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Application of a novel quadratic polynomial discrete grey model to forecast energy consumption of China

X. Ma[a,b,*], W. Wu[a,c], and Y. Zhang[a]

a. *School of Science, Southwest University of Science and Technology, 621010, Mianyang, China.*
b. *State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, 610500, Chengdu, China.*
c. *V.C. & V.R. Key Lab of Sichuan Province, Sichuan Normal University, 610068, Chengdu, China.*

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Abstract. The discrete grey modelling technique is a novel methodology of grey prediction models, which is effective to improve the effectiveness and applicability of grey models. In order to build a more general and effective univariate grey prediction model, the discrete grey modelling technique is utilised in this paper to build a Quadratic Polynomial Discrete Grey Model (QPDGM). The properties of the QPDGM have been discussed, which indicate that the new model can be regarded as an extension of the conventional discrete grey model and nonhomogeneous grey model, and it is also coincidence with three classes of exponential sequences. The QPDGM is finally applied to predict the energy consumption of China, including the electric power, crude oil and natural gas consumptions. The results have been compared to some commonly used univariate grey prediction models, which indicates the QPDGM is generally more accurate than other models.

1. Introduction

Energy consumption is an important indicator reflecting the economic level and also related to environment protection. Future energy consumption is always important for the decision makers to adjust the policies and marketing strategy, as a consequence, energy forecasting and related topics have appealed considerable interest of research in recent years. Many methods have been applied in these fields, such as regression models [1], time series analysis methods [2], data-driven schemes [3], computational intelligence technology [4], hybrid prediction system [5], and grey models [6]. Among these methods, the grey models present a different way of modelling, which are aiming at dealing with the systems with known and unknown information.

In the grey system theory, the grey prediction models play an important role which have been widely applied in many fields. The univariate grey prediction models are most popular due to their effectiveness

*. *Corresponding author.*
E-mail address: cauchy7203@gmail.com (X. Ma)

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and applicability in time series forecasting with small samples. The basic GM(1,1) model represents the key idea of the methodology of the grey prediction models, and a lot of new univariate grey prediction models have been developed based on the similar methodology in the recent few years, such as the Nonhomogeneous Grey Model with exogenous term k (NGM$(1,1,k)$) [7], Euler Polynomial-Driven Grey Model (EPDGM) [8], multiple fractional order time-delayed grey model [9], Grey Model with N-order Polynomial term (GMP$(1,1,N)$) [10], and Nonlinear Multivariate Grey Model (NMGM(1, N)) [11], hyperbolic time delayed grey model [12], grey Verhulst model [13], Nonlinear Grey Bernoulli Model (NGBM(1,1)) [14], etc. Such researches have significantly enriched the grey system model family and provide a wide range of choice for decision-oriented research.

But there still exist issues in the methodology of the grey prediction models. One of the most significant issues is believed to be that the structure of the so called background value of the GM(1,1) model is not accurate. Since this issue was pointed out by Tan [15] in 2000, a lot of researchers have made significant efforts to find an appropriate structure of the GM(1,1) model, and finally an accurate structure of the background value was found by Wang et al. [16] in 2008, with which the GM(1,1) can be available to express arbitrary homogeneous exponential sequences precisely. The improved background restructuring method by Wang et al. [16] have been successfully applied to modify the NGM$(1,1,k)$ [17] and grey Verhulst model [18]. Another issue has been pointed out by Kong and Wei [19], which is the inconsistency between the grey differential model and the solution of the GM(2,1) model. Such inconsistency can be modified using a parameter transform, and the revised GM(2,1) can also be coincidence with pure homogeneous exponential sequences. And this idea has also been successfully applied to improve the GM(1,1) [20], NGM$(1,1,k)$ [21], FAGMO$(1,1,k)$ [22]. The similar issues have also been found in the multivariate grey models in recent research [23], and it was proved that the method provided by Kong [19] is also efficient to improve the GMC$(1,n)$ model. But it should be noticed that although these issues can be modified, the correspondence methods are still complex and the revised models are not easy to be analysed.

A novel methodology of grey prediction models have been developed by Xie and Liu in 2009, which is called the Discrete Grey Modelling Technique (DGMT) in this paper. The initial work was presented to build a novel discrete GM(1,1) model (DGM(1,1)) [17], which has proved that the DGM(1,1) is accurate to predict pure homogenous exponential sequences, and this property is the same to the modified GM(1,1) model with restructured background values by Wang et al. [16] and the improved GM(1,1) by Chen et al. [20]. This indicates that the DGMT is also available to modify the issues of the conventional grey prediction models. And it was shown that the implementation of the DGMT was much simpler than background value reconstruction. Due to its effectiveness and applicability for improving the conventional univariate grey prediction models, the DGMT has also been used to build some novel discrete grey prediction models in recent researches, such as the DGM [17], the discrete grey Verhulst model [24], the NIGM [25], etc. In our previous works, the DGMT has been extended to build the discrete GM$(1,n)$ model with difference formulations [26], the DGMT has also been proved to be efficient to build the nonlinear multivariate grey models [27]. All these discrete models have been proved to be more effective in applications than their correspondence conventional grey models and also easy to use.

This study is aiming at building a more general and effective univariate grey prediction model using the DGMT, which is called the Quadratic Polynomial Discrete Grey Model (QPDGM). The QPDGM can be regarded as an extension of the DGM and NDGM, which contains the properties of coincidence with homogeneous and nonhomogeneous exponential sequences of the DGM and NDGM. The applications of predicting the energy consumption of China, including the electric power consumption, Crude Oil Consumption (COC) and natural gas consumption, are carried out to evaluate the performance of the QPDGM in comparison with some commonly used univariate grey prediction models.

The rest of this paper is organized as follows: Section 2 presents the definition and the properties of the conventional DGM and NDGM. Section 3 gives the definition and properties of the QPDGM along with the discussion on the relationship of the QPDGM, the DGM and NDGM. Section 4 shows the numerical results of the applications of China's energy consumption forecasting and conclusions are drawn in Section 5.

2. The existing DGM(1,1) model and NDGM(1,1) model

In this section, we firstly overview some important definitions of the grey modelling method and the existing DGM(1,1) and NDGM(1,1) models [28].

2.1. The DGM(1,1) model and its properties

Definition 1. Set the original sequence to be:

$$X^{(0)} = \left\{ x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n) \right\}. \quad (1)$$

The first-order accumulated generating operation (1-AGO) sequence of the $X^{(0)}$ is defined as:

$$X^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right\}, \tag{2}$$

where $x^{(1)}(k) = \sum_{j=1}^{k} x^{(0)}(j)$.

The difference equation:

$$x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2 \tag{3}$$

is called the discrete grey model, abbreviated as DGM.

The parameters β_1 and β_2 can be obtained using the least squares estimation method as:

$$[\beta_1, \beta_2]^T = (B^T B)^{-1} BY, \tag{4}$$

where:

$$Y = \begin{pmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{pmatrix}, \qquad B = \begin{pmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \vdots \\ x^{(1)}(n-1) & 1 \end{pmatrix}.$$

The recursive function of the DGM is given as:

$$x^{(1)}(k+1) = \beta_1^k x^{(0)}(1) + \frac{1-\beta_1^k}{1-\beta_1}\beta_2. \tag{5}$$

The predicted values $\hat{X}^{(0)}$ is obtained using the first-order Inverse Accumulated Generating Operation (1-IAGO), which is defined as:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1). \tag{6}$$

The most important property of the DGM is its accuracy of predicting the pure exponential sequence, which is also the most significant advantage over the conventional GM(1,1) model, which can be described by the following Thereom 1.

Theorem 1 [17]. *Assume that the original sequence is:*

$$X^{(0)} = \left\{ac, ac^2, ..., ac^n, ac^{n+1}, ac^{n+2}, ...\right\}, \quad c > 0. \tag{7}$$

One uses the first n points to build the DGM, the parameters will be:

$$[\beta_1, \beta_2]^T = [c, ac]^T,$$

and the predicted values will be:

$$\hat{x}^{(0)}(k) = ac^k, \qquad k = 1, 2, \ldots.$$

The Theorem 1 indicates that the DGM can fit the pure homogeneous exponential sequence without bias. This is a very important property that it will not be limited by the grey development coefficient term like the classical GM(1,1) [17].

2.2. The NDGM(1,1) model and its properties

Definition 2. The definitions of original sequence $X^{(0)}$ and its 1-AGO sequence are the same to Eqs. (1) and (2). The difference equation:

$$x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2 k + \beta_3, \tag{8}$$

is called the NDGM.

The parameters β_1, β_2, and β_3 can be obtained using the least squares method as:

$$[\beta_1, \beta_2, \beta_3]^T = (B^T B)^{-1} BY, \tag{9}$$

where:

$$Y = \begin{pmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{pmatrix},$$

$$B = \begin{pmatrix} x^{(1)}(1) & 1 & 1 \\ x^{(1)}(2) & 2 & 1 \\ \vdots & \vdots & \vdots \\ x^{(1)}(n-1) & n-1 & 1 \end{pmatrix}.$$

The recursive function of the NDGM is given as:

$$x^{(1)}(k+1) = \beta_1^k x^{(0)}(1) + \beta_2 \sum_{j=1}^{k} j\beta_1^{k-j} + \frac{1-\beta_1^k}{1-\beta_1}\beta_3. \tag{10}$$

The predicted values $\hat{X}^{(0)}$ of NDGM can also be obtained using the 1-IAGO in Eq. (6).

It can be seen that when $\beta_2 = 0$, the NDGM model (8) yields to the DGM model (3). The following Theorem 2 is a summary of the results by Xie and Liu [17], which depicts an important property of the NDGM.

Theorem 2. *Assume that the original sequence is:*

$$X^{(0)} = \Big\{ac + b, ac^2 + b, \ldots, ac^n + b, ac^{n+1} + b, ac^{n+2} + b, \ldots,\Big\}, \qquad c > 0. \tag{11}$$

One uses the first n points to build the NDGM, the parameters will be:

$$[\beta_1, \beta_2, \beta_3]^T = [c, b(1-c), ac + b]^T,$$

and the predicted values will be:

$$\hat{x}^{(0)}(k) = ac^k + b. \tag{12}$$

The Theorem 2 indicates that the NDGM is accurate to predict the pure nonhomogeneous exponential sequences. And it is obvious that when $b = 0$ the original sequence (11) turns to be a pure homogeneous exponential sequence, and at this time the NDGM is accurate to predict the pure homogeneous exponential sequences too. Obviously, this property makes the NDGM more flexible to deal with more complex sequence.

3. The proposed QPDGM and its properties

In this section, we will introduce a novel as QPDGM.

3.1. Definition of the proposed QPDGM model

Definition 3. The definitions of original sequence $X^{(0)}$ and its 1-AGO sequence are the same to Eqs. (1) and (2). The difference equation:

$$x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2 k^2 + \beta_3 k + \beta_4, \tag{13}$$

is called the QPDGM.

The system parameters β_1, β_2, β_3, and β_4 can be obtained using the least squares estimation method as:

$$[\beta_1, \beta_2, \beta_3, \beta_4]^T = (B^T B)^{-1} BY, \tag{14}$$

where:

$$Y = \begin{pmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{pmatrix},$$

$$B = \begin{pmatrix} x^{(1)}(1) & 1 & 1 & 1 \\ x^{(1)}(2) & 2^2 & 2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x^{(1)}(n-1) & (n-1)^2 & n-1 & 1 \end{pmatrix}.$$

The recursive function of the NDGM is given as:

$$x^{(1)}(k+1) = \beta_1^k x^{(0)}(1) + \sum_{j=1}^{k} [\beta_2 j^2 + \beta_3 j] \beta_1^{k-j} + \frac{1-\beta_1^k}{1-\beta_1}\beta_4. \tag{15}$$

The predicted values $\hat{X}^{(0)}$ of NDGM can also be obtained using the 1-IAGO in Eq. (6).

It can be seen that the QPDGM yields the NDGM when $\beta_2 = 0$, and it yields to DGM when $\beta_2 = \beta_3 = 0$. Thus it is actually a more general model than these two models. And it is also reasonable to imply that the existing models can be taken place by the QPDGM with proper coefficients.

3.2. Some important properties of the QPDGM

In this subsection we will discuss the properties of the QPDGM, which depicts its accuracy with three classes of exponential sequences. And then make a brief discussion on its flexibility.

Theorem 3. *Assume that the original sequence is:*

$$X^{(0)} = \{ac + b + d, ac^2 + 2b + d, \ldots, ac^n + bn + d, ac^{n+1} + b(n+1) + d, ac^{n+2} + b(n+2) + d, \ldots\}, \tag{16}$$

where $a, b, c, d \in R$ and $c > 0$. One uses the first n points to build the QPDGM, the system parameters will be:

$$[\beta_1, \beta_2, \beta_3, \beta_4]^T = \left[c, \frac{b}{2}(1-c), b + (1-c)\left(\frac{b}{2} + d\right), ac + b + d\right]^T, \tag{17}$$

and the predicted values will be:

$$\hat{x}^{(0)}(k) = ac^k + bk + d. \tag{18}$$

Proof 1. Considering the 1-AGO of the given sequence (16), we have:

$$x^{(1)}(k+1) = \sum_{j=1}^{k+1} x^{(0)}(j) = ac\frac{1-c^{c+1}}{1-c} + b\frac{(k+1)(k+2)}{2} + d(k+1) = -\frac{ac^{k+2}}{1-c} + \frac{b}{2}k^2 + \left(\frac{3b}{2} + d\right)k + \left(b + d + \frac{ac}{1-c}\right). \tag{19}$$

Then we substitute the $x^{(1)}(k)(k = 2, 3, \ldots, n)$ into the right side of the QPDGM model (13), we have:

$$\beta_1 x^{(1)}(k) + \beta_2 k^2 + \beta_3 k + \beta_4 = \beta_1 \left(ac\frac{1-c^k}{1-c} + b\frac{k(k+1)}{2} + dk\right) + \beta_2 k^2 + \beta_3 k + \beta_4 = \left(-\frac{ac^{k+1}}{1-c}\right)\beta_1 + \left(\frac{b}{2}\beta_1 + \beta_2\right)k^2 + \left(\frac{b}{2}\beta_1 + \beta_1 d + \beta_3\right)k + \left(\frac{ac}{1-c}\beta_1 + \beta_4\right). \tag{20}$$

By substituting the parameters in Eq. (17) into Eq. (20), we have:

$$x^{(1)}(k+1) = -\frac{ac^{k+2}}{1-c} + \frac{b}{2}k^2 + \left(\frac{3b}{2} + b\right)k + \left(b + d + \frac{ac}{1-c}\right) = \beta_1 x^{(1)}(k) + \beta_2 k^2 + \beta_3 k + \beta_4. \tag{21}$$

Noticing that the Eq. (21) holds for any $k = 2, 3, \ldots$, which means the parameters β_1, β_2, β_3, and β_4 in Eq. (17) are the solution of the following linear system:

$$B[\beta_1, \beta_2, \beta_3, \beta_4]^T = Y, \tag{22}$$

where B and Y are defined in Eq. (14). Thus, there must be:

$$[\beta_1, \beta_2, \beta_3, \beta_4]^T = (B^T B)^{-1} B^T Y. \tag{23}$$

It is obvious that the matrix B has full column rank when $n > 4$, thus the solution of the least squares method (23) is unique. Thus, if we substitute the original sequence $x^{(0)}(k)$ by k from 1 to n to build the QPDGM, the parameters are the ones in Eq. (17).

Then we substitute the parameters β_1, β_2, β_3, and β_4 in Eq. (17) into the recursive function in Eq. (15), we have:

$$\begin{aligned}\hat{x}^{(1)}(k+1) &= c^k[ac+b+d] \\ &+\sum_{j=1}^{k}\left[\frac{b}{2}(1-c)j^2+\left[b+(1-c)\left(\frac{b}{2}+d\right)\right]j\right]c^{k-j} \\ &+\frac{1-c^k}{1-c}[ac+b+d] = -\frac{ac^{k+1}}{1-c}+\frac{b}{2}k^2 \\ &+\left(\frac{3b}{2}+b\right)k+\left(b+d+\frac{ac}{1-c}\right).\end{aligned} \tag{24}$$

Thus the predicted values $\hat{X}^{(0)}$ can be obtained using 1-IAGO (6) as:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) = ac^k + bk + d. \tag{25}$$

Above all, Theorem 3 is proved.

Theorem 3 represents the most important property of QPDGM, which indicates that the QPDGM is unbiased to the sequence satisfying the formulation (16). This formulation contains an exponential term ac^k, a linear term bk and a constant. Comparing to the properties described in Theorems 1 and 2, it is clear to see that the sequences to which the DGM and NDGM are unbiased are specific formulations of the sequence (16) described above. Thus it is clear that the QPDGM is a more general unbiased model than the existing DGM and NDGM. In fact, it is very easy to prove that the QPDGM can also accurately fit and predict the sequences described in Theorems 1 and 2, the details are presented in the following corollaries.

Corollary 1. Assume that the original sequence is:

$$X^{(0)} = \left\{ac, ac^2, \ldots, ac^n, ac^{n+1}, ac^{n+2}, \ldots\right\}, \tag{26}$$

where $a, c \in R$ and $c > 0$. One uses the first n points to build the QPDGM, the system parameters will be:

$$[\beta_1, \beta_2, \beta_3, \beta_4]^T = [c, 0, 0, ac]^T, \tag{27}$$

and the predicted values will be:

$$\hat{x}^{(0)}(k) = ac^k. \tag{28}$$

Proof 2. The sequence (26) is actually a specific formulation of sequence (16) if we set $b = d = 0$.

Then following the results in Theorem 3, the parameters obtained by QPDGM based on the first n points should be:

$$\begin{aligned}[\beta_1, \beta_2, \beta_3, \beta_4]^T = \Bigg[c, \frac{b}{2}(1-c), b+(1-c)\left(\frac{b}{2}+d\right), \\ ac+b+d\Bigg]^T \underline{\underline{b=d=0}}[c, 0, 0, ac]^T.\end{aligned} \tag{29}$$

Substituting the parameters into the response function of QPDGM like it did in Eq. (24), the restored values of QPDGM can be obtained by:

$$\hat{x}^{(0)}(k) = ac^k. \tag{30}$$

Above all, Corollary 1 is proved.

Corollary 2. Assume that the original sequence is:

$$\begin{aligned}X^{(0)} = \Big\{ac+d, ac^2+d, \ldots, ac^n+d, ac^{n+1}+d, \\ ac^{n+2}+d, \ldots\Big\},\end{aligned} \tag{31}$$

where $a, c, d \in R$ and $c > 0$. One uses the first n points to build the QPDGM, the system parameters will be:

$$[\beta_1, \beta_2, \beta_3, \beta_4]^T = [c, 0, (1-c)d, ac+d]^T, \tag{32}$$

and the predicted values will be:

$$\hat{x}^{(0)}(k) = ac^k + d. \tag{33}$$

Proof 3. The sequence (31) is actually a specific formulation of sequence (16) if we set $b = 0$.

Similarly, following the results in Theorem 3, the parameters obtained by QPDGM based on the first n points should be:

$$\begin{aligned}[\beta_1, \beta_2, \beta_3, \beta_4]^T = \Bigg[c, \frac{b}{2}(1-c), b+(1-c)\left(\frac{b}{2}+d\right), \\ ac+b+d\Bigg]^T \underline{\underline{b=0}}[c, 0, (1-c)d, ac+d]^T.\end{aligned} \tag{34}$$

Similarly, we have the restored values as:

$$\hat{x}^{(0)}(k) = ac^k + d. \tag{35}$$

Above all, Corollary 2 is proved.

On the other hand, it should be noticed that the DGM and NDGM can only predict the monotonous sequences according to their properties Theorems 1 and 2. It is clear that, with more flexible structure, the QPDGM is not limited to the monotonous

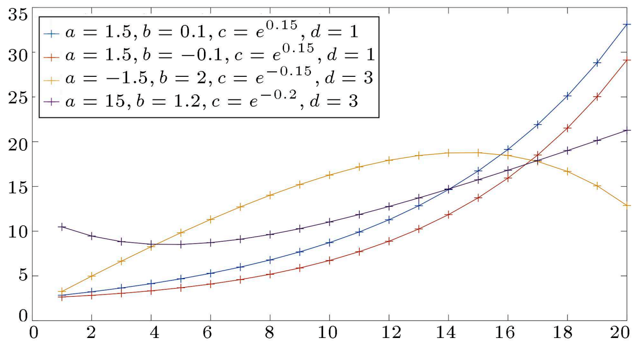

Figure 1. Different curves produced by QPDGM model with different parameters.

sequences. The Figure 1 plots the curves produces by QPDGM, the parameters are the ones defined in Eq. (16). It is clear that if proper parameters are set, the QPDGM can produce more complex curves rather than monotonous sequences. This clearly implies that the QPDGM would can be suitable for more complex data sets, therefor it can be expected to be applied in wider range of variety of fields.

4. Applications

China is now the largest energy consumer in the world, and it consumes 23% world's total primary energy in 2014. But now China is also facing a great challenge with growing scale of energy consumption. Thus it is very important to forecast the future energy consumption accurately in order to make suitable policies and marketing strategies for the decision makers, such as the government and energy companies.

In this section, we use the QPDGM to predict the primary energy consumption of China, including electric power consumption (EPC, 10^9 kilowatt-hour=10^9 kWh), (COC, 10^4 tons of equivalent coal= 10^4 TEC) and natural gas consumption (NGC, 10^9 m^3). The results by QPDGM are compared to the commonly used univariate grey models, including the GM(1,1) [28], DGM [17], NDGM [29], and ONGM [21], in which these grey models are commonly applied in energy consumption forecasting in recent researches. The latest raw data of energy consumption of China from 2000 to 2014 are collected from the official website http://data.stats.gov.cn of National Bureau of Statistics of China. In all the cases the first twelve points are used to build the prediction models, and the rest three points are used for testing.

Three evaluation criteria are used to assess the effectiveness of the prediction models, including the absolute percentage error (ε_k), the Mean Absolute Percentage Error (MAPE) and the maximum absolute percentage error ($\varepsilon_{\max}$), which are defined as:

$$\varepsilon_k = \left| \frac{\hat{x}^{(0)}(k) - x(k)}{x^{(0)}(k)} \right| \times 100(\%),$$

$$k = 2, 3, \ldots, n, \tag{36}$$

$$MAPE = \frac{1}{N-1} \sum_{k=2}^{N} \left| \frac{\hat{x}^{(0)}(k) - x(k)}{x^{(0)}(k)} \right| \times 100(\%), \tag{37}$$

$$\varepsilon_{\max} = \max_{k=1,\ldots,N} \left| \frac{\hat{x}^{(0)}(k) - x(k)}{x^{(0)}(k)} \right| \times 100(\%). \tag{38}$$

4.1. Case 1: Predicting the electric power consumption of China

In this subsection, the case study of electric power consumption of China by forecasted grey models will be presented. The raw data of EPC of China from 2000 to 2014 are listed in the following Table 1.

The predicted values by GM(1,1), DGM, NDGM, ONGM, and QPDGM are listed in Table 2 and Figure 2

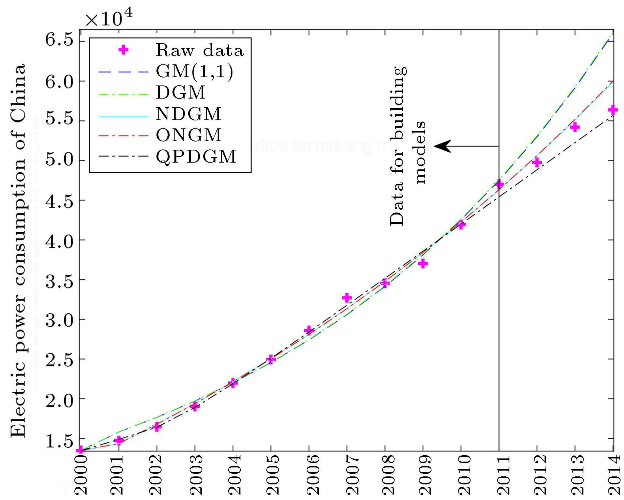

Figure 2. The predicted values of EPC of China by GM(1,1), DGM, NDGM, ONGM, and QPDGM.

Table 1. Raw data of EPC 10^9 kWh of China from 2000 to 2014.

Year	EPC	Year	EPC	Year	EPC
2000	13472.38	2005	24940.32	2010	41934.49
2001	14723.46	2006	28587.97	2011	47000.88
2002	16465.45	2007	32711.81	2012	49762.64
2003	19031.6	2008	34541.35	2013	54203.41
2004	21971.37	2009	37032.14	2014	56383.69

Table 2. Prediction results of EPC of China by GM(1,1), DGM, NDGM, ONGM, and QPDGM.

Year	Raw data	GM(1,1)	DGM	NDGM	ONGM	QPDGM
2000	13472.38	13472.38	13472.38	13472.38	13472.38	13472.38
2001	14723.46	15826.5305	15843.3686	14337.8442	14366.1904	14883.6562
2002	16465.45	17664.2495	17685.0000	16797.2886	16811.0017	16412.6033
2003	19031.60	19715.358	19740.7025	19396.7388	19398.1159	18912.9337
2004	21971.37	22004.6333	22035.3597	22144.1647	22135.8159	21885.9529
2005	24940.32	24559.7311	24596.7475	25047.9901	25032.8669	25088.9893
2006	28587.97	27411.5175	27455.8708	28117.1180	28098.5440	28403.9552
2007	32711.81	30594.4430	30647.3383	31360.9585	31342.6623	31773.3877
2008	34541.35	34146.9581	34209.7816	34789.4572	34775.6081	35169.3243
2009	37032.14	38111.9784	38186.3229	38413.1259	38408.3724	38578.1583
2010	41934.49	42537.4024	42625.0970	42243.0748	42252.5858	41993.2682
2011	47000.88	47476.6905	47579.8336	46291.0467	46320.5559	45411.4322
2012	49762.64	52989.5107	53110.5084	50569.4525	50625.3068	48831.0822
2013	54203.41	59142.4596	59284.0681	55091.4100	55180.6206	52251.4555
2014	56383.69	66009.8663	66175.2418	59870.7836	60001.0815	55672.1806

Table 3. Error values of EPC of China by GM(1,1), DGM, NDGM, ONGM, and QPDGM.

Year	GM(1,1)	DGM	NDGM	ONGM	QPDGM
2000	0.0000	0.0000	0.0000	0.0000	0.0000
2001	7.4919	7.6063	2.6191	2.4265	1.0880
2002	7.2807	7.4067	2.0154	2.0986	0.3210
2003	3.5927	3.7259	1.9186	1.9258	0.6235
2004	0.1514	0.2912	0.7865	0.7485	0.3888
2005	1.5260	1.3776	0.4317	0.3711	0.5961
2006	4.1152	3.9601	1.6470	1.7120	0.6437
2007	6.4728	6.3111	4.1296	4.1855	2.8688
2008	1.1418	0.9599	0.7183	0.6782	1.8180
2009	2.9159	3.1167	3.7292	3.7163	4.1748
2010	1.4377	1.6469	0.7359	0.7586	0.1402
2011	1.0123	1.2318	1.5103	1.4475	3.3817
MAPE	3.3762	3.4213	1.8401	1.8244	**1.4586**
$\varepsilon_{\max}$	7.4919	7.6063	**4.1296**	4.1855	4.1748
2012	6.4845	6.7277	1.6213	1.7336	1.8720
2013	9.1121	9.3733	1.6383	1.8029	3.6012
2014	17.0726	17.3659	6.1846	6.4157	1.2619
MAPE	10.8897	11.1556	3.1481	3.3174	**2.2450**
$\varepsilon_{\max}$	17.0726	17.3659	6.1846	6.4157	**3.6012**

along with the MAPE and $\varepsilon_{\max}$, which are also listed in Table 3.

The numerics of minimum MAPE and $\varepsilon_{\max}$ are all presented in bold font in Table 3. It is clearly to see that the QPDGM has the minimum MAPE for fitting and prediction, which indicates it has the best overall accuracy in this case study. The NDGM has the minimum $\varepsilon_{\max}$ for fitting, which is only 0.0452% smaller than that of QPDGM. Meanwhile the QPDGM still has the minimum $\varepsilon_{\max}$ for prediction, which is much smaller than the other four models, which indicates the QPDGM has the best robustness. In summary, the QPDGM performs best in this case.

4.2. Case 2: Predicting the COC of China

In this subsection, the China's COC will be forecasted by the above five grey models. The raw data of COC of China from 2000 to 2014 and the predicted values by GM(1,1), DGM, NDGM, ONGM, and QPDGM are listed in Table 4 which are also plotted in Figure 3. Meanwhile, the MAPE and the $\varepsilon_{\max}$ are listed in Table 5.

It can be seen in Table 5 that the MAPE from 2000 to 2011 of the GM(1,1) model is 3.6871%, that of the DGM is 3.6918%, that of NDGM is 2.3189%, that of ONGM is 2.4028% and that of QPDGM is 2.7418%. Meanwhile, the MAPE from 2012 to 2014 of the GM(1,1) model is 4.8842%, that of the DGM is 4.9432%, that of NDGM is 2.8825%, that of ONGM is 2.3114% and that of QPDGM is 2.2728%. We can observe that the ONGM has the minimum MAPE for fitting, which is only 0.3390% smaller than that of QPDGM, and the QPDGM has the minimum MAPE for prediction and the minimum $\varepsilon_{\max}$ for fitting and prediction. Thus the QPDGM performs best in this case study as it has the best overall accuracy and robustness.

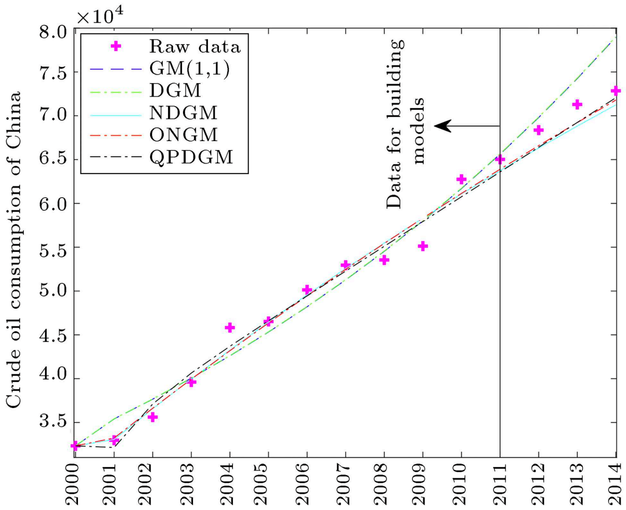

Figure 3. The predicted values of COC of China by GM(1,1), DGM, NDGM, ONGM, and QPDGM.

4.3. Case 3: Predicting the natural gas consumption of China

The raw data of NGC of China from 2000 to 2014 and the predicted values by GM(1,1), DGM, NDGM, ONGM, and QPDGM are listed in Table 6, which are also plotted in Figure 4. Meanwhile, the MAPE and $\varepsilon_{\max}$ are shown in Table 7.

As shown in Table 7, the fitting MAPE and the prediction MAPE of the GM(1,1) are 3.0374% and 5.8277%, those of the DGM are 3.1451% and 6.5468%, those of the NDGM are 3.1668% and 7.4146, those of the ONGM are 3.1563% and 7.6893, those of the QPDGM are 2.1131% and 1.7141%, respectively. It is clearly seen that the QPDGM has the minimum MAPE and $\varepsilon_{\max}$ for fitting and prediction, which indicates it outperforms the other four prediction models in this case study.

Table 4. Prediction results of COC of China by GM(1,1), DGM, NDGM, ONGM, and QPDGM.

Year	Raw data	GM(1,1)	DGM	NDGM	ONGM	QPDGM
2000	32332.08	32332.0800	32332.0800	32332.0800	32332.0800	32332.0800
2001	32975.96	35408.0474	35419.6379	33061.6908	33228.8121	32144.2832
2002	35611.17	37661.6128	37674.6790	36566.1477	36625.9026	37110.8163
2003	39613.68	40058.6076	40073.2903	39965.4017	39945.0585	40639.2110
2004	45825.92	42608.1606	42624.6127	43262.6110	43188.0677	43695.3724
2005	46523.68	45319.9813	45338.3685	46460.8389	46356.6770	46596.4692
2006	50131.73	48204.3974	48224.8994	49563.0569	49452.5934	49446.6483
2007	52945.14	51272.3938	51295.2054	52572.1471	52477.4846	52280.1079
2008	53542.04	54535.6546	54560.9867	55490.9052	55432.9799	55108.0774
2009	55124.66	58006.6074	58034.6886	58322.0429	58320.6714	57934.2442
2010	62752.75	61698.4711	61729.5486	61068.1907	61142.1147	60759.8189
2011	65023.22	65625.3055	65659.6470	63731.8998	63898.8296	63585.1993
2012	68363.46	69802.0655	69839.9607	66315.6450	66592.3010	66410.5159
2013	71292.12	74244.6579	74286.4199	68821.8269	69223.9799	69235.8115
2014	72846.00	78970.0015	79015.9692	71252.7738	71795.2839	72061.1002

Table 5. Error values of COC of China by GM(1,1), DGM, NDGM, ONGM, and QPDGM.

Year	GM(1,1)	DGM	NDGM	ONGM	QPDGM
2000	0.0000	0.0000	0.0000	0.0000	0.0000
2001	7.3753	7.4105	0.2600	0.7668	2.5221
2002	5.7579	5.7946	2.6817	2.8495	4.2112
2003	1.1232	1.1602	0.8879	0.8365	2.5888
2004	7.0217	6.9858	5.5936	5.7562	4.6492
2005	2.5873	2.5478	0.1351	0.3590	0.1565
2006	3.8445	3.8036	1.1344	1.3547	1.3666
2007	3.1594	3.1163	0.7045	0.8833	1.2561
2008	1.8558	1.9031	3.6399	3.5317	2.9249
2009	5.2281	5.2790	5.8003	5.7978	5.0968
2010	1.6801	1.6305	2.6844	2.5666	3.1758
2011	0.9260	0.9788	1.9859	1.7292	2.2115
MAPE	3.6872	3.6918	**2.3189**	2.4028	2.7418
$\varepsilon_{\max}$	7.3753	7.4105	5.8003	5.7562	**5.0968**
2012	2.1043	2.1598	2.9955	2.5908	2.8567
2013	4.1415	4.2000	3.4650	2.9009	2.8843
2014	8.4068	8.4699	2.1871	1.4424	1.0775
MAPE	4.8842	4.9432	2.8825	2.3114	**2.2728**
$\varepsilon_{\max}$	8.4068	8.4699	3.4650	2.9009	**2.8843**

Table 6. Prediction results of NGC of China by GM(1,1), DGM, NDGM, ONGM, and QPDGM.

Year	Raw data	GM(1,1)	DGM	NDGM	ONGM	QPDGM
2000	245.03	245.0300	245.0300	245.0300	245.0300	245.0300
2001	274.30	249.6623	250.2644	253.9693	254.8541	269.6823
2002	291.84	293.8823	294.6979	297.3991	297.9783	298.0926
2003	339.08	345.9345	347.0205	348.7467	349.0296	341.1392
2004	396.72	407.2061	408.6327	409.4556	409.4651	399.3272
2005	467.63	479.3301	481.1839	481.2325	481.0100	473.1790
2006	561.41	564.2287	566.6163	566.0952	565.7062	563.2352
2007	705.23	664.1644	667.2170	666.4293	665.9713	670.0549
2008	812.94	781.8006	785.6789	785.0555	784.6672	794.2165
2009	895.20	920.2725	925.1733	925.3088	925.1818	936.3185
2010	1069.41	1083.2704	1089.4344	1091.1319	1091.5259	1096.9798
2011	1305.30	1275.1383	1282.8594	1287.1865	1288.4474	1276.8410
2012	1463.00	1500.9898	1510.6264	1518.9842	1521.5671	1476.5644
2013	1705.37	1766.8438	1778.8325	1793.0414	1797.5389	1696.8355
2014	1868.94	2079.7858	2094.6577	2117.0626	2124.2400	1938.3633

5. Conclusions

A novel univariate grey prediction model, the Quadratic Polynominal Discrete Grey Model (QPDGM), has been proposed in this paper. The QPDGM is an extension of the conventional DGM and NDGM, as it can be accurate to predict the homogeneous and nonhomogeneous exponential sequences. The QPDGM model is actually coincidence with the sequences in the form of Eq. (18), which is a more general exponential form, and thus the QPDGM is more flexible than the existing DGM and NDGM.

Table 7. Error values of NGC of China by GM(1,1), DGM, NDGM, ONGM, and QPDGM.

Year	GM(1,1)	DGM	NDGM	ONGM	QPDGM
2000	0.0000	0.0000	0.0000	0.0000	0.0000
2001	8.9820	8.7625	7.4119	7.0893	1.6834
2002	0.6998	0.9793	1.9048	2.1033	2.1425
2003	2.0215	2.3418	2.8509	2.9343	0.6073
2004	2.6432	3.0028	3.2102	3.2126	0.6572
2005	2.5020	2.8984	2.9088	2.8612	1.1866
2006	0.5021	0.9274	0.8345	0.7652	0.3251
2007	5.8230	5.3902	5.5019	5.5668	4.9878
2008	3.8305	3.3534	3.4301	3.4778	2.3032
2009	2.8008	3.3482	3.3634	3.3492	4.5932
2010	1.2961	1.8725	2.0312	2.0680	2.5780
2011	2.3107	1.7192	1.3877	1.2911	2.1803
MAPE	3.0374	3.1451	3.1668	3.1563	**2.1131**
$\varepsilon_{\max}$	8.9820	8.7625	7.4119	7.0893	**4.9878**
2012	2.5967	3.2554	3.8267	4.0032	0.9272
2013	3.6047	4.3077	5.1409	5.4046	0.5004
2014	11.2816	12.0773	13.2761	13.6602	3.7146
MAPE	5.8277	6.5468	7.4146	7.6893	**1.7141**
$\varepsilon_{\max}$	11.2816	12.0773	13.2761	13.6602	**3.7146**

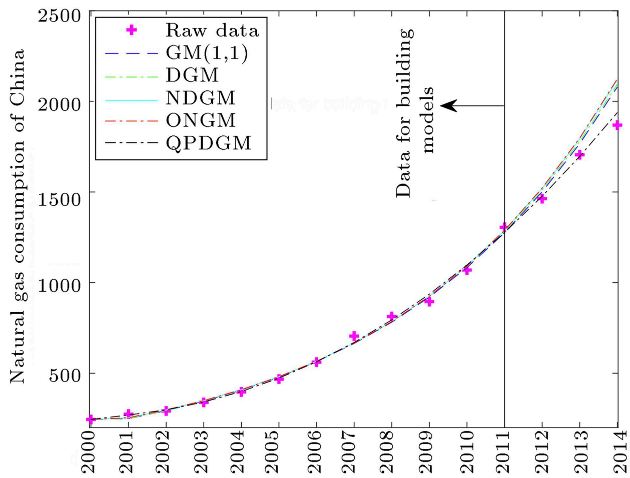

Figure 4. The predicted values of NGC of China by GM(1,1), DGM, NDGM, ONGM, and QPDGM.

The results of the applications of energy consumption forecasts of China indicate that the QPDGM can be more accurate than the GM(1,1), DGM, NDGM, ONGM, which indicates that QPDGM has a high potential in energy consumption forecasts due to its higher flexibility. In summary, the QPDGM is a general and effective univariate grey prediction model, which can be expected to used in wider range of fields in the future studies.

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Biographies

Xin Ma received the PhD degree in Petroleum Engineering from Southwest Petroleum University, Chengdu, China, in 2016. His current research interests include grey forecasting model and petroleum engineering computing technology.

Wenqing Wu obtained both his MSc and DSc degrees in Applied Mathematics from Sichuan Normal University, Chengdu, China, in 2012 and 2015, respectively. His current research interests include grey forecasting model and reliability analysis.

Yuanyuan Zhang obtained her MSc degree in basic mathematics from Sichuan Normal University, Chengdu, China, in 2015. Her major research interest is in forecasting theory, partial differential equation.