Scheduling of transportation fleet based on the customer’s priority in a hub location problem

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Abstract

This study proposes six novel strategies on the customer’s priority while addressing the conventional hub location issue. Each strategy assigns a value to every customer based on distance and demand parameters, in which customers are prioritized based on this value. Then, the vehicle fleet is scheduled according to the customer’s priority. A new mixed-integer linear programming model is presented and applied for each strategy in a new hub location-scheduling problem solved by three approaches. Then, by using the CAB dataset, extensive experiments are designed to evaluate each strategy. The strategies are evaluated with statistical and non-statistical analyses and ranked accordingly. In each case, a comparison of the non-priority strategy with the best customer’s prioritization strategy shows that the non-priority strategy has an adverse effect on the delivery time (i.e., 129.7%, 171.68%, and 161.33% than the best strategy in the case of near, medium, and far nodes, respectively). In addition to the above tests, other tests are conducted to evaluate the optimum number of vehicles for different conditions. The results show that as the distance between customers and hubs increases, reducing the number of vehicles while increasing their capacity is preferable. Also, each strategy requires using a certain number of vehicles.

Keywords: Hub location; Scheduling; Transportation fleet; Vehicles; Customer’s priority.

1. Introduction

Hubs are special intermediary facilities within distribution networks and are necessary for aggregating and consolidating goods and distributing them to destination nodes. The existing literature on the locations of facilities, hubs, distribution centers, warehouses, and assigning customer zones has defined hub placement decisions as strategic decisions that, once made, remain unchanged for three to five years. If these decisions are combined with shorter-term decisions (tactical and operational decisions), the efficiency and responsiveness of distribution networks will increase. One of the most critical tactical decisions is transportation fleet planning. Previous studies have shown that integrating these two issues can reduce long-term costs versus considering each case separately [1].

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The concept of integrating vehicle scheduling and routing choices with facility location extends back almost 50 years. The first to thoroughly examine this hybrid issue was probably Watson-Gandy and Dohrn [2]. Whether dealing with hybrid issues or focusing solely on transport fleet planning, there usually are some constraints for the transportation fleet. Some typical constraints include the type of vehicles, vehicle capacity, time window, number of available vehicles, and precedence restrictions where one request must be answered before another [3].

**Contribution**

In the current effort, we carefully address the precedence limits in addition to the vehicle capacity and the number of available vehicles. Precedence constraints have been used to cover the main contribution of this study, namely customer prioritization. This constraint has been used in other studies. However, three significant differences distinguish this study from similar existing articles.

1. The first difference is related to how the customers are served by vehicles. Almost all the articles deal with the routing and scheduling of transportation fleet, which consider the precedence restriction. The vehicles start moving from the depot and, after servicing a set of customers, return to the depot. Also, each vehicle performs at most one trip; however, in reality, these assumptions cannot yield a solid strategy for the transportation fleet, especially when the vehicles are trucks/trailers, batch sizes are large, or the products have chemical interactions or are perishable. Differently, in this study, the vehicles return to the depot (hub) to reload after servicing one customer before moving on to the next customer.

2. The second distinction - and one of the study’s key findings - relates to customer ratings. In the few articles in which the customer rankings are, the rankings are usually based on arbitrary, authority, evaluation measures, or environmental weights (see [4-6]). In this study, six new innovative ranking formulas are presented. Based on a customer’s geographical location and his/her demand, we assign a score to that customer. Then, using the precedence constraint, we force the transportation fleet to prioritize customers with higher scores. This new planning method is called customer prioritization.

3. This study, like other articles related to location and scheduling problems, minimizes costs and time. However, the main purpose of this study (and the most important contribution) is to answer the following questions:
   - Will shipping costs be reduced by paying attention to customers’ demands and geographical locations? Also, will customer satisfaction be increased (by reducing delivery time)?
   - Based on what order should customers be served to reduce the problem of shortage of available vehicles? (Sometimes, more than one vehicle is needed to meet a customer’s demand, but if there is only one vehicle in the hub, the company would have to wait for the nearest vehicle(s) to return to the hub).

Comprehensive literature reviews on the hybrid issue of facility location and transportation fleet planning can be found in [7] and [1]. Also, Drexel [8] reviewed the routing problem, focusing on synchronization constraints (such as precedence restriction). We also studied related articles published from 2014 to 2023 and concluded that none of these papers addressed the hub location problem related to the issue of prioritizing customers (based on distance and demand parameters) and sending vehicles to them accordingly.
Therefore, this study uses new formulas that prioritize all nodes through six strategies. Then, a novel mixed-integer linear bi-objective single-allocation $p$-hub location mathematical model is presented for each strategy in which the vehicles start departing according to a predetermined hub schedule and then, after delivering the goods, return to the hub and are reloaded for the next trip (next customer). This process is continued until the demands of all customers allocated to the hub are met. Predetermined quantities of vehicles are required at each hub to satisfy the needs of the clients who have been allocated to that hub based on their priority.

In this manner, the sum of the vehicle departure times from the hubs and overall transportation expenses will be kept to a minimum. Unlike traditional models, in this model, more than one vehicle can travel to a single customer zone if needed based on the customer's demand and vehicle capacity. In this study, the number of hubs is fixed, and inter-hub transportation is possible. A mixed-integer non-linear programming (MINLP) model is presented and linearized using several constraints. The presented models are solved threefold, as a single objective, lexicographic, and $\varepsilon$-constraint approach. In this study, meta-heuristic algorithms were not used intentionally because evaluating the main idea of this study requires accurate comparison and ranking of models. So, it seems that optimal global solutions obtained from the exact solutions are a more reliable basis for comparing models.

The rest of this research is divided into the following sections. In Section 2, we examine earlier pertinent investigations. We outline the issue and provide relevant formulae and models in Section 3. We discuss the numerous computational experiments that were carried out utilizing the CAB dataset in Section 4, together with the findings that were subjected to statistical and non-statistical analyses. Also, we provide in-depth comparisons and score the different research methodologies that were used in the study. Finally, Section 5 provides practical findings and management insights.

2. Literature evaluation

2.1. Hub location challenge in transportation fleet planning

The classic hub placement issue is resolved by locating the ideal hub location and allocating client zones to them. O'Kelly's investigations from 1986a, 1986b, and 1987 [9-11] seem to be the first to distinguish between hub-locating and traditional hub-finding problems. He gave an integer programming model with a quadratic objective function and the hub placement issue. Following that, Campbell [12] divided hub placement issues into four groups depending on their aims (i.e., un-capacitated hub location, $p$-hub median, $p$-hub center, and hub coverage issues) and provided linear models for each. The first two issues reduce the overall cost.

The highest service level between any origin-destination pair is minimized by the $p$-hub center issue, while the hub covering problem reduces the number of hub locations. Watson-Gandy and Dohrn [2] coupled the facility location issue with transportation fleet scheduling for the first time. Numerous papers on this subject have since been written. The scheduling of the transportation fleet within the hub location challenge is the main topic of this research.

Thus, we will refrain from discussing other location-routing challenges; instead, we recommend that Prodhon and Prins [1] thoroughly analyzed the various location-routing issues. Campbell [13] presented several $p$-hub median issues with numerous allocations and demonstrated how time-definite service standards affect the architecture of the truck transportation network. Alumur et al. [14] investigated a hierarchical multimodal hub placement issue across a hub network that includes aircraft and road segments with time-definite delivery. Later, Dukkanci and Kara [15] expanded this issue to include routing and
scheduling choices. Ghodratnama et al. [16] proposed a novel hub location model that reduced shipping time, service time, the number of cars, and greenhouse gas (GHG) emissions from many vehicles while considering various modes of transportation.

Serper and Alumur Alev [17] created a mixed-integer programming model for a capacitated intermodal hub network with various vehicle kinds. They wanted to reduce overall costs and determine the ideal number of cars. A novel bi-objective hub-location model was debuted by When modeling the sustainable food hub placement challenge, Musavi and Bozorgi-Amiri [18] presented vehicle allocation and sequencing in a multi-objective linear programming model while considering each hub’s vehicle restrictions.

The ideal number, locations, and capacities of hubs and fleet size were determined concurrently using a multiple assignment model with road congestion restrictions provided by Hu et al. [19]. The goal of the concept was to reduce overall expenses, including those associated with material handling, congestion, fixed hub establishment costs, vehicle acquisition, operation, and transportation costs. Dukkanci et al. [20] used a non-linear programming framework to simulate the green hub placement issue while taking into account the weight and speed of vehicles. A methodology to simultaneously increase network resilience and reduce overall routing costs was put up by Babashahi et al. [21]. The model was then solved using a non-dominated sorting genetic algorithm. Golestani et al. [22] looked at the simultaneous distribution of different perishable goods at varying storage temperatures in a cold supply chain in a bi-objective green hub placement issue. The hub placement, distributing of customers to the hubs, allocating customers to the cars, and the sequence of vehicles were the results of this model’s solution using the ε-constraint approach. To reduce the overall system cost, Wu et al. [23] developed an integrated model of hub-and-spoke network design and fleet planning with the restrictions of the passenger flow requirement and the adoption of various kinds of aircraft for each route. To solve their suggested model, they created a heuristic. In a competitive market where consumer preference is determined using the Logit Model, Mohammadi and Karimi [24] considered the integrated price and hub location dilemma with environmental costs. They solved and observed the effects of the environmental cost, entrant profit, incumbent income, consumer sensitivity, and discount between hubs on the entrant profit using genetic algorithms.

2.2. Application of precedence constraints in transportation fleet planning

The term “precedence constraints” refers to cases where one activity must be accomplished before another. Such constraints often arise in production planning, vehicle routing, and scheduling problems. The following cases highlight the diversity of precedence constraints in different issues.

Žulj et al. [25] used precedence constraints to pick heavy items before light items in a warehouse system. Li et al. [26] incorporated the precedence constraints for a system by which people and parcels share taxis, with passengers prioritized ahead of parcels. To service patients, Abdul Nasir and Kuo [27] applied precedence constraints for picking up and dropping off nurses. De Azevedo et al. [28] used priority constraints to address a project scheduling issue and suggested that each activity had a related early start time and late end time. We research the hybrid issue that includes hubs (not necessarily hub locations), vehicle fleet planning, and precedence constraints. The majority of the papers cover topics related to "pickup and delivery" and "dial-a-ride," both of which involve a restricted traveling salesman issue where one node must be reached by the vehicle(s) before another node. Several articles related to this topic;
however, none have directly tackled this subject ([29-35]). Table 1 lists characteristics of research that are most like our study.

To the best of our knowledge, no attempt has been made to plan a transportation fleet based on the clients’ priority in the hub placement issue, taking into account three of our key methods (see Section 1).

3. Formulation of the issue and mathematical modeling
3.1. Problem formulation
3.1.1. Scheduling for vehicle fleets

In classical hybrid problems of locating facilities and transportation systems (e.g., routing location problems), Typically, a vehicle with limited or unlimited capacity is believed to deliver to clients in such a manner that each customer is only served once and by one vehicle. Also, each vehicle performs at most one trip; however, in reality, these following assumptions cannot yield a solid strategy for the transportation fleet, especially when the vehicles are trucks/trailers, batch sizes are large, or the products have chemical interactions or are perishable.

In this study, we assume that each hub has a predetermined number of vehicles so that, based on the customer’s demand, one or more vehicles can be loaded simultaneously into the hub and begin their trip to the customer zone. After the goods have been delivered to the customer, the vehicles return to the same hub. As required, one or more of the returning vehicles are loaded again and sent to subsequent customers. This process continues until the demands of all assigned customers are met. Also, we assume that the goods are transported from the origin nodes to the hub by vehicles provided in the origin nodes (inbound vehicles). Meanwhile, vehicles assigned to the hub only transport the goods to destination nodes (i.e., outbound vehicles).

The following are the basic presumptions behind the vehicle fleet scheduling:

- Each hub receives an equal quantity of the predefined vehicles (of course, an equal or unequal number of vehicles allocated to hubs does not affect customer prioritization).
- Each hub has the same number of vehicles allocated to it as the highest demand. If it were any lower, the customer’s demand could not be delivered in one shipment; if it were any higher, a vehicle might be left unused, meaning that the transportation fleet capacity would not be properly utilized. In fact, with this assumption, the customer prioritization issue is evaluated in the most challenging condition of vehicle inventory.
- All vehicles assigned to hubs are homogeneous and have a limited capacity.
- The customers’ demand is delivered at once.
- The function \( \text{roundup} \left( \frac{\text{Customer’s Demand}}{\text{vehicle capacity}} \right) \) is used to determine how many vehicles are required to meet customer demand. It should be noted that the Excel function “roundup()” rounds its input value to the next integer greater than or equal to it (Ceil() is the GAMS counterpart).
- Each vehicle travels to each customer zone only one time at most.
• The delivery of products to the client and the time it takes for vehicles to load at the hub are both regarded as zero.
• Vehicles do not stop along their route.

According to the last two assumptions, the start time of vehicles departing can be used instead of the delivery time for simplifying the objective function equations. This replacement will not affect the results of the computational studies because the aim is to compare six strategies (when one factor has the same effect on all strategies, it is ineffective in comparison).

3.1.2. Customer prioritization

As explained in the previous section, customers are best served when the vehicle(s) return to the hub after delivering one customer’s shipment and then reload for the next trip. The decision-maker (i.e., hub manager) must consider the following crucial issue due to the restricted number of vehicles and the difficulty that the customer’s order must be delivered immediately: Which client should be attended to first to avoid any issues with vehicles not being available to assist the next customers?

We aim to minimize the total departing start time of outbound vehicles. This issue may be solved by considering the consumers’ needs and their distance to the hub. The decision-maker may choose to take the following actions, for instance. (a) The closest customers to the hub should be served before others because this will allow vehicles to return to the hub sooner and go to the next customer(s) with less start time. (b) Customers with smaller demands should be served before others; this way, more customers can be performed at the beginning of service. Suppose that the decision-maker makes decision b. Then, a new challenge arises: If customers with smaller demands are far from the hub, should they still be served before others?

This study seeks to determine how customers should be prioritized in various scenarios to serve all customers in the shortest amount of time. We have introduced six goods delivery strategies to meet this goal and prioritized customers based on these strategies. Whenever any customer has a higher priority than others, that customer’s delivery must be sent earlier. In the proposed model, each customer’s priority is based on their geographical location and demand value, with lower values indicating higher priority. All strategies are formulated such that each customer’s priority value does not have a measurement unit.

Six prioritization strategies and the used parameters for prioritizing the customers are as follows:

\[
W_{ij} \quad \text{Demand came from node } i \text{ and was intended for node } j \text{ (Kg)}
\]

\[
T_{ij} \quad \text{Distance between nodes } i \text{ and } j \text{ in time (h)}
\]

\[
m_1, m_2 \quad \text{Weighting coefficients}
\]

\[
\varepsilon \quad \text{A very small positive number}
\]

\[
priority_\varphi \quad \text{Customer priority } \varphi
\]

**Strategy 1:** In this strategy, the lower the customer’s demand, the higher his/her priority.

\[
priority_\varphi = \frac{\sum_{i=1}^{n} W_{\varphi i}}{\sum_{i=1}^{n} \sum_{\varphi=1}^{n} W_{i\varphi}} \quad \varphi = \{1, 2, ..., n\} \tag{1}
\]

6
It should be noted that the total products that must be delivered to customer \( \varphi \) have been considered as the demand of customer \( \varphi \).

**Strategy 2:** In this strategy, the more the customer’s demand, the higher his/her priority is, as follows.

\[
priority_\varphi = \frac{1}{\varepsilon + \sum_{\varphi=1}^{n} W_{\varphi\varphi}} \quad \varphi = \{1, 2, \ldots, n\}
\]  

(2)

**Strategy 3:** This strategy aims to serve customers closer to the hub before those further from the hub. However, since the hub’s location is not yet known, we use Formula (3) to assign priority to customers whose total distance to other customers is small (i.e. the shorter the customer’s total distance to others, the higher the customer’s priority). This is predicated on the idea that hubs are most likely situated close to these clients, given the main aim of hub location issues, which is to lower transportation costs.

\[
priority_\varphi = \frac{\sum_{\varphi=1}^{n} T_{\varphi\varphi}}{\sum_{\varphi=1}^{n} \sum_{\varphi=1}^{n} T_{\varphi\varphi}} \quad \varphi = \{1, 2, \ldots, n\}
\]  

(3)

**Strategy 4:** In this strategy, both more demand and less distance are considered simultaneously.

\[
priority_\varphi = \frac{m_1 \sum_{\varphi=1}^{n} T_{\varphi\varphi} + m_2}{\frac{1}{\varepsilon + \sum_{\varphi=1}^{n} W_{\varphi\varphi}}} + m_2 \sum_{\varphi=1}^{n} \sum_{\varphi=1}^{n} T_{\varphi\varphi} \quad m_1, m_2 > 0, \varphi = \{1, 2, \ldots, n\}
\]  

(4)

**Strategy 5:** This strategy considers lower demand and less distance simultaneously.

\[
priority_\varphi = \frac{m_1 \sum_{\varphi=1}^{n} T_{\varphi\varphi} + m_2}{\sum_{\varphi=1}^{n} \sum_{\varphi=1}^{n} T_{\varphi\varphi}} \sum_{\varphi=1}^{n} \sum_{\varphi=1}^{n} W_{\varphi\varphi} \quad m_1, m_2 > 0, \varphi = \{1, 2, \ldots, n\}
\]  

(5)

All denominators in Equations (1) to (5) are ineffective in the customer’s priority and are solely used to eliminate their unit.

**Strategy 6:** Customers are prioritized based on their location codes (the numbers randomly assigned to each customer zone on the map). The lower the customer’s zone code, the higher their priority. We call this a non-priority strategy because neither demand nor distance plays a
role. This strategy is presented to provide a baseline for measuring the efficiency of the customer prioritization policy.

After using one of the mentioned strategies, we consider an ordered couple for each node. The first component of each couple expresses the node’s location code \( \phi \), and the second component shows the priority of that node (that is the same \( \text{priority}_\phi \) and named \( \xi \) in the ordered couple). All ordered couples are unique and are displayed as follows.

\[
\{(\phi, \xi) | \phi = \{1,2,...,n\}, \xi = \{1,2,...,n\}\}
\]  

(6)

In the above set, \( \phi \) and \( \xi \) are the node’s location and priority, respectively, and \( n \) is the number of network nodes. To satisfy some of the constraints of the proposed model, we use two dummy nodes: \((0,0)\) and \((n+1, n+1)\).

### 3.2. Mathematical modeling

**Indices:**

\[
N = \{(\phi, \xi) | \phi = \{1,2,...,n\}, \xi = \{1,2,...,n\}\}
\]

Nodes of the network, along with their priority

\[
(\phi, \xi) = \{(0,0),(n+1,n+1)\}
\]

Dummy nodes

\[
i, j, j_1, j_2 \in N_i = N \cup \{(0, 0), (n+1, n+1)\}
\]

Customer zone nodes

\[
k, l \in N
\]

Possible zones for locating the hubs

\[
v, v_i \in \{1,2,...,V\}
\]

Vehicles

It should be noted that the indices associated with customers and hubs (i.e., \( i, j, j_1, j_2, k \) and \( l \)) are placed according to the first component of the ordered pair (i.e., \( \phi \)). In all constraints of the proposed model, except constraint of customer’s priority (18).

**Parameters:**

| \( \alpha \) | Discount factor for inter-hub transit, \( 0 \leq \alpha \leq 1 \) |
| \( P \) | Number of hubs to find |
| \( M \) | A large number |
| \( \varepsilon \) | A very small positive number |
| \( m_1, m_2 \) | Weighting coefficients |
| \( CT \) | Cost of transportation for moving goods per unit of distance (\$/km) |
| \( Vc \) | Vehicle capacity (Kg) |
| \( DI_{ij} \) | Node \( i \) to node \( j \) distance in terms of travel (Km) |
| \( W_{ij} \) | Demand came from node \( i \) and was directed toward node \( j \). (Kg) |
| \( T_{ij} \) | Distance between nodes \( i \) and \( j \) in time (h) |
\( F_k \) Hub’s fixed location cost in node \( k \).

**Decision variables:**
Decision variables are divided into four categories, which are as follows:

**Activation variables:**
\( X_{ik} \) 0 if node \( k \) doesn’t have a hub; 1, otherwise

**Allocation variables:**
\( X_{ij} \) 1 if a node \( i \) is allocated to hub \( k \); 0, otherwise
\( Z_{kj} \) 1 if a product is sent from hub \( k \) to destination \( j \); 0, otherwise

**Flow variables:**
\( F_{il} \) Flow originated at node \( i \) and routed on the inter-hub link from hub \( k \) to hub \( l \)

**Vehicle fleet scheduling variables:**
\( ST_{kj}^v \) Start time of traveling of loaded vehicle \( v \) from the hub \( k \) to the destination node \( j \)
\( Y_{kj}^v \) 1 if hub \( k \) serves destination node \( j \) with vehicle \( v \); 0 otherwise.
\( X_{k,j,j}^v \) If vehicle \( v \) serves the customer \( j \) shortly after returning from customer \( j \), and both of those customers are allocated to hub \( k \) and \( j \neq j_1 \), then the result is 1; otherwise, the result is 0

The hub location-scheduling issue is modeled using bi-objective MINLP as follows:

**Minimizing the costs:**
\[
\text{Min} Z_1 = \sum_{k \in N} F_k X_k + \sum_{k \in N} \sum_{i \in N} \sum_{j \in N} CT \cdot DI_{ij} W_{ij} X_{ij} + \sum_{k \in N} \sum_{i \in N} \sum_{j \in N} CT \cdot DI_{ij} W_{ij} X_{kj} + \alpha \sum_{k \in N} \sum_{i \in N} \sum_{j \in N} CT \cdot DI_{ij} F_{ij}
\]  \( \alpha \in \mathbb{R} \) \hspace{1cm} (7)

**Minimizing the transportation fleet times:**
\[
\text{Min} Z_2 = \sum_{k \in N} \sum_{j \in N} \sum_{v} ST_{kj}^v \left\lceil \frac{\sum_{g} W_g}{V_c} \right\rceil
\] \hspace{1cm} (8)

\[
\text{s.t.}
\]
**Constraints of hub location-allocation:**
\[
\sum_{k \in N} X_k = P
\]  \( P \in \mathbb{N} \) \hspace{1cm} (9)
\[
\sum_{k \in N} X_{ik} = 1 \hspace{1cm} i \in N
\] \hspace{1cm} (10)
\[
X_{k,j} \leq X_i \hspace{1cm} k, j \in N
\] \hspace{1cm} (11)
\[ \sum_{i=0}^{\infty} W_{ij} X_{ij} \leq M Z_{ij} \quad k \neq j \quad k, j \in N \]  

(12)

\[ Z_{ij} \leq \sum_{i=0}^{\infty} W_{ij} X_{ij} \quad k \neq j \quad k, j \in N \]  

(13)

\[ \sum_{i=0}^{\infty} F_{ij} = \left( \sum_{i=0}^{\infty} W_{ij} \right) X_{ij} - \sum_{i=0}^{\infty} W_{ij} X_{ij} \quad i, k \in N \]  

(14)

**Constraints of vehicle fleet scheduling:**

\[ Y_{ij}^{\nu} \leq Z_{ij} \quad k \neq j \quad k, j \in N \quad \nu = \{1, 2, \ldots, V\} \]  

(15)

\[ \sum_{j'=N \setminus \{t+1 \}} X_{k,j',ij} = V_{ij} \quad k \neq j \quad k \in N \]  

(16)

\[ \sum_{j'=N \setminus \{t+1 \}} X_{k,j',ij} \leq Y_{ij}^{\nu} \quad k \in N \quad \nu = \{1, 2, \ldots, V\} \]  

(17)

\[ ST_{ij}^{\nu} \leq ST_{ij}^{\nu} + M \left( 1 - Y_{ij}^{\nu} \right) \quad k \neq j \neq j_i \quad k \in N \quad \nu, v_i = \{1, 2, \ldots, V\} \]  

(18)

\[ \sum_{\nu} Y_{ij}^{\nu} = \text{roundup} \left( \frac{\sum_{i=0}^{\infty} W_{ij}}{\text{Vc}} \right) Z_{ij} \quad k, j \in N \quad k \neq j \]  

(19)

\[ \sum_{\nu} \sum_{j'=N \setminus \{t+1 \}} X_{k,j',ij} = \text{roundup} \left( \frac{\sum_{i=0}^{\infty} W_{ij}}{\text{Vc}} \right) Z_{ij} \quad k, j \in N \quad k \neq j \neq j_i \]  

(20)

\[ ST_{ij}^{\nu} \leq M Y_{ij}^{\nu} \quad k, j \in N \quad k \neq j \quad \nu = \{1, 2, \ldots, V\} \]  

(21)

\[ ST_{ij}^{\nu} \leq M \left( 1 - X_{k,j',ij}^{\nu} \right) \quad k, j \in N \quad k \neq j \quad \nu = \{1, 2, \ldots, V\} \]  

(22)

\[ Y_{ij}^{\nu} ST_{ij}^{\nu} = Y_{ij}^{\nu} ST_{ij}^{\nu} \quad k \in N \quad k \neq j \quad \nu, v_i = \{1, 2, \ldots, V\} \]  

(23)

\[ X_{k,j,ij}^{\nu} \left[ T_{ij}^{\nu} + 2T_{ij}^{\nu} \right] \leq ST_{ij}^{\nu} \quad k, j, j_i \in N \quad k \neq j \neq j_i \quad \nu = \{1, 2, \ldots, V\} \]  

(24)

\[ X_{k,j,ij}^{\nu} \leq Y_{ij}^{\nu} \quad k, j, j_i \in N \quad k \neq j \neq j_i \quad \nu = \{1, 2, \ldots, V\} \]  

(25)

\[ X_{k,j,ij}^{\nu} \leq Y_{ij}^{\nu} \quad k, j, j_i \in N \quad k \neq j \neq j_i \quad \nu = \{1, 2, \ldots, V\} \]  

(26)

\[ X_{k,j,ij}^{\nu} + X_{k,j,ij}^{\nu} + Y_{ij}^{\nu} \leq 3 \quad j \in N \setminus \{0, 0\} \quad k \in N \quad k \neq j \neq j_i \quad j_i, j_i \in N \setminus \{(n+1,n+1)\} \quad \nu = \{1, 2, \ldots, V\} \]  

(27)
\[
\sum_{j=1}^{v} \sum_{k,j=1}^{v} X'_{k,j} = V \cdot X_k \quad \forall k \in N \tag{28}
\]

**Variables constraints:**

\[
X_i, X_j, Z_{ij} \in \{0,1\} \quad i, j, k \in N \tag{29}
\]

\[
ST_v \geq 0 \quad j,k \in N_i \quad k \neq j \quad v = \{1,2,...,V\} \tag{30}
\]

\[
X'_{i,j,k} \in \{0,1\} \quad j,k \in N_i \quad k \neq j \quad v = \{1,2,...,V\} \tag{31}
\]

\[
Y_{ij} \in \{0,1\} \quad j,k \in N_i \quad k \neq j \quad v = \{1,2,...,V\} \tag{32}
\]

\[
F_{ikl} \geq 0 \quad i,k,l \in N \quad i \neq k \neq l \tag{33}
\]

The first objective function (7) has four terms: minimizing the costs of hub establishment in the first component; minimizing the costs of shipping goods from origin nodes to hubs and from hubs to destination nodes in the second and third components, respectively; and minimizing the costs of shipping goods between hubs while taking the discount factor into account in the fourth component.

The second objective function (8) reduces the sum of vehicle start times and the sum of customer product delivery times. Expression of \(\text{roundup} \left( \sum_{i \in N} \frac{W_{ij}}{V_c} \right)\) has been used to eliminate the effect of the customer's demand value (number of vehicles sent to the customer zone) on the objective function. Constraint (9) ensures that exactly \(P\) nodes are selected as the hub for this network. Constraint (10) indicates that each customer zone should merely be allocated to one hub node (single-allocation assumption). Condition \(i \neq k\) under the summation sign emphasizes that vehicles scheduling is specific for outbound vehicles (i.e. \(X_{ik} = 0\) then \(Z_{ik} = 0\) and consequently \(Y_{ij} = 0\)). Constraint (11) states that if a hub is established, customer \(i\) can be assigned to it.

Constraints (12) and (13) guarantee that the products will be shipped from origins to destination \(j\) through hub \(k\) only when there is a direct link between destination \(j\) and hub \(k\) (\(X_{ij} = 1\)), and in such a situation, there is no limit to supply the demand for the destination \(j\). Constraint (14) guarantees flow balance at each node. According to Constraint (15), hub \(k\) will not allocate any vehicles to destination \(j\) if no commodities are sent there through hub \(k\). Constraint (16) ensures that all vehicles are available at the beginning of the scheduling. Constraint (17) expresses that in each schedule, the vehicle \(v\) departs to the customer \(j\) at most once. Constraint (18) ensures the prioritization of allocated customers to the hub \(k\) for service delivery. Be aware that by applying this constraint, we may determine either an upper limit, a lower bound, or both for any \(ST_v\). The right side of this inequality must be deactivated if the vehicle \(v\) does not go to the customer zone \(j_i\), which is why the formula \(M \cdot (1 - Y_{ij})\) is used there.

Constraint (19) indicates that at any schedule, the number of \(\text{roundup} \left( \sum_{i \in N} \frac{W_{ij}}{V_c} \right)\) vehicles must exactly depart to the customer zone \(j\). It is crucial to remember that while using the phrase
we consider the vehicles’ capacity limit \((Vc \text{ (kg)})\); if the demand of destination \(j\) be \(\sum_{i \in N} W_{ij}\), this constraint states that as \(\text{roundup} \left( \frac{\sum_{i \in N} W_{ij}}{Vc} \right)\) as vehicles have departed to the destination \(j\). This constraint also satisfies the customers’ demand for each allocated customer to hub \(k\) \(\left( \sum_{v} Y_{vj} \geq \text{roundup} \left( \frac{\sum_{i \in N} W_{ij}}{Vc} \right) Z_{ijk} \right)\), and guarantees that no empty vehicles will depart to the destination \(j\) \(\left( \sum_{v} Y_{vj} \leq \text{roundup} \left( \frac{\sum_{i \in N} W_{ij}}{Vc} \right) Z_{ijk} \right)\). Constraint (20) states that for each customer zone \(j\), \(\text{roundup} \left( \frac{\sum_{i \in N} W_{ij}}{Vc} \right)\) vehicles must return from the previous trip, and all depart immediately to the customer zone \(j\). Constraint (21), along with Constraint (28) state, if the vehicle \(v\) has not traveled from the hub \(k\) to the customer zone \(j\), the start time of this trip is necessarily zero. Constraint (22) refers to the base time for the transportation fleet; in reality, each vehicle begins its first journey at the beginning of time. To meet the demand of client zone \(j\), loaded vehicles are required to begin their journey concurrently, according to constraint (23). If the vehicle \(v\) travels from the hub \(k\) to the customer zone \(j\), then the start time of the traveling is a non-negative value; if the vehicle \(v_j\) travels from the hub \(k\) to the customer zone \(j\), according to Constraint (23), its traveling start time must be equal to the start time of vehicle \(v\) otherwise the two sides of Equation (23) be zero (that is, the traveling start time of two vehicles \(v\) and \(v_j\) is not related together). Constraint (24) specifies a lower bound for the traveling start time of vehicle \(v\) to customer zone \(j\) (for each vehicle \(v\)). This lower bound is the return time of vehicle \(v\) from its previous trip \((ST_{ij}^v + 2T_{ij})\). It is crucial to remember that the maximum lower bound \(ST_{ij}^v\) \((\forall v)\), corresponds to the traveling start time of vehicles to the client zone \(j\), Constraints (25) and (26) indicate that variable \(X_{i,j-1-k}\) will receive value one if the vehicle \(v\) has traveled to both destinations \(j\) and \(j_i\), Constraint (27) states that vehicle \(v\) that has come back from customer zone \(j\) cannot serve two customers simultaneously. Considering dummy customer zones \((0,0)\) and \((n+1,n+1)\) this constraint also satisfies even the first customers and the latest customers who served from hub \(k\). The following Constraint (28) (the dummy customer \((n+1,n+1)\) is the same hub \(k\)), all vehicles must return to hub \(k\) after the proposed scheduling of vehicles. Constraints (29)-(32) specify types of investigated variables. Constraints 23 and 24 are nonlinear. So it is necessary to linearize them using linear auxiliary variables.

3.3. Organizing the proposed mathematical model based on six prioritization strategies

In this section, the prioritization strategies introduced in Section 3.1.2 are used in the proposed mathematical model. Each model is referred to as “pri.” Models differ only in Constraint (18).
• Mathematical model pri.1: To make this model, we first obtain the index $N$ based on the proposed strategy (1) and then apply it to the model presented in Section 3.2.

• Mathematical model pri.2: To make this model, we first obtain the index $N$ based on the proposed strategy (2) and then apply it to the model presented in Section 3.2.

• Mathematical model pri.3: To make this model, we first obtain the index $N$ based on the proposed strategy (3) and then apply it to the model presented in Section 3.2.

• Mathematical model pri.4: To make this model, we first obtain the index $N$ based on the proposed strategy (4) and then apply it to the model presented in Section 3.2.

• Mathematical model pri.5: To make this model, we first obtain the index $N$ based on the proposed strategy (5) and then apply it to the model presented in Section 3.2.

• Mathematical model pri.6: In this model, which we call the non-priority model, the index $N$ is obtained as follows:

$$N = \left\{ (\phi, \xi) \mid \xi = \{1, 2, ..., n\} \& \phi = \xi \right\} \quad (34)$$

In this ordered couple, the priority component ($\xi$) is based on the customer's location component ($\phi$), and demand and distance parameters are not included in this model.

### 3.4. Solution approach

To compare strategies and present accurate results accurately, we have applied three approaches for solving the mathematical models.

#### 3.4.1. Single objective approach

With this method, we must consider the cost objective function and evaluate the value of the time objective function as a variable. At the same time, we solve the models pri.1 through pri.6. We use this approach to remove the effect of time optimization on the results.

#### 3.4.2. Lexicography approaches

The lexicography approach is an exact solution method used to obtain one solution of multi-objective models. This approach first optimizes the important objective, and by fixing this value, it seeks to optimize the second objective [39]. In this study, the lexicography approach first minimizes the cost objective function and then minimizes the time objective function.

#### 3.4.3. $\varepsilon$-constraint approach

The $\varepsilon$-constraint approach is an exact solution that solves the multi-objective models and gives the optimal Pareto frontier. In this study, The enhanced $\varepsilon$-constraint technique (AUGMECON), introduced by Mavrotas [40], is an upgraded version of the $\varepsilon$-constraint method we utilize. It should be noted that although the models are solved with three exact approaches, each provides a unique answer.
4. Computational study

We provide a thorough computational analysis of the six techniques described in this section. The CAB dataset utilized in computational studies is first presented. The results of the six suggested solutions are then assessed and compared across 102 small-scale studies and 27 medium-scale trials that we design. Also, we rank the proposed strategies using the Friedman statistical test and the criterion of the relative percentage deviation (RPD). Then, we introduce the best and worst strategies for different cases. Also, we investigate the relationship between strategies and the number of vehicles using the Friedman test. We introduce the best policy for the number of vehicles for different strategies and situations. We also provide some managerial insights from our comparisons of the strategies on the CAB dataset.

A server with an Intel Core i9-9900K processor running at 5.0 GHz and 64 GB of RAM was used for small-scale computational studies. A server running Linux OS with dual AMD Opteron 6134 8Core processors running at 2.3 GHz and 64 GB of RAM was used for computational medium-scale research. With the aid of the GAMS program CPLEX 12.6.3.0, the mathematical models were solved. All the instances were solved to optimality (gap 0.00).

4.1. CAB dataset and problem size challenge for designing the experiments

Based on airline passenger contacts between 25 US cities in 1970, the CAB dataset was created [11]. Fig. 1 displays the locations of the 25 cities that serve as the demand nodes and prospective hubs. The OR Library provides the transportation distances \(D_{ij}\) and demands between each pair of cities \(W_{ij}\) for this dataset. Other parameters that are not included in the CAB dataset are listed in Table 2.

To conduct computational studies, we faced the following major challenges that made it impossible to solve the large-sized problem.

1- According to O'Kelly [11], the hub placement issue is an NP-hard problem. Meanwhile, the precedence constraint makes the problem \(NP\)-hard [41]. In addition to the abovementioned issue, two objective functions dramatically increase the model-solving time, even in a small size. For example, solving a case with 25 nodes, seven hubs, and four vehicles using the GAMS multi-threading solve option took longer than five days; a single-objective similar experiment takes about 40 minutes (on a Linux OS environment with Dual AMD Opteron 6134 8Core 2.3GHz processors with 64GB RAM).

2- The strategies are compared and ranked using the Friedman test. To the best of our knowledge, this test needs at least 15 instances to provide an acceptable ranking. As a result, extensive experiments had to be designed to examine all the factors affecting this ranking.

3- To serve as a potent foundation for an accurate analysis, rankings must be based on the global optimum solution produced from the precise solution techniques. Heuristic and meta-heuristic algorithms, which are capable of solving large-scale issues, cannot be helpful in this respect since they provide reasonably but not always optimum solutions.
Therefore, based on the above problems, 102 experiments in small size ($\{5,...,12\}$ nodes, $\{2,3,4\}$ hubs and $\{2,3,4\}$ vehicles) and 27 experiments in medium size ($\{23,25\}$ nodes, $\{6,7,10\}$ hubs, and $\{2,4,5\}$ vehicles) have been examined. Almost all of the presented models became infeasible in all instances with more than five vehicles, so a maximum of five vehicles was tested.

4.2. statistical and non-statistical analyses

According to the United States map (Fig. 1), cities in the Northeast were considered as near distances, central cities (and some border cities) as medium distances, and border cities (and some central cities) as far distances, which we call “near nodes,” “medium nodes,” and “far nodes”, respectively. The candidate cities in each category (near, medium, and far) have been marked with red stars in Fig. 2. In addition to the above cases, more nodes have been tested, regardless of the distance of the nodes. This case (“mediocre size”) contains 23 and 25 nodes.

4.2.1. Near nodes analyses

As described in Section 3.4, the instances were solved using three approaches. We analyze the results of each approach separately. We offer the overall findings after each subsection.

4.2.1.1. Single objective approach

The rankings of the strategies are presented in Table 3 (relations (35) and (36)). This ranking and their related mean ranks (i.e., the number above each strategy) indicate that the best (or worst) strategy in terms of time is not the best (or worst) strategy in terms of cost. Therefore, to more precisely evaluate the methods, we consider criterion $RPD$. We consider strategy $pri.3$ to be the best strategy for the time goal and compute the mean $RPD$ of strategy $pri.2$, which is the best strategy for costs based on the Friedman test. The results show that, on average, strategy $pri.3$ is 66.15% stronger and more efficient than strategy $pri.2$ in terms of time.

We then consider $RPD$ again (this time, considering cost) to evaluate the efficiency of strategy $pri.2$ in relation to strategy $pri.3$ (which is the best strategy in terms of time). The results show that strategy $pri.2$ is, on average, 2.05% superior to strategy $pri.3$, which is not a significant amount. Therefore, strategy $pri.3$ is assumed to be the best overall strategy.

However, when considering the relation (35), we conclude that the mean rank of strategy $pri.5$ is not much different than that of strategy $pri.3$. Also, strategy $pri.5$ is better than strategy $pri.3$ in terms of cost. Therefore, we evaluate the efficiency of these two strategies.

The results show that, in terms of time, strategy $pri.3$ is only 5.35% stronger than strategy $pri.5$ and, in terms of cost, is only 1.12% weaker than it. It should be noted that strategy $pri.5$ is also slightly different from the best strategy in terms of cost (0.74%). Therefore, we present strategies $pri.3$ and $pri.5$ as the best strategies. Investigating the top strategies shows that in
the case of “near nodes”, giving priority to close customers with low-demand is the best decision. By comparing the two strategies pri.1 and pri.4, it can be understood that the close distance factor is more effective in reducing time than the low-demand factor. Because the worst strategy (pri.2) when combined with the close distance factor, takes a better place in the ranking (i.e. immediately after the strategy of prioritizing customers with low-demand (pri.1). The non-priority strategy ranks poorly in terms of time and is not in a good position in terms of cost. Therefore, this strategy is introduced as the worst strategy in this section.

4.2.1.2. Lexicography approaches

The outcomes of the Friedman test for this data are shown in this section using the results of solving the instances using the lexicography technique (see expressions (37) and (38) in Table 3). The strategies’ ranks did not considerably deviate from the rankings shown in the preceding section. This little difference is due to the time goal function’s inclusion in this method. Based on expressions (37) and (38), we used RPD to compare the efficiency of better techniques in terms of time and cost. In terms of time and cost, strategy pri.5 outperforms strategy pri.2 on average by 43.2% and 0.8%, respectively. The cost advantage of strategy pri.2 over strategy pri.5 is so marginal that it may be disregarded. Therefore, the optimum method is adjudged to be pri.5.

4.2.1.3. $\epsilon$-constraint approach

The experiments and analyses described in previous sections have shown the superiority of strategy pri.5, followed by strategy pri.3. In this section, a limited number of instances are selected from among 34 instances and solved by the $\epsilon$-constraint approach. Then, the Pareto frontier for each strategy is plotted (Fig. 3). Analysis and comparison of Pareto frontiers can confirm or deny the presented results in the previous two sections.

Expression $A\#B\#C$ in the captions of the figures indicates the numbers of nodes, hubs, and vehicles, respectively, from left to right.

Although it is practically impossible to compare Pareto frontiers that represent time and cost factors simultaneously, we conclude that frontiers closer to the origin of the coordinates represent better strategies because they represent shorter times and lower costs. Fig. 3 illustrates that as the nodes increase, the Pareto frontiers of strategies pri.3 and pri.5 become shorter and closer to the origin of the coordinates. Meanwhile, the Pareto frontiers of strategies pri.2 and pri.4 (which coincides) becomes far from of origin of the coordinates. Of course, the Pareto frontier of pri.4 is much better than that one.

The Pareto frontier for pri.1 is in a worse situation than the Pareto frontiers for pri.3, pri.4, and pri.5. This is exacerbated when the nodes are increased. The most interesting point in this figure is related to the Pareto frontier for pri.6 (non-priority strategy) has higher time and cost values than the other strategies. Only the cost of this strategy seems to improve by increasing the nodes, but its time remains longer than those of other strategies. Relations (35) and (37) also confirm that the non-priority strategy (pri.6) is the worst in terms of time (129.7% worse than
in terms of cost, it cannot be claimed as the worst strategy, although it is certainly not the best strategy.

These results indicate that in cases where customers are very close to each other, prioritizing delivery to customers with lower demands and shorter distances to the hub has beneficial consequences when giving service. Meanwhile, the non-priority strategy and prioritizing high-demand customers will increase the delivery time of goods and reduce customer satisfaction.

4.2.2. Medium nodes analyses

Like the "near nodes" section, the instances were solved using three approaches. We analyze the results of each approach separately. We offer the overall findings after each subsection.

4.2.2.1. Single objective approach

The rankings of strategies using the Friedman test are presented in Table 3 (relations (39) and (40)). Rankings show that, as with the case of "near nodes," strategies $pri.3$ and $pri.5$ are the best in terms of time. Of course, strategy $pri.5$ is slightly better. In terms of cost, strategies $pri.2$ and $pri.4$ achieved the same mean ranking and are ranked first. Since strategy $pri.4$ is better than strategy $pri.2$ in terms of time, we measure the efficiency of this strategy against strategy $pri.5$ using $RPD$.

The findings indicate that, on average, approach $pri.5$ is 19.3% faster and 3.2% cheaper than strategy $pri.4$ in terms of time. Therefore, there is very little difference between strategy $pri.5$ and strategy $pri.4$. Taking a deeper look at the mean ranking of strategies (Expression 39), it can be seen that $pri.5$, $pri.3$, and $pri.4$ have a slight difference with each other. This means that in the case of "medium nodes," customers should be prioritized only based on the distance parameter, and close customers should be given a higher priority. The position of $pri.2$ and $pri.3$ together confirms that the criterion of demand value (both low and high) in the case of "medium nodes" will not positively affect the delivery time and customer satisfaction. Non-priority strategy is still the worst strategy. But the important point is that compared to the case of "near nodes", this strategy has taken a worse place. This means that as the distance between the nodes increases, the importance of prioritizing customers increases. This indicates that prioritizing consumers becomes more crucial as the distance between nodes grows.

4.2.2.2. Lexicography approaches

The rankings of strategies derived from the Friedman test are presented in Table 3 (relations (41) and (42)). The rankings are the same as in the previous section. Therefore, we re-evaluate the efficiency of strategies $pri.4$ and $pri.5$ in relation to each other based on the $RPD$. The findings indicate that, on average, strategy $pri.5$ outperforms strategy $pri.4$ in terms of time and cost by 11.4% and 2.8%, respectively.
To conclude this section, neither strategy \textit{pri.4} nor \textit{pri.5} has a significant advantage over the other. Therefore, both can be considered optimal strategies. The rest of the analyzes are the same as the single objective approach section.

4.2.2.3. \(\epsilon\)-constraint approach

Among the 34 designed instances in this section, three instances are selected and solved by the \(\epsilon\)-constraint approach. Fig. 4 displays the Pareto frontiers of the various techniques.

The Pareto frontiers indicate that strategies \textit{pri.3}, \textit{pri.4}, and \textit{pri.5} are better than the others, while strategies \textit{pri.1}, \textit{pri.2}, and \textit{pri.6} are not useful in terms of time or cost. This is especially true of the non-priority strategy (\textit{pri.6}), which is a significant distance away from the others (171.68\% worse than \textit{pri.5} in terms of time).

Based on the results of these three solution approaches, we conclude that strategies \textit{pri.3}, \textit{pri.4} and \textit{pri.5} are the best strategies, while the non-priority strategy is the worst. According to the viewpoint of strategies \textit{pri.4} and \textit{pri.5}, which have different approaches to customers' demands (one prioritizes high-demand customers and the other prioritizes low-demand customers) but a similar approach about customers' distance, the distance parameter is determined as the most significant factor in the case of "medium nodes".

4.2.3. Far nodes analyses

In this section, the instances were solved using three approaches. We analyze the results of each approach separately. We offer the overall findings after each subsection.

4.2.3.1. Single objective approach

The rankings of strategies using the Friedman test are presented in Table 3 (relations (43) and (44)). In this section, as in the previous cases, \textit{pri.5} and \textit{pri.2} are ranked first in terms of time and cost, respectively. The \textit{RPD} indicates that strategy \textit{pri.5} is 1.8\% weaker in terms of cost and 28.4\% stronger in terms of time than strategy \textit{pri.2}. By looking more closely at the position of \textit{pri.3} in relation (43), it can be understood that in the case of "far nodes", the criterion of close distance should not be used as the basis for prioritizing customers. Only close customers who have little demand can be serviced first. Instead, customers who have more demand should have more priority. Like the previous cases, the non-priority strategy is again introduced as the worst strategy in this section.

4.2.3.2. Lexicography approaches

Based on the results of the Friedman test rankings (expressions (45) and (46) in Table 3), we compare the efficiencies of strategies \textit{pri.2} and \textit{pri.5}. The findings indicate that strategy \textit{pri.5} is 1.7\% weaker in terms of cost and 17.9\% stronger in terms of time than strategy \textit{pri.2}. Relations (43)-(46), along with the \textit{RPD} results, indicate that, unlike the near and medium
cases, in terms of time, strategy \texttt{pri.5} is not significantly superior to strategy \texttt{pri.2}. Additionally, approach \texttt{pri.1} is rated second in this instance in terms of time and cost, when it was not a worthy strategy in the other examples. Meanwhile, strategy \texttt{pri.3}, a desirable strategy in previous cases, is not desirable in this case.

It seems that in the case of “far nodes”, prioritization strategies that consider the customer’s demand (\texttt{pri.1, pri.2, pri.4 and pri.5}) are valuable strategies in terms of time. Their mean ranks are very close together, especially when considering strategies \texttt{pri.1, pri.2, and pri.4}. The $\varepsilon$-constraint approach is considered in the next part to provide more reliable judgments of these four strategies.

\textbf{4.2.3.3. $\varepsilon$-constraint approach}

Three of the 34 designed instances in this section are selected and solved using the $\varepsilon$-constraint approach. Fig. 5 displays the Pareto frontiers of the various techniques.

As can be seen, for time and cost, increasing the nodes results in coinciding with strategies \texttt{pri.2} and \texttt{pri.4} with outperforming \texttt{pri.1} and \texttt{pri.5}. Therefore, we ignore the very slight time superiority of strategies \texttt{pri.1} and \texttt{pri.5} in the lexicography approach, thus considering strategies \texttt{pri.2} and \texttt{pri.4} as the best strategies. Also, the non-priority strategy (\texttt{pri.6}) is once again the worst (161.33% worse than \texttt{pri.5} in terms of time). This section's most significant consequence occurs when customers are very scattered. In such cases, customers' demands must be considered above all else when prioritizing customers. Preferably, high-demand customers should be served before others.

\textbf{4.2.4. Effect of the numbers of nodes, hubs, and vehicles on strategies rankings}

To ascertain the impacts of important factors on the rankings of strategies, we evaluate the lexicography approach’s findings in this section. According to Figs. 6 and 7, the number of nodes does not substantially alter the ranks of strategies in any of the three scenarios (near, medium, and remote nodes, which are based on average findings).

We investigate the effect of the number of hubs and vehicles on the rankings of strategies. Each result obtained from the instances is assayed and compared. According to Fig. 8a, in terms of time, changing the number of hubs has had the greatest effect on the far node case (35.0%) and the weakest effect on the medium node case (16.7%). We look at how the quantity of hubs and vehicles affects the ranks of tactics. The outcomes from the examples are all evaluated and contrasted.

Changing the number of hubs has had the most impact on the distant node scenario (35.0%) and the least impact on the medium node case (16.7%) over time, according to Fig. 8a. Changing the number of vehicles has a similar effect in all three cases (22.6% to 27.1%).
The number of hubs and vehicles has a minor influence on the cost rankings of strategies (see Fig. 8b). Changes in the number of hubs (in the case of distant nodes) and close nodes, respectively, have the biggest (20.0%) and lowest (1.7%) impacts. According to the time objective, the smallest change in the rankings is related to the non-priority strategy, which is the worst strategy in all instances except one. Our research demonstrates that under strategy pri.3 (which prioritizes near clients), the impact of adjusting the number of hubs and vehicles is larger the closer the nodes are to one another. The ranks are most significantly displaced as the distance between nodes grows when comparing methods pri.1 and pri.5 with pri.2 and pri.4. In this instance, the ranks are switched around for strategies that have divergent consumer demand policies.

Contrary to the previous instance, the non-priority strategy (followed by the priority strategy) pri.3 is the one that is most significantly impacted by altering the number of hubs and vehicles for the rankings of strategies in terms of cost.

The rest of the strategies experience negligible displacements in the rankings.

The results of this section show that the numbers of nodes, hubs, and vehicles do not profoundly affect the rankings of strategies (especially in terms of cost). The most influential parameter is the distance of nodes (that is, the same distance as near, medium, and far).

4.2.5. Analyses of the mediocre size case

The results of the previous sections show that when the nodes are close to each other, giving priority to customers with lower demands and shorter distances is advantageous in terms of the time objective. When the distance between nodes increases past a certain point, it is better to emphasize customers’ demands instead of their distance to the hub.

In this section, we will study the rankings of strategies in a greater number of nodes when the dispersion of nodes combines close, medium, and distant distances. This is based on the conclusion of the previous section. In this regard, in eight modes, two nodes have been purposely removed from the CAB dataset, and the remaining 23 nodes in three conditions \{10 hubs, 2 vehicles\}, \{7 hubs, 4 vehicles\}, and \{6 hubs, 5 vehicles\} are considered to evaluate and compare strategies.

By purposeful removal, we mean, for example, that two nodes with the highest demand (nodes 4 and 17), the two nodes that are farthest from the rest (nodes 22 and 23), or the two nodes with the most convenient access for others (nodes 11 and 21) are removed. In addition to the 23-node instances, all nodes of the CAB dataset have been tested in the three mentioned conditions.

Therefore, in line with the above explanations, 27 experiments are designed, and only the lexicography approach is used to solve them. We designed other experiments, including those that included more than five vehicles; unfortunately, they became infeasible. Even most of the experiments involving five vehicles were deemed infeasible.

The rankings of strategies using the Friedman test are as follows.

<table>
<thead>
<tr>
<th>Ranking in terms of time:</th>
<th>2.05</th>
<th>2.17</th>
<th>3.25</th>
<th>4.15</th>
<th>4.33</th>
<th>5.05</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>pri.4 (\gg) pri.2 (\gg) pri.3 (\gg) pri.6 (\gg) pri.1</td>
<td>(47)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ranking in terms of cost:</td>
<td>1.58</td>
<td>2.73</td>
<td>3.50</td>
<td>4.08</td>
<td>4.53</td>
<td>4.60</td>
</tr>
<tr>
<td></td>
<td>pri.2 (\gg) pri.4 (\gg) pri.1 (\gg) pri.6 (\gg) pri.3 (\gg) pri.5</td>
<td>(48)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The best choice in terms of time and money is to give high-demand clients (pri.2 and pri.4) priority, according to Expressions (47) and (48). In this case, when the variety of customer scattering is great (i.e., some customers are very close together and others are very distant from any other customers), it is better to prioritize the customers closer to the hub (pri.3) after serving high-demand customers. Note that the worst strategies, in this case, are those which prioritize low-demand customers (pri.1 and pri.5) as well as the non-priority strategy (pri.6). The optimum strategies in terms of both time and cost are regarded as being pri.2 and pri.4. Thus, there is no need to consider the RPD.

4.2.6. Analyses of vehicles

In the previous sections, strategies were ranked in cases of the near, medium, and far nodes and mediocre size cases. In this section, we intend to answer the following two questions;

- How many vehicles are appropriate for near, medium, and far node cases?
- How many vehicles are appropriate for each strategy?

All statistical analyses and graphs in this section are based on the results of the lexicography approach.

4.2.6.1. Relationship between the distance of nodes and the number of vehicles

Figs. 9 and 10 show that distinguished policies should be adopted to allocate vehicles to hubs in near, medium, and far cases. Considering these figures, in the case of far nodes, allocating fewer vehicles to the hubs (two vehicles) optimizes customer service time and cost. That means using vehicles with greater capacities is advantageous. In the case of near nodes, using more (low-capacity) vehicles is very useful for satisfying the time objective while keeping cost at an acceptable level (except when considering strategies pri.3 and pri.6). In the case of medium nodes, using three vehicles optimizes cost and provides service in a reasonable time.

As the distance between the nodes increases, allocating fewer vehicles (but with greater capacities) to the hubs becomes a more beneficial strategy.

4.2.6.2. Relationship between strategies and the number of vehicles

For a certain commodities delivery system, we need to determine how many vehicles need to be allotted to each hub. To do this, we use the Friedman test for each strategy. We combine the results of near, medium, and far cases for each test. According to the Friedman test results (Fig. 11), for strategies pri.1, pri.4, and pri.5, the best strategy is to allocate more vehicles to each hub. Meanwhile, for strategies pri.2, pri.3 and pri.6, it is best to allocate fewer vehicles to each hub. Interestingly, using three vehicles (a medium number of vehicles) is not the best choice.

The quantity of vehicles significantly impacts how well each of these tactics performs. Therefore, we must align our strategy with the number and capacity of vehicles to achieve the
best results. For example, strategies that prioritize customers with lower demands (\textit{pri.1} and \textit{pri.5}) should involve more vehicles. In other words, if the hubs have few high-capacity vehicles, strategies \textit{pri.1} and \textit{pri.5} are not recommended.

The basic concept of this study (i.e., customer prioritizing based on demand and location) has not previously been explored. However, the findings of computer experiments generally indicate that it may have a considerable impact on reducing transportation costs, particularly delivery time. In each case, a comparison of the non-priority strategy with the best customer prioritization strategy shows that the non-priority strategy has had an adverse effect on the delivery time (129.7\%, 171.68\%, and 161.33\% than the best strategy in the case of “near nodes”, “medium nodes” and “far nodes”, respectively).

5. Conclusion

In this work, we created a conventional hub location model for fleet planning in transportation. Vehicle restrictions and a customer priority policy were taken into account throughout the development phase. We started by outlining six priority techniques. Then, a new mixed-integer non-linear programming (MINLP) model was devised, considering the hub location, round-trip vehicle travel, client prioritizing, and other key research assumptions. Then, we linearized the proposed model using alternative restrictions. Then employing each strategy in the proposed model, six new models were introduced that differed only in the constraint of customer prioritization. The introduced models are as follows.

\textit{pri.1}: Customers prioritization based on low-demand
\textit{pri.2}: Customers prioritization based on high-demand
\textit{pri.3}: Customers prioritization based on short distance
\textit{pri.4}: Customers prioritization based on high-demand and short distance
\textit{pri.5}: Customers prioritization based on low-demand and short distance
\textit{pri.6}: Non-priority strategy

After that, we conducted a thorough computational analysis using the CAB dataset to contrast the six suggested solutions. In this respect, sensitivity assessments were carried out by varying the problem's essential variables, namely the separation between nodes and the quantities of nodes, hubs, and vehicles. As a result, 102 instances in small-sized cases and 27 instances in medium-sized cases were designed. We used these instances to compare the six strategies and solve them through three different approaches. We then analyzed the results with several statistical and non-statistical tests and used the test results to rank the six strategies. We conducted further statistical tests to evaluate the number of vehicles. Here, the authors summarize the results for readers. As such, the findings of this study are presented in the form of managerial insights.

- Transportation centers and distributors should not provide services without prioritizing customers; this was the worst strategy in all of our experiments, especially in terms of time.
- If distribution centers are very close to their customers (e.g., hypermarket stores, catering centers), it is best to prioritize customers based on their distance from a distribution center. In this regard, we recommend strategies \textit{pri.3} and \textit{pri.5}.
When customers are not very close to a distribution center and are not too scattered (normal distribution), decision-makers should consider the distance and demand parameters simultaneously. We suggest strategies \textit{pri.4} and \textit{pri.5} in such cases.

If the considered locations do not have a favorable geographical location (i.e., they are far from each other and scattered), the best delivery policy is to prioritize customers with higher demands. In this case, we propose strategies \textit{pri.2} and \textit{pri.4}.

In cases where the variety of customer scattering is great (i.e., some customers are very close together and others are very distant from any other customers), it is better to prioritize the customers closer to the hub (\textit{pri.3}) after serving high-demand customers.

Distribution centers located close to customers can save time and money by using relatively low-capacity vehicles. Conversely, scattered locations should use a few high-capacity vehicles.

Distribution centers with many available vehicles should prioritize customers with lower demands.

Distributors concerned only with the cost of shipping and delivering goods should prioritize the customers with higher demands. We suggest using strategies \textit{pri.2} and \textit{pri.4}.

When time is much more important than cost (as is the case for hospitals, fire stations, centers that deliver highly perishable goods to customers, among other organizations), centers are advised to use strategy \textit{pri.5}. They should never use a non-priority strategy.

Finally, it is crucial to remember that using a particular strategy may not meet all of a distribution network's objectives; instead, it should adopt one or more of the most practical methods based on its capabilities and the needs of its consumers.

References


Biography

**Zahra Motamedi** received her PhD in Industrial Engineering from Kharazmi University, Tehran, Iran in 2021. In her thesis, at the level of operational decision-making, she proposed the innovative idea of customer prioritization, and by implementing this idea in the raw milk supply chain, she achieved wonderful results regarding the improvement of product quality. Her research interests include mathematical modeling, operational research and scheduling.

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operational research, mathematical modeling and multi-criteria decision-making approaches. Many students were graduated at the Kharazmi University under his supervisory. He also published many papers in reputable journals and conferences.

**Seyed Hamid Reza Pasandideh** is an Associate Professor in the Department of Industrial Engineering at the Kharazmi University, Tehran, Iran. He received his B.Sc., M.Sc., and Ph.D. in Industrial Engineering from the Sharif University of Technology, Tehran, Iran. Moreover, he conducted the profound research on the Cold Supply Chain at the University of Nebraska-Lincoln, Lincoln, US. His research interests include optimizing inventory control, multi-objective optimization and application of queuing theory. He has widely published in those fields, and he is the editor of some journals such as International Journal of Supply and Operations Management (IJ SOM).

**Reza Tavakkoli-Moghaddam** is a Professor of Industrial Engineering at the College of Engineering, University of Tehran in Iran. He obtained his Ph.D., M.Sc. and B.Sc. degrees in Industrial Engineering from Swinburne University of Technology in Melbourne (1998), University of Melbourne in Melbourne (1994), and Iran University of Science and Technology in Tehran (1989), respectively. He serves as the Editor-in-Chief of Journal of Industrial Engineering published by the University of Tehran and the Editorial Board member of nine reputable academic journals. He is the recipient of the 2009 and 2011 Distinguished Researcher Awards and the 2010 and 2014 Distinguished Applied Research Awards at University of Tehran, Iran. He has been selected as the National Iranian Distinguished Researcher in 2008 and 2010 by the MSRT (Ministry of Science, Research, and Technology) in Iran. He obtained the outstanding rank as the top 1% scientist and researcher in the world elite group. He has published 5 books, 30 book chapters, and more than 1000 journal and conference papers.

**Figures**

Fig. 1. Map of the US with 25 cities

Fig. 2. Selected cities from the US map as near, medium and far cases

Fig. 3. Pareto frontier of some instances of near nodes

Fig. 4. Pareto frontier of some instances of medium nodes

Fig. 5. Pareto frontier of some instances of far nodes

Fig. 6. Number of nodes' effect on the rankings of strategies in terms of time

Fig. 7. Number of nodes' effect on the rankings of strategies in terms of cost

Fig. 8. effect of the numbers of nodes, hubs, and vehicles on strategies rankings

Fig. 9. Relationship between the distance of nodes and the number of vehicles (in terms of time)

Fig. 10. Relationship between the distance of nodes and the number of vehicles (in terms of cost)

Fig. 11. Relationship between strategies and the number of vehicles

**Tables**

26
Table 1. Comparison of features of the related papers

Table 2. Parameters setting

Table 3. Strategies ranking results for cases of "near nodes", "medium nodes" and "far nodes" based on Friedman test

Figures

Fig. 1. Map of the US with 25 cities

(a) Near nodes

(b) Medium nodes

(c) Far nodes
Fig. 2. Selected cities from the US map as near, medium and far cases

(a) Pareto frontier of instance 8#2#2
(b) Pareto frontier of instance 10#2#2
(c) Pareto frontier of instance 12#2#2

Fig. 3. Pareto frontier of some instances of near nodes
Fig. 4. Pareto frontier of some instances of medium nodes

(c) Pareto frontier of instance 12#2#2
Fig. 5. Pareto frontier of some instances of far nodes

Fig. 6. Number of nodes’ effect on the rankings of strategies in terms of time
Fig. 7. Number of nodes’ effect on the rankings of strategies in terms of cost

(c) Far nodes

Fig. 8. Effect of the numbers of nodes, hubs, and vehicles on strategies rankings

(a) In terms of time

(b) In terms of cost
Fig. 9. Relationship between the distance of nodes and the number of vehicles (in terms of time)
Far nodes

**Fig. 10.** Relationship between the distance of nodes and the number of vehicles (in terms of cost)

**Fig. 11.** Relationship between strategies and the number of vehicles

### Tables

**Table 1**

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Hub network design</th>
<th>Scheduling of fleet</th>
<th>Multi objective</th>
<th>Limitation in number of vehicles</th>
<th>Precedence restriction</th>
<th>customers prioritization</th>
<th>The number of new presented models</th>
<th>Introducing the best/worst strategies in each situation (for future management decisions)</th>
<th>Planning of the vehicles number and capacity</th>
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* Serving one customer and Returning to the hub (to load the next customer's shipment)
**Pickup and Delivery
### Table 2

Parameters setting

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### Table 3

Strategies ranking results for cases of "near nodes", "medium nodes" and "far nodes" based on Friedman test

** The expression $A \gg B$ means that $A$ is better than $B$

** $pri.1$: Customers prioritization based on low-demand; $pri.2$: Customers prioritization based on high-demand; $pri.3$: Customers prioritization based on short distance; $pri.4$: Customers prioritization based on high-demand and short distance; $pri.5$: Customers prioritization based on low-demand and short distance; $pri.6$: Non-priority strategy

| Near nodes | Single objective approach | Ranking in terms of time | 2.20 | 2.47 | 3.08 | 4.00 | 4.53 | 4.72 |
|            |                         | Ranking in terms of cost | $pri.3 \gg pri.5 \gg pri.1 \gg pri.4 \gg pri.6 \gg pri.2$ | 2.53 | 2.91 | 3.16 | 3.84 | 3.49 | 4.67 |
| Lexicography approaches | Ranking in terms of time | $pri.2 \gg pri.4 \gg pri.6 \gg pri.5 \gg pri.1 \gg pri.3$ | 2.42 | 2.58 | 3.08 | 3.30 | 2.47 | 5.56 |
| Medium nodes | Single objective approach | Ranking in terms of time | $pri.5 \gg pri.3 \gg pri.1 \gg pri.4 \gg pri.2 \gg pri.6$ | 2.55 | 2.64 | 3.22 | 3.92 | 3.97 | 4.70 |
| Lexicography approaches | Ranking in terms of time | $pri.2 \gg pri.4 \gg pri.6 \gg pri.1 \gg pri.5 \gg pri.3$ | 2.20 | 2.64 | 3.39 | 3.86 | 4.36 | 4.55 |
|              | Ranking in terms of cost | $pri.5 \gg pri.3 \gg pri.4 \gg pri.1 \gg pri.2 \gg pri.6$ | 2.27 | 3.17 | 3.76 | 4.66 | 4.63 | 4.89 |
|              | Ranking in terms of cost | $pri.2, pri.4 \gg pri.6 \gg pri.3 \gg pri.5 \gg pri.1$ | 2.19 | 2.64 | 2.75 | 3.52 | 3.95 | 5.95 |
|              | Ranking in terms of time | $pri.5 \gg pri.3 \gg pri.4 \gg pri.1 \gg pri.2 \gg pri.6$ | 2.28 | 2.38 | 3.20 | 3.81 | 4.45 | 4.91 |
Table 3
Strategies ranking results for cases of "near nodes", "medium nodes" and "far nodes" based on Friedman test

** The expression $A \gg B$ means that $A$ is better than $B$

**

pri.1: Customers prioritization based on low-demand; pri.2: Customers prioritization based on high-demand; pri.3: Customers prioritization based on short distance; pri.4: Customers prioritization based on high-demand and short distance; pri.5: Customers prioritization based on low-demand and short distance; pri.6: Non-priority strategy

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<th>Far nodes</th>
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(43) \hspace{1cm} (44) \hspace{1cm} (45) \hspace{1cm} (46)