A novel mathematical model to minimize the total cost of the hospital and COVID-19 outbreak concerning waiting time of patients using Jackson queueing networks, a case study

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Abstract:

One of the patients' basic needs when referring to the hospital is to access doctors as soon as possible at a low cost. In this regard, many hospital managers aim to improve healthcare quality. They strive to plan and perform better patient flow in different parts of hospitals. With the widespread of Covid-19, the importance of this matter has become more apparent. Queueing systems are one of the methods that help recognize delays and help to identify bottlenecks. This paper has extended a queue theory model to measure the number of servers in each part of the hospital. The model aims to reduce the hospital's expected total cost, including the waiting time cost of the patients in queues, idle server cost, operating, and the marginal cost of the servers, in a covid-19 pandemic. The proposed model has been solved with Grasshopper Optimization Algorithm (GOA) for large-scale data. Then sensitivity analysis is presented to understand the model better and identify effective parameters.

Keywords: Jackson queueing networks, covid-19, hospital, waiting time, mathematical model, healthcare

1. Introduction

Healthcare is considered a crucial part of any country where its expenditures and revenue are substantial parts of any economy. Hospital is acknowledged as a critical department in the healthcare system, affecting patients' health and lives. Formal statistics indicate a vital need for hospital services; however, there is a considerable gap between supply and demand in hospitals because of medical staff and the facility's shortage, which causes queuing in different hospital sections [1]. Hospital overcrowding indicates a general characteristic that influences the quality and access to health care services. The most critical hospitals challenge is decreasing the patients' waiting time, providing proper care, and enhancing patient satisfaction. Long waiting time is essential in patients' dissatisfaction and complaints about medical centers and increases the disease's severity [2]. In this regard, a slight improvement in managing patients' waiting time can increase this sector's stakeholders' revenue, efficiency, and satisfaction.

Queuing in a hospital has a volatile condition that induces delay, decreases service effectiveness, and causes severe consequences. In some cases, it leads to complications and eventual death (asthma, diabetes, and cardiac disease patients). Besides, we see long queues for hospital services due to the Coronavirus (COVID-19) presence and the inadequacy of ICU beds or other hospital services. Outbreak response to COVID-19 should be considered as a significant project and its opportunity window for planning is often quite limited [3]. So, it is essential to employ the queuing model to improve hospital efficiency and decrease waiting time costs. A hospital's system can be considered a queue network consisting of different sections with various servers . Patients arrive at these sections with multiple probabilities, wait to get their services, get a result, and then according to their condition, go home or be admitted to a related hospital ward. The resources (or servers) in these queuing systems also vary in type and scope. They consist of trained personnel and specialized equipment that patients require. To better manage a hospital, it is necessary to precisely determine the number of servers in each part to avoid long queues to receive services.

This study investigates queuing theory in all different parts of a hospital in Iran, Tehran such as Triage, Specialist, Laboratory, Radiology, MRI, General practitioner, Pharmacy, ICU, CT scan, Pulmonologist, Coronary ward, and Corona laboratory. We aim to determine the optimal number of servers in each section to minimize different hospital costs (patients' waiting time cost in queues, idle server cost, and operating cost of servers) using the M/M/C queueing formulation. We consider the probability of patients moving between different wards and the probability of corona patients entering the hospital in our model. To the best of our knowledge, no study considered applying queueing models in different wards of hospitals during the COVID-19 pandemic as a Jackson queue network; by determining the number of optimal servers in each ward to reduce the total cost; so, our contribution is novel.

The remainder of this paper is organized as follows; first, section 2 addresses related work in the scope of application queueing theory in healthcare and hospitals. In section 3, we present our problem description and mathematical model. Our Solution approach is presented in section 4. Sections 5, 6, and 7 represent our real case study, the comparative study of solution methods, and sensitivity analysis. Finally, the conclusion and managerial insights are presented in section 8.

2 Literature review

The long waiting time in hospitals endangers patients' lives and causes dissatisfaction. A vast amount of research has been conducted to improve various hospital wards' performance [4-9]. They attempt to reduce patients' waiting time through accurate planning and scheduling and the optimal allocation of resources [10] [11-14]; however, hospitals have long queues. It is essential to employ Queueing Theory to improve the hospital's performance and increase patient satisfaction.

Agner Krarup Erlang first introduced the Queuing Theory in 1913 in the concept of telephone traffic. Application of this theory employed in various fields like commercial banks [15-17], food delivery [18], the production-inventory problem [19, 20], cellular manufacturing [21, 22] telecommunications [23, 24], etc. Recently, researchers have applied the queuing theory in the health care system. There is a detailed survey on queuing theory in the health care system, proving it's beneficial. McClain (1976) investigated the research on applying the queueing system in bed assignment policies and its effect on utilization, waiting time, and the probability of turning away patients [25]. The application of this theory in pharmacy is reviewed by [26]. Here, we review just some of the studies related to this research applying queueing theory in hospitals.

Most of the papers study queueing models in emergency departments based on the waiting time in this part of the hospital, which is critical [27, 28]. [29] mentions that queueing model presents applicable models to minimize patients' waiting time and maximize emergency department performance. Queueing models can also help estimate the number of staff members in each server to satisfy demand. [30] employs a queuing model as a tool to optimize screening services by considering the Covid-19 pandemic. [31]proposed a GI/G/C model to find the efficient number of servers to decrease waiting time in an emergency department. The limited waiting time and patients' priorities also have been considered. [32] have analyzed the emergency service of a hospital by utilizing concepts and relationships of the queueing system. [33] used a queueing model to optimize the waiting time and the entrance rate of the patients in the Fortis Escorts Hospital in Jaipur. As a result, to improve patient satisfaction, they proposed some suggestions to improve the efficiency of the outpatient department and delays. [34] considered patient's registration department, outpatients department, and pharmacy parts of hospitals to applying queueing theory. In the past years, several works have been done in health care systems and optimization [35-37]. Most of the papers considered emergency departments, but we consider the whole hospital to study in this paper.

Due to the COVID-19 pandemic, the demand for treatment services increased, leading to increased waiting times in different hospital wards. Considering the critical situation of the COVID-19 pandemic, we have proposed a queueing model to define the number of doctors and servers in each part of the hospital to reduce the total cost of the hospital. The objective function involves the patients' waiting time cost in queues, idle server cost, operating, and the marginal cost of the servers. To obtain the optimal number of servers in each part, we have considered the individual probability of patient arrival rate at each part by considering historical data in one of the Tehran hospitals. To the best of our knowledge, no study applies queueing theory in all different sections of hospitals by considering patients who are struggling with COVID-19, so our contribution is novel.

3. Problem description

In this research, we study twelve critical parts in a hospital in Tehran, where the patients form a queue in each one. The hospital accepts both coronary and non-coronary patients. The relationships between the different departments of the hospital are shown in Figure 1. In this section, we define the problem and the relationships between these departments.

In the hospital, the patients arrive at the triage part (T) according to a Poisson process with a rate λ_t . We assume that each server servers at node T Need an exponential time with mean $\frac{1}{\mu_t}$ to examine patients and prioritize them based on their condition. Therefore, if the servers are busy in triage, the patients should wait in the queue. In this node, at first, the patients' temperature is assessed. Suppose symptoms occur due to coronavirus (or according to the patient's desire); in this case, the patients will be sent to a corona test laboratory (K), with the probability of P_{tk} to test and diagnose the disease, which we will discuss the process in detail later.

In a situation where there is no evidence of coronavirus, patients will be sent to a general practitioner (G) with the probability of P_{tg} . The Service times at node G are independent of other nodes and exponentially distributed with a mean of $\frac{1}{\mu_g}$. At the doctor's discretion, the patient who has ended the service in node G may need a test, x-ray, or MRI to make a more accurate diagnosis. In these cases, they will refer to a pharmacy (node D), laboratory (node L), radiology (node R), MRI (node M), and specialist (node E) with the probabilities of $P_{gd}, P_{gl}, P_{gr}, P_{gm}$, and P_{ge} respectively. Otherwise, they leave the hospital with the probability of P_{go} for performing outpatient treatment .It is assumed that the Service times at nodes D, L, R, M, and E are independent of other nodes and exponentially distributed with a means of $\frac{1}{\mu_d}, \frac{1}{\mu_l}, \frac{1}{\mu_r}, \frac{1}{\mu_m}$ and $\frac{1}{\mu_e}$ respectively. The patients who have completed service at node E, at the specialist's discretion, will refer to a pharmacy (node D), laboratory (node L), radiology (node R), and MRI (node M) with the probabilities of P_{ed}, P_{el}, P_{er}, and P_{em} respectively, or they will leave the hospital with the probability of P_{eo}. Also, the patients who have finished their service at the laboratory (node L), radiology (node R), and MRI (node M) refer to the general practitioner or the specialist to show their medical report. Hence, they refer to the former with probabilities P_{lg}, P_{rg} and P_{mg} and the latter with probabilities P_{le}, P_{re}, and P_{me}. Alternatively, they may exit the hospital with the probability of P_{lo}, P_{ro}, and P_{mo}.

As mentioned at the beginning of this section, after initial thermometry in the triage section (T) and observation of the patient's symptoms, if the patient suspects coronavirus, they should be referred to the Corona test laboratory (K) with the probability of P_{tk}. Service time at node K is independent of other nodes and exponentially distributed with a mean of $\frac{1}{\mu_k}$. After determining the test result, if the result is negative, the patient will leave the hospital with a probability of P_{ko}. Otherwise, the patient will be referred to a pulmonologist (B) with the probability of P_{kb}. The service times in node B are independent of other nodes and exponentially distributed with a mean of $\frac{1}{\mu_b}$. Then patients who have completed service in node B, according to the diagnosis of a pulmonologist and the severity of the disease, will be sent to one of the nodes, coronary ward (S), ICU (U), and CT-scan (Z) with the probabilities of P_{bs}, P_{bu} and P_{bz} Respectively. Alternatively, it is possible that the patient needs care at home, so the patient may go to the pharmacy (D) to get their medicine or leave the hospital, which each one occurs with probabilities of P_{bd} and P_{bo} , respectively. It is assumed that the service times at nodes S, Z, and U are independent of other nodes and exponentially distributed by means of $\frac{1}{\mu_s}, \frac{1}{\mu_z}$, and $\frac{1}{\mu_u}$. When patients receive their CT-scan result, they may go to the pulmonologist (to show the results) or exit the hospital, with probabilities of P_{zb} , and P_{zo} , respectively. When the patient's service is complete in node S, according to the patient's condition and the pulmonologist's diagnosis, they will be sent to CT-scan and ICU with the probabilities of P_{sz} , and P_{su} , respectively. Alternatively, they will discharge from the hospital with the probability of P_{so} . Depending on the condition of the patients hospitalized in the ICU, they may be referred to the coronary ward with a probability of P_{us} . Patients may also die in this node and be discharged with a probability of P_{uo}. We assume that there is no capacity limit in the nodes, and also, the arrival process from the outside to node i for $i = L, R, M, D, Z$, and K , follow a Poisson process with a mean rate λ_{io} .

Given that the proposed network characteristics are precisely the same as Jackson's open network assumptions, we have a network that acts as if each node is an independent $M/M/C$ queueing system [38]. Therefore, each node's steady-state probability values can be calculated using the steady-state probability of the $M/M/C$ queueing system. We can also compute expected performance measures for each node, the same as $M/M/C$ [38].

3.1. Traffic equations of the proposed network

As mentioned before, a patient may go back or refer to a node several times. So, we have to solve the following system of equations to obtain the total expected flow rate into each node ("from outside and from other nodes" [38]). Note that the following equations are obtained according to the traffic equation [38].

$$\lambda_i = \lambda_{io} + \sum_{i=1}^{k} \lambda_i \times p_{ij} \quad (1)$$

As mentioned in the previous section, we have assumed the entry rate to triage, and external entry rates to nodes D, R, M, L, K, and Z are given.

$$\lambda_g = \lambda_t \times P_{tg} + \lambda_l \times P_{lg} + \lambda_r \times P_{rg} + \lambda_m \times P_{mg} \quad (2)$$

$$\lambda_d = \lambda_{do} + \lambda_g \times P_{gd} + \lambda_b \times P_{bd} + \lambda_e \times P_{ed} \quad (3)$$

$$\lambda_e = \lambda_g \times P_{ge} + \lambda_l \times P_{le} + \lambda_r \times P_{re} + \lambda_m \times P_{me} \quad (4)$$

$$\lambda_l = \lambda_g \times P_{gl} + \lambda_e \times P_{el} + \lambda_{lo} \quad (5)$$

$$\lambda_r = \lambda_g \times P_{gr} + \lambda_e \times P_{er} + \lambda_{ro} \quad (6)$$

$$\lambda_m = \lambda_g \times P_{gm} + \lambda_e \times P_{em} + \lambda_{mo} \quad (7)$$

$$\lambda_b = \lambda_k \times P_{kb} + \lambda_z \times P_{zb} \quad (8)$$

$$\lambda_k = \lambda_t \times P_{tk} + \lambda_{ko} \quad (9)$$

$$\lambda_s = \lambda_b \times P_{bs} + \lambda_u \times P_{us} \quad (10)$$

$$\lambda_u = \lambda_b \times P_{bu} + \lambda_s \times P_{su} \quad (11)$$

$$\lambda_z = \lambda_s \times P_{sz} + \lambda_b \times P_{bz} + \lambda_{zo} \quad (12)$$

We can rewrite the above system of equations in a standard matrix form A.X=B as follows:

$$\begin{bmatrix} 1 & 0 & 0 & -P_{\text{lg}} & -P_{r\text{g}} & -P_{m\text{g}} & 0 & 0 & 0 & 0 & 0 \\ -P_{gd} & 1 & -P_{ed} & 0 & 0 & 0 & -P_{bd} & 0 & 0 & 0 & 0 \\ -P_{\text{ge}} & 0 & 1 & -P_{\text{le}} & -P_{re} & -P_{me} & 0 & 0 & 0 & 0 & 0 \\ -P_{gl} & 0 & -P_{el} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -P_{gr} & 0 & -P_{er} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -P_{gm} & 0 & -P_{em} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -P_{kb} & 0 & 0 & -P_{zb} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -P_{bs} & 0 & 1 & -P_{us} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -P_{bu} & 0 & -P_{su} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -P_{bz} & 0 & -P_{sz} & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_G \\ \lambda_D \\ \lambda_E \\ \lambda_L \\ \lambda_R \\ \lambda_M \\ \lambda_B \\ \lambda_K \\ \lambda_S \\ \lambda_U \\ \lambda_Z \end{bmatrix} = \begin{bmatrix} \lambda_t P_{tg} \\ \lambda_{do} \\ 0 \\ \lambda_{lo} \\ \lambda_{ro} \\ \lambda_{mo} \\ 0 \\ \lambda_t P_{tk} + \lambda_{ko} \\ 0 \\ 0 \\ \lambda_{zo} \end{bmatrix} \quad (13)$$

In which the coefficient matrix (14) and column vector (15) of the equations' right-hand-sides are as follows:

$$A = \begin{bmatrix} 1 & 0 & 0 & -P_{\mathrm{lg}} & -P_{r\,\mathrm{g}} & -P_{m\,\mathrm{g}} & 0 & 0 & 0 & 0 & 0 \\ -P_{gd} & 1 & -P_{ed} & 0 & 0 & 0 & -P_{bd} & 0 & 0 & 0 & 0 \\ -P_{\mathrm{g}e} & 0 & 1 & -P_{\mathrm{l}e} & -P_{re} & -P_{me} & 0 & 0 & 0 & 0 & 0 \\ -P_{gl} & 0 & -P_{el} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -P_{gr} & 0 & -P_{er} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -P_{gm} & 0 & -P_{em} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -P_{kb} & 0 & 0 & -P_{zb} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -P_{bs} & 0 & 1 & -P_{us} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -P_{bu} & 0 & -P_{su} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -P_{bz} & 0 & -P_{sz} & 0 & 1 \end{bmatrix} \qquad (14)$$

$$B = \begin{bmatrix} \lambda_t P_{tg} \\ \lambda_{do} \\ 0 \\ \lambda_{lo} \\ \lambda_{ro} \\ \lambda_{mo} \\ 0 \\ \lambda_t P_{tk} + \lambda_{ko} \\ 0 \\ 0 \\ \lambda_{zo} \end{bmatrix} \qquad (15)$$

Then we should solve the system of the above equations to obtain the values of the input rates, which are used in the next section in solving the mathematical model.

3.2. Mathematical model

This section develops a mixed-integer non-linear programming model to minimize the hospital's expected total cost, including the cost of the patients' waiting time in queues, idle server cost, operating, and the marginal cost of the servers. Before presenting the model, we express the notions as follows:

Set	
i	Different hospital's wards index
parameters	
λ_i	Arrival rate of part i
u_i	Service rate of each server in part i
c_{pi}	cost per unit time when a patient waits in the formed queue in part i
c_{si}	cost per unit time when a server in part i is idle
c_{osi}	Operating cost of a busy server in part i per unit time
c_{asi}	Adding server cost in part i
s_i	Upper bound for the number of servers in part i
$budget$	Maximum budget
Decision Variables	
c_i	Number of servers in each part
π_0^i	Fraction of time that part i is empty
π_n^i	Fraction of time that there is n patient in part i
ρ^i	Traffic intensity of $M/M/C$ queue of part i
L^i	Expected total number of patients in part i
L_q^i	Expected total number of patients in the formed queue in part i
w_q^i	Expected time a patient spends in the formed queue in part i

The proposed MINLP model is developed as follows:

$$\min \sum_i c_{pi} L_{qi} w_q^i + c_{si}(c_i - (L_i - L_{qi})) + c_{osi}(L_i - L_{qi}) + c_{asi} c_i \tag{16}$$

Subjected to:

$$\pi_0^i = \left[1 + \sum_{n=1}^{c_i-1} (\frac{\lambda_i}{\mu_i})^n . \frac{1}{n!} + \sum_{n=c_i}^{\infty} (\frac{\lambda_i}{\mu_i})^n . \frac{1}{c_i\,!} . \frac{1}{c_i^{n-c_i}\,!}\right]^{-1} \qquad for \quad i \in \{1, 2, ..., 12\} \qquad (17)$$

$$L_q^i = \frac{\pi_0^i}{c_i\,!} . (\frac{\lambda_i}{\mu_i})^{c_i} . \frac{\rho_i}{(1-\rho_i)^2} \qquad for \quad i \in \{1, 2, ..., 12\} \qquad (18)$$

$$\rho_i = \frac{\lambda_i}{(c_i \times \mu_i)} \qquad for \quad i \in \{1, 2, ..., 12\} \qquad (19)$$

$$L^i = \frac{\pi_0^i}{s_i\,!} . (\frac{\lambda_i}{W_q})^{c_i} . \frac{\rho_i}{(1-\rho_i)^2} + \frac{\lambda_i}{\mu_i}$$

$$z \qquad for \quad i \in \{1, 2, ..., 12\} \qquad (20)$$

$$Z = 9516.652$$

$$Z = 10471.03$$

$$w_q^i = \frac{\pi_0^i}{(c_i \mu_i) c_i\,!} . (\frac{\lambda_i}{\mu_i})^{c_i} . \frac{1}{(1-\rho_i)^2} \qquad for \quad i \in \{1, 2, ..., 12\} \qquad (21)$$

$$\sum_i c_{asi} c_i \le budget \qquad for \quad i \in \{1, 2, ..., 12\} \qquad (22)$$

$$\rho_i \le 1 \qquad for \quad i \in \{1, 2, ..., 12\} \qquad (23)$$

$$c_i \le s_i \qquad for \quad i \in \{1, 2, ..., 12\} \qquad (24)$$

$$\pi_n^i = \begin{cases} (\frac{\lambda_i}{\mu_i})^n . \frac{1}{n!} \pi_0^i & n \langle c_k \\ (\frac{\lambda_i}{\mu_i})^n . \frac{1}{c_i\,!} . \frac{1}{c_i^{n-c_i}\,!} \pi_0^i & n \rangle c_k \end{cases} \qquad for \quad i \in \{1, 2, ..., 12\} \qquad (25)$$

$$L^i = \sum_{n=0}^{\infty} n \pi_n^i \qquad for \quad i \in \{1, 2, ..., 12\} \qquad (26)$$

The objective function (16) seeks to minimize the expected total cost of the hospital. It includes the cost of the patients' waiting time in queues, idle server cost, operating, and the marginal cost of the servers. Constraint (17) calculates the fraction of time that part i is empty. Constraint (18) illustrates the expected total number of patients in the formed queue in part$i$. Constraint (19) shows traffic intensity of the $M / M / C$ queue of part$i$, and the total number of patients in the hospital in different servers is calculated by equation (20). The expected waiting time a patient spends in the formed queue in part i is proposed in constraint (21). Constraint (22) indicates that the cost of adding a server in part i must be less than our budget. Constraint (23) presents the condition of stability of the $M/M/C$ queue formed in part i. Constraint (24) states that the number of servers in each part must be less than the upper bound for the number of servers in these parts. Constraint (25) calculates the fraction of time that there is n patient in part$i$. Constraint (26) calculates the expected total number of patients in part$i$.

4. Solution approach

To evaluate the performance of the proposed model, we solved the problem in various sizes with the help of GAMS software, which is considered an exact tool to solve optimization problems. Due to the nonlinearity of the problem, solving it in GAMS is very time-consuming; Moreover, by increasing the upper bound of servers, the problem's answer space becomes more extensive, significantly increasing the number of feasible solutions. So, it is necessary to employ other methods to solve the problem in large size as well as faster. So, we apply the Grasshopper Optimization Algorithm (GOA) which will be fully explained in Sections 4-2. Furthermore, the case study and several test problems are prepared and solved with GAMS and MATLAB simultaneously to indicate GOA algorithm efficiency.

4.1. Cramer's rule

As we know, patients may go back or refer to a node several times. Afterward, we should find the entry rate of each node. So, we have to solve the system of equations presented in section 3-1. For this purpose, we use Cramer's rule, which is an explicit formula for the solution of a system of linear equations. It expresses the solution in terms of the determinants of the coefficient matrix and matrices obtained from it by replacing one column with the column vector of the equations' right-hand sides [39]. In the Cramer method, we should represent our equation in matrix multiplication form $A.x = B$, as it shown in section 3-1. As it proposed in section 3-1. The vector $x = (x_1.x_2.x_3.x_4)^T$ is the column vector of the variables. In our problem, the variables are the entry rate of each node, $\lambda = (\lambda_g.\lambda_d.\lambda_e.\lambda_l.\lambda_r.\lambda_m.\lambda_b.\lambda_k.\lambda_s.\lambda_u.\lambda_z)^T$. Then the Cramer rule states that in this case, the system has a unique solution, whose individual values for the unknowns are given by:

$$x_i = \frac{\det(A_i)}{\det(A)} \qquad for \quad i \in \{1,2,...,12\} \tag{27}$$

Where A_i is the matrix formed by replacing the i-th column of A with the column vector b.

4.2. Grasshopper optimization algorithm (GOA)

The GOA is a meta-heuristic method presented by Saremi et al. in 2017 [40]. Slow movement and small grasshopper steps are the swarm's principal specifications. Although, long-term and sudden motion is the necessary trait of the swarm in adulthood. Seeking food is one of the main feathers of grasshoppers' swarming [40]. The GOA algorithm simulates grasshoppers' swarming behavior in nature and tries to find the optimal solutions Swarm intelligence techniques [41]. GOA imitates grasshoppers' behavior and their movements for seeking food sources. The search process in nature-inspired algorithms is divided into two propensities: exploration and exploitation, which are done naturally by grasshoppers. The search factors are influenced to move suddenly, while grasshoppers want to move locally during the exploitation state. Saremi et al. (2017) found a way to model this behavior mathematically and proposed a powerful nature-inspired algorithm for optimizing problems [40].

As mentioned before, the proposed model is non-linear, which remarkably increases the solution time. Solving this model with an exact method like GAMS is time-consuming, even for a medium scale. So, it is necessary to employ the GOA algorithm as a metaheuristic approach. GOA is a kind of constructive algorithm capable of large-scale and real-world problems on any scale. GOA has prepared for continuous spaces, so it is essential to continuously create initial solutions in discrete spaces so that exploration and exploitation operators work better [42, 43]. Due to the mentioned reasons, this algorithm has been used to solve this study's model.

5. Case Study

Due to the COVID-19 pandemic, more than 215,655,811 cases of COVID-19 have been reported globally, and Iran has declared 4,869,414 patients and 105,287 deaths. Besides this statistic is increasing daily due to different types of coronavirus, so hospitals and the medical community in Iran are in a severe crisis. Tajrish Hospital is one of the most important hospitals in Iran in terms of receiving coronary patients. In the fifth corona peak, this hospital has allocated more than its capacity to coronary patients. It has turned many other wards into coronary wards and ICUs, even postponing unnecessary surgeries and imposing costs on the hospital. These issues have caused economic problems along with problems related to virus management and control in the hospital. In addition, hospital managers are always trying to determine the optimal number of servers in different wards of the hospital in a way that patients' waiting time and the costs incurred by the hospital are minimized. Therefore, Tajrish Hospital has been considered our case study in this ward. We attempt to reduce the total cost of the hospital and determine the optimal number of servers in each ward, by employing the proposed model and the Cramer method. We will also solve the model by GAMS software and the GOA algorithm, discussing the results later.

According to the Cramer method, the system of equations mentioned in sections 3-1 has an answer if the determinant of matrix A is not zero. To calculate A's determinant, we should know the probability values for the entry and exit of each node. These probabilities and the rate of external entry into the system are obtained from the previous years of the Tajrish hospital. Regarding the probabilities related to the coronary department and the studied hospital data, the statistics published by the Ministry of Health have been used for more accurate research. The default values are shown in Table 1.

Therefore, we have the numerical coefficient matrix as follows:

$$A=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.4 & 1 & 0 & 0 & -0.2 & -0.2 & -0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1 & 1 & -0.15 & 0 & 0 & 0 & -0.4 & 0 & 0 & 0 & 0 \\ 0 & -0.2 & 0 & 1 & -0.3 & -0.3 & -0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2 & 0 & -0.25 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2 & 0 & -0.15 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2 & 0 & -0.25 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -0.8 & 0 & 0 & -1 \\ -0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.1 & 0 & 1 & -0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.1 & 0 & -0.1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.1 & 0 & -0.3 & 0 & 1 \end{bmatrix}$$

Before solving equations, we determine the determinant of matrix A whose values were 0.4688, so we can use the Cramer method. After solving the equations with MATLAB software's help, the following results will be obtained for each node's entry rates, which are shown in Table 2.

To solve the model in the studied hospital, the following data was collected from the hospital's historical data (Table 3). We solved the problem with the help of GAMS software, which is considered an exact tool for solving optimization problems. The output is shown in Table 4.

As shown in table 4, the most significant servers are related to the general laboratory, corona test laboratory, and inpatient wards of coronary patients, which have more arrival patients. On the other hand, inpatient wards for coronary patients, ICU, and CT scans are among the most critical wards of the hospital because they are directly related to patient's health and lives and have the most arrival rate. Furthermore, a slight delay in patient admission in such wards will cause irreparable damage. Accordingly, we try to determine the number of servers optimally by employing the proposed model while minimizing the total cost of waiting time for patients in each ward. By utilizing this model in the Tajrish hospital, waiting times have been significantly reduced compared to before. In contrast, the total cost is reduced, which is less than the total hospital budget. These results demonstrate the advantage of our model for hospital managers in making decisions.

6. Comparative study of solution methods

To evaluate the performance of the proposed GOA algorithm, first, we solved our case study in MATLAB software by employing GOA, then we compare the obtained results with GAMS's output (Table 4). The following formulation is used for calculating the gap rate.

$$Gap = (\frac{GOA \quad result - GAMS \quad result}{GAMS} result)*100 \tag{28}$$

As shown in Table 5, the gap between values is 2.008% on average, indicating the GOA's efficiency. Three test problems have been solved simultaneously by these two softwares with predetermined parameters to ensure the GOA algorithm's accuracy and efficiency. The values of variables derived from GAMS and MATLAB for the test problems are shown in Tables 6, 7, 8. Our result shows that there is just a slight deviation between results gained from these two softwares, which has proven the efficiency of the proposed GOA algorithm. It is worth noting that by increasing the upper bound of servers, the problem's answer space becomes more extensive, significantly increasing the number of feasible solutions. It leads to a significant increase in the problem-solving time with GAMS; in this case, decision-makers can use the proposed GOA algorithm.

7. Sensitivity analysis

According to the developed model, we aim to survey the influence of various parameters on the model variable by changing them and evaluating their effect on the objective function. Parameter λ_i, which represents the arrival rate to each node, will be surveyed first. To determine the effect of the arrival rate on the model variables, we try to find trends and analysis through several runs of GAMS software by gradually increasing the arrival rate. We recorded some critical variables' values in all sections gathered from GAMS software in Table 9. Then, for simplicity in analysis, we considered the sum of these values in all twelve hospital sections in Table 10. Figure 2 indicates the effect of increasing the total entry rate parameter on the number of servers in all the sections. The higher arrival rate will increase the total number of servers. Increasing the arrival rate will increase the waiting time of patients until the hospital adds to its servers which causes to reduce the waiting time. As shown in Figure 3, with the increase of λ from 127 to 151, w_q increased with a more precipitous slope than the same increase from 151 to 175, which is due to the increase in the number of servers from 22 to 25. However, increasing the number of servers reduces the waiting time, it will cause the cost of adding a server to the hospital. Furthermore, in some nodes, we may even encounter server idle, which is costly. Therefore, it is essential to know the effect of each parameter on the total cost to determine the optimal values for the number of servers in each section. Figures 4 indicate the effect of arrival rate on the total number of patients in the queue. During the corona epidemic, one of the most critical wards of hospitals in the inpatient ward for coronary people. The higher number of nurses and beds in this ward leads to faster service and reduced waiting time for patients, which decreases the corona mortality rate. For this purpose, Table 11 has been designed to show the importance of this issue in this ward. Figures 5 and 6 indicate the impact of service rate on the number of servers and the coronary ward waiting time, respectively.

8. Conclusions and managerial insights

This article discusses the application of queueing theory in one of the medical centers in Tehran (Tajrish hospital) during the pandemic of covid-19. As we know, in coronary conditions, hospitals, especially those that accept coronary patients, face an increase in patient admissions, resulting in increased overcrowding in various wards . In this situation, analyzing the performance of different departments and calculating the number of employees required in each department will be very helpful. Also, the knowledge of hospital managers about the service status of the medication system to patients will help a lot in developing an appropriate strategy to improve service.

To the best of our knowledge, this is the first paper presenting the covid-19 pandemic in Iran, twelve wards of hospital conditions, and their analysis using queue theory. For this purpose, Jackson queueing network is used. As shown in the numerical results, the results of this model indicate that there are long queues in the ICU and heart departments. Therefore, the number of servers in these sections should be increased compared to before.

On the other hand, due to the high prevalence of covid-19 and bed shortage in the coronary ward, a percentage of beds in non-coronary wards should be allocated to this section. Also, a large number of coronary patients in the ICU are waiting to receive services. While these patients do not have to wait long, the number of beds and equipment in this ward needs to increase compared to before.

As mentioned earlier, this article can help hospital managers in many ways. Here are some of the suggestions based on sensitivity analysis:

- According to Table 9, managers are suggested that when patients' arrival rates increase, increasing the number of servers in that part to reduce patient density and waiting time will be helpful.
- According to Table 10, managers are advised that if more patients visit the system, they should consider a larger budget for the system to increase the number of servers in each department to the appropriate level at the right time.

- According to Figures 3 and 4, it can be seen that with the increase of patients entering the hospital, if the managers do not have the necessary measures to increase the server of each ward, the waiting time of patients has increased significantly, which can have irreparable consequences. So, the model helps managers when to increase the number of servers in each department.
- According to Table 11, in the case of the possibility, managers advised increasing interns' numbers instead of increasing the server to reduce the total cost. Furthermore, if it is impossible to increase the number of servers, managers can increase the number of assistants instead. The exact amount of increase is determined by the proposed model.
- According to Figure 6, managers can reduce patients' waiting time in the system in case of hospital congestion by increasing the number of assistants.

For future studies, we suggest the following directions extend the current research:

- Consider more sections in the hospital.
- Using fuzzy service rate for each node.
- Consider priority for emergency patients.
- Using correlation analysis between different ward
- Considering patient priority and satisfaction

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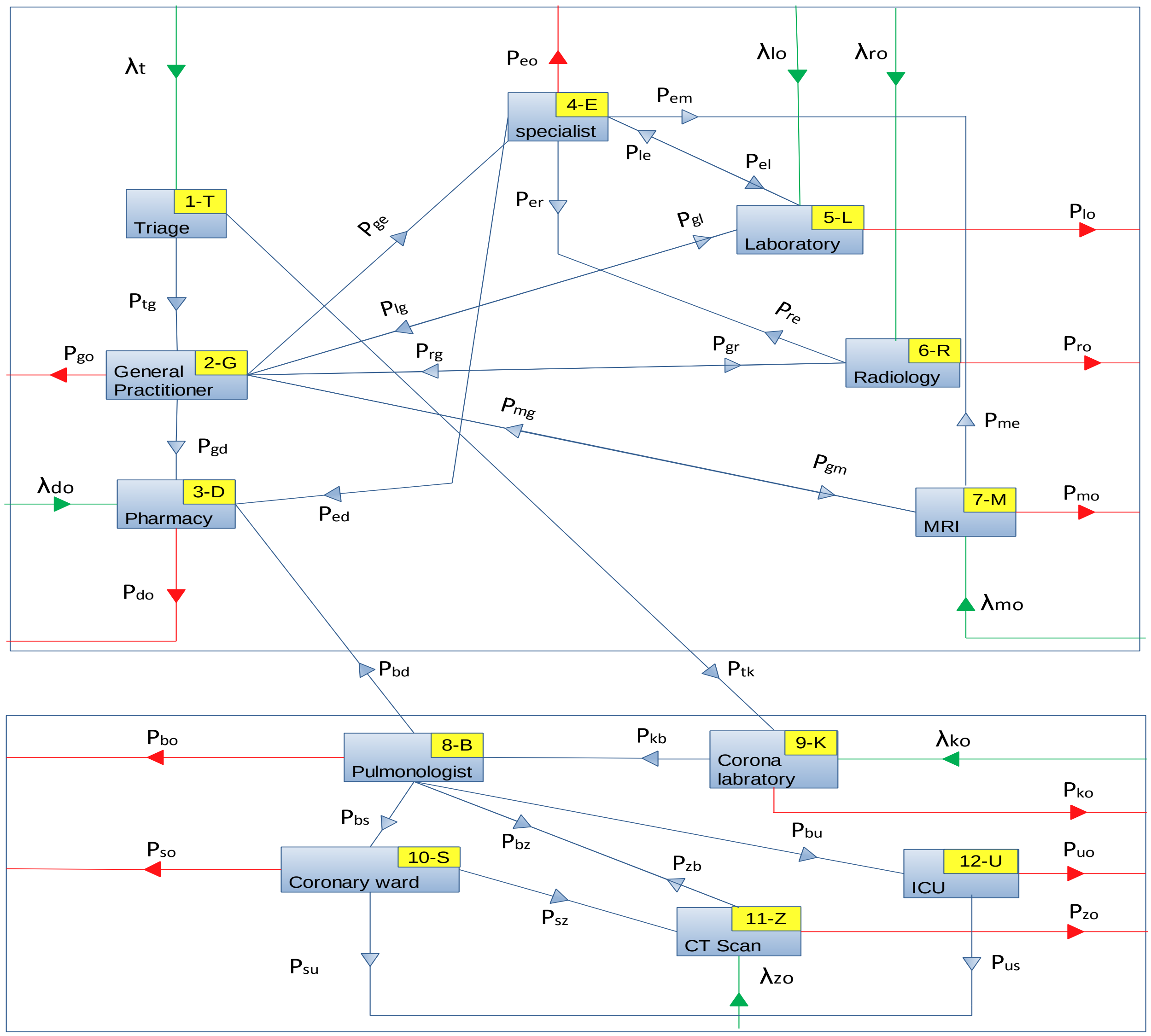

Figure 1. Relationships between different parts in the hospital

Patient departure

Patient arrival

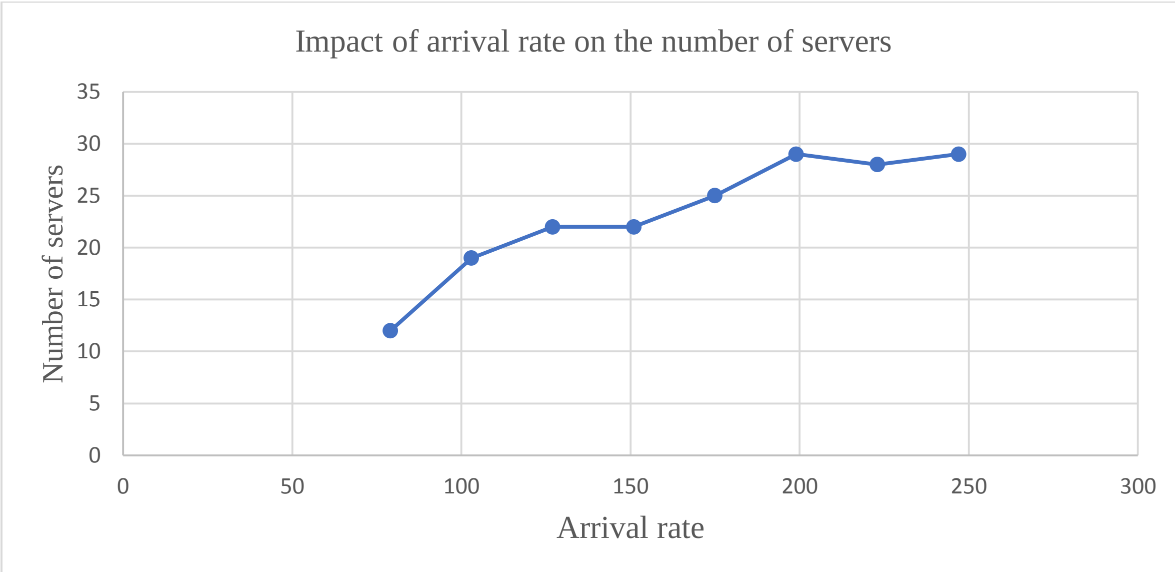

Fig 2. Impact of arrival rate on the number of servers

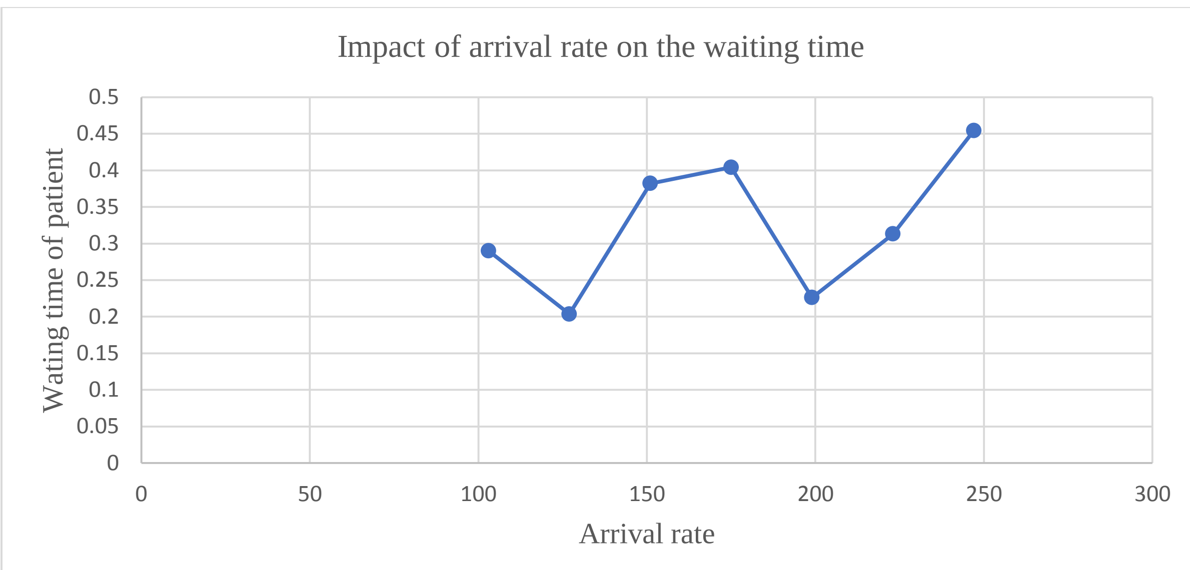

Fig 3. Impact of arrival rate on the waiting time

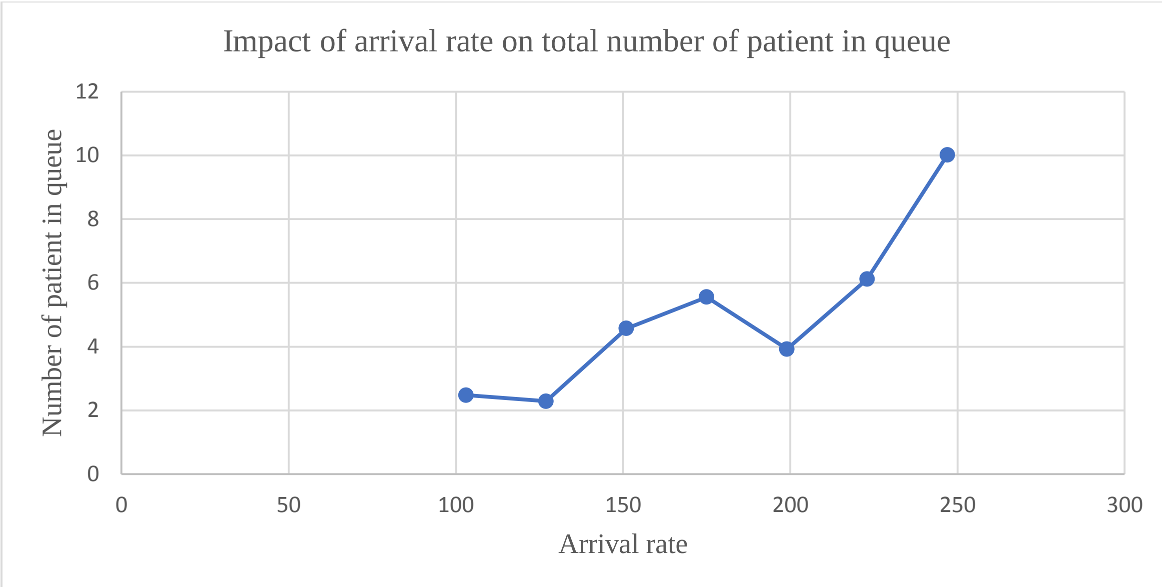

Fig 4. Impact of arrival rate on total number of patients in queue

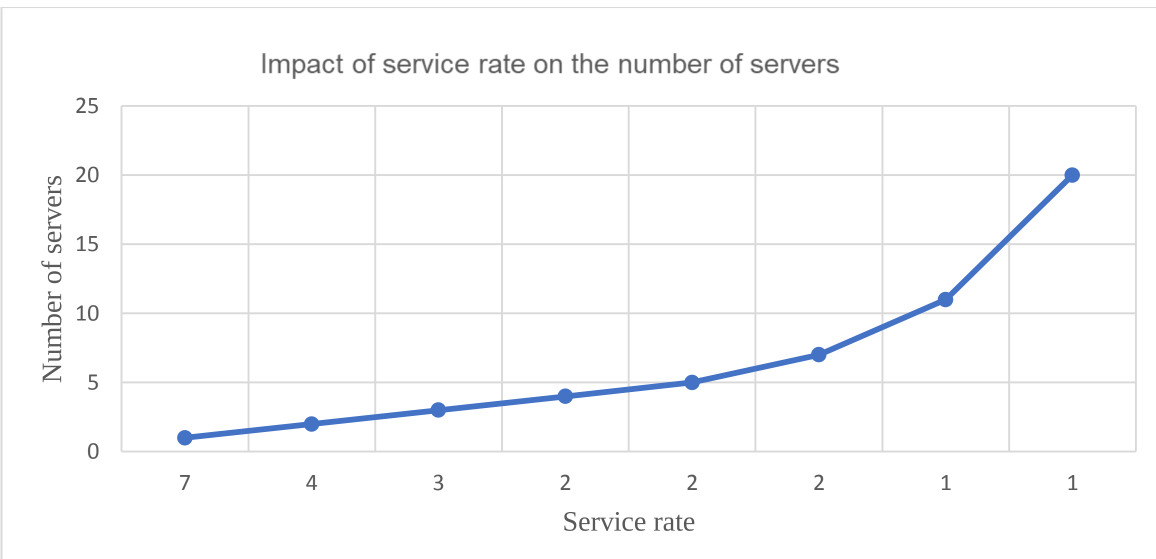

Fig 5. Impact of service rate on the number of servers in the coronary ward

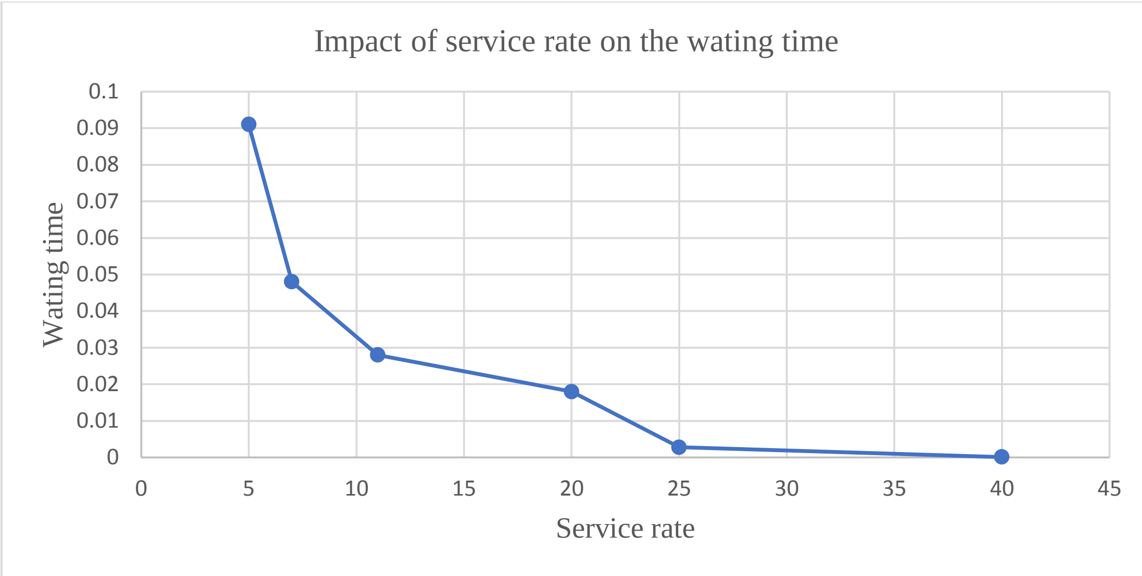

Fig 6. Impact of service rate on the waiting time in the coronary ward

Table 1. The default values of probabilities and external rate of arrivals

$P_{tg}=0.4$	$P_{tk}=0.6$	$P_{gm}=0.2$	$\lambda_{do}=10$
$P_{gd}=0.1$	$P_{ge}=0.2$	$P_{ed}=0.15$	$\lambda_{lo}=5$
$P_{do}=1$	$P_{gl}=0.2$	$P_{eo}=0.2$	$\lambda_{mo}=1$
$P_{em}=0.25$	$P_{el}=0.25$	$P_{er}=0.15$	$\lambda_{ro}=1$
$P_{le}=0.3$	$P_{\lg}=0.2$	$P_{lo}=0.5$	$\lambda_{ko}=9$
$P_{re}=0.3$	$P_{rg}=0.2$	$P_{ro}=0.5$	$\lambda_{T}=5$
$P_{me}=0.3$	$P_{mg}=0.2$	$P_{mo}=0.5$	$\lambda_{zo}=3$
$P_{bd}=0.4$	$P_{bs}=0.1$	$P_{bu}=0.1$	
$P_{kd}=0.8$	$P_{bo}=0.3$	$P_{go}=0.1$	
$P_{sz}=0.3$	$P_{su}=0.1$	$P_{so}=0.6$	
$P_{us}=0.6$	$P_{uo}=0.4$	$P_{bz}=0.1$	
$P_{zb}=0.6$			
$\sum_{i=1}^{n} X_i^2=5.52$	$P_{gr}=0.2$		
C_{asi}			
W^{i}			

Table 2. The entry rates of each node

$\lambda_Z=5.52$	$\lambda_R=7.5$	$\lambda_L=5.18$	$\lambda_E=9.81$	$\lambda_D=17.44$	$\lambda_G=15.12$
$\lambda_U=1.85$	$\lambda_S=3.36$	$\lambda_K=12$	$\lambda_B=10.95$	$\lambda_M=3.5$	

Parameters (budget=100000 $)							
section	λ_i (per hour)	μ_i (per hour)	C_{pi} ($)	C_{si} ($)	C_{osi} ($)	C_{asi} ($)	S_i
1	5	8	3000	1	3	100	3
2	15.12	25	3000	7	4	180	4
3	17.44	33	1000	8	2	110	3
4	5.81	10	4000	2	5	300	3
5	9.18	8	2000	1	2	400	4
6	7.5	10	2000	1	2	500	3
7	5.52	8	2000	2	2	600	4
8	3.5	11	5000	4	3	270	3
9	10.95	21	4000	5	2	350	4
10	12	16	6000	2	1	260	8
11	3.36	7	3000	4	2	350	3
12	1.85	5	9000	9	2	220	6

Table 3. The value of parameters for the test problem

Table 4. The result of GAMS for the case study

section	C_i	π_0^i	L_q^i	L^i	W_q^i	ρ_i
1	4	0.535	0.00001	0.625	0.00012	0.104
2	3	0.546	0.00006	0.605	0.12961	0.121
3	4	0.582	0.03967	0.568	0.00227	0.264
4	2	0.550	0.05355	0.635	0.00922	0.291
5	6	0.353	1.18938	1.837	0.00381	0.648
6	2	0.455	0.12273	0.873	0.01636	0.375
7	2	0.487	0.09322	0.783	0.01689	0.345
8	3	0.727	0.00052	0.319	0.00015	0.106
9	5	0.790	0.00016	0.236	0.00032	0.079
10	5	0.647	0.35423	0.354	0.02952	0.600
11	2	0.520	0.44308	0.923	0.13187	0.480
12	2	0.688	0.01311	0.383	0.00709	0.185
Total cost: 9783.1047						

Table 5. The result of GAMS and GOA algorithm for the case study

c_i			π_0^i		L_q^i		L^i		W_q^i		ρ_i		Objective function	
section	GAMS	GOA	GAMS	GOA	GAMS	GOA	GAMS	GOA	GAMS	GOA	GAMS	GOA	GAMS	GOA
1	4	5	0.535	0.496	0.00001	0.00002	0.625	0.613	0.00012	0.00015	0.104	0.238	9783.1047	9787.013
2	3	4	0.546	0.596	0.00006	0.00005	0.605	0.715	0.12961	0.17852	0.121	0.281		
3	4	3	0.582	0.432	0.03967	0.04657	0.568	0.495	0.00227	0.00365	0.264	0.136		
4	2	3	0.55	0.487	0.05355	0.05125	0.635	0.545	0.00922	0.00869	0.291	0.315		
5	6	5	0.353	0.401	1.18938	1.1954	1.837	2.352	0.00381	0.00456	0.648	0.715		
6	2	3	0.455	0.52	0.12273	0.1123	0.873	0.921	0.01636	0.0156	0.375	0.415		
7	2	2	0.487	0.452	0.09322	0.0965	0.783	0.635	0.01689	0.0188	0.345	0.395		
8	3	3	0.727	0.735	0.00052	0.00049	0.319	0.378	0.00015	0.00024	0.106	0.286		
9	5	5	0.79	0.63	0.00016	0.00012	0.236	0.401	0.00032	0.00045	0.079	0.085		
10	5	4	0.647	0.652	0.35423	0.3412	0.354	0.752	0.02952	0.0322	0.6	0.521		
11	2	3	0.52	0.58	0.44308	0.4374	0.923	1.364	0.13187	0.1551	0.48	0.5		
12	2	2	0.688	0.612	0.01311	0.0112	0.383	0.465	0.00709	0.0095	0.185	0.301		
Average Gap	1.46		0.22		0.42		2.03		3.05		4.87		0.04	

Table 6. The result of test problems 1 with GAMS and GOA algorithm

Parameters				W_q^i			c_i			Objective function		
section	λ_i	μ_i	s_i	GAMS	GOA	Gap	GAMS	GOA	Gap	GAMS	GOA	Gap
1	10	12	5	0.018	0.018	0.03	2	2	0.00	8953.889	9601.707	0.07
2	5	9	5	0.01	0.009	0.07	2	2	0.00			
3	5	8	6	0.014	0.208	0.83	2	1	0.50			
4	4	7	5	0.013	0.190	0.69	2	1	0.50			
5	4	8	5	0.14	0.125	0.11	1	1	0.00			
6	6	7	5	0.034	0.032	0.05	2	2	0.00			
7	5	6	5	0.037	0.035	0.05	2	2	0.00			
8	5	7	5	0.022	0.021	0.05	2	2	0.00			
9	4	8	6	0.14	0.125	0.11	1	1	0.00			
10	2	2	8	0.023	0.167	0.47	3	2	0.33			
11	4	2	4	0.227	0.043	0.81	3	4	0.33			
12	2	1	8	0.088	0.001	0.99	4	7	0.75			

Table 7. The result of test problems 2 with GAMS and GOA algorithm

Parameters				W_q^i			c_i			Objective function		
section	λ_i	μ_i	s_i	GAMS	GOA	Gap	GAMS	GOA	Gap	GAMS	GOA	Gap
1	5	8	3	0.014	0.014	0.03	2	2	0.00	6669.876	6520.360	0.02
2	15	25	4	0.024	0.060	1.50	2	1	0.50			
3	17	33	3	0.036	0.032	0.11	1	1	0.00			
4	6	10	3	0.01	0.010	0.01	2	2	0.00			
5	5	8	4	0.014	0.014	0.03	2	2	0.00			
6	7	10	3	0.015	0.014	0.07	2	2	0.00			
7	4	8	4	0.14	0.125	0.11	1	1	0.00			
8	5	11	3	0.085	0.076	0.11	1	1	0.00			
9	12	21	4	0.071	0.063	0.11	1	1	0.00			
10	3	4	8	0.043	0.041	0.05	2	2	0.00			
11	2	4	3	0.017	0.017	0.02	2	2	0.00			
12	6	9	6	0.014	0.014	0.01	2	2	0.00			

Table 8. The result of test problems 3 with GAMS and GOA algorithm

Parameters				W_q^i			c_i			Objective function		
section	λ_i	μ_i	s_i	GAMS	GOA	Gap	GAMS	GOA	Gap	GAMS	GOA	Gap
1	40	48	20	0.005	0.004	0.20	2	3	0.50	7549.379	8432.325	0.12
2	20	36	20	0.039	0.025	0.36	1	5	4.00			
3	20	32	24	0.058	0.047	0.19	1	5	4.00			
4	16	28	20	0.053	0.051	0.04	1	2	1.00			
5	16	32	20	0.035	0.021	0.40	1	3	2.00			
6	24	28	20	0.008	0.006	0.33	2	2	0.00			
7	20	24	20	0.009	0.007	-0.40	2	3	0.50			
8	20	28	20	0.005	0.012	0.66	2	3	0.50			
9	16	32	24	0.035	0.035	0.20	1	5	4.00			
10	8	8	32	0.044	0.039	0.32	2	6	2.00			
11	16	8	16	0.057	0.043	0.25	3	7	1.33			
12	8	4	32	0.022	0.018	0.18	4	5	0.25			

Table 9. The value of some crucial variables in a sensitivity analysis

section	$Z=6290.727$				$Z=6611.085$			
	λ	c	W_q	L_q	λ	c	W_q	L_q
1	5	1	0.037	0.185	7	1	0.065	0.456
2	10	1	0.056	0.559	12	2	0.005	0.062
3	4	1	0.074	0.297	6	2	0.01	0.062
4	3	1	0.01	0.029	5	1	0.018	0.092
5	6	1	0.168	1.008	8	2	0.02	0.159
6	7	1	0.261	1.83	9	2	0.027	0.239
7	10	1	0.149	1.494	12	2	0.013	0.159
8	8	1	0.037	0.297	10	1	0.056	0.559
9	7	1	0.065	0.456	9	2	0.007	0.062
10	4	1	0.008	0.033	6	1	0.014	0.084
11	6	1	0.168	1.008	8	2	0.02	0.159
12	9	1	0.025	0.225	11	1	0.035	0.385
section	$Z=7350.924$				$Z=7350.782$			
	λ	c	W_q	L_q	λ	c	W_q	L_q
1	9	2	0.00069	0.006	11	2	0.011	0.119
2	14	2	0.007	0.102	16	2	0.01	0.159
3	8	2	0.02	0.159	10	2	0.035	0.349
4	7	2	0.002	0.011	9	1	0.046	0.411
5	10	2	0.035	0.349	12	2	0.059	0.709
6	11	2	0.045	0.5	13	2	0.077	1
7	14	1	0.019	0.272	16	2	0.028	0.445
8	12	2	0.005	0.062	14	2	0.007	0.102
9	11	2	0.011	0.119	13	2	0.016	0.209
10	8	1	0.021	0.167	10	1	0.03	0.297
11	10	2	0.035	0.149	12	2	0.059	0.709
12	13	2	0.003	0.39	15	2	0.004	0.062
section	$Z=8439.166$				$Z=9312.14$			
	λ	c	W_q	L_q	λ	c	W_q	L_q
1	13	2	0.016	0.209	15	2	0.023	0.349
2	18	2	0.013	0.239	20	2	0.017	0.349
3	12	2	0.059	0.709	14	3	0.013	0.181
4	11	2	0.068	0.752	13	2	0.006	0.08
5	14	2	0.101	1.416	16	3	0.02	0.32

6	15	3	0.016	0.243	17	3	0.025	0.418
7	18	2	0.039	0.709	20	2	0.056	1.122
8	16	2	0.01	0.159	18	3	0.002	0.031
9	15	2	0.023	0.349	17	2	0.033	0.566
10	12	1	0.041	0.495	14	2	0.004	0.05
11	14	3	0.013	0.181	16	3	0.02	0.32
12	17	2	0.005	0.093	19	2	0.007	0.134

section	$Z = 9516.652$				$Z = 10471.03$			
	λ	c	W_q	L_q	λ	c	W_q	L_q
1	17	3	0.004	0.076	19	3	0.006	0.12
2	22	2	0.023	0.5	24	2	0.03	0.709
3	16	3	0.02	0.32	18	3	0.03	0.544
4	15	2	0.009	0.128	17	2	0.01	0.159
5	18	3	0.03	0.544	20	3	0.045	0.908
6	19	2	0.037	0.703	21	3	0.056	1.374
7	22	2	0.082	1.798	24	2	0.125	3.002
8	20	2	0.017	0.349	22	2	0.023	0.5
9	19	2	0.047	0.892	21	2	0.067	1.416
10	16	2	0.005	0.076	18	2	0.006	0.112
11	18	3	0.03	0.544	20	3	0.045	0.908
12	21	2	0.009	0.188	23	2	0.011	0.258

Table 10. The sum of some crucial variables in a sensitivity analysis

λ	c	W_q	L_q	L	Total cost
79	12	1.058	7.421	12.756	6290.727
103	19	0.29	2.478	9.474	6611.085
127	22	0.20369	2.286	10.791	7350.924
151	22	0.382	4.571	14.887	7350.782
175	25	0.404	5.554	17.53	8439.166
199	29	0.226	3.92	17.556	9312.14
223	28	0.313	6.118	21.415	9516.652
247	29	0.454	10.01	26.716	10471.03

Table 11. The value of the variable related to the coronary ward

λ	c	W_q	z
1	7	0.161	11166.612
2	4	0.108	9626.216
3	3	0.077	9193.518
4	2	0.168	9607.262
5	2	0.037	8535.079
7	2	0.022	8509.274
11	1	0.085	8447.527
20	1	0.018	8243.650