Multi response optimization of friction stir welding in air and water by analytic hierarchy process and VIKOR method

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\textbf{Abstract} Friction stir welding of Titanium sheets was carried out under air and water environment and the tensile properties of the joints made were measured. The tool rotational speed and tool traversing speed which significantly influence the tensile properties of the welded joints were considered as input process parameters. This work deals with the application of Analytic hierarchy process to calculate the weights of the relative importance of the output responses using the pairwise comparison of responses and checked for the consistency and acceptability of the assumed comparison. Also, the VIKOR optimization technique, a multi-response multi criterion method, was used for determining the optimum process parameters. From the VIKOR optimization method, it is observed that the higher tool rotational speed and lower tool traversing speed are the optimum process parameters in both conventional and underwater FSW. The results from the experimental measurements and the study of microstructure supports the results obtained from VIKOR optimization method.

1. Introduction

Friction stir welding (FSW) is a eco-friendly solid state welding process due to the absence of fumes and arcs which are common in conventional welding processes\cite{1}. Attempts have been made to perform FSW underwater (UFSW), also referred as submerged FSW (SFSW), to explore the possible applications under different environment than that of air\cite{2}. Generally, the process parameters used in FSW process are tool rotational speed, feed, tool pin shape, pin size, shoulder size, tilt angle of the tool. These parameters were optimized to attain the improvement in different mechanical properties. Ghiasvand et al. \cite{3} performed a novel method known as parallel-FSW and the resulting mechanical properties were compared with that of conventional FSW. It was concluded that there is a significant improvement in mechanical properties of the welded joints made by parallel-FSW over the conventional method. Lombard et al. \cite{4} tried to optimize the process parameters in order to achieve a weldment having higher fatigue life with negligible defects in aluminium alloy AA5083-H321. The mechanical properties of the joint made by FSW in AM60 and AZ31 magnesium alloys were optimized by Zhang et al.\cite{5} by considering the tool rotational speed. Similar work was carried out by Sevvel and Jaiganesh \cite{6} using AZ31B taking the effect of tool rotational speed and feed on the output results. Four different shapes of the tool pin such as triangle, frustum, hexagonal and cubic were used to study their influence on the mechanical and microstructural properties of the Aluminum-Steel welded joints made in UFSW. From the investigation, it was presented that the internal material flow is better in a pin having a greater number of edges and the descending order in which the generation of frictional heat is hexagonal, cubic, triangle and frustum pin shape\cite{7}.UFSW of same combination of metals under different cooling atmosphere like low temperature water (LTW), room temperature water(RTW), high temperature water(HTW) and air was investigated by Derazkola and Khodabakhshi \cite{8}.

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It was concluded that excellent results were observed in tensile strength and elongation at RTW cooling atmosphere where as the higher and lower cooling rate reduced the transverse tensile properties and also noticed the reduced formation of intermetallic compound (IMC) when the cooling medium temperature is decreased. The thermal and material flow analysis of UFSW of aluminum-magnesium alloy using the computational fluid dynamic (CFD) has revealed that the maximum heat generated in FSW is 7% more than the UFSW and the increased cooling due to the surrounding water reduced the flow rate of heat [9].

Researchers have attempted to develop a mathematical model for the output responses and optimized the process parameters by response surface graphs [10,11]. Also, successful optimization of FSW of dissimilar aluminium alloys [12], ferritic steels [13] and stainless steel AISI 316L [14] was carried out using response surface methodology. Different approaches have been made in the past by the researchers to optimize the process parameters such as Taguchi method [15], modelling and testing with analysis of variants (ANOVA) [16], RSM in conjunction with central composite design (CCD) [17], and Taguchi–ANOVA-RSM [18]. The optimization of FSW process was performed by analysing the heat transfer using the sequential quadratic programming gradient (SQP) algorithm coupled with computational fluid dynamics (CFD) code for thermal model [19]. Artificial neural network is one of the techniques for developing the predictive models and able to solve the problems where uncertainty and nonlinearity is at higher level [20]. Jamalian et al. [21] used ANN to choose the best profile of the pin so as to maximize the ultimate tensile strength of the friction stir welded joints. To investigate the influence of the process parameters on the tensile properties of the welds made in copper material, Heidarzadeh et al. Employed fuzzy logic-based models [22]. Optimization of the process parameters was carried out by a hybrid method which comprises of Taguchi-grey relation analysis (GRA)- ANN by Waktehaure et al. [23] to attain the optimum mechanical properties. Shehabeldeen et al. [24] developed a predictive model for mapping the input and output parameters using adaptive neuro-fuzzy inference system (ANFIS), which is an integration of ANN and fuzzy logic, and optimized the parameters by Harris Hawks Optimizer (HHO). Banik et al. [25] presented a multi-objective hybrid optimization method utilizing Technique for order preference by similarity to ideal solution (TOPSIS) and Principal component analysis (PCA). Hybrid Differential evolution (DE) and Particle swarm optimization (PSO) approach was applied by Caseiro et al. [26] to find out the optimal configuration of integrally stiffened panels (ISP) joined by FSW and proved that the hybrid method is so effective than the other optimization techniques. Pitchippoo et al. [27] proposed a Dragonfly optimization algorithm (DFA), a technique which is better than the other optimizing methods due to its efficiency, speed of convergence etc., to optimize the input parameters so as to achieve an optimal tensile strength of the welds.

The Analytic Hierarchy Process (AHP) is one of the multi-criteria decision-making (MCDM) methods in which the problem under consideration is broken into hierarchy of interrelated elements and provide a comprehensive structure during the process of decision making. The AHP has the following advantages (i)Flexibility; (ii)Easy to handle; (iii) Capable of indicating the measures of consistency in judgement made and (iv) Takes into account of tangible and intangible factors [28]. The AHP uses a pairwise comparison, at all levels, of the hierarchy of elements and find out the preferences of the set criteria [29].

Visekriterijumska Optimizacija i Kompromisno Resenje (VIKOR), which means multi-criteria optimization and compromise solution, is one of the popular multi-criteria decision making tools to solve problems with conflicting factors. Lee-Ing Tong et al. compared the conventional Taguchi method and VIKOR method of optimization. The study has revealed the following conclusions:

(i) Taguchi SN ratio takes into account of mean and variance of a characteristic of the quality and can be used for single response optimization process effectively; and (ii) VIKOR method takes into account of the measure of utility and regret and is very much effective in multi-response optimization [30]. VIKOR optimization technique is considered to be simple, effective and suitable method for optimizing the process parameters of any type of welding processes. Due to these advantages, Aravind et al. [31] executed VIKOR optimization method to optimize the parameters used for cold metal transfer welding for achieving the set criteria for size of the reinforcement, penetration depth, width of the weld bead and width of the heat affected zone in Al5083 aluminium alloy welded joints. To optimize the width of the bead, penetration depth and microhardness of the laser welded joints of Titanium Ti6Al4V sheets, Aravind et al. employed VIKOR method and verified with experimental results [32]. Also, VIKOR optimization is adopted for the comprehensive analysis and evaluation in different fields such as crashworthiness performance in trains [33], electrical discharge machining process parameter optimization [34], material selection for thermal energy storage system [35], ecological security of water [36] and selection of vaccine for COVID-19 [37].

Different techniques of optimization were performed in FSW in the past. From the review of literature, it is observed that most of the researchers used these techniques to optimize either the single response or multi-responses individually in FSW carried out in air. But, there is a gap in optimizing the multi-responses in FSW performed under water and also there is no comparison was made between optimized parameters obtained in air and water FSW. Also, VIKOR optimization method is one of the simple and efficient techniques to identify the optimum process parameters in multi objective functions. To fill this gap, the multi-response optimization is carried out using AHP and VIKOR methods to optimize the process parameters used in conventional FSW and Underwater FSW to achieve the optimum tensile properties. The results obtained may open a new process window to achieve good welded joints in titanium sheets by FSW performed in air and water.

2. Materials and methods

Experimentation

The Friction stir welding experiments were conducted using 1mm thick titanium grade 1 sheet in air and water. Due to lack of availability of literature on FSW of 1 mm titanium sheet in air and water, the selection of the process window was a challenge.
The following initial range of process parameters were fixed based on the literature on the FSW of other materials: Tool rotational speed range = 100 - 1000 rpm and tool travelling speed range = 40 to 200 mm/min. Trial and error experimentations within this range narrowed down the process parameters selection to rotational speed of 400 and 500 rpm and tool travelling speed of 80 – 160 mm/min. Low rotational speed below 400 rpm and high travelling speed above 160 mm/min produced incomplete welding due to the poor heat generation in both air and water. In both FSW, higher rotational speed above 500 rpm and below 80 mm/min produced overheating in the stir region and burnt the sheet material and also damaged the tool. The various process parameters used in these experiments are presented in the Table 1.

High carbon high chromium (HCHCr) is the tool material used. The shape of the pin in the stirring tool is cylindrical with pin diameter 10 mm and pin length 0.8 mm. The shoulder diameter of the tool is 16 mm and overall length is 120mm. The FSW in air was performed by fixing the plates on the machine bed with a backup plate. Specially designed container with fixture was used for underwater friction stir welding process. The water is stagnant and the initial temperature of water in the container is 25°C. The FSW tool used for air and water is presented in the Figure 1 and the welding setup for air and water is shown in Figure 2.

The operation was carried out in CNC vertical machining centre (LITZ MV-800) Figure 3. Ten experiments each are carried out in air and water respectively based on L9 orthogonal array from Taguchi method with additional one more experiment. The output responses Yield strength (YS), Tensile strength (TS) and % Elongation were measured using tensile testing of the welded joints according to the ASTM-E8 standard. The tensile specimen is shown in Figure 4.

2.1 ANALYTIC HIERARCHY PROCESS (AHP)

AHP is used for finding the weights of the relative importance of the factors used. It decides the preferences among the criteria that is set using pairwise comparisons. The quality of importance of the factors according to Saaty (1980) is presented in the Table 2. The matrix A, using the comparison, will be formed where $a_{ij}$ is the element of the matrix obtained by comparing $A_i$ of the $i^{th}$ row with the $A_j$ of the $j^{th}$ column. The pairwise comparison is shown in Table 3.

In the Table 3, the value 1 indicates equal importance and the value 7 indicates the YS is 7 times important than the TS. Hence, the TS is 1/7 times important than the YS.

Therefore,

\[
A = \begin{bmatrix}
1 & 7 & 3 \\
0.1428 & 1 & 0.2 \\
0.333 & 5 & 1 \\
\end{bmatrix}
\]

The weightage is calculated using the equations (1) and (2) from [32].

For a given $i^{th}$ row,

\[
GM_i = \left\{ \prod_{j=1}^{b} a_{ij} \right\}^{1/b} \tag{1}
\]

\[
W_j = \frac{GM_j}{\sum_{i=1}^{N} GM_i} \tag{2}
\]

Where $a_{ij}$ is the element in $i^{th}$ row and $j^{th}$ column of matrix A.

In order to check the consistency of comparison made among the responses shown in Table 3, the following procedure is adopted: First, the weighted sum of the responses and the eigen value ($\lambda$) is calculated for each row based on the given formula (3). For a given $i$,
\[
\lambda_i = \frac{\sum_{j=1}^{b} W_j x_{ij}}{W_i}
\]  

(3)

Then, the maximum eigen value (\(\lambda_{\text{max}}\)) is calculated using equation (4)

\[
\lambda_{\text{max}} = \frac{\sum_{i=1}^{a} \lambda_i}{a}
\]

(4)

According to Satty (1977), a consistency index (CI) can be calculated using the formula (5),

\[
\text{CI} = \frac{\lambda_{\text{max}} - n}{n-1}
\]

(5)

Where, \(n\) is the order of the matrix

The calculated CI value is compared with the appropriate consistency index known as Random consistency Index (RI).

The RI values are taken from the Table 4 (Satty 1980). The comparison between the CI and RI, known as Consistency Ratio (CR), is calculated using the formula shown in equation (6),

\[
\text{CR} = \frac{\text{CI}}{\text{RI}}
\]

(6)

If the calculated CR value is less than 10\% (0.1), the pairwise comparison matrix (A) formed is consistent and acceptable. The pairwise comparison matrix is not consistent and reliable when the CR value is more than 10\% and the pairwise comparison values need to be altered to achieve the consistency.

2.2 VIKOR Optimization method

To identify the optimum process parameters, VIKOR optimization method is used which is one of the MCDM techniques. The procedure includes the calculation of utility factor and regret factor from which the VIKOR index is determined. Then the ranking is done based on the ascending values of the VIKOR index and the optimal solution is derived from the smallest VIKOR index.

The following steps are employed in VIKOR optimization:

Step 1: Determination of Normalized decision matrix.

Let X be the decision matrix formed with the values of output responses under consideration. Then, the elements \(p_{ij}\) of the normalized decision matrix \(P\) [32], can be calculated from the decision matrix X using the equations (7) & (8),

\[
p_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{a} x_{ij}^2}}
\]

(7)

And,

\[
P = \left[ p_{ij} \right]_{a \times b}
\]

(8)

Where \(i = 1, 2, 3, \ldots, a\); \(j = 1, 2, 3, \ldots, b\)

Step 2: Determination of the weightages of the comparative significance of the factors.

The weightages are calculated and the pairwise comparison value made are checked for their consistency and acceptability using AHP procedure explained in the previous section.

Step 3: Calculation of utility factor (S) and regret factor (R).

The selection of the optimal parameters are based on the criteria that the welded joints must have higher YS and TS. Whereas the...
%EL must be lower.

The following equations (9-12) are used for calculation:

For maximum criteria,

\[ S_i = \sum_{j=1}^{b} W_j \frac{(p_{max} - p_j)}{(p_{max} - p_{min})} \]  

(9)

\[ R_i = \max_{t} \left[ W_j \frac{(p_{max} - p_j)}{(p_{max} - p_{min})} \right] \]  

(10)

For minimum criteria,

\[ S_i = \sum_{j=1}^{b} W_j \frac{(p_j - p_{min})}{(p_{max} - p_{min})} \]  

(11)

\[ R_i = \max_{t} \left[ W_j \frac{(p_j - p_{min})}{(p_{max} - p_{min})} \right] \]  

(12)

Step 4: Determination of VIKOR constant.

Using the equation (13) VIKOR constant can be calculated.

\[ Q_i = V \left[ \frac{S_i - S_{min}}{S_{max} - S_{min}} \right] + (1-V) \left[ \frac{R_i - R_{min}}{R_{max} - R_{min}} \right] \]  

(13)

Where, \( i = 1, 2, 3, \ldots, a \).

\( V \), a constant, is the weight incorporated to keep up the plan of action of maximum group utility and its value ranges from 0 to 1; but, in general, the value is considered as 0.5.

Step 5: Ranking of alternatives.

Based on the calculated \( Q_i \) values, the alternatives are ranked in the ascending order.

Step 6: Proposing the optimal solution.

The optimal parameters for the best responses correspond to the lowest value of the VIKOR constant.

3. Results and Discussion

The friction stir welding was carried out using the Ti sheets in air and water environment and the joints are tested for the output responses such as YS, TS, %EL which are tabulated in the Table 5. The welded samples made in air and water at different process parameters are shown in Figure 5.

The value 0 indicates that the welded joints are broken by hand force or gripping in the tensile testing machine. The calculated weightage of the output responses using the equations (1) and (2) are presented in Table 6.

To check the consistency of the pairwise comparison, the following calculations (Table 7) are made:

\[ CI = \frac{(3.066-3)}{2} = 0.033, \text{ From Table 4, } RI = 0.58, \text{ for } n = 3, CR = \frac{0.58}{0.033} = 0.0568 \]

Since CR < 0.1, the weightage and hence the pairwise comparison is reliable and acceptable.

The normalized decision matrix of the output responses for both conventional FSW and underwater FSW are provided in the following Table 8. The calculated Utility factor (\( S_i \)), Regret factor (\( R_i \)), VIKOR constant (\( Q_i \)), and the corresponding ranking for
both air and water FSW are shown in Table 9.

The tool rotational speed of 500 rpm and tool traversing speed of 80 mm/min are the optimal process parameters observed from the Table 9 in case of both conventional FSW and UFSW. This results show that the higher rotational speed of the tool and lower travelling speed of the tool helps to achieve the best tensile properties of the welded joints made in both air and water within the experimented process parameters due to the generation of higher amount of heat and the proper stirring of the material by the rotating tool. The optimum result obtained at higher tool rotational speed and lower travelling speed for the different output responses by other researchers \[5,6,23\] using different optimization techniques supports the present result. The tensile properties of the base metal are as follows: Yield strength = 288MPa; Tensile strength = 311MPa and % Elongation = 34.5. The observed % elongation in both FSW in air and under water FSW is very low when compared with the % elongation of the base metal (i.e.) 1.5% and 3% of the base metal in air and underwater FSW respectively. From the experimental results, it is also noticed that the yield strength, ultimate tensile strength and % elongation of the joint made in water FSW using the optimal process parameters are 18.7%, 48.7% and 100% respectively higher than the conventional FSW. The improvement in the strength of the welded joint in under water may be attributed to the hardening effect caused due to the drastic cooling by the surrounding water. But, in case of air FSW the softening effect is caused by the slow cooling in air. Further increase in the tool travelling speed decreases the tensile properties drastically due to lower heat generation and improper stirring action by the tool.

Thus, the hardness is higher in the stir zone than the parent metal which increases the yield and ultimate tensile strength of the joint. More finer grains are formed in UFSW when compared to FSW due to the drastic cooling by the surrounding water which results in higher tensile properties in UFSW than FSW at optimum process parameter. Also, there is no evidence of defects such as crack, void etc noticed in the observed microstructure of the joints formed in both air and water. The macro structural view of cross section of the welded joint fabricated using the tool rotational speed 500rpm and tool travelling speed 80mm/min under water is shown in Figure 7. Since the thickness of the sheet material is only 1mm, the macrostructure does not show clearly the boundary layer between the different zones.

Overall, the strength of the welded joint depends on the following: (1) Sufficient generation of heat due to friction between the tool and the workpiece; (2) Proper mixing of plastized material due to the stirring of the rotating tool; (3) The rate of cooling; (4) Formation of the grains. The dominating combination of the above said factors decides the strength of the welded joints in both air and underwater FSW.

4. Conclusion

The friction stir welding of titanium sheets were performed under air and water environment and tested for the joint tensile properties. The analytic hierarchy process was used for calculating the weights of the relative importance of the factors and VIKOR method of optimization was applied for obtaining the optimum process parameters. From the obtained results, the following conclusions are derived:

- In AHP, the pairwise comparison of output responses for their quality of significance were made and the weights of the relative importance was calculated and successfully checked for the acceptability and consistency of the assumed pairwise comparison of the responses.
- The VIKOR method, a multi criterion and multi response optimization technique, was implemented successfully in conventional and under water FSW to obtain the optimum process parameters.
- By VIKOR optimization method, the optimal process parameters are identified in air FSW are 500 rpm of tool rotation speed and 80 mm/min tool traversing speed.
- In case of submerged FSW process, the optimal process parameters are 500 rpm and 80 mm/min from VIKOR method.
- The results of the VIKOR method are supported by the results of the microstructural study.

Conflict of interest and funding

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References

1. Memon, S., Tomków, J. and Derazkola, H.A. “Thermo-Mechanical Simulation of Underwater Friction Stir Welding of


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Figure 2 Welding Setup

Figure 3. CNC Vertical machining centre

Figure 4. Specimen for tensile testing.
Figure 5. Welded joints at different process parameters

(a)  
(b)  
(c)  
(d)  

Figure 6 Microstructure of the welded joints: a) FSW at 500 rpm 80 mm/min; b) FSW 400 rpm 120 mm/min; c) UFSW at 500 rpm 80 mm/min and d) UFSW 500 rpm 140 mm/min

Figure 7 Macrostructure of the welded joints of UFSW 500 rpm 80 mm/min

Table 1. FSW parameters used in air and water

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Process Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Tool rotational speed</td>
<td>400; 500 rpm</td>
</tr>
<tr>
<td>2.</td>
<td>Tool traversing speed</td>
<td>80; 100; 120; 140; 160 mm/min</td>
</tr>
</tbody>
</table>

Table 2. Quality of Significance of the factors [32].

<table>
<thead>
<tr>
<th>Description</th>
<th>Quality level of significance</th>
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<tbody>
<tr>
<td>Intermediate values</td>
<td>2, 4, 6, 8</td>
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<tr>
<td>Absolute</td>
<td>9</td>
</tr>
<tr>
<td>Very much strong</td>
<td>7</td>
</tr>
<tr>
<td>Essential/strong</td>
<td>5</td>
</tr>
<tr>
<td>Moderate</td>
<td>3</td>
</tr>
<tr>
<td>Equal importance</td>
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</table>
### Table 3. Pairwise comparison

<table>
<thead>
<tr>
<th>Output Responses</th>
<th>YS</th>
<th>TS</th>
<th>% EL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield strength (YS)</td>
<td>$1$</td>
<td>$7$</td>
<td>$3$</td>
</tr>
<tr>
<td>Tensile strength (TS)</td>
<td>$1/7$</td>
<td>$1$</td>
<td>$1/5$</td>
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<tr>
<td>% Elongation (% EL)</td>
<td>$1/3$</td>
<td>$5$</td>
<td>$1$</td>
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### Table 4. Random Consistency Index (RI)

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<th>$4$</th>
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<th>$8$</th>
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<tr>
<td>RI</td>
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<td>$0.90$</td>
<td>$1.12$</td>
<td>$1.24$</td>
<td>$1.32$</td>
<td>$1.41$</td>
<td>$1.45$</td>
<td>$1.49$</td>
<td></td>
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</table>

### Table 5 Output Responses

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Speed (rpm)</th>
<th>Feed (mm/min)</th>
<th>Air</th>
<th>Water</th>
<th>YS (MPa)</th>
<th>TS (MPa)</th>
<th>% EL</th>
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<tbody>
<tr>
<td>1</td>
<td>400</td>
<td>80</td>
<td>220</td>
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<td>292</td>
<td>437</td>
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<td>219</td>
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</table>

### Table 6 Weightage of the Output Responses using AHP

<table>
<thead>
<tr>
<th>Output Responses</th>
<th>YS</th>
<th>TS</th>
<th>% EL</th>
<th>Product of the values</th>
<th>$GM_i$</th>
<th>Weightage $W_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>YS</td>
<td>$1$</td>
<td>$7$</td>
<td>$3$</td>
<td>$21$</td>
<td>$2.758924$</td>
<td>$0.649$</td>
</tr>
<tr>
<td>TS</td>
<td>$0.1428$</td>
<td>$1$</td>
<td>$0.2$</td>
<td>$0.0286$</td>
<td>$0.30567$</td>
<td>$0.072$</td>
</tr>
<tr>
<td>% EL</td>
<td>$0.333$</td>
<td>$5$</td>
<td>$1$</td>
<td>$1.6650$</td>
<td>$1.185236$</td>
<td>$0.279$</td>
</tr>
</tbody>
</table>

### Table 7 Calculation for checking the consistency

<table>
<thead>
<tr>
<th>Output Responses</th>
<th>YS</th>
<th>TS</th>
<th>% EL</th>
<th>Weightage $W_j$</th>
<th>$\lambda$</th>
<th>$\lambda_{max} = Mean of \lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>YS</td>
<td>$1$</td>
<td>$7$</td>
<td>$3$</td>
<td>$0.649$</td>
<td>$3.066$</td>
<td>$3.066$</td>
</tr>
<tr>
<td>TS</td>
<td>$0.1428$</td>
<td>$1$</td>
<td>$0.2$</td>
<td>$0.072$</td>
<td>$3.069$</td>
<td>$3.066$</td>
</tr>
<tr>
<td>% EL</td>
<td>$0.333$</td>
<td>$5$</td>
<td>$1$</td>
<td>$0.279$</td>
<td>$3.064$</td>
<td>$3.066$</td>
</tr>
</tbody>
</table>

### Table 8 Normalized Decision matrix

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Air</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YS</td>
<td>TS</td>
</tr>
<tr>
<td>1</td>
<td>0.486994</td>
<td>0.48426</td>
</tr>
<tr>
<td>2</td>
<td>0.416158</td>
<td>0.401582</td>
</tr>
<tr>
<td>3</td>
<td>0.146098</td>
<td>0.139766</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.591034</td>
<td>0.60631</td>
</tr>
<tr>
<td>7</td>
<td>0.449363</td>
<td>0.446858</td>
</tr>
<tr>
<td>8</td>
<td>0.130603</td>
<td>0.131892</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 9 Calculated Si, Ri,Qi and Corresponding Ranking

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Air</th>
<th>Water</th>
<th>Rank</th>
<th>Air</th>
<th>Water</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_i$</td>
<td>$R_i$</td>
<td>$Q_i$</td>
<td></td>
<td>$S_i$</td>
<td>$R_i$</td>
</tr>
<tr>
<td>1</td>
<td>0.198487</td>
<td>0.114243</td>
<td>0.121874</td>
<td>2</td>
<td>0.059563</td>
<td>0.056337</td>
</tr>
<tr>
<td>2</td>
<td>0.355838</td>
<td>0.192026</td>
<td>0.291032</td>
<td>4</td>
<td>0.112862</td>
<td>0.096899</td>
</tr>
<tr>
<td>3</td>
<td>0.822976</td>
<td>0.488573</td>
<td>0.849884</td>
<td>5</td>
<td>0.315431</td>
<td>0.279431</td>
</tr>
<tr>
<td>4</td>
<td>0.721000</td>
<td>0.649000</td>
<td>0.922244</td>
<td>7</td>
<td>0.406141</td>
<td>0.299711</td>
</tr>
<tr>
<td>5</td>
<td>0.721000</td>
<td>0.649000</td>
<td>0.922244</td>
<td>7</td>
<td>1</td>
<td>0.649000</td>
</tr>
<tr>
<td>6</td>
<td>0.069750</td>
<td>0.069750</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.314001</td>
<td>0.155566</td>
<td>0.232437</td>
<td>3</td>
<td>0.168597</td>
<td>0.148729</td>
</tr>
<tr>
<td>8</td>
<td>0.840926</td>
<td>0.505588</td>
<td>0.876209</td>
<td>6</td>
<td>0.183091</td>
<td>0.159997</td>
</tr>
<tr>
<td>9</td>
<td>0.721000</td>
<td>0.649000</td>
<td>0.922244</td>
<td>7</td>
<td>0.330634</td>
<td>0.227600</td>
</tr>
<tr>
<td>10</td>
<td>0.721000</td>
<td>0.649000</td>
<td>0.922244</td>
<td>7</td>
<td>0.460356</td>
<td>0.315486</td>
</tr>
</tbody>
</table>

Appendix

Nomenclature

- **FSW**: Friction stir welding
- **UFSW**: Underwater friction stir welding
- **SFSW**: Submerged friction stir welding
- **AHP**: Analytic hierarchy process
- **VIKOR**: Vlsekriterijumska Optimizacija I Kompromisno Resenje
- **LTW**: Low temperature water
- **RTW**: Room temperature water
- **HTW**: High temperature water
- **AISI**: American iron and steel institute
- **ANOVA**: Analysis of variance
- **RSM**: Response surface methodology
- **CCD**: Central composite design
- **SQP**: Sequential quadratic programming gradient
- **CFD**: Computational fluid dynamics
- **GRA**: Grey relational analysis
- **ANN**: Artificial neural network
- **ANFIS**: Adaptive neuro-fuzzy inference system
- **HHO**: Harris hawks optimizer
- **TOPSIS**: Technique for order preference by similarity to ideal solution

**Nomenclature**

- **PCA**: Principal component analysis
- **DE**: Differential evolution
- **PSO**: Particle swarm optimization
- **MCDM**: Multi-criteria decision-making
- **HCHCr**: High carbon high chromium
- **YS**: Yield strength
- **TS**: Tensile strength
- **ASTM**: American society of testing and materials
- **GM**: Geometric mean of $i^{th}$ row
- **$W_j$**: Weightage of $j^{th}$ column
- **$\lambda_i$**: Eigen value of $i^{th}$ column
- **CI**: Consistency index
- **RI**: Random consistency index
- **CR**: Consistency ratio
- **P**: Normalized decision matrix
- **$S_i$**: Utility factor of $i^{th}$ row
- **$R_i$**: Regret factor of $i^{th}$ row
- **$Q_i$**: VIKOR constant