2-Tuple Linguistic q-Rung Orthopair Fuzzy Power MSM Approach for Choosing Sustainable Waste Disposal Technology

Sumera Naz[a], **Muhammad Akram**[b], **Iqra Naseer**[a], **A. Borumand Saeid**[c*] **and Areej Fatima**[a]

a. Department of Mathematics, Division of Science and Technology, University of Education, Lahore, Pakistan.

E-mail: sumera.naz@ue.edu.pk

E-mail: iqranaseer1922000@gmail.com, areejfatima077@gmail.com

b. Department of Mathematics, University of the Punjab, New Campus, Lahore, Pakistan.

E-mail: m.akram@pucit.edu.pk

c. Department of Pure Mathematics, Faculty of Mathematics and Computer, Shahid Bahonar university of Kerman, Kerman, Iran.

E-mail: arsham@uk.ac.ir

Abstract

The municipal solid waste (MSW) generation rate in Asian cities is dramatically increasing with rising urbanization and conventional interaction. In many urban areas, the strategy for MSW disposal is neither environmentally sustainable nor appropriate for human health. Based on the awareness and the implementation of appropriate technology, the selection of waste disposal technology is a sensitive issue. This ultimate decision could have a deep-rooted effect on the environment and commercial development. This study considers the multi-attribute group decision-making (MAGDM) process that also handles uncertainty to select a sustainable waste disposal technology. For this purpose, 2-tuple linguistic q-rung orthopair fuzzy sets (2TLq-ROFSs) are used to permit decision-makers to make assessments in a broader space and to depict the unreliability and vagueness in making real decisions. Firstly, we introduce novel aggregation operators, such as 2TLq-ROF power Maclaurin symmetric mean (2TLq-ROFPMSM) and 2TLq-ROF power weighted Maclaurin symmetric mean (2TLq-ROFPWMSM) operators. We develop an elegant MAGDM approach using the 2TLq-ROFPWMSM operator. Finally, to demonstrate the practicability and efficacy of the proposed method, we consider an MAGDM problem for selecting a sustainable waste disposal technology. The effectiveness of the proposed approach is evaluated further through comparison with different approaches and sensitivity analysis.

Keywords: Municipal solid waste · 2-Tuple linguistic q-rung orthopair fuzzy set · Power Maclaurin symmetric mean operator · Multi-attribute group decision-making.

1 Introduction

Municipal solid waste (MSW) production is skyrocketing as a consequence of modern and urban lifestyle behaviors. The rate of demographic growth has led to an enormous increase in the urban population and, as a result, MSW generation. The world's population is growing at a rate of roughly 1.05% per year (2020), and it is expected to surpass 10 billion people in 2021 [1]. Annually, the worldwide production of MSW is roughly 2.01 billion tonnes, of which 33% is mismanaged. The increasing volume of this waste has sparked concerns about its environmental, economical, and social factors [2]. Due to the environmental impacts of its management, MSW is still an indispensable by-product of daily life as well as a complex and serious problem for our society. Trash management is primarily the moral obligation of local authorities in low- and middle-income countries, which

*Corresponding Author: A. Borumand Saeid;
E-mail: arsham@mail.uk.ac.ir
Mobile: +98-9133425138

have limited resources to deal with the dilemma [3]. Managing the growing amount of MSW sustainably has become a more challenging task for local governments. Asian countries are facing challenges in MSW management. MSW generated in Asia's developing countries is high in organic fraction. Still, there is no reliable waste management system in Pakistan. Karachi is Pakistan's largest and most demographic city, situated in South Asia. In Karachi, a significant amount of MSW is generated and mismanaged, leading to drastic environmental and public health issues. Karachi, Pakistan's industrial and finance core, produces a large portion of MSW, with a high potential for energy production. Sustainable waste management measures must be amended to effectively solve MSW management problems. Such measures must consider a variety of factors of waste management, from waste collection to the final stage of reducing MSW mass. Given the potentially significant consequences, a sustainable plan for identifying waste disposal sustainable technology must be implemented. The selection of the optimal technology is problematic because each technology has its own benefits and down sides. A wrong choice of waste disposal technology, which is emerging to be a major concern in MSW management, can have everlasting harmful effects on social and environmental development [4].

Zadeh [5] devised the fuzzy set (FS) theory to deal with the uncertainties in decision-making. Based on Zadeh's FSs, Atanassov [6] introduced intuitionistic fuzzy sets (IFSs). The restrictions of FS and IFS, specifically their inability to express uncertain data, make it complicated to assess the strategies under complex information conditions. Yager [7], then enlarged the traditional IFS by introducing the q-rung orthopair fuzzy set (q-ROFS). The q-ROFS is only valid if the sum of the qth-powers of membership degree (MD) and non-membership degree (NMD) is limited to 1. The q-ROFS perceives a broader range of data than the IFS when the parameter $q = 1$. Xu [8] proposed IF aggregation operators (AOs). Qin and Liu [9] presented the MSM operators for IFSs. Rahman et al. [10] proposed the Pythagorean fuzzy weighted averaging AOs. MAGDM methods have been identified as one of the most effective applications for real-world problems. Therefore, MAGDM approaches can be used to alleviate the MSW management issues. Naz and Akram [11] presented a decision-making approach based on hesitant fuzzy sets and graph theory. Akram and Sitara [12] proposed a decision-making methodology with q-ROF graph structures. Although, decision-makers (DMs) portray their thoughts as a crisp number in the MAGDM approach, when ambiguous and unclear information is integrated into decision-making, these crisp numbers are often imprecise and insufficient for solving real-world decision-making situations [13]. Yager [14] developed the power average operators. Garg et al. [15] proposed the Muirhead mean and dual Muirhead mean operators within the complex interval-valued q-ROF framework to more effectively acquire DMs' assessment information in the complex MAGDM process. Researchers presented MSM operators for q-ROFSs [16,17]. In the light of the Hamacher operator, Naz et al. [18] introduced the new concept of interval-valued q-rung orthopair dual hesitant fuzzy graphs, named interval-valued q-rung orthopair dual hesitant fuzzy Hamacher graphs, and estimate their energy. Akram et al. [19] developed complex q-ROF Hamachar graphs (Cq-ROFHGs), a new Cq-ROF graph concept based on the Hamacher operator. Kumar and Chen [20] developed an MAGDM approach based on the advanced linguistic IF weighted aggregation operator (AO) of linguistic IF numbers. Gong et al. [21] presented a novel method for MAGDM with multiplicative linguistic data. Concretely, owing to its symbolic translation, the 2TL representation model presented by Herrera and Martinez [22], stands out for its versatility, accuracy, and willingness to perform precise linguistic computations without any assumptions. The 2TL model is useful for processing information with linguistic patterns and can help to prevent data loss throughout the information aggregation process. Naz et al. [23] developed a 2TL bipolar fuzzy geometric Heronian mean operator and a 2TL bipolar weighted geometric Heronian mean operator. Akram and Niaz [24] presented a DM methodology based on COCOSO (Combined Compromise Solution) and CRITIC (Criteria Interrelation Through Intercriteria Correlation) methods. Akram et al. [25, 26] developed certain DMs techniques under the background of 2TL Fermatean FSs. To enhance the 2TL framework, Akram et al. [27, 28] inserted the 2TL multipolar data in different fuzzy methodologies. Naz et al. [29] proposed a DM strategy and its application in arc welding robot selection for 2TL q-ROFSs. Yuan [30] presented several basic theories, combining the Pythagorean interval 2TL numbers with the EDAS (Evaluation based on Distance from Average Solution) method. He et al. [31] presented a method that combined the Pythagorean fuzzy 2TL set with the qualitative flexible multiple criteria (QUALIFLEX) method. In this paper, a 2TLq-ROFS, which is an innovative strategy to tackle the fuzziness in a decision-making framework that combines a 2TL term set with a q-ROF set, is utilized. Information aggregation operator (AO) is an imperative method in the domain of data integration. Information AOs can address MAGDM problems more efficiently by not only providing the sorted alternatives but also providing an overall assessment result for each alternative. Naz et al. [32] evaluated network security service provider using 2-TL complex-ROF COPRAS method, modified EDAS method based on MSM operators with 2-TL-spherical FSs [33], and presented models for MAGDM with dual hesitant q-ROF 2-TL MSM operators and their application to COVID-19 pandemic [34]. Feng et al. [35] proposed novel score functions of generalized orthopair fuzzy membership grades with application to MADM. Maclaurin [36] was a pioneer in developing a conventional modeled type AO. The MSM operator can dynamically depict correlations with any multiple attributes by changing the range of a parameter κ, allowing it to deal with decision-making issues. Qin and Liu [37] extended the MSM operator to the dual MSM (DMSM) operator.

In practical decision-making contexts, the interaction among multiple attributes is ubiquitous. Both the Bonferroni mean (BM) and Heronian mean (HM) operators are efficient in negotiating this circumstance. Anyhow, the BM and HM operators can articulate the interrelation between the two attributes, which is insufficient and limited in responding to some practical MAGDM complications. By changing parametric values, the MSM operator depicts a correlation within any number of attributes, which is more realistic and adaptive when interacting with MADM or MAGDM issues. Furthermore, it must be observed that due to the bias of DMs with various preference perceptions in actual decision-making, DMs may provide extreme assessment values. The PA operator, using the weighted vector, can remove the effect of absolute values from the bias DMs. Further, many AOs are used to solve MAGDM problems in IF and Pythagorean fuzzy frameworks and are unable to deal with imprecise information fully and effectively. 2TLq-ROFSs can help to avoid data loss during the aggregation process and can describe fuzzy information more comprehensively and effectively. We intended to develop an algorithm merging PA and MSM operators with the 2TLq-ROFS to provide an appropriate strategy for the selection of sustainable waste disposal technology for MSW management. Liu et al. [38] proposed a group decision-making analysis based on Lq-ROF generalized point weighted AOs. Due to the exceptional reliability of PMSM in fusing fuzzy information, we generalize PMSM into 2TLq-ROFSs and present 2TLq-ROFPMSM and 2TLq-ROFPWMSM operators. An MAGDM approach is established to assess and choose the best waste disposal technology for MSW management based on the 2TLq-ROFPWMSM operator.

The rest of the article is structured as follows: Section 2 introduces a 2TL representation model as well as some fundamental concepts of the q-ROFS, 2TLq-ROFS, PA operator and MSM operator. In Section 3, we develop the 2TLq-ROFPMSM and 2TLq-ROFPWMSM operators. Furthermore, some applicable properties of the proposed operators are discussed. A new approach for MAGDM problems has been developed by utilizing the 2TLq-ROFPWMSM operator in Section 4. Section 5 extends the proposed method to a case study to emphasize its reliability and effectiveness. Section 6 encloses a summary and future research directions.

2 Preliminaries

Definition 2.1. [7] For any universal set Y, a q-ROFS is of the form

$$T = \{\langle y, (p_T(y), l_T(y))\rangle | y \in Y\},$$

where $p_T, l_T : Y \to [0,1]$ represent the MD and NMD, respectively, with the condition $0 \leq (p_T(y))^q + (l_T(y))^q \leq 1$ for positive number $q \geq 1$, and $r(y) = \sqrt[q]{1 - ((p_T(y))^q + (l_T(y))^q)}$ is known as the degree of refusal of y in T. To express information conveniently, the pair (p, l) is known as a q-ROF number (q-ROFN).

A q-ROFN is a generalized form of existing fuzzy framework and it reduces to:

(i) Pythagorean fuzzy number (PFN); by taking q as 2.

(ii) Intuitionistic fuzzy number (IFN); by taking q as 1.

(iii) Fuzzy number (FN); by taking l as zero and q as 1.

Motivated by the idea of 2TL terms and q-ROF sets, Naz et al. [29] established the novel concept of 2TLq-ROFSs by incorporating both the features of 2TL terms and q-ROF sets, as an expansion of 2TLIFSs and 2TLPFSs.

Definition 2.2. [29] Let $S = \{s_t | t = 0, 1, \ldots, \tau\}$ be an LTS with odd cardinality. If $((s_p(y), \wp(y)), (s_l(y), \pounds(y)))$ is defined for $s_p(y), s_l(y) \in S$, $\wp(y), \pounds(y) \in [-0.5, 0.5)$, where $(s_p(y), \wp(y))$ and $(s_l(y), \pounds(y))$ represent the MD and NMD by 2TLSs, respectively. A 2TLq-ROFS is defined as:

$$\aleph = \{\langle y, ((s_p(y), \wp(y)), (s_l(y), \pounds(y)))\rangle | y \in Y\}, \tag{2.1}$$

where $0 \leq \Delta^{-1}(s_p(y), \wp(y)) \leq \tau, 0 \leq \Delta^{-1}(s_l(y), \pounds(y)) \leq \tau$, and $0 \leq (\Delta^{-1}(s_p(y), \wp(y)))^q + (\Delta^{-1}(s_l(y), \pounds(y)))^q \leq \tau^q$.

To compare any two 2TLq-ROFNs, their score and accuracy values are defined as follows:

Definition 2.3. [29] Let $\eta = ((s_p, \wp), (s_l, \pounds))$ be a 2TLq-ROFN. Then the score value Sc of a 2TLq-ROFN η, can be represented as

$$Sc(\eta) = \Delta\left(\frac{\tau}{2}\left(1 + \left(\frac{\Delta^{-1}(s_p, \wp)}{\tau}\right)^q - \left(\frac{\Delta^{-1}(s_l, \pounds)}{\tau}\right)^q\right)\right), \; Sc(\eta) \in [0, \tau], \tag{2.2}$$

and its accuracy value Ac is defined as

$$Ac(\eta) = \Delta\left(\tau\left(\left(\frac{\Delta^{-1}(s_p, \wp)}{\tau}\right)^q + \left(\frac{\Delta^{-1}(s_l, \pounds)}{\tau}\right)^q\right)\right), \; Ac(\eta) \in [0, \tau]. \tag{2.3}$$

Definition 2.4. [29] Let $\eta_1 = ((s_{p_1}, \wp_1), (s_{l_1}, \pounds_1))$ and $\eta_2 = ((s_{p_2}, \wp_2), (s_{l_2}, \pounds_2))$ be two 2TLq-ROFNs, then these two 2TLq-ROFNs can be assessed according to the following rules:

(1) If $Sc(\eta_1) > Sc(\eta_2)$, then $\eta_1 > \eta_2$;

(2) If $Sc(\eta_1) = Sc(\eta_2)$, then

- If $Ac(\eta_1) > Ac(\eta_2)$, then $\eta_1 > \eta_2$;
- If $Ac(\eta_1) = Ac(\eta_2)$, then $\eta_1 \sim \eta_2$.

Definition 2.5. [29] Let $\eta_1 = ((s_{p_1}, \wp_1), (s_{l_1}, \pounds_1))$ and $\eta_2 = ((s_{p_2}, \wp_2), (s_{l_2}, \pounds_2))$ be two 2TLq-ROFNs. We define the 2TLq-ROF normalized Hamming distance (HD) as:

$$d(\eta_1, \eta_2) = \Delta\left(\frac{\tau}{2}\left(\left|\left(\frac{\Delta^{-1}(s_{p_1}, \wp_1)}{\tau}\right)^q - \left(\frac{\Delta^{-1}(s_{p_2}, \wp_2)}{\tau}\right)^q\right| + \left|\left(\frac{\Delta^{-1}(s_{l_1}, \pounds_1)}{\tau}\right)^q - \left(\frac{\Delta^{-1}(s_{l_2}, \pounds_2)}{\tau}\right)^q\right| \right)\right). \tag{2.4}$$

The novel operational laws based on 2TLq-ROFNs, such as addition, multiplication, scalar multiplication, power, and ranking rules are as follows:

Definition 2.6. [29] Let $\eta = ((s_p, \wp), (s_l, \pounds))$, $\eta_1 = ((s_{p_1}, \wp_1), (s_{l_1}, \pounds_1))$ and $\eta_2 = ((s_{p_2}, \wp_2), (s_{l_2}, \pounds_2))$ be three 2TLq-ROFNs, $q \geq 1$, then

1. $\eta_1 \oplus \eta_2 = \begin{pmatrix} \Delta\left(\tau\sqrt[q]{1 - \left(1 - \left(\frac{\Delta^{-1}(s_{p_1}, \wp_1)}{\tau}\right)^q\right)\left(1 - \left(\frac{\Delta^{-1}(s_{p_2}, \wp_2)}{\tau}\right)^q\right)}\right), \\ \Delta\left(\tau\left(\frac{\Delta^{-1}(s_{l_1}, \pounds_1)}{\tau}\right)\left(\frac{\Delta^{-1}(s_{l_2}, \pounds_2)}{\tau}\right)\right) \end{pmatrix}$;
2. $\eta_1 \otimes \eta_2 = \begin{pmatrix} \Delta\left(\tau\left(\frac{\Delta^{-1}(s_{p_1}, \wp_1)}{\tau}\right)\left(\frac{\Delta^{-1}(s_{p_2}, \wp_2)}{\tau}\right)\right), \\ \Delta\left(\tau\sqrt[q]{1 - \left(1 - \left(\frac{\Delta^{-1}(s_{l_1}, \pounds_1)}{\tau}\right)^q\right)\left(1 - \left(\frac{\Delta^{-1}(s_{l_2}, \pounds_2)}{\tau}\right)^q\right)}\right) \end{pmatrix}$;
3. $\lambda\eta = \left(\Delta\left(\tau\sqrt[q]{1 - \left(1 - \left(\frac{\Delta^{-1}(s_p, \wp)}{\tau}\right)^q\right)^\lambda}\right), \Delta\left(\tau\left(\frac{\Delta^{-1}(s_l, \pounds)}{\tau}\right)^\lambda\right) \right), \; \lambda > 0$;
4. $\eta^\lambda = \left(\Delta\left(\tau\left(\frac{\Delta^{-1}(s_p, \wp)}{\tau}\right)^\lambda\right), \Delta\left(\tau\sqrt[q]{1 - \left(1 - \left(\frac{\Delta^{-1}(s_l, \pounds)}{\tau}\right)^q\right)^\lambda}\right) \right), \; \lambda > 0.$

Definition 2.7. [14] Let $a_\jmath$ $(\jmath = 1, 2, \ldots, \mathfrak{n})$ be a set of non-negative real numbers then PA aggregation operator is defined as follows:

$$PA(a_1, a_2, \ldots, a_\mathfrak{n}) = \frac{\sum_{\jmath=1}^{\mathfrak{n}}(1 + \mathcal{T}(a_\jmath))a_\jmath}{\sum_{\jmath=1}^{\mathfrak{n}}(1 + \mathcal{T}(a_\jmath))},$$

where $\mathcal{T}(a_\jmath) = \sum_{\jmath,t=1,\jmath\neq t}^{\mathfrak{n}} \mathfrak{S}(a_\jmath, a_t)$, $\mathfrak{S}(a_\jmath, a_t) = 1 - d(a_\jmath, a_t)$ and $\mathfrak{S}(a_\jmath, a_t)$ is the support for $a_\jmath$ from a_t that satisfies the three characteristics:

(1) $\mathfrak{S}\ (a_\jmath, a_t)\ \in\ [0,1]$;

(2) $\mathfrak{S}\ (a_\jmath, a_t) = \mathfrak{S}\ (a_t, a_\jmath)$;

(3) $\mathfrak{S}\ (a_\jmath, a_t)\ \geq\ \mathfrak{S}\ (b_\jmath, b_t),\ if\ |a_\jmath - a_t| < |b_\jmath - b_t|$.

Evidently, the support ($\mathfrak{S}$) function is a similarity factor. The closer two values are, and therefore the more they support each other, the more identical they are. And, let $\frac{(1+\mathcal{T}(a_\jmath))}{\sum_{\jmath=1}^{\mathfrak{n}}(1+\mathcal{T}(a_\jmath))}$ can be written as $\lambda_\jmath$.

3 The 2TLq-ROFPMSM aggregation operators

Definition 3.1. Let $\eta_\jmath = \left(\left(s_{p_\jmath}, \wp_\jmath\right), \left(s_{l_\jmath}, \pounds_\jmath\right)\right)(\jmath = 1,2,\ldots,\mathfrak{n})$ be a set of 2TLq-ROFNs; then the 2TLq-ROFPMSM operator is a mapping $\mathbb{R}^{\mathfrak{n}} \to \mathbb{R}$ described as follows:

$$\text{2TL}q\text{-ROFPMSM}^{(\kappa)}(\eta_1,\eta_2,\ldots,\eta_{\mathfrak{n}}) = \left(\frac{\oplus_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(\otimes_{\jmath=1}^{\kappa}\mathfrak{n}\lambda_{\imath_\jmath}\eta_{\imath_\jmath}\right)}{C_{\mathfrak{n}}^{\kappa}}\right)^{\frac{1}{\kappa}},$$

where $\lambda_\jmath = \frac{(1+\mathcal{T}(\eta_\jmath))}{\sum_{\imath=1}^{\mathfrak{n}}(1+\mathcal{T}(\eta_\imath))}$ means the power weights of $\eta_\jmath$, $\mathcal{T}(\eta_\jmath) = \sum_{\jmath,\mathrm{t}=1,\jmath\neq\mathrm{t}}^{\mathfrak{n}}\mathfrak{S}(\eta_\jmath,\eta_{\mathrm{t}})$, $\mathfrak{S}(\eta_\jmath,\eta_{\mathrm{t}}) = 1 - d(\eta_\jmath,\eta_{\mathrm{t}})$.

By utilizing the novel operational laws of 2TLq-ROFNs (see Definition 2.6), we can obtain Theorem 3.1.

Theorem 3.1. *Let $\eta_\jmath = \left(\left(s_{p_\jmath}, \wp_\jmath\right), \left(s_{l_\jmath}, \pounds_\jmath\right)\right)(\jmath = 1,2,\ldots,\mathfrak{n})$ be a set of 2TLq-ROFNs, then their aggregated result utilizing the 2TLq-ROFPMSM operator is also a 2TLq-ROFN, and*

$$\begin{aligned}&\textit{2TLq-ROFPMSM}^{(\kappa)}(\eta_1,\eta_2,\ldots,\eta_{\mathfrak{n}}) = \left(\frac{\oplus_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(\otimes_{\jmath=1}^{\kappa}\mathfrak{n}\lambda_{\imath_\jmath}\eta_{\imath_\jmath}\right)}{C_{\mathfrak{n}}^{\kappa}}\right)^{\frac{1}{\kappa}}\\ &= \left(\begin{array}{c}\Delta\left(\tau\left(\sqrt[q]{1-\left(\prod_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(1-\prod_{\jmath=1}^{\kappa}\left(1-\left(1-\left(\frac{\Delta^{-1}\left(s_{p_{\imath_\jmath}},\wp_{\imath_\jmath}\right)}{\tau}\right)^{q}\right)^{\mathfrak{n}\lambda_{\imath_\jmath}}\right)\right)\right)^{\frac{1}{C_{\mathfrak{n}}^{\kappa}}}}\right)^{\frac{1}{\kappa}}\right),\\ \Delta\left(\tau\sqrt[q]{1-\left(1-\prod_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(1-\prod_{\jmath=1}^{\kappa}\left(1-\left(\frac{\Delta^{-1}\left(s_{l_{\imath_\jmath}},\pounds_{\imath_\jmath}\right)}{\tau}\right)^{q\mathfrak{n}\lambda_{\imath_\jmath}}\right)\right)^{\frac{1}{C_{\mathfrak{n}}^{\kappa}}}\right)^{\frac{1}{\kappa}}}\right)\end{array}\right).\end{aligned} \tag{3.1}$$

Proof. By utilizing the novel operational laws of 2TLq-ROFNs (see Definition 2.6), we have

$$\otimes_{\jmath=1}^{\kappa}\mathfrak{n}\lambda_{\imath_\jmath}\eta_{\imath_\jmath} = \left(\Delta\left(\tau\prod_{\jmath=1}^{\kappa}\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(s_{p_{\imath_\jmath}},\wp_{\imath_\jmath})}{\tau}\right)^{q}\right)^{\mathfrak{n}\lambda_{\imath_\jmath}}}\right),\Delta\left(\tau\left(\sqrt[q]{1-\prod_{\jmath=1}^{\kappa}\left(1-\left(\frac{\Delta^{-1}(s_{l_{\imath_\jmath}},\pounds_{\imath_\jmath})}{\tau}\right)^{q\mathfrak{n}\lambda_{\imath_\jmath}}\right)}\right)\right)\right)$$

And,

$$\oplus_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(\otimes_{\jmath=1}^{\kappa}\mathfrak{n}\lambda_{\imath_\jmath}\eta_{\imath_\jmath}\right) = \left(\begin{array}{c}\Delta\left(\tau\sqrt[q]{1-\left(\prod_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(1-\prod_{\jmath=1}^{\kappa}\left(1-\left(1-\left(\frac{\Delta^{-1}\left(s_{p_{\imath_\jmath}},\wp_{\imath_\jmath}\right)}{\tau}\right)^{q}\right)^{\mathfrak{n}\lambda_{\imath_\jmath}}\right)\right)\right)}\right),\\ \Delta\left(\tau\left(\prod_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\sqrt[q]{1-\prod_{\jmath=1}^{\kappa}\left(1-\left(\frac{\Delta^{-1}\left(s_{l_{\imath_\jmath}},\pounds_{\imath_\jmath}\right)}{\tau}\right)^{q\mathfrak{n}\lambda_{\imath_\jmath}}\right)}\right)\right)\end{array}\right)$$

Thus we obtain,

$$\frac{1}{C_{\mathfrak{n}}^{\kappa}}\oplus_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(\otimes_{\jmath=1}^{\kappa}\mathfrak{n}\lambda_{\imath_\jmath}\eta_{\imath_\jmath}\right) = \left(\begin{array}{c}\Delta\left(\tau\sqrt[q]{1-\left(\prod_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(1-\prod_{\jmath=1}^{\kappa}\left(1-\left(1-\left(\frac{\Delta^{-1}\left(s_{p_{\imath_\jmath}},\wp_{\imath_\jmath}\right)}{\tau}\right)^{q}\right)^{\mathfrak{n}\lambda_{\imath_\jmath}}\right)\right)\right)^{\frac{1}{C_{\mathfrak{n}}^{\kappa}}}}\right),\\ \Delta\left(\tau\left(\prod_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\sqrt[q]{1-\prod_{\jmath=1}^{\kappa}\left(1-\left(\frac{\Delta^{-1}\left(s_{l_{\imath_\jmath}},\pounds_{\imath_\jmath}\right)}{\tau}\right)^{q\mathfrak{n}\lambda_{\imath_\jmath}}\right)}\right)^{\frac{1}{C_{\mathfrak{n}}^{\kappa}}}\right)\end{array}\right)$$

Accordingly,

$$
\begin{aligned}
&\text{2TL}q\text{-ROFPMSM}^{(\kappa)}(\eta_1,\eta_2,\ldots,\eta_{\mathfrak{n}})\\
&=\left(\begin{array}{l}
\Delta\left(\tau\left(\sqrt[q]{1-\left(\prod\limits_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(1-\prod\limits_{\jmath=1}^{\kappa}\left(1-\left(1-\left(\frac{\Delta^{-1}\left(s_{p_{\imath_\jmath}},\wp_{\imath_\jmath}\right)}{\tau}\right)^{q}\right)^{\mathfrak{n}\lambda_{\imath_\jmath}}\right)\right)\right)^{\frac{1}{C_{\mathfrak{n}}^{\kappa}}}}\right)^{\frac{1}{\kappa}}\right),\\
\Delta\left(\tau\sqrt[q]{1-\left(1-\prod\limits_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(1-\prod\limits_{\jmath=1}^{\kappa}\left(1-\left(\frac{\Delta^{-1}\left(s_{l_{\imath_\jmath}},\pounds_{\imath_\jmath}\right)}{\tau}\right)^{q\mathfrak{n}\lambda_{\imath_\jmath}}\right)\right)\right)^{\frac{1}{C_{\mathfrak{n}}^{\kappa}}}}^{\frac{1}{\kappa}}\right)
\end{array}\right).
\end{aligned}
$$

□

Theorem 3.2. *Let* $\eta_\jmath = \left(\left(s_{p_\jmath},\wp_\jmath\right),\left(s_{l_\jmath},\pounds_\jmath\right)\right)$ *and* $\eta'_\jmath = \left(\left(s'_{p_\jmath},\wp'_\jmath\right),\left(s'_{l_\jmath},\pounds'_\jmath\right)\right)$ $(\jmath = 1,2,\ldots,\mathfrak{n})$ *be two sets of 2TLq-ROFNs, then the 2TLq-ROFPMSM operator has the following properties:*

1. *(Idempotency) If all* $\eta_\jmath$ $(\jmath = 1,2,\ldots,\mathfrak{n})$ *are equal, i.e.,* $\eta_\jmath = \eta$ *for all* $\jmath$*, then*

$$\textit{2TLq-ROFPMSM}^{(\kappa)}(\eta_1,\eta_2,\ldots,\eta_{\mathfrak{n}}) = \eta.$$

2. *(Commutativity) Let* $\eta_\jmath$ $(\jmath = 1,2,\ldots,\mathfrak{n})$ *be any set of 2TLq-ROFNs, and* ${\eta'}_\jmath$ $(\jmath = 1,2,\ldots,\mathfrak{n})$ *be a permutation of* $\eta_\jmath$ $(\jmath = 1,2,\ldots,\mathfrak{n})$*, then*

$$\textit{2TLq-ROFPMSM}^{(\kappa)}(\eta_1,\eta_2,\ldots,\eta_{\mathfrak{n}}) = \textit{2TLq-ROFPMSM}^{(\kappa)}(\eta'_1,\eta'_2,\ldots,\eta'_{\mathfrak{n}}).$$

3. *(Monotonicity) Let* $\eta_\jmath$ *and* $\eta'_\jmath$ $(\jmath = 1,2,\ldots,\mathfrak{n})$ *be two sets of 2TLq-ROFNs; if* $\eta_\jmath \geq {\eta'}_\jmath$ *for all* $\jmath$*, then*

$$\textit{2TLq-ROFPMSM}^{(\kappa)}(\eta_1,\eta_2,\ldots,\eta_{\mathfrak{n}}) \geq \textit{2TLq-ROFPMSM}^{(\kappa)}(\eta'_1,\eta'_2,\ldots,\eta'_{\mathfrak{n}}).$$

4. *(Boundedness) Let* $\eta_\jmath$ $(\jmath = 1,2,\ldots,\mathfrak{n})$ *be any set of 2TLq-ROFNs, suppose*

$$\eta^- = \min_\jmath \eta_\jmath = \left(\min_\jmath(s_{p_\jmath},\wp_\jmath), \max_\jmath(s_{l_\jmath},\pounds_\jmath)\right),$$

$$\eta^+ = \max_\jmath \eta_\jmath = \left(\max_\jmath(s_{p_\jmath},\wp_\jmath), \min_\jmath(s_{l_\jmath},\pounds_\jmath)\right).$$

Then,

$$\eta^- \leq \textit{2TLq-ROFPMSM}^{(\kappa)}(\eta_1,\eta_2,\ldots,\eta_{\mathfrak{n}}) \leq \eta^+.$$

The attribute weights are important in actual decision-making situations, and have an impact on the final decision outcomes. As a consequence, in the information fusion process, it is essential to consider attribute weights. So, we will introduce the 2TLq-ROFPWMSM operator, which is as follows:

Definition 3.2. Let $\eta_\jmath = \left(\left(s_{p_\jmath},\wp_\jmath\right),\left(s_{l_\jmath},\pounds_\jmath\right)\right)$ $(\jmath = 1,2,\ldots,\mathfrak{n})$ be a set of 2TLq-ROFNs with weight vector $\omega = (\omega_1,\omega_2,\ldots,\omega_{\mathfrak{n}})^T$, satisfying $\omega_\jmath \in [0,1]$, $\sum\limits_{\jmath=1}^{\mathfrak{n}} \omega_\jmath = 1$, then the 2TL$q$-ROFPWMSM operator is a mapping $\mathbb{R}^{\mathfrak{n}} \to \mathbb{R}$ described as follows:

$$\text{2TL}q\text{-ROFPWMSM}_{\omega}^{(\kappa)}(\eta_1,\eta_2,\ldots,\eta_{\mathfrak{n}}) = \left(\frac{\bigoplus_{1\leq\imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(\bigotimes_{\jmath=1}^{\kappa}\mathfrak{n}\gamma_{\imath_\jmath}\eta_{\imath_\jmath}\right)}{C_{\mathfrak{n}}^{\kappa}}\right)^{\frac{1}{\kappa}},$$

where $\gamma_\jmath = \frac{\omega_\jmath(1+\mathcal{T}(\eta_\jmath))}{\sum\limits_{\imath=1}^{\mathfrak{n}}\omega_\imath(1+\mathcal{T}(\eta_\imath))}$ means the power weights of $\eta_\jmath$, $\mathcal{T}(\eta_\jmath) = \sum\limits_{\jmath,\mathfrak{t}=1,\jmath\neq\mathfrak{t}}^{\mathfrak{n}} \omega_\jmath \mathfrak{S}(\eta_\jmath,\eta_{\mathfrak{t}})$, $\mathfrak{S}(\eta_\jmath,\eta_{\mathfrak{t}}) = 1 - d(\eta_\jmath,\eta_{\mathfrak{t}})$.

By utilizing the novel operational laws of 2TLq-ROFNs (see Definition 2.6), we can obtain Theorem 3.3.

Theorem 3.3. *Let $\eta_\jmath = \left(\left(s_{p_\jmath}, \wp_\jmath\right), \left(s_{l_\jmath}, \pounds_\jmath\right)\right) (\jmath = 1, 2, \ldots, \mathfrak{n})$ be a set of 2TLq-ROFNs, then their aggregated result utilizing the 2TLq-ROFPWMSM operator is also a 2TLq-ROFN, and*

$$\text{2TLq-ROFPWMSM}_{\omega}^{(\kappa)}(\eta_1, \eta_2, \ldots, \eta_{\mathfrak{n}}) = \left(\frac{\oplus_{1\leq \imath_1 < \ldots < \imath_\kappa \leq \mathfrak{n}} \left(\otimes_{\jmath=1}^{\kappa} \mathfrak{n}\gamma_{\imath_\jmath}\eta_{\imath_\jmath}\right)}{C_{\mathfrak{n}}^{\kappa}}\right)^{\frac{1}{\kappa}}$$

$$= \left(\begin{array}{c} \Delta\left(\tau\left(\sqrt[q]{1-\left(\prod\limits_{1\leq \imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(1-\prod\limits_{\jmath=1}^{\kappa}\left(1-\left(1-\left(\frac{\Delta^{-1}\left(s_{p_{\imath_\jmath}},\wp_{\imath_\jmath}\right)}{\tau}\right)^{q}\right)^{\mathfrak{n}\gamma_{\imath_\jmath}}\right)\right)\right)^{\frac{1}{C_{\mathfrak{n}}^{\kappa}}}}\right)^{\frac{1}{\kappa}}\right), \\ \Delta\left(\tau\sqrt[q]{1-\left(1-\prod\limits_{1\leq \imath_1<\ldots<\imath_\kappa\leq\mathfrak{n}}\left(1-\prod\limits_{\jmath=1}^{\kappa}\left(1-\left(\frac{\Delta^{-1}\left(s_{l_{\imath_\jmath}},\pounds_{\imath_\jmath}\right)}{\tau}\right)^{q\mathfrak{n}\gamma_{\imath_\jmath}}\right)\right)^{\frac{1}{C_{\mathfrak{n}}^{\kappa}}}\right)^{\frac{1}{\kappa}}}\right) \end{array}\right). \quad (3.2)$$

Theorem 3.4. *Let $\eta_\jmath = \left(\left(s_{p_\jmath}, \wp_\jmath\right), \left(s_{l_\jmath}, \pounds_\jmath\right)\right)$ and $\eta'_\jmath = \left(\left(s'_{p_\jmath}, \wp'_\jmath\right), \left(s'_{l_\jmath}, \pounds'_\jmath\right)\right) (\jmath = 1, 2, \ldots, \mathfrak{n})$ be two sets of 2TLq-ROFNs, then the 2TLq-ROFPWMSM operator has the following properties:*

1. *(Idempotency) If all $\eta_\jmath$ $(\jmath = 1, 2, \ldots, \mathfrak{n})$ are equal, i.e., $\eta_\jmath = \eta$ for all $\jmath$, then*

$$\text{2TLq-ROFPWMSM}_{\omega}^{(\kappa)}(\eta_1, \eta_2, \ldots, \eta_{\mathfrak{n}}) = \eta.$$

2. *(Commutativity) Let $\eta_\jmath$ $(\jmath = 1, 2, \ldots, \mathfrak{n})$ be any set of 2TLq-ROFNs, and $\eta'_\jmath$ $(\jmath = 1, 2, \ldots, \mathfrak{n})$ be a permutation of $\eta_\jmath$ $(\jmath = 1, 2, \ldots, \mathfrak{n})$, then*

$$\text{2TLq-ROFPWMSM}_{\omega}^{(\kappa)}(\eta_1, \eta_2, \ldots, \eta_{\mathfrak{n}}) = \text{2TLq-ROFPWMSM}_{\omega}^{(\kappa)}(\eta'_1, \eta'_2, \ldots, \eta'_{\mathfrak{n}}).$$

3. *(Monotonicity) Let $\eta_\jmath$ and $\eta'_\jmath$ $(\jmath = 1, 2, \ldots, \mathfrak{n})$ be two sets of 2TLq-ROFNs; if $\eta_\jmath \geq \eta'_\jmath$ for all $\jmath$, then*

$$\text{2TLq-ROFPWMSM}_{\omega}^{(\kappa)}(\eta_1, \eta_2, \ldots, \eta_{\mathfrak{n}}) \geq \text{2TLq-ROFPWMSM}_{\omega}^{(\kappa)}(\eta'_1, \eta'_2, \ldots, \eta'_{\mathfrak{n}}).$$

4. *(Boundedness) Let $\eta_\jmath$ $(\jmath = 1, 2, \ldots, \mathfrak{n})$ be any set of 2TLq-ROFNs, suppose*

$$\eta^- = \min_\jmath \eta_\jmath = \left(\min_\jmath(s_{p_\jmath}, \wp_\jmath), \max_\jmath(s_{l_\jmath}, \pounds_\jmath)\right),$$

$$\eta^+ = \max_\jmath \eta_\jmath = \left(\max_\jmath(s_{p_\jmath}, \wp_\jmath), \min_\jmath(s_{l_\jmath}, \pounds_\jmath)\right).$$

Then,

$$\eta^- \leq \text{2TLq-ROFPWMSM}_{\omega}^{(\kappa)}(\eta_1, \eta_2, \ldots, \eta_{\mathfrak{n}}) \leq \eta^+.$$

4 An MAGDM method based on the 2TLq-ROFPWMSM operator

We establish an MAGDM method utilizing the 2TLq-ROFPWMSM operator.

4.1 Basic description of the MAGDM problem

2TLq-ROFSs can help to avoid data loss during the aggregation process and can describe fuzzy information more comprehensively and effectively. The MAGDM issue with 2TLq-ROF information is specified as: let $\{\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_{\mathfrak{m}}\}$ be a set of $\mathfrak{m}$ alternatives, $\{\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{\mathfrak{n}}\}$ be a set of $\mathfrak{n}$ attributes with weighting vector $\omega = (\omega_1, \omega_2, \ldots, \omega_{\mathfrak{n}})^T$, satisfying $\omega_\jmath \in [0, 1]$, $\sum\limits_{\jmath=1}^{\mathfrak{n}} \omega_\jmath = 1$. The DMs' group is denoted by $\{\mathfrak{D}_1, \mathfrak{D}_2, \ldots, \mathfrak{D}_e\}$ with weighting vector $\omega' = (\omega'_1, \omega'_2, \ldots, \omega'_e)^T$, satisfying $\omega'_h \in [0, 1]$, $\sum\limits_{h=1}^{e} \omega'_h = 1$. For attributes $\mathcal{N}_\jmath$ $(\jmath = 1, 2, \ldots, \mathfrak{n})$, the evaluation value of each alternative $\mathcal{M}_\imath$ $(\imath = 1, 2, \ldots, \mathfrak{m})$ given by DM $\mathfrak{D}_h$ $(h = 1, 2, \ldots, e)$ is expressed by the 2TLq-ROFN $\eta^h_{\imath\jmath} = ((s_{p^h_{\imath\jmath}}, \wp^h_{\imath\jmath}), (s_{l^h_{\imath\jmath}}, \pounds^h_{\imath\jmath}))$.

4.2 Calculation steps of the MAGDM method

The steps of the novel MAGDM approach are enumerated below:

Step 1. Normalize the decision matrices by translating the cost-type attributes to benefit-type utilizing Eq. (4.1) if the attributes are cost-type.

$$\eta_{\imath\jmath}^{h} = ((s_{p_{\imath\jmath}^{h}}, \wp_{\imath\jmath}^{h}), (s_{l_{\imath\jmath}^{h}}, \pounds_{\imath\jmath}^{h})), \begin{cases} ((s_{p_{\imath\jmath}^{h}}, \wp_{\imath\jmath}^{h}), (s_{l_{\imath\jmath}^{h}}, \pounds_{\imath\jmath}^{h})), \ \mathcal{N}_{\jmath} \in I_1 \\ ((s_{l_{\imath\jmath}^{h}}, \pounds_{\imath\jmath}^{h}), (s_{p_{\imath\jmath}^{h}}, \wp_{\imath\jmath}^{h})), \ \mathcal{N}_{\jmath} \in I_2 \end{cases} \tag{4.1}$$

where I_1 and I_2 are the indices of the benefit-type and the cost-type attributes, respectively.

Step 2. Compute the support degrees $\mathfrak{S}\ (\eta_{\imath\jmath}^{h}, \eta_{\imath\jmath}^{d})$ using Eq. (4.2):

$$\mathfrak{S}(\eta_{\imath\jmath}^{h}, \eta_{\imath\jmath}^{d}) = 1 - d(\eta_{\imath\jmath}^{h}, \eta_{\imath\jmath}^{d}), \ h, d = 1, 2, \ldots, e, \ h \neq d \tag{4.2}$$

where $d(\eta_{\imath\jmath}^{h}, \eta_{\imath\jmath}^{d})$ represents the HD between $\eta_{\imath\jmath}^{h}$ and $\eta_{\imath\jmath}^{d}$.

Step 3. Determine the weighted support matrices $[\mathcal{T}(\eta_{\imath\jmath}^{h})]_{\mathfrak{m}\times\mathfrak{n}}$ using Eq. (4.3) and the power weight matrices $[\lambda_{\imath\jmath}^{h}]_{\mathfrak{m}\times\mathfrak{n}}(h = 1, 2, \ldots, e)$ using Eq. (4.4):

$$\mathcal{T}(\eta_{\imath\jmath}^{h}) = \sum_{h,d=1;h\neq d}^{e} \mathfrak{S}(\eta_{\imath\jmath}^{h}, \eta_{\imath\jmath}^{d}) \tag{4.3}$$

$$\gamma_{\imath\jmath}^{h} = \frac{\omega_h' \left(1 + \mathcal{T}(\eta_{\imath\jmath}^{h})\right)}{\sum\limits_{h=1}^{e} \omega_h' \left(1 + \mathcal{T}(\eta_{\imath\jmath}^{h})\right)} \tag{4.4}$$

Step 4. Utilize the 2TLq-ROFPWMSM operator from Eq. (3.2) to aggregate all decision matrices $\eta_{\imath\jmath}^{h}$ $(h = 1, 2, \ldots, e)$ into a collective matrix $(\eta_{\imath\jmath})_{\mathfrak{m}\times\mathfrak{n}}$.

$$\eta_{\imath\jmath} = 2\text{TL}q\text{-ROFPWMSM}(\eta_{\imath\jmath}^{1}, \eta_{\imath\jmath}^{2}, \ldots, \eta_{\imath\jmath}^{e}). \tag{4.5}$$

Step 5. Compute the support degree $\mathfrak{S}\ (\eta_{\imath\jmath}, \eta_{\imath k})$ using Eq. (4.6):

$$\mathfrak{S}(\eta_{\imath\jmath}, \eta_{\imath k}) = 1 - d(\eta_{\imath\jmath}, \eta_{\imath k}), \ \jmath, k = 1, 2, \ldots, \mathfrak{n}, \ \jmath \neq k \tag{4.6}$$

Step 6. Calculate the comprehensive weighted support matrix $[\mathcal{T}(\eta_{\imath\jmath})]_{\mathfrak{m}\times\mathfrak{n}}$ using Eq. (4.7) and the comprehensive power weight matrix $[\gamma_{\imath\jmath}]_{\mathfrak{m}\times\mathfrak{n}}$ using Eq. (4.8):

$$\mathcal{T}(\eta_{\imath\jmath}) = \sum_{\jmath,k=1,\jmath\neq k}^{\mathfrak{n}} \omega_{\jmath} \mathfrak{S}(\eta_{\imath\jmath}, \eta_{\imath k}) \tag{4.7}$$

$$\gamma_{\imath\jmath} = \frac{\omega_{\jmath} \left(1 + \mathcal{T}(\eta_{\imath\jmath})\right)}{\sum\limits_{\jmath=1}^{\mathfrak{n}} \omega_{\jmath} \left(1 + \mathcal{T}(\eta_{\imath\jmath})\right)} \tag{4.8}$$

Step 7. Utilize the 2TLq-ROFPWMSM operator from Eq. (3.2) to calculate the comprehensive value $\eta_{\imath}$ of each alternative $\mathcal{M}_{\imath}$.

$$\eta_{\imath} = 2\text{TL}q\text{-ROFPWMSM}(\eta_{\imath 1}, \eta_{\imath 2}, \ldots, \eta_{\imath\mathfrak{n}}) \tag{4.9}$$

Step 8. Compute the score values $Sc(\eta_{\imath})$ $(\imath = 1, 2, \ldots, \mathfrak{m})$ of the calculated comprehensive values using Eq. (2.2), and if the score values are equal then compute the accuracy values $Ac(\eta_{\imath})$ $(\imath = 1, 2, \ldots, \mathfrak{m})$ using Eq. (2.3).

Step 9. Sort the scores of alternatives in the descending order, the largest score value represents the best choice.

Figure 1 describes the framework of the proposed approach.

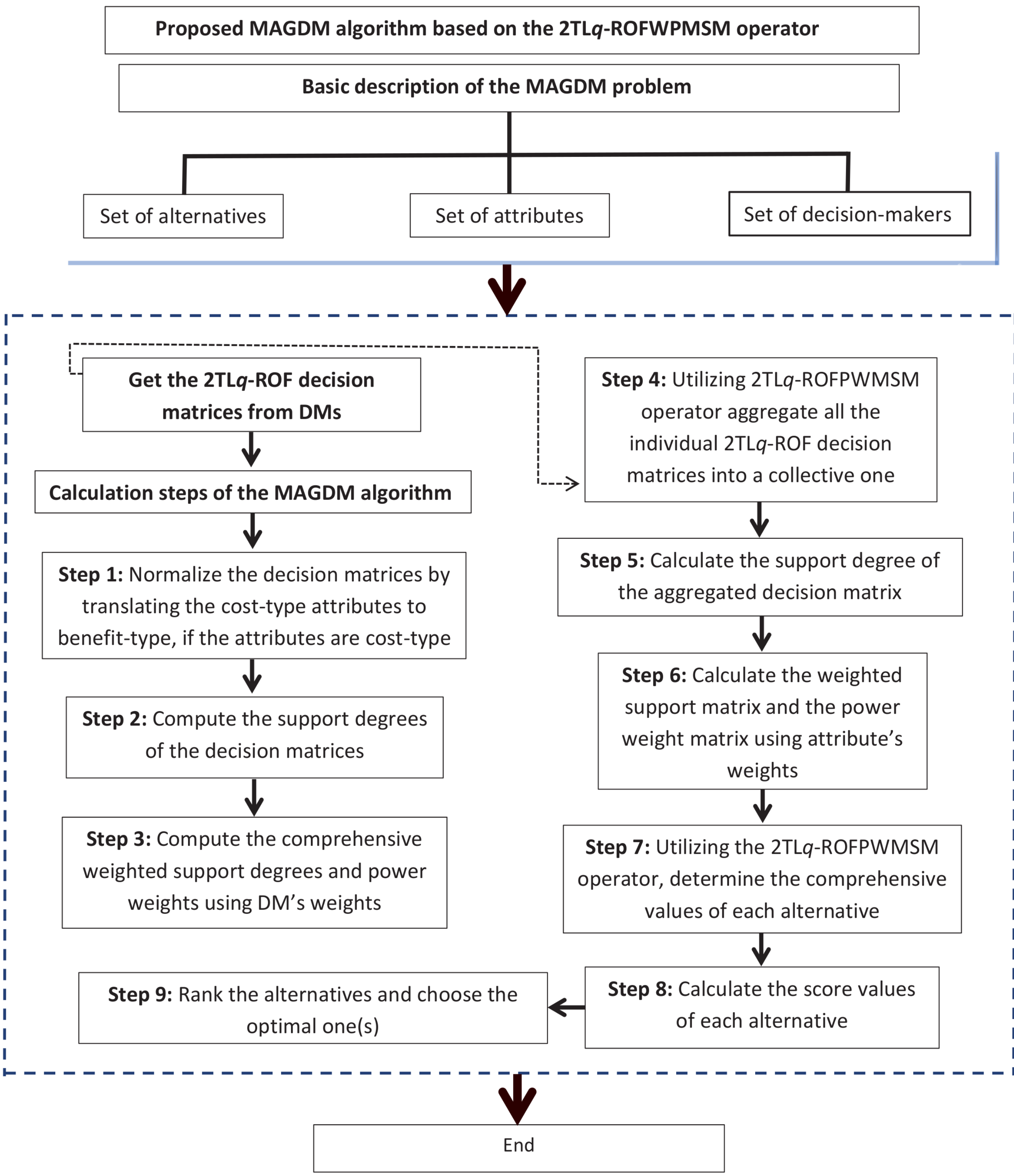

Figure 1: Framework of proposed approach.

5 Numerical analysis to select a technology for MSW management

Municipal solid waste (MSW), also known as trash or crap, is a mixture of everyday products that we use and then discard, such as product labels, yard waste, interiors, clothing, bottles, food scraps, newspapers, electronics, and paint. Our homes, schools, hospitals, and workplaces are the main sources of MSW. In many developing countries, sustainable MSW management is a major concern. The state of solid waste management in Pakistan is a major source of concern, as over 5 million people die each year as a consequence of garbage-related diseases. All major cities, including Islamabad, Lahore, Peshawar, and Karachi, face significant challenges in dealing with urban trash. Even though numerous studies have been conducted to address Pakistan's MSW problem, new tools and strategies can still be used to provide more effective solutions. In order to rate the validity of our proposed approach, a numerical example is illustrated.

5.1 Numerical example

Karachi is Pakistan's largest while the world's twelfth-largest city. The population of Karachi is approximately 20 million (20,678,896) in 2020. Karachi has industrial areas that generate a tremendous volume of MSW and have high energy production potential. However, due to the country's industrial zone and rapid population growth, Karachi is Pakistan's largest MSW-producing city. Every day, the city generates 12,000 tonnes of MSW, with 52% of it being organic trash. In Pakistan, there is currently no comprehensive waste management system in place. Lack of urban planning, ancient infrastructure, a lack of public perception, and widespread corruption are the major causes of Karachi's exacerbated garbage problem. 60% of MSW generated in Karachi is compiled and publicly disposed of in so-called landfill sites situated beyond the city. While the remaining 40% of waste is placed over streets and along roadways, endangering public health.

The management of a large volume of MSW generated on daily basis has become a serious problem for Karachi's municipal authority. The majority of municipal organizations do not take charge of the volume, composition, or characteristics of garbage. One of the biggest obstacles to adequately planning waste treatment and disposal facilities is a lack of essential information. Despite the fact that many strategies have been implemented to reduce MSW in Karachi. Since, the new strategies are being considered to deal with MSW problems, the aim is to choose the optimal technique from a range of solutions. We have a set of four DMs $\{\mathfrak{D}_1, \mathfrak{D}_2, \mathfrak{D}_3, \mathfrak{D}_4\}$. They worked on several multi-criteria decision-making initiatives. Here, DMs provide nine technologies (alternatives) for MSW management: (1) Landfills ($\mathcal{M}_1$); (2) Anaerobic digestion ($\mathcal{M}_2$); (3) Incineration ($\mathcal{M}_3$); (4) Pyrolysis ($\mathcal{M}_4$); (5) Plasma ($\mathcal{M}_5$); (6) Gasification ($\mathcal{M}_6$); (7) Recycling ($\mathcal{M}_7$); (8) Waste to energy ($\mathcal{M}_8$); (9) Composting ($\mathcal{M}_9$).

To analyze waste disposal technologies, thereby incorporating the aforementioned factors, DMs identify four attributes for the sustainable MSW management issue in terms of technical, environmental, economical, and social factors. They are as follows: emission ($\mathcal{N}_1$) reflects the amount of gas released into the atmosphere, land, and water as a result of the disposal process; energy recovery ($\mathcal{N}_2$) reflects the rate at which waste disposal technology can be used to generate electricity or heat; technology accessibility ($\mathcal{N}_3$) considers the accessibility and limitations while examining the accessibility to waste disposal technology from national or foreign organizations; social acceptance ($\mathcal{N}_4$) is a criterion for evaluating the public perception of waste disposal methods regarding social scenarios. The nine waste disposal technologies $\mathcal{M}_\imath$ $(\imath = 1, 2, \ldots, 9)$ are evaluated under the 2TLq-ROFNs by DMs according to four attributes $\mathcal{N}_\jmath$ $(\jmath = 1, 2, 3, 4)$ with weighting vector $\omega = (0.25, 0.24, 0.26, 0.25)^T$. Decision matrices provided by four DMs with weighting vector $\omega' = (0.3, 0.1, 0.2, 0.4)^T$ are presented in Table 1. Suppose $q = 4,\ \kappa = 3$ and $\tau = 8$.

Table 1: 2TLq-ROF decision matrix of each DM.

$\mathfrak{D}_1$				
	$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$	$\mathcal{N}_4$
$\mathcal{M}_1$	$((s_5,0),(s_3,0))$	$((s_7,0),(s_3,0))$	$((s_6,0),(s_6,0))$	$((s_4,0),(s_7,0))$
$\mathcal{M}_2$	$((s_5,0),(s_3,0))$	$((s_4,0),(s_4,0))$	$((s_7,0),(s_1,0))$	$((s_3,0),(s_5,0))$
$\mathcal{M}_3$	$((s_7,0),(s_2,0))$	$((s_8,0),(s_3,0))$	$((s_3,0),(s_4,0))$	$((s_5,0),(s_7,0))$
$\mathcal{M}_4$	$((s_5,0),(s_5,0))$	$((s_2,0),(s_7,0))$	$((s_7,0),(s_4,0))$	$((s_6,0),(s_5,0))$
$\mathcal{M}_5$	$((s_6,0),(s_5,0))$	$((s_4,0),(s_4,0))$	$((s_8,0),(s_3,0))$	$((s_7,0),(s_1,0))$
$\mathcal{M}_6$	$((s_5,0),(s_3,0))$	$((s_5,0),(s_6,0))$	$((s_3,0),(s_6,0))$	$((s_3,0),(s_7,0))$
$\mathcal{M}_7$	$((s_6,0),(s_7,0))$	$((s_6,0),(s_4,0))$	$((s_5,0),(s_5,0))$	$((s_2,0),(s_1,0))$
$\mathcal{M}_8$	$((s_3,0),(s_5,0))$	$((s_4,0),(s_4,0))$	$((s_6,0),(s_2,0))$	$((s_7,0),(s_3,0))$
$\mathcal{M}_9$	$((s_4,0),(s_7,0))$	$((s_7,0),(s_2,0))$	$((s_2,0),(s_4,0))$	$((s_4,0),(s_6,0))$
$\mathfrak{D}_2$				
	$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$	$\mathcal{N}_4$
$\mathcal{M}_1$	$((s_1,0),(s_8,0))$	$((s_2,0),(s_8,0))$	$((s_5,0),(s_3,0))$	$((s_4,0),(s_5,0))$
$\mathcal{M}_2$	$((s_6,0),(s_4,0))$	$((s_5,0),(s_2,0))$	$((s_3,0),(s_8,0))$	$((s_7,0),(s_1,0))$
$\mathcal{M}_3$	$((s_5,0),(s_3,0))$	$((s_5,0),(s_6,0))$	$((s_3,0),(s_6,0))$	$((s_3,0),(s_8,0))$
$\mathcal{M}_4$	$((s_6,0),(s_5,0))$	$((s_1,0),(s_7,0))$	$((s_6,0),(s_4,0))$	$((s_2,0),(s_7,0))$
$\mathcal{M}_5$	$((s_3,0),(s_5,0))$	$((s_6,0),(s_5,0))$	$((s_8,0),(s_0,0))$	$((s_3,0),(s_6,0))$
$\mathcal{M}_6$	$((s_6,0),(s_4,0))$	$((s_5,0),(s_3,0))$	$((s_6,0),(s_5,0))$	$((s_7,0),(s_2,0))$
$\mathcal{M}_7$	$((s_5,0),(s_7,0))$	$((s_6,0),(s_6,0))$	$((s_5,0),(s_3,0))$	$((s_7,0),(s_5,0))$
$\mathcal{M}_8$	$((s_5,0),(s_3,0))$	$((s_4,0),(s_4,0))$	$((s_7,0),(s_1,0))$	$((s_3,0),(s_5,0))$
$\mathcal{M}_9$	$((s_6,0),(s_4,0))$	$((s_1,0),(s_4,0))$	$((s_5,0),(s_7,0))$	$((s_3,0),(s_6,0))$
$\mathfrak{D}_3$				
	$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$	$\mathcal{N}_4$
$\mathcal{M}_1$	$((s_5,0),(s_8,0))$	$((s_3,0),(s_7,0))$	$((s_2,0),(s_6,0))$	$((s_5,0),(s_5,0))$
$\mathcal{M}_2$	$((s_4,0),(s_4,0))$	$((s_8,0),(s_3,0))$	$((s_6,0),(s_8,0))$	$((s_7,0),(s_4,0))$
$\mathcal{M}_3$	$((s_6,0),(s_4,0))$	$((s_2,0),(s_8,0))$	$((s_3,0),(s_7,0))$	$((s_6,0),(s_6,0))$
$\mathcal{M}_4$	$((s_6,0),(s_2,0))$	$((s_4,0),(s_6,0))$	$((s_7,0),(s_1,0))$	$((s_6,0),(s_5,0))$
$\mathcal{M}_5$	$((s_5,0),(s_5,0))$	$((s_4,0),(s_4,0))$	$((s_6,0),(s_3,0))$	$((s_7,0),(s_1,0))$
$\mathcal{M}_6$	$((s_5,0),(s_3,0))$	$((s_5,0),(s_6,0))$	$((s_3,0),(s_5,0))$	$((s_6,0),(s_6,0))$
$\mathcal{M}_7$	$((s_6,0),(s_2,0))$	$((s_2,0),(s_6,0))$	$((s_7,0),(s_3,0))$	$((s_1,0),(s_7,0))$
$\mathcal{M}_8$	$((s_6,0),(s_4,0))$	$((s_4,0),(s_4,0))$	$((s_5,0),(s_7,0))$	$((s_3,0),(s_6,0))$
$\mathcal{M}_9$	$((s_6,0),(s_5,0))$	$((s_0,0),(s_7,0))$	$((s_7,0),(s_4,0))$	$((s_3,0),(s_5,0))$
$\mathfrak{D}_4$				
	$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$	$\mathcal{N}_4$
$\mathcal{M}_1$	$((s_8,0),(s_3,0))$	$((s_1,0),(s_7,0))$	$((s_6,0),(s_5,0))$	$((s_3,0),(s_5,0))$
$\mathcal{M}_2$	$((s_4,0),(s_4,0))$	$((s_5,0),(s_7,0))$	$((s_5,0),(s_8,0))$	$((s_7,0),(s_5,0))$
$\mathcal{M}_3$	$((s_7,0),(s_4,0))$	$((s_8,0),(s_0,0))$	$((s_7,0),(s_3,0))$	$((s_1,0),(s_7,0))$
$\mathcal{M}_4$	$((s_6,0),(s_2,0))$	$((s_2,0),(s_8,0))$	$((s_7,0),(s_1,0))$	$((s_6,0),(s_6,0))$
$\mathcal{M}_5$	$((s_5,0),(s_5,0))$	$((s_4,0),(s_4,0))$	$((s_8,0),(s_3,0))$	$((s_7,0),(s_1,0))$
$\mathcal{M}_6$	$((s_5,0),(s_3,0))$	$((s_7,0),(s_3,0))$	$((s_3,0),(s_5,0))$	$((s_4,0),(s_6,0))$
$\mathcal{M}_7$	$((s_4,0),(s_5,0))$	$((s_2,0),(s_6,0))$	$((s_6,0),(s_6,0))$	$((s_1,0),(s_7,0))$
$\mathcal{M}_8$	$((s_5,0),(s_5,0))$	$((s_1,0),(s_8,0))$	$((s_7,0),(s_4,0))$	$((s_3,0),(s_5,0))$
$\mathcal{M}_9$	$((s_8,0),(s_5,0))$	$((s_2,0),(s_8,0))$	$((s_5,0),(s_7,0))$	$((s_3,0),(s_5,0))$

5.2 Problem solution

In this subsection, we will apply our scheme based on 2TLq-ROFPWMSM operator to evaluate MSW management technologies.

5.2.1 Decision-making procedure based on the 2TLq-ROFPWMSM operator

Step 1. Normalize the decision matrices by translating the cost-type attributes to benefit-type utilizing Eq. (4.1) if the attributes are cost-type. It is explicit that all attributes are benefit-type, so the decision matrices do not require normalization.

Step 2. For each decision matrix η_{ij}^h, compute the support degrees using Eq. (4.2), $[\mathfrak{S}(\eta_{ij}^h, \eta_{ij}^d)]_{9\times 4}$ be the support degree between η_{ij}^h and η_{ij}^d. The computed results are as follows:

$$\mathfrak{S}^{12} = \mathfrak{S}^{21} = \begin{bmatrix} 0.4337 & 0.2188 & 0.7698 & 0.7832 \\ 0.8967 & 0.9257 & 0.2169 & 0.6406 \\ 0.7753 & 0.4280 & 0.8730 & 0.7267 \\ 0.9181 & 0.9982 & 0.8651 & 0.6270 \\ 0.8517 & 0.8280 & 0.9901 & 0.5587 \\ 0.8967 & 0.8517 & 0.7698 & 0.4257 \\ 0.9181 & 0.8730 & 0.9336 & 0.6327 \\ 0.8672 & 1.0000 & 0.8633 & 0.6504 \\ 0.6112 & 0.6777 & 0.6638 & 0.9786 \end{bmatrix} \quad \mathfrak{S}^{13} = \mathfrak{S}^{31} = \begin{bmatrix} 0.5099 & 0.4336 & 0.8438 & 0.7382 \\ 0.9336 & 0.5099 & 0.3652 & 0.6718 \\ 0.8358 & 0.0118 & 0.7382 & 0.7832 \\ 0.8438 & 0.8358 & 0.9689 & 1.0000 \\ 0.9181 & 1.0000 & 0.6582 & 1.0000 \\ 1.0000 & 1.0000 & 0.9181 & 0.7168 \\ 0.7089 & 0.7168 & 0.7168 & 0.7052 \\ 0.8066 & 1.0000 & 0.6270 & 0.5685 \\ 0.6563 & 0.4158 & 0.7089 & 0.8967 \end{bmatrix}$$

$$\mathfrak{S}^{14} = \mathfrak{S}^{41} = \begin{bmatrix} 0.5763 & 0.4238 & 0.9181 & 0.7618 \\ 0.9336 & 0.6931 & 0.2833 & 0.7168 \\ 0.9707 & 0.9901 & 0.6954 & 0.9238 \\ 0.8438 & 0.7931 & 0.9689 & 0.9181 \\ 0.9181 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0.6349 & 0.9181 & 0.8438 \\ 0.6563 & 0.7168 & 0.8362 & 0.7052 \\ 0.9336 & 0.5001 & 0.8358 & 0.6504 \\ 0.3145 & 0.2108 & 0.6638 & 0.8967 \end{bmatrix} \quad \mathfrak{S}^{23} = \mathfrak{S}^{32} = \begin{bmatrix} 0.9238 & 0.7852 & 0.7773 & 0.9550 \\ 0.8730 & 0.5684 & 0.8517 & 0.9689 \\ 0.8967 & 0.5839 & 0.8651 & 0.5099 \\ 0.9257 & 0.8340 & 0.8340 & 0.6270 \\ 0.9336 & 0.8280 & 0.6483 & 0.5587 \\ 0.8967 & 0.8517 & 0.8517 & 0.7089 \\ 0.6270 & 0.8438 & 0.7832 & 0.4902 \\ 0.8967 & 1.0000 & 0.4902 & 0.9181 \\ 0.9550 & 0.7380 & 0.5214 & 0.9181 \end{bmatrix}$$

$$\mathfrak{S}^{24} = \mathfrak{S}^{42} = \begin{bmatrix} 0.0100 & 0.7913 & 0.8517 & 0.9786 \\ 0.8730 & 0.7089 & 0.9336 & 0.9238 \\ 0.7618 & 0.4181 & 0.5685 & 0.7833 \\ 0.9257 & 0.7913 & 0.8340 & 0.7089 \\ 0.9336 & 0.8280 & 0.9901 & 0.5587 \\ 0.8967 & 0.7832 & 0.8517 & 0.5819 \\ 0.7382 & 0.8438 & 0.7698 & 0.4902 \\ 0.9336 & 0.5001 & 0.9689 & 1.0000 \\ 0.6132 & 0.5294 & 1.0000 & 0.9181 \end{bmatrix} \quad \mathfrak{S}^{34} = \mathfrak{S}^{43} = \begin{bmatrix} 0.0862 & 0.9902 & 0.7618 & 0.9336 \\ 1.0000 & 0.2931 & 0.9181 & 0.9550 \\ 0.8651 & 0.0020 & 0.4336 & 0.7070 \\ 1.0000 & 0.6289 & 1.0000 & 0.9181 \\ 1.0000 & 1.0000 & 0.6582 & 1.0000 \\ 1.0000 & 0.6349 & 1.0000 & 0.8730 \\ 0.7987 & 1.0000 & 0.7168 & 1.0000 \\ 0.8730 & 0.5001 & 0.5214 & 0.9181 \\ 0.6582 & 0.7911 & 0.5214 & 1.0000 \end{bmatrix}.$$

Step 3. Compute the comprehensive weighted support matrices $\mathcal{T}(\eta_{ij}^h)$ using Eq. (4.3) and power weight matrices γ_{ij}^h using Eq. (4.4). The calculated outcomes are given below:

$$\mathcal{T}^1 = \begin{bmatrix} 0.3759 & 0.2781 & 0.6130 & 0.5307 \\ 0.6498 & 0.4718 & 0.2081 & 0.4851 \\ 0.6330 & 0.4412 & 0.5131 & 0.5988 \\ 0.5981 & 0.5842 & 0.6678 & 0.6299 \\ 0.6360 & 0.6828 & 0.6307 & 0.6559 \\ 0.6897 & 0.5391 & 0.6278 & 0.5234 \\ 0.4961 & 0.5174 & 0.5712 & 0.4864 \\ 0.6215 & 0.5000 & 0.5460 & 0.4389 \\ 0.3182 & 0.2353 & 0.4737 & 0.6359 \end{bmatrix} \quad \mathcal{T}^2 = \begin{bmatrix} 0.3189 & 0.5392 & 0.7271 & 0.8174 \\ 0.7928 & 0.6749 & 0.6089 & 0.7555 \\ 0.7167 & 0.4124 & 0.6623 & 0.6333 \\ 0.8308 & 0.7828 & 0.7599 & 0.5970 \\ 0.8157 & 0.7452 & 0.8227 & 0.5028 \\ 0.8071 & 0.7391 & 0.7419 & 0.5022 \\ 0.6961 & 0.7682 & 0.7446 & 0.4839 \\ 0.8129 & 0.7000 & 0.7446 & 0.7787 \\ 0.6196 & 0.5627 & 0.7034 & 0.8444 \end{bmatrix}$$

$$\mathcal{T}^3 = \begin{bmatrix} 0.2798 & 0.6047 & 0.6356 & 0.6904 \\ 0.7674 & 0.3270 & 0.5620 & 0.6804 \\ 0.6865 & 0.0627 & 0.4814 & 0.5688 \\ 0.7457 & 0.5857 & 0.7741 & 0.7299 \\ 0.7688 & 0.7828 & 0.5256 & 0.7559 \\ 0.7897 & 0.6391 & 0.7606 & 0.6351 \\ 0.5948 & 0.6994 & 0.5801 & 0.6606 \\ 0.6809 & 0.6000 & 0.4457 & 0.6296 \\ 0.5557 & 0.5150 & 0.4733 & 0.7608 \end{bmatrix} \quad \mathcal{T}^4 = \begin{bmatrix} 0.1911 & 0.4043 & 0.5130 & 0.5131 \\ 0.5674 & 0.3374 & 0.3620 & 0.4984 \\ 0.5404 & 0.3392 & 0.3522 & 0.4969 \\ 0.5457 & 0.4428 & 0.5741 & 0.5299 \\ 0.5688 & 0.5828 & 0.5307 & 0.5559 \\ 0.5897 & 0.3958 & 0.5606 & 0.4859 \\ 0.4304 & 0.4994 & 0.4712 & 0.4606 \\ 0.5480 & 0.3001 & 0.4519 & 0.4787 \\ 0.2873 & 0.2744 & 0.4034 & 0.5608 \end{bmatrix}$$

$$\gamma^1 = \begin{bmatrix} 0.3232 & 0.2700 & 0.3045 & 0.2899 \\ 0.2991 & 0.3133 & 0.2625 & 0.2862 \\ 0.3033 & 0.3271 & 0.3115 & 0.3084 \\ 0.2941 & 0.3071 & 0.3013 & 0.3043 \\ 0.2968 & 0.3025 & 0.3079 & 0.3065 \\ 0.3015 & 0.3034 & 0.2980 & 0.2990 \\ 0.2973 & 0.2896 & 0.3040 & 0.2952 \\ 0.2997 & 0.3082 & 0.3075 & 0.2827 \\ 0.2858 & 0.2766 & 0.3011 & 0.2971 \end{bmatrix} \quad \gamma^2 = \begin{bmatrix} 0.1033 & 0.1084 & 0.1087 & 0.1147 \\ 0.1084 & 0.1188 & 0.1165 & 0.1128 \\ 0.1063 & 0.1069 & 0.1141 & 0.1050 \\ 0.1123 & 0.1152 & 0.1060 & 0.0994 \\ 0.1098 & 0.1046 & 0.1147 & 0.0927 \\ 0.1075 & 0.1143 & 0.1063 & 0.0983 \\ 0.1124 & 0.1125 & 0.1125 & 0.0982 \\ 0.1117 & 0.1164 & 0.1157 & 0.1165 \\ 0.1171 & 0.1167 & 0.1160 & 0.1117 \end{bmatrix}$$

$$\gamma^3 = \begin{bmatrix} 0.2004 & 0.2260 & 0.2059 & 0.2134 \\ 0.2136 & 0.1883 & 0.2263 & 0.2159 \\ 0.2088 & 0.1608 & 0.2033 & 0.2017 \\ 0.2142 & 0.2049 & 0.2136 & 0.2153 \\ 0.2139 & 0.2136 & 0.1920 & 0.2167 \\ 0.2129 & 0.2154 & 0.2149 & 0.2139 \\ 0.2113 & 0.2163 & 0.2038 & 0.2198 \\ 0.2071 & 0.2192 & 0.1917 & 0.2134 \\ 0.2249 & 0.2262 & 0.2007 & 0.2132 \end{bmatrix} \gamma^4 = \begin{bmatrix} 0.3731 & 0.3956 & 0.3809 & 0.3820 \\ 0.3789 & 0.3796 & 0.3946 & 0.3851 \\ 0.3815 & 0.4053 & 0.3711 & 0.3849 \\ 0.3793 & 0.3729 & 0.3791 & 0.3809 \\ 0.3795 & 0.3793 & 0.3853 & 0.3840 \\ 0.3782 & 0.3669 & 0.3809 & 0.3888 \\ 0.3790 & 0.3816 & 0.3796 & 0.3867 \\ 0.3815 & 0.3562 & 0.3851 & 0.3874 \\ 0.3722 & 0.3805 & 0.3823 & 0.3780 \end{bmatrix}.$$

Step 4. Utilizing 2TLq-ROFPWMSM operator from Eq. (4.5), aggregate all the individual 2TLq-ROF decision matrices η_{ij}^h $(h = 1, 2, \ldots, 4)$ into a collective one η_{ij} given in Table 2.

Table 2: Collective 2TLq-ROF decision matrix based on 2TLq-ROFPWMSM operator.

	$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$	$\mathcal{N}_4$
$\mathcal{M}_1$	$((s_5, -0.0018), (s_8, 0.0000))$	$((s_3, -0.2493), (s_7, 0.2367))$	$((s_5, -0.3610), (s_6, -0.0041))$	$((s_4, -0.3533), (s_6, 0.4066))$
$\mathcal{M}_2$	$((s_4, 0.3282), (s_5, 0.0681))$	$((s_5, 0.2223), (s_5, 0.4695))$	$((s_5, 0.1881), (s_8, 0.0000))$	$((s_6, -0.2072), (s_5, -0.1522))$
$\mathcal{M}_3$	$((s_6, -0.0836), (s_5, -0.2245))$	$((s_6, -0.4050), (s_7, -0.0835))$	$((s_4, -0.4042), (s_6, 0.4723))$	$((s_3, 0.4930), (s_7, 0.3547))$
$\mathcal{M}_4$	$((s_5, 0.3036), (s_5, 0.2608))$	$((s_2, 0.1712), (s_8, -0.4420))$	$((s_6, 0.3581), (s_5, -0.3871))$	$((s_5, 0.2049), (s_6, 0.4870))$
$\mathcal{M}_5$	$((s_5, -0.3726), (s_6, -0.1125))$	$((s_4, 0.0688), (s_5, 0.4386))$	$((s_8, 0.0000), (s_3, 0.3833))$	$((s_6, 0.1469), (s_4, 0.1924))$
$\mathcal{M}_6$	$((s_5, -0.1988), (s_5, -0.2618))$	$((s_5, 0.0970), (s_6, -0.1570))$	$((s_3, 0.3334), (s_6, 0.1065))$	$((s_5, -0.4125), (s_6, 0.3706))$
$\mathcal{M}_7$	$((s_5, -0.1591), (s_7, -0.3924))$	$((s_4, -0.4388), (s_6, 0.3627))$	$((s_5, 0.3833), (s_5, 0.3685))$	$((s_2, 0.0711), (s_7, -0.2989))$
$\mathcal{M}_8$	$((s_4, 0.3893), (s_5, 0.2639))$	$((s_3, 0.1467), (s_7, -0.3843))$	$((s_6, -0.1684), (s_5, 0.2309))$	$((s_4, -0.3888), (s_6, -0.0633))$
$\mathcal{M}_9$	$((s_6, -0.4571), (s_6, 0.2297))$	$((s_2, -0.0309), (s_7, 0.1290))$	$((s_5, -0.4686), (s_7, -0.4172))$	$((s_3, -0.0005), (s_6, 0.2889))$

Step 5. Compute the support degree $\mathfrak{S}(\eta_{ij}, \eta_{ik})$ of collective 2TLq-ROF decision matrix between η_{ij} and η_{ik} using Eq. (4.6).

$$\mathfrak{S} = \begin{bmatrix} 0.7656 & 0.6381 & 0.6510 & 0.7734 & 0.8563 & 0.9172 \\ 0.9233 & 0.5349 & 0.8923 & 0.6069 & 0.9115 & 0.5184 \\ 0.7542 & 0.7201 & 0.5749 & 0.8356 & 0.8207 & 0.8548 \\ 0.6013 & 0.8589 & 0.8703 & 0.4602 & 0.7310 & 0.7292 \\ 0.9376 & 0.4253 & 0.7727 & 0.4427 & 0.7901 & 0.6526 \\ 0.9017 & 0.8420 & 0.8497 & 0.9052 & 0.9129 & 0.9297 \\ 0.9200 & 0.8332 & 0.9218 & 0.8184 & 0.9365 & 0.7550 \\ 0.8265 & 0.9018 & 0.9175 & 0.7284 & 0.9090 & 0.8193 \\ 0.7552 & 0.8909 & 0.8876 & 0.8643 & 0.8676 & 0.9201 \end{bmatrix}$$

Step 6. Calculate the comprehensive weighted support matrix $[\mathcal{T}(\eta_{ij})]_{9\times 4}$ using Eq. (4.7) and comprehensive power weight matrix $\gamma = [\gamma_{ij}]_{9\times 4}$ using Eq. (4.8).

$$\mathcal{T} = \begin{bmatrix} 0.5137 & 0.5749 & 0.6055 & 0.6061 \\ 0.5876 & 0.5860 & 0.4317 & 0.5805 \\ 0.5123 & 0.5785 & 0.6267 & 0.5626 \\ 0.5826 & 0.4302 & 0.5325 & 0.5826 \\ 0.5339 & 0.5209 & 0.3953 & 0.5539 \\ 0.6483 & 0.6528 & 0.6960 & 0.6731 \\ 0.6687 & 0.6420 & 0.6257 & 0.6533 \\ 0.6615 & 0.5913 & 0.6369 & 0.6615 \\ 0.6334 & 0.5969 & 0.6956 & 0.6688 \end{bmatrix} \gamma = \begin{bmatrix} 0.2402 & 0.2399 & 0.2650 & 0.2549 \\ 0.2569 & 0.2464 & 0.2409 & 0.2558 \\ 0.2407 & 0.2412 & 0.2693 & 0.2487 \\ 0.2581 & 0.2239 & 0.2599 & 0.2581 \\ 0.2557 & 0.2434 & 0.2419 & 0.2590 \\ 0.2471 & 0.2378 & 0.2644 & 0.2508 \\ 0.2533 & 0.2392 & 0.2566 & 0.2509 \\ 0.2535 & 0.2331 & 0.2598 & 0.2535 \\ 0.2475 & 0.2323 & 0.2672 & 0.2529 \end{bmatrix}.$$

Step 7. Utilize the 2TLq-ROFPWMSM operator from Eq. (4.9), to fuse all η_{ij} into the comprehensive value η_i of the alternative $\mathcal{M}_i$. The results are shown in Table 3.

Table 3: Aggregated 2TLq-ROF decision matrix using 2TLq-ROFPWMSM operator.

Alternatives	2TLq-ROFNs
$\mathcal{M}_1$	$((s_4, -0.2108), (s_7, 0.4016))$
$\mathcal{M}_2$	$((s_5, -0.1833), (s_7, -0.2501))$
$\mathcal{M}_3$	$((s_4, 0.3696), (s_7, -0.0474))$
$\mathcal{M}_4$	$((s_5, -0.2560), (s_7, -0.2698))$
$\mathcal{M}_5$	$((s_5, 0.4969), (s_6, -0.4183))$
$\mathcal{M}_6$	$((s_4, 0.1867), (s_6, 0.3315))$
$\mathcal{M}_7$	$((s_4, -0.1825), (s_7, -0.2828))$
$\mathcal{M}_8$	$((s_4, -0.0164), (s_6, 0.3299))$
$\mathcal{M}_9$	$((s_4, -0.4071), (s_7, -0.0726))$

Step 8. Compute the score values $Sc(\eta_\imath)$ $(\imath = 1, 2, \ldots, 9)$ of all the calculated comprehensive values according to Definition 2.3.

Step 9. Sort the scores of alternatives in the descending order and the best technology for waste disposal is plasma $\mathcal{M}_5$. Score values and ranking results are shown in Table 4.

Table 4: Scores and ranking results of alternatives.

	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$
Scores	0.1588	0.3123	0.2593	0.3114	0.4930	0.3413	0.2774	0.3348	0.2392
Ranking	9	4	7	5	1	2	6	3	8

5.3 The effects of parameters q and κ on experimental results

In this portion, sensitivity analysis is carried out to evaluate the robustness of the proposed decision-making outcomes. We examine the relevance of q and κ to MSW management technologies. The parameter κ in the 2TLq-ROFPWMSM operator is noted to have a considerable impact on the decision-making results. In the proposed method, we rank the waste disposal technologies using different values of parameter κ. First, we establish the ranking results of the alternatives when the values of κ vary $(\kappa = 1, 2, 3, 4)$ (suppose $q = 4$). Table 5 shows the ranking results for different values of κ based on 2TLq-ROFPWMSM operator. The ranking results of the alternatives differ when κ is changed. However, the optimal alternative obtained using various values of κ is the same, namely, plasma $(\mathcal{M}_5)$.

Table 5: Ranking with different parameter κ utilizing 2TLq-ROFPWMSM operator (q=4).

Parameter	2TLq-ROFPWMSM operator
κ=1	$\mathcal{M}_5 > \mathcal{M}_3 > \mathcal{M}_2 > \mathcal{M}_1 > \mathcal{M}_4 > \mathcal{M}_8 > \mathcal{M}_6 > \mathcal{M}_7 > \mathcal{M}_9$
κ=2	$\mathcal{M}_5 > \mathcal{M}_3 > \mathcal{M}_4 > \mathcal{M}_8 > \mathcal{M}_2 > \mathcal{M}_6 > \mathcal{M}_7 > \mathcal{M}_1 > \mathcal{M}_9$
κ=3	$\mathcal{M}_5 > \mathcal{M}_6 > \mathcal{M}_8 > \mathcal{M}_2 > \mathcal{M}_4 > \mathcal{M}_7 > \mathcal{M}_3 > \mathcal{M}_9 > \mathcal{M}_1$
κ=4	$\mathcal{M}_5 > \mathcal{M}_6 > \mathcal{M}_7 > \mathcal{M}_2 > \mathcal{M}_4 > \mathcal{M}_3 > \mathcal{M}_8 > \mathcal{M}_1 > \mathcal{M}_9$

The parameter q in the 2TLq-ROFPWMSM operator is noted to have a considerable impact on the decision-making results. In the proposed method, we rank the waste disposal technologies using different values of parameter q. First, we establish the ranking of the alternatives when the values of q vary $(q = 1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21)$ (suppose $\kappa = 3$). Table 6 shows the ranking results for different values of q based on 2TLq-ROFPWMSM operator. The ranking results of the alternatives differ when q is changed. However, the optimal alternative obtained using various values of q is the same, namely, $\mathcal{M}_5$.

Depending on their risk aversion, DMs can select an adequate parameter value κ. For instance, if a DM prefers risk, he/she can set the parameter κ to a higher value. This research study is in full concordance with Qin and Liu's MAGDM technique [9], when the parameter $\kappa = \lfloor \frac{\mathfrak{m}}{2} \rfloor$, the risk attitude of the DM is neutral, and interrelations among the attributes are completely taken into consideration, where $\mathfrak{m}$ is the total number of alternatives, and the symbol $\lfloor \cdot \rfloor$ denotes the standard round function.

Table 6: Ranking with different parameter q utilizing 2TLq-ROFPWMSM operator (κ=3).

Parameter	2TLq-ROFPWMSM operator
q=1	$\mathcal{M}_5>\mathcal{M}_2>\mathcal{M}_6>\mathcal{M}_4>\mathcal{M}_8>\mathcal{M}_3>\mathcal{M}_7>\mathcal{M}_9>\mathcal{M}_1$
q=2	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_2>\mathcal{M}_8>\mathcal{M}_4>\mathcal{M}_3>\mathcal{M}_7>\mathcal{M}_9>\mathcal{M}_1$
q=3	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_2>\mathcal{M}_8>\mathcal{M}_4>\mathcal{M}_7>\mathcal{M}_3>\mathcal{M}_9>\mathcal{M}_1$
q=5	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_8>\mathcal{M}_4>\mathcal{M}_2>\mathcal{M}_7>\mathcal{M}_3>\mathcal{M}_9>\mathcal{M}_1$
q=7	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_8>\mathcal{M}_4>\mathcal{M}_2>\mathcal{M}_7>\mathcal{M}_9>\mathcal{M}_3>\mathcal{M}_1$
q=9	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_8>\mathcal{M}_7>\mathcal{M}_4>\mathcal{M}_2>\mathcal{M}_9>\mathcal{M}_3>\mathcal{M}_1$
q=11	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_8>\mathcal{M}_7>\mathcal{M}_2>\mathcal{M}_4>\mathcal{M}_9>\mathcal{M}_3>\mathcal{M}_1$
q=13	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_8>\mathcal{M}_7>\mathcal{M}_2>\mathcal{M}_4>\mathcal{M}_9>\mathcal{M}_3>\mathcal{M}_1$
q=15	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_8>\mathcal{M}_7>\mathcal{M}_2>\mathcal{M}_4>\mathcal{M}_9>\mathcal{M}_3>\mathcal{M}_1$
q=17	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_8>\mathcal{M}_7>\mathcal{M}_2>\mathcal{M}_4>\mathcal{M}_9>\mathcal{M}_3>\mathcal{M}_1$
q=19	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_8>\mathcal{M}_7>\mathcal{M}_2>\mathcal{M}_4>\mathcal{M}_9>\mathcal{M}_3>\mathcal{M}_1$
q=21	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_8>\mathcal{M}_7>\mathcal{M}_2>\mathcal{M}_4>\mathcal{M}_9>\mathcal{M}_3>\mathcal{M}_1$

5.4 Comparative analysis

We used several pre-existing algorithms to solve the above-shown numerical problem to validate the feasibility of our model, and we compared the experimental results. We tested the intuitionistic fuzzy weighted average (IFWA) operator [8], Pythagorean fuzzy weighted average (PFWA) operator [10], q-ROFWMSM operator [16], q-ROFPWMSM operator [17], 2TLq-ROF power weighted average (2TLq-ROFPWA) operator [14], 2TLq-ROF power weighted Bonferroni mean (2TLq-ROFPWBM) operator, 2TL intuitionistic fuzzy PWMSM (2TLIF-PWMSM) operator, 2TL Pythagorean fuzzy PWMSM (2TLPFPWMSM) operator and 2TLq-ROFPWMSM operator. Using these techniques, we systematically calculated the evaluation results to determine the best disposal technology. We set $q = 4$ and $\kappa = 3$, and Table 7 demonstrates the comparison.

Table 7: Alternatives ranking by various approaches.

Methods	Score values	Ranking
The IFWA operator proposed by Xu [8].		No
The PFWA operator proposed by Rahman et al. [10].		No
The q-ROFWMSM operator proposed by Wei et al. [16].		No
The q-ROFPWMSM operator proposed by Liu et al. [17].		No
The 2TLq-ROFPWA operator proposed by Yager [14].	$Sc(\mathcal{M}_1) = 0.5533$, $Sc(\mathcal{M}_2) = 0.5761$, $Sc(\mathcal{M}_3) = 0.9999$, $Sc(\mathcal{M}_4) = 0.0996$, $Sc(\mathcal{M}_5) = 1.0000$, $Sc(\mathcal{M}_6) = 0.0741$, $Sc(\mathcal{M}_7) = 0.0723$, $Sc(\mathcal{M}_8) = 0.0827$, $Sc(\mathcal{M}_9) = 0.5505$.	$\mathcal{M}_5>\mathcal{M}_3>\mathcal{M}_2>\mathcal{M}_1>\mathcal{M}_9>\mathcal{M}_4>\mathcal{M}_8>\mathcal{M}_6>\mathcal{M}_7$
The 2TLq-ROFPWBM operator (s = 1, r = 1, q = 4).	$Sc(\mathcal{M}_1) = 0.1578$, $Sc(\mathcal{M}_2) = 0.2138$, $Sc(\mathcal{M}_3) = 0.2643$, $Sc(\mathcal{M}_4) = 0.2245$, $Sc(\mathcal{M}_5) = 0.3879$, $Sc(\mathcal{M}_6) = 0.2088$, $Sc(\mathcal{M}_7) = 0.1770$, $Sc(\mathcal{M}_8) = 0.2235$, $Sc(\mathcal{M}_9) = 0.1554$.	$\mathcal{M}_5>\mathcal{M}_3>\mathcal{M}_4>\mathcal{M}_8>\mathcal{M}_2>\mathcal{M}_6>\mathcal{M}_7>\mathcal{M}_1>\mathcal{M}_9$
The 2TLIFPWMSM operator ($q = 1$ and $\kappa = 3$).	$Sc(\mathcal{M}_1) = 0.1936$, $Sc(\mathcal{M}_2) = 0.3135$, $Sc(\mathcal{M}_3) = 0.2690$, $Sc(\mathcal{M}_4) = 0.2983$, $Sc(\mathcal{M}_5) = 0.4464$, $Sc(\mathcal{M}_6) = 0.3026$, $Sc(\mathcal{M}_7) = 0.2517$, $Sc(\mathcal{M}_8) = 0.2898$, $Sc(\mathcal{M}_9) = 0.2138$.	$\mathcal{M}_5>\mathcal{M}_2>\mathcal{M}_6>\mathcal{M}_4>\mathcal{M}_8>\mathcal{M}_3>\mathcal{M}_7>\mathcal{M}_9>\mathcal{M}_1$
The 2TLPFPWMSM operator ($q = 2$ and $\kappa = 3$).	$Sc(\mathcal{M}_1) = 0.1570$, $Sc(\mathcal{M}_2) = 0.2999$, $Sc(\mathcal{M}_3) = 0.2501$, $Sc(\mathcal{M}_4) = 0.2914$, $Sc(\mathcal{M}_5) = 0.4694$, $Sc(\mathcal{M}_6) = 0.3012$, $Sc(\mathcal{M}_7) = 0.2411$, $Sc(\mathcal{M}_8) = 0.2933$, $Sc(\mathcal{M}_9) = 0.2036$.	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_2>\mathcal{M}_8>\mathcal{M}_4>\mathcal{M}_3>\mathcal{M}_7>\mathcal{M}_9>\mathcal{M}_1$
The 2TLq-ROFPWMSM operator proposed in this paper ($q = 4$ and $\kappa = 3$).	$Sc(\mathcal{M}_1) = 0.1588$, $Sc(\mathcal{M}_2) = 0.3123$, $Sc(\mathcal{M}_3) = 0.2593$, $Sc(\mathcal{M}_4) = 0.3114$, $Sc(\mathcal{M}_5) = 0.4930$, $Sc(\mathcal{M}_6) = 0.3413$, $Sc(\mathcal{M}_7) = 0.2774$, $Sc(\mathcal{M}_8) = 0.3348$, $Sc(\mathcal{M}_9) = 0.2392$.	$\mathcal{M}_5>\mathcal{M}_6>\mathcal{M}_8>\mathcal{M}_2>\mathcal{M}_4>\mathcal{M}_7>\mathcal{M}_3>\mathcal{M}_9>\mathcal{M}_1$

In various linguistic decision models, information aggregation operators such as the BM, HM, and MSM are widely used. In comparison to the BM and HM operators, the MSM can describe the relationships between any number of input arguments in a simplified mathematical form. Furthermore, the PA operator is a traditional and powerful aggregation technique that can reduce the impact of unreasonable data on ranking results by measuring the similarity of arguments and assigning smaller weights to data that is too high or too low.

It has been shown that the outcomes obtained using prevalent approaches are equivalent to the suggested strategy. This fact indicates that the outcomes induced by the proposed operators are precise and logical.

We derive the conclusion that while ranking lists given by the specified approach, the compared approaches are marginally different and the best option across all the approaches is the same based on the results of the proposed operators and the other conventional models. In particular, the IFWA operator is unable to solve the problem of complex evaluation information where the sum of MD and NMD exceeds 1. Also, 2TLq-ROFPWA operator fails to explain the interdependence between criteria or their impact on the ranking outcomes since they presume that the criteria are independent of one another. Furthermore, the q-ROFWMSM and q-ROFPWMSM operators are incapable of resolving the linguistic information problem.

Despite being adaptable and competent to capture the correlation between attributes, the 2TLq-ROFPWBM operator cannot allocate weights to correlations. The proposed approach also can take into account the relationships between two or three attributes, and it can explore a range of ranking outcomes. Since there is a lot of interdependence in actual MAGDM issues, the ranking results given by the suggested method for $\kappa = 3$ or $q = 4$ are significantly more reasonable than those obtained by other approaches. The best option remains the same for approaches based on dual operators. In Figure 2, a graphic representation of the ranking results of the alternatives based on their score values using various approaches is shown.

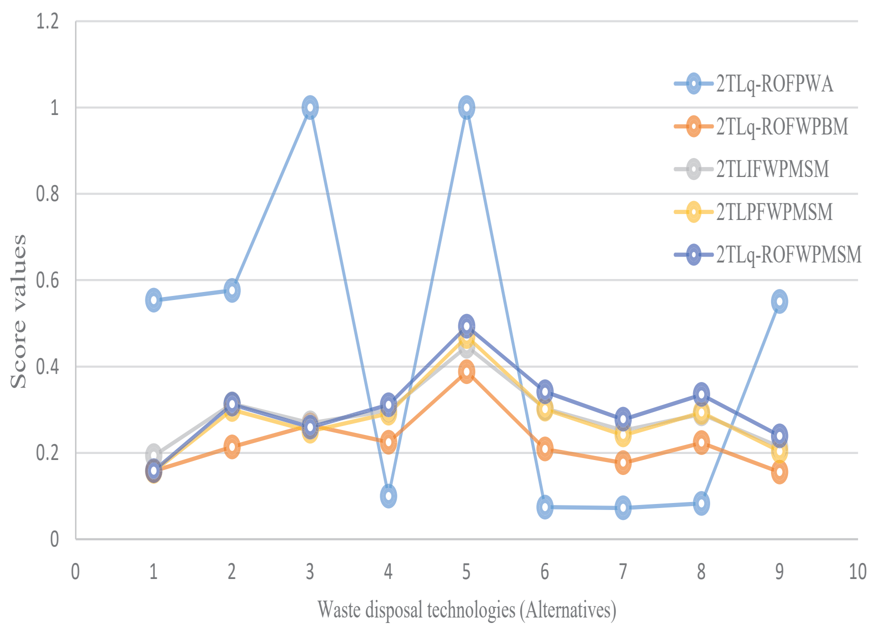

Figure 2: Comparative evaluation results.

6 Concluding remarks

It is imperative to identify the peculiarities, compilation, and heterogeneity of the generated trash to manage MSW efficiently and identify the best technology for waste management. The preference for MSW disposal technology is a critical concern in waste management. Different MAGDM approaches are widely employed to handle this issue, different criteria must be examined while selecting a technology. However, the implemented strategies were either unsuccessful or turned out to be harmful rather than helpful. To address the complexities of the environment, this research proposed an MAGDM framework for validating and selecting a disposal technology for MSW management based on the 2TLq-ROFPWMSM operator.

In this research work, the MSM and PA operators are integrated with the 2TLq-ROFS to provide (the 2TLq-ROFPMSM and 2TLq-ROFPWMSM) aggregation operators that access an efficient way to aggregate evaluation data. By expressing information more flexibly, 2TLq-ROFSs offer a higher ability to contain large amounts of information and are more effective at addressing problems in fuzzy and ambiguous situations. Furthermore, we examined some relevant properties of the proposed operators. To select a technology for the municipal solid waste management system, we developed a new MAGDM algorithm that employs the 2TLq-ROFPWMSM operator. By applying it to a real-life example, we confirmed our method's validity and effectiveness. Finally, we looked at the effects of the parameters and compared the proposed approach to different approaches. In other words, the proposed operators are generalization of the averaging aggregation operator based on the 2TLq-ROF information. The illustration statistics indicate that the technique simultaneously offers the essential characteristics: reducing the impacts of biassed values; considering the interrelationships between attributes, and overcoming the limits of existing operational rules for 2TLq-ROFSs. In the future, we will integrate the proposed aggregation operations with depth learning and apply them to text classification and recommendation systems. Furthermore, we will explore some new operators based on 2TLq-ROFNs and also the proposed method based on real data can be applied to various fields.

Data availability: No data were used to support this study.
Conflict of Interest: The authors declare no conflict of interest.
Ethical Approval: This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] Shah, A. V., Srivastava, V. K., Mohanty, S. S., et al. "Municipal solid waste as a sustainable resource for energy production: State-of-the-art review", *Journal of Environmental Chemical Engineering*, 9(4), p. 105717 (2021).

[2] Torkayesh, A. E., Malmir, B., and Asadabadi, M. R. "Sustainable waste disposal technology selection: The stratified best-worst multi-criteria decision-making method", *Waste Management*, 122, pp. 100-112 (2021).

[3] Mascarenhas, L. C., Ness, B., Oloko, M., et al. "Multi-criteria analysis of municipal solid waste treatment technologies to support decision-making in Kisumu, Kenya", *Environmental Challenges*, 4, p. 100189 (2021).

[4] Suh, J. W., Sohn, S. Y., and Lee, B. K. "Patent clustering and network analyses to explore nuclear waste management technologies", *Energy Policy*, 146, p. 111794 (2020).

[5] Zadeh, L. A. "Fuzzy logic and approximate reasoning", *Synthese*, 30(3), pp. 407-428 (1975).

[6] Atanassov, K.T. "Intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, 20, pp. 87-96 (1986).

[7] Yager, R. R. "Generalized orthopair fuzzy sets", *IEEE Transactions on Fuzzy Systems*, 25(5), pp. 1222-1230 (2016).

[8] Xu, Z. "Intuitionistic fuzzy aggregation operators", *IEEE Transactions on fuzzy systems*, 15(6), pp. 1179-1187 (2007).

[9] Qin, J., and Liu, X. "An approach to intuitionistic fuzzy multiple attribute decision making based on Maclaurin symmetric mean operators", *Journal of Intelligent and Fuzzy Systems*, 27(5), pp. 2177-2190 (2014).

[10] Rahman, K., Ali, A., Shakeel, M., et al. "Pythagorean fuzzy weighted averaging aggregation operator and its application to decision making theory", *The Nucleus*, 54(3), pp. 190-196 (2017).

[11] Naz, S., and Akram, M. "Novel decision-making approach based on hesitant fuzzy sets and graph theory", *Computational and Applied Mathematics*, 38(1), p. 7 (2019).

[12] Akram, M., and Sitara, M. "Decision-making with q-rung orthopair fuzzy graph structures", *Granular Computing*, 7(3), pp. 505-526 (2022).

[13] Buyukozkan, G., and Gocer, F. "A novel approach integrating AHP and COPRAS under Pythagorean fuzzy sets for digital supply chain partner selection", *IEEE Transactions on Engineering Management*, 68(5), pp. 1486-1503 (2019).

[14] Yager, R. R. "The power average operator", *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 31(6), pp. 724-731 (2001).

[15] Garg, H., Naz, S., Ziaa, F., et al. "A ranking method based on Muirhead mean operator for group decision making with complex interval-valued q-rung orthopair fuzzy numbers", *Soft Computing*, 25(22), pp. 14001-14027 (2021).

[16] Wei, G., Wei, C., Wang, J., et al. "Some q-rung orthopair fuzzy Maclaurin symmetric mean operators and their applications to potential evaluation of emerging technology commercialization", *International Journal of Intelligent Systems*, 34(1), pp. 50-81 (2019).

[17] Liu, P., Chen, S. M., and Wang, P. "Multiple-attribute group decision-making based on q-rung orthopair fuzzy power Maclaurin symmetric mean operators", *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 50(10), pp. 3741-3756 (2018).

[18] Naz, S., Akram, M., Alsulami, S., et al. "Decision-making analysis under interval-valued q-rung orthopair dual hesitant fuzzy environment", *International Journal of Computational Intelligence Systems*, 14(1), pp. 332-357 (2021).

[19] Akram, M., Naz, S., and Ziaa, F. "Novel decision-making framework based on complex q-rung orthopair fuzzy information", *Scientia Iranica*, (2021), DOI: https://doi. org/10.24200/SCI

[20] Kumar, K., and Chen, S. M. "Multiple attribute group decision making based on advanced linguistic intuitionistic fuzzy weighted averaging aggregation operator of linguistic intuitionistic fuzzy numbers", *Information Sciences*, 587, pp. 813-824 (2022).

[21] Gong, Z., Lin, J., and Weng, L. "A novel approach for multiplicative linguistic group decision making based on symmetrical linguistic chi-square deviation and vikor method", *Symmetry*, 14(1), p. 136 (2022).

[22] Herrera, F., and Martinez, L. "A 2-tuple fuzzy linguistic representation model for computing with words", *IEEE Transactions on Fuzzy Systems*, 8(6), pp. 746-752 (2000).

[23] Naz, S., Akram, M., Al-Shamiri, M. M. A., et al. "A new MAGDM method with 2-tuple linguistic bipolar fuzzy Heronian mean operators", *Mathematical Biosciences and Engineering*, 19(4), pp. 3843-3878 (2022).

[24] Akram, M., and Niaz, Z. "2-Tuple linguistic Fermatean fuzzy decision-making method based on COCOSO with CRITIC for drip irrigation system analysis", *Journal of Computational and Cognitive Engineering*, (2022), DOI: https://doi.org/10.47852/bonviewJCCE2202356

[25] Akram, M., Niaz, Z. and Feng, F. "Extended CODAS method for multi-attribute group decision-making based on 2-tuple linguistic Fermatean fuzzy Hamacher aggregation operators", *Granular Computing*, (2022), DOI: https://doi.org/10.1007/s41066-022-00332-3

[26] Akram, M., Ramzan, N., and Feng, F. "Extending COPRAS method with linguistic Fermatean fuzzy sets and Hamy mean operators", *Journal of Mathematics*, (2022), DOI: https://doi.org/10.1155/2022/8239263

[27] Akram, M., Noreen, U. and Deveci, M. "Enhanced ELECTRE II method with 2-tuple linguistic m-polar fuzzy sets for multi-criteria group decision making", *Expert Systems with Applications*, 213, p. 119237 (2023).

[28] Akram, M., Noreen, U., Al-Shamiri, M.M.A., et al. "Integrated decision-making methods based on 2-tuple linguistic m-polar fuzzy information", *AIMS Mathematics*, 7(8), pp.14557-14594 (2022).

[29] Naz, S., Akram, M., Sattar, A., et al. "2-tuple linguistic q-rung orthopair fuzzy CODAS approach and its application in arc welding robot selection", *AIMS Mathematics*, 7(9), pp. 17529-17569 (2022).

[30] Yuan, F. "Method for Pythagorean interval 2-tuple linguistic multi attribute group decision making and its application to the ship navigation environment safety assessment", *Discrete Dynamics in Nature and Society*, (2022), DOI: https://doi.org/10.1155/2022/6881900

[31] He, T., Wei, G., Wu, J., et al. "QUALIFLEX method for evaluating human factors in construction project management with Pythagorean 2-tuple linguistic information", *Journal of Intelligent and Fuzzy Systems*, 40(3), pp. 4039-4050 (2021).

[32] Naz, S., Akram, M., Al-Shamiri, M. M. A., et al. "Evaluation of network security service provider using 2-tuple linguistic complex-rung orthopair fuzzy COPRAS method", *Complexity*, 2022, pp. 1-27 (2022).

[33] Naz, S., Akram, M., Muhiuddin, G., et al. "Modified EDAS method for MAGDM based on MSM operators with 2-tuple linguistic-spherical fuzzy sets", *Mathematical Problems in Engineering*, 2022, pp. 1-34 (2022).

[34] Naz, S., Akram, M., Saeid, A. B., et al. "Models for MAGDM with dual hesitant q-rung orthopair fuzzy 2-tuple linguistic MSM operators and their application to COVID-19 pandemic", *Expert Systems*, 39(8), e13005, (2022), DOI: https://doi.org/10.1111/exsy.13005

[35] Feng, F., Zheng, Y., Sun, B., et al. "Novel score functions of generalized orthopair fuzzy membership grades with application to multiple attribute decision making", *Granular Computing*, 7(1), pp. 95-111 (2022).

[36] Maclaurin, C. "A second letter to Martin Folkes, Esq.; concerning the roots of equations, with demonstration of other rules of algebra", *Philosophical Transactions of the Royal Society of London*, 1729(36), pp. 59-96 (1729).

[37] Qin, J., and Liu, X. "Approaches to uncertain linguistic multiple attribute decision making based on dual Maclaurin symmetric mean", *Journal of Intelligent and Fuzzy Systems*, 29(1), pp. 171-186 (2015).

[38] Liu, P., Naz, S., Akram, M., et al. "Group decision-making analysis based on linguistic q-rung orthopair fuzzy generalized point weighted aggregation operators", *International Journal of Machine Learning and Cybernetics*, 13(4), pp. 883-906 (2022).

Authors' Biographies

Sumera Naz completed her MS and PhD in Mathematics from University of the Punjab, Lahore, Pakistan. Currently, she is serving as an Assistant Professor at University of Education, Lahore, Pakistan. She has published 35 research articles in international well recognized peer-reviewed journals. Some of her papers have been published in top rank journals, including Soft Computing, Complex & Intelligent Systems, Computational and Applied Mathematics, International Journal of Computational Intelligence Systems and Symmetry. Her research interest includes fuzzy graph theory and decision analysis. Her current H-index on Google scholar is 16 and i10-index is 21. She has been acting as a reviewer for International journals. She has produced 15 M.Phil. students. Currently, she is supervising more five M.Phil. students.

Muhammad Akram has received MSc degrees in Mathematics and Computer Science, MPhil in (Computational) Mathematics and PhD in (Fuzzy) Mathematics. He is currently a Professor in the Department of Mathematics at the University of the Punjab, Lahore. He has also served the Punjab University College of Information Technology as Assistant Professor and Associate Professor. Dr. Akrams research interests include fuzzy numerical methods, fuzzy graphs, fuzzy algebras, and fuzzy decision support systems. He has published 10 monographs and 485 research articles in international scientific journals. Some of his papers have been published in high impact journals, including Knowledge-Based Systems, Information Sciences, Expert systems with applications, Applied Soft Computing, International Journal of Intelligent Systems, Artificial Intelligence Review, Neural Computing and Applications. Dr Akrams total impact factor is more than 1100. His current H-index on Google scholar is 54 and i10-index is 355. According to reports from Stanford University for years 2020, 2021 and 2022, Dr. Akram is ranked in the top 2% of scientists for years 2020, 2021 and 2022 in the world in Artificial Intelligence and Image Processing. He has been an Editorial Member of several international academic journals. Reviewer/Referee for 160 International Journals including Mathematical Reviews (USA) and Zentralblatt MATH (Germany). Sixteen students have successfully completed their Ph.D. research work under his supervision. Currently, he is supervising 5 PhD students.

Iqra Naseer received her BS degree in Mathematics from the University of Education, Lahore, Pakistan. She is currently pursuing her MS degree in Mathematics from the University of Education, Lahore, Pakistan. Her research interests are in the field of fuzzy aggregation operators and multi-attribute decision analysis.

Arsham Borumand Saeid] received his Ph.D. in Mathematics, Iran in 2003. He is currently a professor at the Department of Pure Mathematics in Shahid Bahonar University of Kerman, Iran. He has published about 385 research papers in international journals and conference proceedings. His current research interests include fuzzy sets and applications, fuzzy logic, algebraic logic and algebraic structures.

Areej Fatima received her BS and MS degrees in Mathematics from the University of Education, Lahore, Pakistan. She has published one book chapter in a in international well recognized peer-reviewed publisheer. Her research interests are in the field of fuzzy aggregation operators and multi-attribute decision analysis.