Comprehensive Process of Multiportfolio Selection and Ordering

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Abstract

Portfolio optimization studies have traditionally assumed that portfolio managers manage only one portfolio. However, in reality, managers often manage multiple portfolios that can impact each other. This creates a need for fairness to all customers, which has led to the emergence of a new topic called "multiportfolio optimization". Previous studies have paid little attention to this issue, and the models used were not developed using real stock market data. These models were also limited to the selection phase and did not consider the ordering phase.

This research provides a comprehensive process for addressing the multiportfolio problem, covering all sections from selection to ordering. It also implements the process using real stock market data. During this process, the market impact function is estimated using the I-STAR model for different stocks. The proposed model for market impact costs includes both permanent and temporary sections. The proposed models were tested using the Tehran Stock Exchange data in 2019.

A comparison of the MPO model output with classical models indicates that the proposed model improves utility by an average of 15%. In the next phase, comparing the proposed ordering model with other models shows a reduction in market impact costs by an average of 26%.

Keywords: Multiportfolio selection, optimization, Market impact, Fair allocation, Optimal execution

1. Introduction

To date, regarding the financial computations and stock selection for creating a portfolio, the existing investments are taken into account in terms of risk degree and return rate so that the investor can create his desirable portfolio according to his financial resources and other policies.

However, most previous studies on portfolio optimization assume that the portfolio manager (PM) manages only one account. In reality, PMs often manage multiple accounts simultaneously. optimizing each account independently and isolated from the others means disregarding the market dynamics and correlation between decisions on one account and the outcome and performance of other accounts [1].

One of the most significant advancements in modern portfolio optimization theory is the focus on maximizing the interests of multiple accounts under a single management. Joint portfolio optimization, which is known as a multiportfolio optimization (MPO), considers the effectiveness and mutual dependency of the accounts and imports the optimal portfolio determination problem from an independent optimization space to a multi-component space. In traditional models, it was assumed that a PM would implement an optimization model for each account independently. However, MPO models take into account the effectiveness of decisions and market realities.

Although MPO models attempt to capture the reality of mutual dependency, a crucial aspect of these models is often ignored - modeling the transaction costs resulting from the effect of a trade on the market. Market impact cost (MI), a specific type of transaction cost, is defined as the change in an asset's market price due to a trade in that asset. MI can impact the return of all portfolios under management that hold that asset as part of their portfolio (in this study, the "assets" included in the portfolio are stocks).

Recent studies have highlighted the problem of MPO, but there are still weaknesses, particularly the lack of a suitable model for the MI function that can accurately estimate this cost while considering the characteristics of different stocks. All previous studies have considered the MI function for all stocks to be the same as a linear or quadratic function of the trading volume. Since the different stock MI functions are considered to be the same, no real data is used and, consequently, the financial market conditions under study are not included in the study results. So, the results are not reliable enough for policy-making and evaluation.

This study estimates the MI function using the I-STAR model proposed by Kissel in 2013 for each stock and has entered the MPO model. The problem of MI arises from the study of market microstructure. So, it is very important to pay attention to the depth of the market. The I-STAR model considers not only trading volume but also the depth of orders and risk (variance) to bring the model closer to reality.

The proposed model is based on a framework developed by Iancu and Trichakis in 2014 and has been modified to solve the MPO problem. For estimating the MI functions and solving the MPO model, the real data of Tehran Stock Exchange (TSE) in 2019 has been used and then the proposed model has been evaluated and validated.

This article provides a comprehensive process that includes optimal ordering in addition to portfolio optimization. The process encompasses all stages from selection to ordering and implementation using real stock market data. Since market impact costs are important factors in transaction execution, the model is designed based on minimizing these costs. This study considers the total market impact, including permanent and temporary impacts, in the optimal ordering model. For this purpose, these impacts and parameters must first be estimated using real data. It is worth noting that other studies consider the parameter of temporary impact as a range of different values or scenarios, leading to serious ambiguity in the results. Finally, the results of the model implementation are compared with other widely-used ordering models (one-time ordering and splitting the order equally) to evaluate the effectiveness of the proposed model.

The results of the present study and the proposed integrated decision-making process from selection through execution can be helpful to financial institutions such as portfolio management companies in optimizing their customers' portfolios. The efficiency of the transaction strategy increases profitability, while its inefficiency brings about costs.

2. Literature Review

This section includes studies on the MI models and ordering strategies, in addition to researches in the field of portfolio optimization and MPO. Finally, an evaluation and conclusion to seek out research gap has been provided.

2.1. Portfolio Optimization

Harry Markowitz [2] proposed a basic portfolio optimization model, which has become the foundation of modern portfolio theory. While the basic Markowitz model is recognized as the starting point of portfolio optimization modeling, it lacks in some respects.

Moon and Yao [3], using a robust optimization approach, solved the problem with the mean absolute deviation (MAD) risk measure, while Huang [4] and Guastaroba [5] solved it with a conditional value at risk (CVaR) measure.

Many researchers, including Xidonas [6], Hadavandi [7], Yunusoglu [8], Kamley [9], Dymova [10], and Amin Naseri [11], have employed rule-based expert systems for optimal portfolio selection. These systems attempt to select portfolios by considering the degree of risk tolerance of the investors and employing both fundamental and technical indices.

Bermudez [12] proposed a genetic algorithm that was constrained by a maximum number of stocks included in the portfolio. He modeled the uncertainty of the return rates as a trapezoidal fuzzy number and applied low risks for the decision-maker's risk aversion criterion. Chen [13], by considering the return rates of the problem as fuzzy, solved the problem using the ABC algorithm. Rostami [14] applied the entropy criterion, which is not dependent on the assets' return distribution symmetry, in contrast to the variance, as the risk measure to optimize the fuzzy portfolio.

Sun [15], Davari Ardakani [16] and Liu [17] have addressed the problem from a different perspective. They addressed the problem as a multi-period one. Furthermore, Mehlawat [18] has solved the problem as a multi-period multi-objective fuzzy problem and Lezmi et al. [19] added several formulations of the objective function, constraints and coupling relationships to multi-period portfolio optimization problem.

Kaucic et al. [20] introduced a strategy for portfolio selection. They used semi-variance, conditional value-at-risk, and a combination of both as the risk criteria for loss-averse investors.

Yeh and Liu [21] considered the challenges of a weight-scoring approach in stock selection models. Their study employed a mixture of experimental designs to collect the weights of stock-picking concepts and portfolio performance data to predict portfolio performance.

Rahiminezhad et al. [22] developed a method for applying multiple criteria to evaluate and select portfolios. The FANP approach was used to rank portfolios in consideration of uncertain conditions and decision-makers' judgments. Ghahtarania et al. [23] developed a mathematical model transformed into an integer linear programming. The novelty of the research is risk criteria which is measured based on the difference between fundamental value and the market value of stocks.

Memarpour et al. [24] created the optimal portfolios of two players in the banking system in two-level game, based on the Markowitz model. Optimal investment portfolios of the players were first determined using GAMS and genetic algorithm. Next, the problem was solved again using the meta-heuristic algorithms of PSO and IWO.

2.2. Market Impact Models and ordering Strategies

There are variety of studies focusing on transaction costs in the market microstructure literature. Some include MI cost models. For example, the primary theoretic models introduced by Kyle and Hasbrouck [25] focused on microstructure models that describe the MI of asymmetric information. Patzelt and Bouchaud [26] investigated whether the basic MI functions can explain the concavity and nonlinearity of the MI.

Kissell et al. [27] introduced the I Star model, an approach which is a top-to-bottom allocation of the costs. The I-Star function includes liquidity, fluctuations, imbalances, and in-day transactions.

Lin et al. [28] in their research solved the problem of trading by controlling the cost of transactions and in situations where the transaction must be completed within a certain time.

Rastegar and Eghbalreihani [29] aimed at offering an order splitting strategy to divide a large order into a number of smaller orders to reduce Market Impact cost and imbalances created by Large orders in the market.

Yamada and Mizuno [30] analyzed the Tokyo Stock Exchange and confirmed through a simple statistical test that the market impact depends on each stock.

Emilio Said [31] proposed a theory of the market impact of metaorders based on a coarse-grained approach where the microscopic details of supply and demand is replaced by a single parameter $\rho \in [0,+\infty]$ shaping the supply-demand equilibrium and the market impact process during the execution of the metaorder.

2.3. Multiportfolio Optimization

The MPO problem was first proposed by O'Cinneide et al. [32]. In their model, the cost of MI is considered as a linear function of trading volume and no method is provided for its division, but they argue that the introduced multiple optimization is able to solve the problem of joint trading and also fairness is observed;

Stubbs and Vandenbussche [33], Savelsbergh [34], and Yang et al. [35] conducted comprehensive investigations of the issues surrounding the MPO techniques. They discussed the advantages and disadvantages of the Cournot-Nash equilibrium economic approach and the collusive solution and, thereby, presented an integrated framework that could solve the problem using both methods.

Iancu and Trichakis [1] proved that the Cournot-Nash equilibrium method not only isn't suitable for the establishment of fairness but, also, it doesn't necessarily yield the optimal solution, because the accounts participate in a fake game in which the Securities and Exchange Commission's rules are violated and, thus, the obtained results cannot be reliable. In their model, the cost of market impact is not exogenous as in previous studies and is considered as a variable in the model. Yang et al. [35] also developed the Nash equilibrium problem and modeled the generalized Nash equilibrium.

Jing Fu [36] proposed an information pooling game for MPO with a key distinction of allowing the clients to decide whether and to what extent their private trading information is shared with others, which directly affects the MI cost split ratio.

Zhang et al. [37] presented a 5-step model for the MPO problem, which in the steps there is linear, nonlinear programming and multi-objective optimization models. They considered max-min objective function with both variance and Conditional Value at Risk (CvaR) risk measures. MI function for all the stocks in this model is the same and in quadratic form of trading volume.

Yu et al. [38] developed a target-oriented framework that optimizes the rebalancing trades and the MI costs incurred by trading jointly with consideration of target and distributional uncertainty.

Lampariello et al. [39] analyzed a Nash equilibrium problem arising when trades from different accounts are pooled for execution. They state conditions for the monotonicity of the underlying Nash equilibrium problem.

Tamoor Khan et al. [40] proposed a variant of Beetle Antennae Search (BAS) known as Distributed Beetle Antennae Search (DBAS) to solve MPO problems without violating the privacy of individual portfolios. DBAS is a swarm-based optimization algorithm that solely shares the gradients of portfolios among the swarm without sharing private data or portfolio stock information.

Khalil Moghadam et al. [41] solved the MPO problem for two accounts and four stocks using the max-min model. The model of market impact implemented in the present study was based on the Istar model (with some modifications). The study went as far as the stock selection step, and the ordering step was not considered.

2.3. Sum-up of literature review

Developments in the field of MPO can be classified as follows:

- Diversity in solution methods of the optimization problem.
- Development of models that take the market microstructure into consideration.
- Diversity in risk and return estimation indices.
- Diversity in modeling constraints or other indices that can be determined proportionate to the behavior of investors and markets.

A review of the literature in MPO indicates a serious gap in modelling the MI as a key factor in the MPO problem. Accordingly, the MI is generally based on simplifying assumptions and is considered similar for all stocks in the form of a linear or quadratic function of the trading volume. Moreover, assuming the same MI functions eliminates the possibility of examining the efficiency of the MPO model in real markets. This study aims to provide a more practical MPO model by estimating MI costs with the I-STAR model. Another difference between current article and previous ones is the comprehensive view of decision-making in the field of portfolio management. MI costs are important in all phases of MPO problem and should be given special attention. So, this research attempts to conduct transactions in such a way that MI costs are minimized and, consequently, the utility is increased.

The general framework of the research, which is presented in the continuation of the article, is illustrated in Figure 1.

3. Modeling

In this section, a MPO model with 4-step optimization schemes, based Iancu and Trichakis [1], will be introduced; in the proposed model, market impact function is I-Star model (The model is proposed by Robert Kissell [27]. Moreover, optimal ordering model is presented in the end of the section.

3.1. *Market Impact Model*

I-Star model is a cost allocation approach where participants incur costs based on the size of their order and the overall participation with market volumes. The idea behind the model follows from economic supply-demand equilibrium starting at the total cost level. The model is broken down into two components: Instantaneous Impact denoted as I-Star or I* and Total Market Impact which denoted as MI which represents impact cost due to the specified trading strategy. This impact function is broken down into a temporary and permanent terms.

The models used in this study are as follows:

$$I^{*} = (Q / ADV)^{y} \sigma^{(a_1)} \tag{1}$$

$$MI = b.Istar.POV^{(a_2)} + (1-b).Istar \tag{2}$$

Q= imbalance is defined as the difference between volume of buy-initiated and sell-initiated trades in the trading interval (Based on the Lee & Ready tick rule).

$I^* =$ The difference between the execution price and midpoint of the bid-ask spread when the order was released to the market.

ADV = The average daily volume of the trades in T days (during transactional hours).

σ (Annualized volatility) = The standard deviation of the close-to-close logarithmic price change scaled for a full year using a factor of 240 days (number of working days in the whole year).

POV = percentage of volume trading rate

b= temporary impact parameter

θ, y= model parameters (via non-linear regression analysis).

According to a research (Kiesel Research Group 2015) parameters "a_1" and "a_2" in the stock market of all countries are numbers very close to 1. So in this research are assumed to be 1 and therefore, the relationship between MI and POV can be considered linear.

3.2. *Multiportfolio Optimization Model*

To construct the MPO model, the following assumptions and symbols are considered:

- The problem is considered and executed in a single-period framework.
- Short selling is not possible.

k: index of portfolio or user account, k=1,2, …, m

i.j: index of stocks

n: number of stocks

x_{ki}: a volume of the i^{th} stock, which is selected for the k^{th} account

X_{ki}: vector of the volume of the selected stocks (x_{ki})

$C_{k:}$ capital of the k^{th} user account

$\bar{r}_i$: expected return of the i^{th} stock

$\bar{r}_{p_k}$: expected return of the k^{th} individual's portfolio

R: vector of the expected return

p_i: price of the i^th stock

σ_i: annualized volatility of the i^th stock

ρ_k: lowest risk level for k^th account that its value is the result of solving the first step

k_k: risk-aversion coefficient of the k^th account (individual)

θ_i: MI coefficient

y_i: MI parameter

ADV_i: average daily volume of the i^th stock

u_k: utility of the k^th account

$f(U_1, U_2, \ldots, U_m)$: welfare function

A. Market Impact Cost

According to the function of market impact that has been introduced in section 3.1, the market impact cost of the whole volume of the i^th stock, which has been purchased by all portfolios, is equal to:

$$t_i\left(\sum_{k=1}^{m} x_{ki}\right) = \theta_i \sigma_i \left(\sum_{k=1}^{m} x_{ki} / ADV_i\right)^{y_j} \tag{3}$$

So the total MI cost of all the stocks is obtained from eq.4:

$$t_T = \sum_{i=1}^{n} \theta_i \sigma_i \left(\sum_{k=1}^{m} x_{ki} / ADV_i\right)^{y_j} \tag{4}$$

B. Method of Cost Division Among the Accounts

In the proposed model, the total MI cost is divided among all the accounts using the pro-rata method. Thus, the total MI cost imposed on the k^th account is:

$$t_k = \sum_{i=1}^{n} \left(x_{ki} / \sum_{a=1}^{m} x_{ai}\right) \theta_i \sigma_i \left(\sum_{a=1}^{m} x_{ai} / ADV_i\right)^{y_j} \tag{5}$$

C. Utility Functions

The most common description for the quantification of utility is to consider the return. Thus, utility of the k^th account is:

$$u_k = R^T X_k \tag{6}$$

The net expected utility (U_k) for the k^th account is equal to the total expected return (u_k) for the k^th account that an amount of it is deducted as MI cost.

$$U_k = u_k - t_k \tag{7}$$

D. The Final Model

The MPO model used in this study has been designed in four steps, at the end, the results of each step are compared. In this model, the variance is used as the risk measure.

Step-1: The portfolio optimization problem is solved for each account independently, and the objective function is the variance of the account that is minimized.

$$minZ_1 = \sum_{i=1}^{n}\sum_{j=1}^{n}((x_{ki}p_i)/C_k)((x_{kj}p_j)/C_k)cov(\bar{r}_i,\bar{r}_j) \qquad \forall k \tag{8}$$

$s.t$:

$$\sum_{i=1}^{n} x_{ki}p_i\bar{r}_i \geq \bar{r}_{pk}C_k \qquad \forall k$$

$$\sum_{i=1}^{n} x_{ki}p_i \leq C_k$$

$$x_{ki} \geq 0$$

The first step is the same as the classic Markowitz model, based on which the minimization of the account's variance considering three constraints, including the minimum expected return, available capital, and the minimum value of the variable. At this step MI costs are ignored and each account is optimized independently.

Step-2: At this step, again, the accounts are optimized independently, and the utility of each account is maximized individually.

$$maxZ_2 = \sum_{i=1}^{n}\bar{r}_i((x_{ki}p_i)/C_k) - \sum_{i=1}^{n}\theta_i\sigma_i(x_{ki}/ADV_i)^{y_j} \qquad \forall k \tag{9}$$

s.t:

$$((x_{ki}p_i)/C_k)((x_{kj}p_j)/C_k)cov(\bar{r}_i,\bar{r}_j) \le k_k \rho_k \qquad \forall k$$

$$\sum_{i=1}^{n} x_{ki}p_i \le C_k$$

$$x_{ki} \ge 0$$

The model is executed based on three constraints, including the maximum portfolio risk, available capital and minimum value of the variable. At this step, the MI cost is estimated, but it is assumed that the transactions of different accounts are independent of each other, and the MI cost of each account is considered in the utility of that account.

The implementation output of the model at this step is the optimal vector x_i, which is represented by X_{ki}^{IND} due to the independence of the accounts.

Step-3. At this step, despite the independent optimization of the accounts, the effect of the transactions of different accounts on each other is taken into account. To do this, the total MI (eq.4) is distributed among all the accounts using the pro-rata method (eq.5), and the net utility calculated at the previous step becomes more real due to the correction of the costs. The net utility for the k[th] account is calculated using eq. (10):

$$U_k^{IND} = \sum_{i=1}^{n} \bar{r}_i((X_{ki}^{IND}p_i)/C_k) - \sum_{i=1}^{n}(X_{ki}^{IND}/\sum_{a=1}^{m}X_{ai}^{IND})\theta_i\sigma_i(\sum_{a=1}^{m}X_{ai}^{IND}/ADV_i)^{y_i} \qquad \forall k \qquad (10)$$

Step-4. At this step, MPO is carried out through integrating the MI costs that have been modified at the third step. This step is aimed at the jointly optimization of the accounts and at the same time split the MI cost across all the accounts. A max-min function (objective function, eq11) describes the trade-off between social welfare (sum of utilities) and fairness (fair allocation of the utilities).

$$f(U_1,U_2,...,U_m) = min\left\{(U_k - U_k^{IND})/U_K^{IND}\right\} \qquad (11)$$

U_k^{IND} is the net utility of the k[th] account, which has been derived from the independent framework while U_k is the net utility determined from the framework of joint optimization. Variance is considered as risk measure. The MPO model is in the form of the eq.12.

$$maxf = max(min\left\{\begin{matrix}(\sum_{i=1}^{n}\bar{r}_i((x_{1i}p_i)/C_1) - \sum_{i=1}^{n}(x_{1i}/\sum_{a=1}^{m}x_{ai})\theta_i\sigma_i(\sum_{a=1}^{m}x_{ai}/ADV_i)^{y_i} - U_1^{IND})/U_1^{IND},..., \\ ((\sum_{i=1}^{n}\bar{r}_i((x_{mi}p_i)/Cm) - \sum_{i=1}^{n}(x_{mi}/\sum_{a=1}^{m}x_{ai})\theta_i\sigma_i(\sum_{a=1}^{m}x_{ai}/ADV_i)^{y_i} - U_m^{IND})/U_m^{IND}\end{matrix}\right\}) \tag{12}$$

$s.t.$ $\forall k$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}((x_{ki}p_i)/C_k)((x_{kj}p_j)/C_k)cov(\bar{r}_i,\bar{r}_j) \le k_k\rho_k$$

$$\sum_{i=1}^{n}\bar{r}_i((x_{ki}p_i)/C_k) - \sum_{i=1}^{n}(x_{ki}/\sum_{a=1}^{m}x_{ai})\theta_i\sigma_i(\sum_{a=1}^{m}x_{ai}/ADV_i)^{y_i} - U_k^{IND}$$

$$\sum_{i=1}^{n}x_{ki}p_i \le C_k$$

$$x_{ki} \ge 0$$

The model's constraints include the risk level of each account regarding the minimum risk level and the increase in the utility compared to the utility in its independent mode.

3.3 Optimal Ordering Model

The results of solving the problem presented in the previous section determine the optimal amount of each stock. In this stage, an optimization model is presented by having information about the required amount of each stock in such a way that the lowest possible cost is imposed on customers by splitting the order into several parts.

To this end, the total MI function of Eq.2, which was introduced in Section 3.1 and includes both permanent and temporary impact is used. The optimization problem can be expressed as eq.13:

$$MinMI(X_i) = \sum_{t=1}^{T}(b.Istar_i.x_{it}^{2}/(X_i.V_{it})) + \sum_{t=1}^{T}(1-b).Istar_i \tag{13}$$

$st.$

$$X_i = \sum_{t=1}^{T}x_{it}$$

Assuming t is the order period, x_{it} is the order amount of ith stock in period t, X_i The total order amount of ith stock, and V_t is the expected trading volume of ith stock in the period.

$$X_i = \sum_{k=1}^{m} X_{ki} \tag{14}$$

$$\forall i = 1,...,n \qquad \forall k = 1,...,m \qquad \forall t = 1,...,T$$

For each stock i, X_{ki} is obtained from solving the MPO model.

4. Numerical Analysis

In this section, first, characteristics of the sample used to implement the models are introduced. Then the MI functions are estimated and using these functions and real data of the TSE, the MPO model introduced in the third section is implemented and the relevant results are extracted under different assumptions. Afterwards, the results obtained from different models are analyzed and compared.

At the end, the output of the previous stage is considered as the input of the ordering model and after estimating the variables and implementing the model, the results are presented. Finally, the results of the proposed model are compared with the results of other widely-used models.

4.1. Input Data

The present work was conducted using the data of the transactions of TSE in 2019. On this basis, initially, 50 stocks with the highest liquidity during the research period were extracted. Characteristics of the used sample along with the data of average return, and standard deviation are presented in Table1.

Due to the large volume of the data of the market microstructure (tick-by-tick transactions), the estimation of the parameters utilized in the MI model, MPO model and ordering model were carried out by using merely the data of the first six months of the given year. Also, the data related to before 9:00:00 (pre-opening time) and after 12:00:00 were not taken into account.

The price used in this study was the closing price of the last trading day, and the logarithmic average of the closing prices was assumed as the average return. The number of working days in the first 6 months of the year was T = 116 and that of the whole year was 240. Given these values, the average daily volume of each stock will be as given in Table 2.

In the present work, to achieve results with higher reliability, the data of the final step were monitored and the case data events were excluded from the study. For this purpose, the filters proposed in Kissell's [27] were used:

$$Daily\ volume \ \leq\ 3 * ADV \tag{15}$$

$$\frac{-4\sigma}{\sqrt{240}} \leq \log\ pricechange(close - to - close) \leq \frac{4\sigma}{\sqrt{240}} \tag{16}$$

4.2. Market Impact Function Estimation

Once the average daily volume of each stock was obtained, the data of imbalance (Q) and market impact (I*) of different stocks were prepared regarding the explanations given in Section 3 and the function was estimated using the ordinary least square (OLS) method in EVIEWS software.

4.3. Implementation of MPO Model

The MPO model presented in Section 3 was implemented assuming 3 accounts (m) and 10 stocks (n). The total capital (C_k) and risk-aversion coefficient (K_k) of each customer were assumed as the model's inputs. The input parameters are presented in Table 3. The input values for the risk-aversion coefficient and the initial capital of the account holders are selected by the customer or PM within a certain range. In the present work, these values were determined randomly in GAMS.

In this research, the model was implemented using GAMS software, which is an exact solution method, the obtained results of which are presented and analyzed below. Tables 4,5 and Figures 2-5 show the variations of risk, return, market impact, and utility from Step 1 to Step 4.

Analysis of Results

The variance values for each account have been calculated separately at different steps, which are given in Table 4 and Figure2. Also, the values of return, MI, and utility are presented in Table 5 and Figures 3-5. As mentioned earlier, at the first step, for calculating the variance and return, the accounts were considered independent and the MI was ignored. The computational variance at this step is, indeed, the minimum variance in the independent optimization of each account. This can be also inferred from the smaller values in the first step of Figure2.

At the second step, since the mutual effects of different accounts were not considered, the results seem to be different from what happens in reality, and the obtained utility is unreal and deviated due to the underestimation of costs and ignoring the correlation of accounts, and since the MI has been taken into account, the risk is increased compared to the previous step; on the other hand, since the effects of the transactions of different accounts on each other have been ignored, the MI cost is underestimated and the return and, consequently, the utility are overestimated in comparison the reality. These results are represented in Tables 4 and 5.

In the third step, the effects of transactions of different accounts on each other were taken into account despite the independent optimization of the accounts. The total MI was split among all the accounts using the pro-rata method, and the obtained value was subtracted from the return of the independent transactions calculated in the second step. As shown in Table 5, the utility decreased as the MI cost increased, while the return remained unchanged, and the results were closer to reality since the effects of transactions of the accounts on each other were considered.

At the fourth step, the problem of the simultaneous optimization of the accounts (MPO) has been discussed, and the proposed max-min function represents the trade-off between welfare (sum of utilities) and fairness (fair allocation of utilities). As indicated by the obtained results, the final utility of all accounts in this method has been increased compared to the utility of the independent mode (third step). The increased utility confirms the better performance of the proposed model and MPO model compared to the independent optimization of the accounts (classic portfolio optimization). The main reason for such a change is the reduced MI for the case of aggregated transactions.

Furthermore, the increase in utility for all accounts based on their initial conditions indicates fairness in this method. Based on these results, optimizing multiple accounts by one portfolio manager in the proposed framework can lead to more appropriate and fair results for all customers.

4.4 *Implementation of Ordering Model*

In this section the optimal ordering model introduced in Section 3-3, is implemented to know how place the orders of the values obtained from solving the MPO model proposed in the previous section. The values obtained can be seen in Table 6.

The whole day will be divided into half-hour intervals (T =7), and the total MI function will be minimized, with limit of completing orders.

Forecasting trading volumes

The V_{it} values in the model indicate the expected trading volume in the period. To this end, the data for the first 6 months of 2019 are used and the trading volume at half-hour intervals per day is separated. Finally, the trading volume in each interval is forecasted by the available historical data using the exponential smoothing method.

Estimation of temporary impact parameter (b)

In this research, the value of “b” has been estimated with high accuracy and is not limited to determining the specific range or scenario analysis like previous studies.

By obtaining the values of MI and POV and their regression (eq.2), the value of “b” can be estimated for each stock (Table 7).

Results of implementation

The proposed optimization problem is solved by the Lagrange multiplier technique and MATLAB program. Table 8 provides the results.

In this way, the total order is split into 7 parts (half-hour intervals) and ordering is done at the beginning of each interval. Amounts of the order that are not executed in each interval will be added to the next interval.

Comparison of the proposed model with other patterns

In this section, the results are compared with two widely-used and common patterns to evaluate the performance of the proposed model. The first one is one-time ordering, so that the total specified amounts of each stock will be ordered as a lump sum at the beginning of the trading day. The second one is splitting the order equally in such a way that the total values are equally split into several daily intervals and ordered in each interval (7 half-hour intervals are considered here). Table 9 shows the results.

According to Table 9 and Figure 6, proposed model performs better than the others, leading to lower MI costs. So, the proposed model has high validity and splits large orders well based on the market behaviour and stock characteristics.

5. Conclusion

Multiportfolio problem is a relatively new issue, and there are still shortcomings in relevant studies, and accordingly. The main challenge in this regard is modeling the MI function as a basic factor of MPO models. Past studies generally modeled the MI function with simplified assumptions and a function of the trading volume is equally considered for all stocks. The mentioned function is estimated to be linear or quadratic form with lab data, so the results of models based on these assumptions cannot be generalized to real stock market conditions.

Accordingly, this study focuses on modeling the MI function and providing a comprehensive process of MPO from selection to execution. As an operational tool for PMs, the proposed process can provide a good reflection of market realities. Proposed models are implemented using GAMS and MATLAB software programs with TSE data in 2019.

The results indicate that, portfolio selection with the proposed model will reduce MI costs and increase utility compered to classical models (15% on average). The increase in utility proportional to the initial conditions of each account indicates the observance of fairness in account management. The process takes a comprehensive view of the MPO problem, considers MI costs from the beginning of the stock selection phase to ordering and execution, and proposes a model to split orders to reduce MI costs.

It should be noted that both permanent and temporary impacts are considered in decision-making at this stage. Similar studies do not estimate the exact value of temporary impact and only provide the final analysis for a range of values of this parameter or use the scenario analysis model and determine the final strategy based on different scenarios on this parameter. This study, however, estimates the exact value of temporary impact. Finally, after implementing the model, the results are compared with other common and widely-used patterns and it is concluded that the proposed model performs better than the others in reducing costs and increasing utility (26% on average).

Availability of data and materials

All Data are taken from Tehran Securities Exchange Technology Management Company. The company is subsidiary Tehran Securities Exchange Company and is responsible for collecting, analysing, publishing and providing database for all activists.

http://en.tsetmc.com/Site.aspx?ParTree=121C

Competing interests

The authors declare that they have no competing interests.

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Authors' contributions

All authors have equally contributed to this work and approved the final manuscript.

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Biographies

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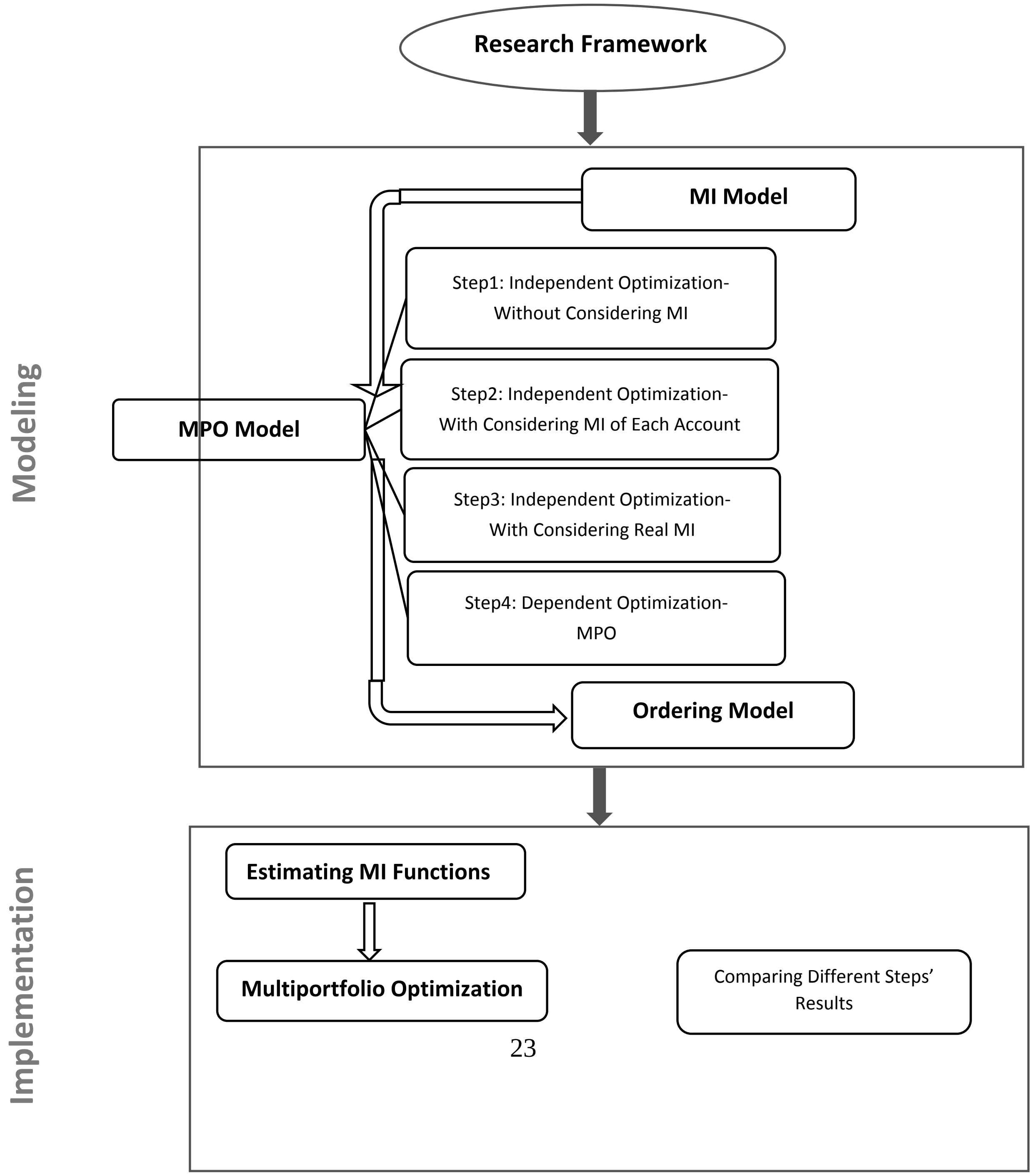


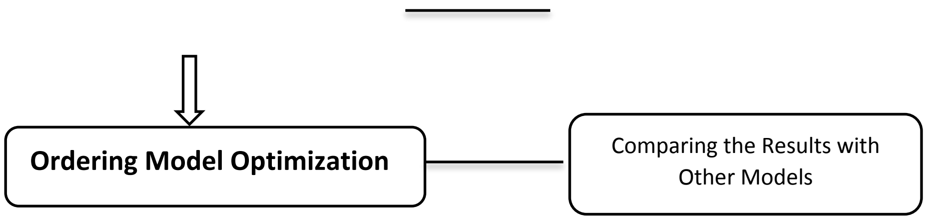


Figure1. Research Framework

Table1 Characteristics of the research sample

No.	Company Name	Stock Symbol	Standard Deviation	Average Return
1	Saipa Automotive Group	Khesapa	0.442836358	0.004167274
2	Melat Bank	Vabemelat	0.35805007	0.004820278
3	Isfahan Mobarakeh Steel	Foolad	0.376654044	0.005135377
4	Pars Khodro	Khepars	0.39716239	0.00176162
5	National Iran Cooper Industries	Femeli	0.507297023	0.003383883
6	Isfahan Oil Refinery	Shepna	0.432466584	0.004372896
7	Tamin Petroleum & Petrochemical Investment	Tapiko	0.361901142	0.001699412
8	Zamyad	Khezamiya	0.43624912	0.001897637
9	Iran Khodro	Khodro	0.359222547	0.000940988
10	Kharazmi Investment	Vekharazm	0.355608557	0.001986713

Table2 Average daily value of each stock

Stock Symbol	ADV	Stock Symbol	ADV
Foolad	65422172.5	Khesapa	74844547
Khepars	33068266.8	Vabemelat	59840242
Vekharazm	26371710.8	Khodro	11541061.2
Khezamiya	36039549.2	Femeli	46801703.3
Tapiko	31547762.8	Shepna	21719012.8

Table3 Model's inputs

Initial Capital (Rial)		Risk Tolerance		Initial Capital (Rial)	
C_1	120,000,000	K_1	5	R_1	8%
C_2	100,000,000	K_2	16	R_2	12%
C_3	140,000,000	K_3	8	R_3	10%

Table4 Variance of different accounts in modelling steps (To make the table easier to read, all data is multiplied by 10^7)

Account	Variance		
	Step1	Step2	Step3
1	9.297514	55.78508	55.78508
2	30.47395	365.6874	365.6874
3	19.50333	175.5299	175.5299

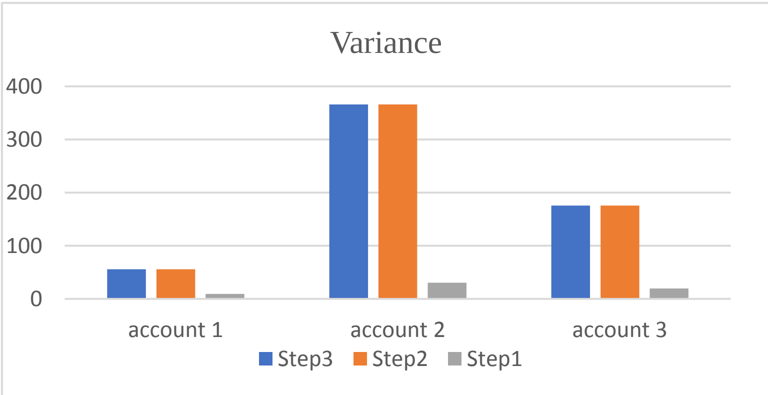
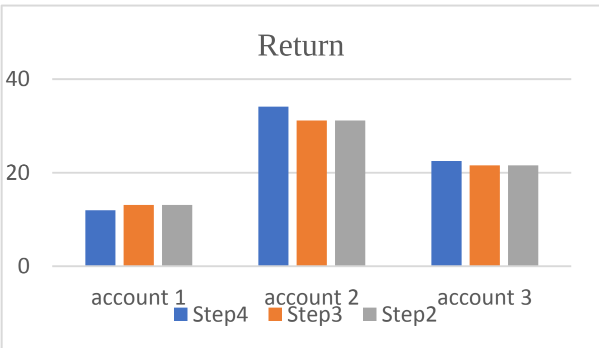


Figure2. Variance of different accounts in modelling steps

Table5 Outputs of the model in different steps

account	Return			MI			Utility			Improve (%)
	Step2	Step3	Step4	Step2	Step3	Step4	Step2	Step3	Step4	
1	13.104	13.104	11.955	0.661	3.737	1.659	12.443	9.367	10.296	9%
2	31.142	31.142	34.110	0.683	3.840	1.210	30.359	27.302	32.9	20%
3	21.546	21.546	22.542	0.519	2.827	0.974	21.047	18.719	21.569	15%



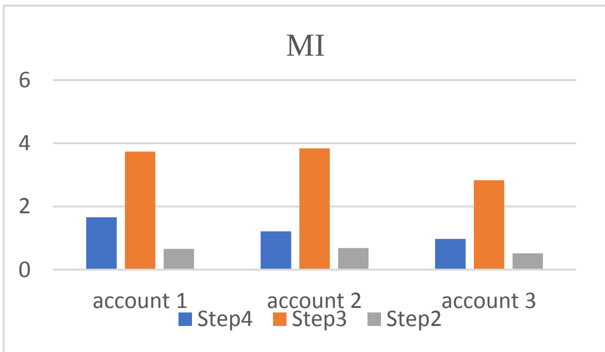


Figure3. Return of different accounts in modelling steps

Figure4. Market Impact of different accounts in modelling steps

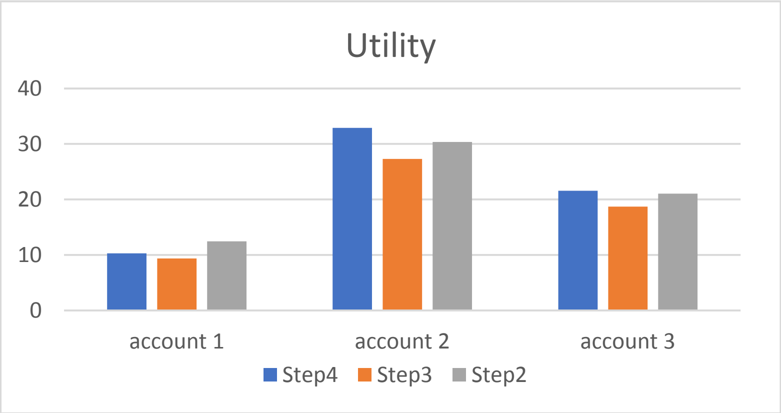


Figure5. Utility of different accounts in modelling steps

Table 6 Various stock values to determine the optimal portfolio for all accounts

Stock Symbol	Account 1	Account 2	Account 3
Khesapa	2870	-	-
Vabemelat	24910	12195	21156
Foolad	1356	2031	4932
Khepars	-	-	-
Femeli	2104	1217	-
Shepna	1702	3970	2517
Tapiko	-	-	-
Khezamiya	-	-	-
Khodro	-	-	-
Vekharazm	-	-	-

Table7 estimated temporary MI parameter

Stock Symbol	$C(1)$	$C(2)$	$\hat{b}$
Khesapa	0.0028	0.0312	0.917647059
Vabemelat	0.0007	0.0429	0.983944954
Foolad	0.0008	0.0269	0.971119134
Femeli	0.00017	0.0222	0.992400536
Shepna	0.000969	0.030776	0.969475508

Table 8 The number of orders for each stock in half-hour intervals and MI cost

MI	x_7	x_6	x_5	x_4	x_3	x_2	x_1	Stock symbol
1.774187474	476	420	414	358	400	431	371	Khesapa
5.677201038	9557	8792	7044	7882	7503	8348	9134	Vabemelat
1.477313773	1219	1076	1195	1179	1177	1187	1286	Foolad
0.494933802	451	496	455	398	419	480	623	Femeli
2.901522602	1146	346	1220	1154	717	1360	2246	Shepna

Table 9 Comparing the performance of the proposed model and other patterns

Stock symbol	Proposed model	One-time ordering		Splitting the order equally	
	MI	MI	Percentage Reduction	MI	Percentage Reduction
Khesapa	1.774187474	2.162210151	17.95%	1.77463431	0.025%
Vabemelat	5.677201038	17.38819871	67.33%	5.719106175	0.733%
Foolad	1.477313773	3.434888225	56.99%	1.478522187	0.082%
Femeli	0.494933802	1.195414122	58.60%	0.49748647	0.513%
Shepna	2.901522602	5.078542766	42.87%	3.182543367	8.83%

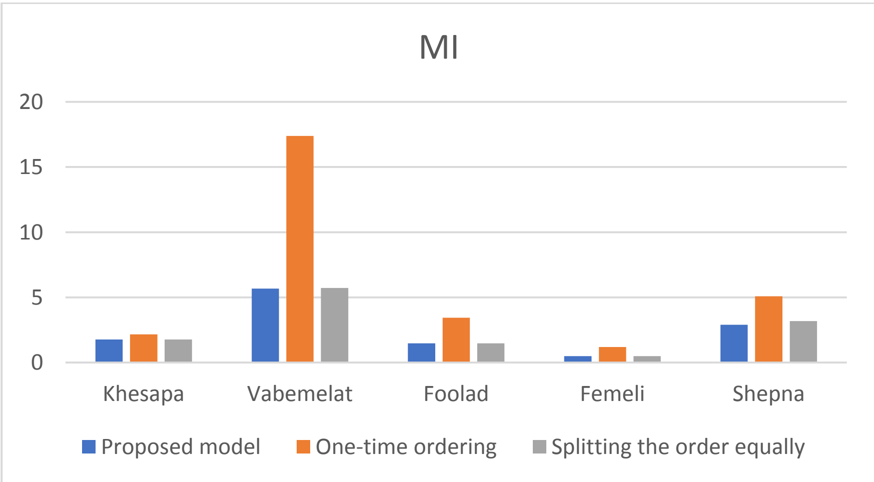


Figure6. Comparing the performance of the proposed model and other patterns