4.2027	4.2026
4	5.3461	5.3460	5.3459

Table 2. Grid Point Stability and Convergence Analysis**.**

$h=0.001$					$h=0.0001$				
η	$f'(\eta)$	$\theta(\eta)$	$\phi(\eta)$	$\chi(\eta)$	η	$f'(\eta)$	$\theta(\eta)$	$\phi(\eta)$	$\chi(\eta)$
0	0.5000	0.8000	0.7000	0.8000	**0**	0.4000	0.2000	0.7000	0.8000
0.4990	0.0714	0.5409	0.4621	0.4004	**0.4999**	0.0291	0.1291	0.4929	0.4530
0.8990	0.0181	0.3651	0.3099	0.2192	**0.8999**	-0.0068	0.0847	0.3457	0.2720
1.4990	0.0029	0.1462	0.1265	0.0680	**1.4999**	-0.0044	0.0328	0.1482	0.0947
1.7990	0.0007	0.0550	0.0487	0.0232	**1.7999**	-0.0013	0.0121	0.0578	0.0336
1.9000	0.0003	0.0268	0.0239	0.0110	**1.9000**	-0.0006	0.0059	0.0286	0.0162
1.9790	0.0001	0.0055	0.0050	0.0022	**1.9799**	-0.0001	0.0012	0.0057	0.0032
1.9830	0.0000	0.0045	0.0040	0.0018	**1.9839**	-0.0001	0.0009	0.0046	0.0025
1.9850	0.0000	0.0039	0.0035	0.0016	**1.9859**	-0.0001	0.0008	0.0040	0.0022
1.9870	0.0000	0.0034	0.0031	0.0014	**1.9879**	-0.0001	0.0007	0.0034	0.0019
1.9880	0.0000	0.0032	0.0028	0.0013	**1.9889**	-0.0001	0.0006	0.0032	0.0017
2.0000	0	0	0	0	**2.0000**	0	0	0	0

Table 3. Influence of some noteworthy parameters on $Re_x^{\frac{1}{2}}C_{fx}$, $Re_x^{-\frac{1}{2}}Nu_x$, $Re_x^{-\frac{1}{2}}Sh_x$ and $Re_x^{-\frac{1}{2}}Nn_x$.

δ	F_r	R_b	Ω	$Re_x^{\frac{1}{2}}C_{fx}$	$Re_x^{-\frac{1}{2}}Nu_x$	$Re_x^{-\frac{1}{2}}Sh_x$	$Re_x^{-\frac{1}{2}}Nn_x$
0.2	0.5	0. 2	0.5	….	….	….	….
0.2				2.3270	0.5377	0.8132	1.9897
0.4				1.9700	0.5437	0.8238	2.0166
0.6				1.5547	0.5479	0.8325	2.0383
	0.5			1.3202	0.0937	0.7793	1.9897
	1.0			1.4050	0.0842	0.7792	1.9896
	1.5			1.5703	0.0709	0.7791	1.9894
		0.2		1.0393	0.0734	0.2265	0.4596
		0.4		1.0230	0.0804	0.2362	0.4804
		0.6		1.0205	0.0924	0.2382	0.5059
			0.5	1.0230	0.1052	0.2253	1.7297
			0.7	1.0159	0.1055	0.2302	1.9093
			0.9	1.0061	0.1059	0.2369	2.0890

Author Contributions
S. Ahmed has contributed to draw the flow diagram and all the figures, also verify the manuscript, while S. Hazarika have explored the result and discussion section and other mathematical part. Both the authors have discussed the results, reviewed and approved the final version of the manuscript.

Conflict of Interest
The authors proclaim no potential conflict of interest with respect to the research, authorship and publication of this article.

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