Modified adolescent identity search algorithm for optimization of steel skeletal frame structures

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Abstracts

In this study, the Modified Adolescent Identity Search Algorithm (MAISA) is proposed to optimize the weight of skeletal structures. The MAISA is a population-based method, similar to other metaheuristic methods. The most advantages of the proposed algorithm are its simplicity and having only one setting parameter. This research aims to increase the balance between exploration and exploitation, improve the convergence rate, and reduce the possibility of being trapped in local points. The applied changes extend the global search at the beginning of the optimization process, and as the number of iterations increases, the possibility of local search increases non-linearly. To evaluate the performance of the proposed method, several benchmark skeletal structure problems have been designed and optimized using the LRFD method under the requirements of the AISC design regulations. The objective function is to calculate the minimum weight of a structure by selecting appropriate discrete sections and considering the deformation and stress constraints. To demonstrate the superiority of the MAISA algorithm, its results have been compared with some popular metaheuristic algorithms. The results show that the proposed algorithm performs better than other metaheuristic methods.

Keywords: Modified Adolescent Identity Search Algorithm, Metaheuristic Method, Optimal Design, Optimization of Skeletal Structures, Optimization of Frame Structures.

1. Introduction

In general, optimization is the process of minimizing or maximizing the solution of real problems, while considering the limitations and constraints of the problem. This response in skeletal structures includes the minimum weight of structures, the best possible configuration and the best connection between nodes. The use of gradient-based methods imposes heavy costs on the system, so researchers have resorted to the use of metaheuristic methods. During the last three decades, these methods have received special attention. The main difference between metaheuristic methods is the relationship between the obtained answers from previous iterations and finding new answers. In other words, because metaheuristic methods are probabilistic, researchers are looking for relations that increase the chance of finding the best solution. In metaheuristic methods, exploration and exploitation, which are two essential components, are directly related to the search ability of an algorithm. The exploration component leads to more exploration in the global search space to find better solutions. The exploitation component also improves their quality by extending the search around the solutions which are obtained from exploration. The main challenge of metaheuristic methods is to establish a balance between exploration and exploitation.

Recently, many metaheuristic algorithms are proposed which shown good performance in solving optimization problems. These algorithms can be classified into three general categories: Standard algorithms, Modified or Upgraded algorithms, and Hybrid algorithms, which will be explained in the following paragraphs.

Standard algorithms: Standard algorithms refer to the primary methods that were developed for solving optimization problems. The following algorithms are among the Standard algorithms: The Genetic Algorithm (GA) [1], which was developed by Holland using Darwin's theory of evolution. In this algorithm, cross-mechanisms and mutations are used to produce a new generation. Goodarzimehr et al. [2] proposed a novel metaheuristic method called Special Relativity Search (SRS) for solving optimization problems. The Cat Swarm Optimization algorithm (CSO) [3] that was developed by Chu et al. based on two major behaviors of cats: seeking and tracking baits. The Chemical Reaction Optimization algorithm (CRO) [4], which was introduced by Lam and Li, It is inspired by the processes of molecular reactions. The molecules decomposition or synthesis with each other till stopping criteria satisfy. The Charged System Search algorithm (CSS) [5] that was developed by Kaveh and Talatahari, which uses Colomb law of electrostatics and Newtonian laws of mechanics. The Teaching–Learning Based Optimization (TLBO) [6] algorithm, which is developed by Rao et al. it’s inspired by observing the relationship between teacher and students and the level of student learning. The Mine Blast Algorithm (MBA) [7] that was introduced by Sadollah et al. by observing the method of mine blasting, in which some parts are thrown as a result of a mine explosion that collides with other mines, causing them to explode. The Vibrating Particles System (VPS) [8] optimization method that was presented by Kaveh and Ghazaan, it’s inspired by the free vibration of single degree of freedom systems. The Grasshopper Optimization Algorithm (GOA) [9], which was designed by Mirjalili et al. is one of the basic metaheuristic algorithms that was inspired by the social behavior of grasshoppers and how affected by its surrounding. In this algorithm, updating the position of each grasshopper depends on the distance of each grasshopper from the entire population in the current iteration, and the position of the best grasshopper. The Black Widow Optimization (BWO) [10] algorithm was proposed by Hayyolalam and Pourhaji Kazem. The BWO is inspired by the Cannibalism behavior of a special type of spider. The black widow spider kills its male partner after mating and considers it as bait for herself. This behavior represents the elimination of inappropriate responses that lead to the convergence of the proposed algorithm.

Modified or Upgraded algorithms: Some researchers have identified the strengths and weaknesses of Standard algorithms, and modified or improved them. The purpose is to increase the convergence speed, not to get stuck in the local optimal points and also to increase the run speed of the algorithm. Therefore, it is possible that a basic method has been upgraded by several researchers in different ways. For these algorithms, the words improved, modified, enhanced, or advanced are commonly used for naming. It is expected that classical algorithms such as PSO and GA have attracted the attention of many scientists to improve their performance, and based on these methods, many modified algorithms have been formed. An example of a modified particle swarm algorithm was the MPSO algorithm proposed by Yitong et al. [11]. Also, the modified particle swarm algorithm introduced by Tian and Shi [12], Goodarzimehr et al. [13] developed a hybrid PSOGA for solving geometrically nonlinear space structures, and PSOST developed by Baghlani and Makiabadi [14], and the improved particle swarm algorithm developed by Liu [15], are all examples of the modified methods based on the PSO algorithm. Also Shuffled Shepherd Optimization Method (SSOA) [16] is inspired by the instrumental use of human beings from the instinct of animals to achieve their goals, and then the authors introduced Enhanced Shuffled Shepherd Optimization Algorithm (ESSOA) [17] by modifying initialization phase and adding new statistically equation to main body. Also, Kaveh and zaerreza utilized ESSOA algorithm for evaluating a new framework for reliability-based design optimization (RBDO) [18]. Goodarzimehr et al. [19] developed weighted chaos game optimization (WCGO) for solving structural problems with dynamic constraints. The Improved Grey Wolf Optimizer algorithm (I-GWO) [20] was proposed by Nadimi-Shahraki et al.. This algorithm has used an instinctive strategy in wolves for individual attacks and prey hunting for improvement. Topal et al. [21] proposed hybrid method based on PSO and GA for solving maximization of the fundamental frequency of the FG-CNTRC quadrilateral plates. Kumari et al. developed the Modified Grasshopper Optimization Algorithm (MGOA) [22] by introducing new coefficient for balancing exploration and exploitation. Enhanced Rao Algorithm (ERao) [23] developed by revising the statistical mechanism and modifying the boundary control mechanism.

Hybrid Algorithms: Applying several algorithms in a hybrid process is one of the most effective methods for solving optimization problems. In this approach, by examining two or more metaheuristic algorithms and identifying their strengths and weaknesses, a new hybrid algorithm will be developed. The aim is to strike a balance between global search capability (exploration) and local search capability (exploitation), that has led researchers to propose and develop many hybrid algorithms over the past decade. Shojaee et al. [24] introduced the MMA-IDPSO algorithm, which optimizes the topology and size of truss structures. Talatahari et al. proposed the Hybrid Teaching-Learning-Based Optimization and Harmonic Search algorithm (TLBO-HS) [25] for optimizing large-scale structures. Also, Talatahari et al. [26] used the Hybrid Symbiotic Organisms Search and Harmony Search algorithm (SOS-HS) to optimize the size of structures. Omidinasab and Goodarzimehr [27] proposed a Hybrid Particle Swarm and Genetics algorithm (PSO-GA) for the optimal design of truss structures with discrete variables. Jiang et al. [28] used the STSA algorithm, which combines the TSA and SCA algorithms, to optimize and solve large-scale and complex problems. Kaveh and Rajabi developed Hybrid Imperialist Competitive Algorithm (ICA) and Biogeography Based Optimization (BBO) that was named Migration-Based Imperialist Competitive Algorithm (MBICA) [29].

Optimization is used in most engineering fields such as mechanics, economics, electronics, civil engineering, geophysics, molecular modeling, and etc. For example, Wang et al. introduced two gradient methods for the inverse wave problem [30,31]. Perin et al. [32] utilized GWO for optimum design of synchronous generator. Optimization has been used specifically in civil engineering to design structures. For example, Tan and Lahmer [33] used Robust Design Optimization (RDO) for the shape design of arch dams. Frame and truss structures are among the common and widely used structures in civil engineering, which are economical and safe. Frames are one of the most widely used structures and the reduction of their weight saves resources and reduces costs. Two-dimensional frames consist of many straight members connected by rigid joints. The stiffness relations of frames can be achieved by combining the stiffness relations of trusses and beams, because if a frame is affected by external forces, its members can be exposed to axial forces, bending moments, and shear forces. The objective function in these problems is the weight of the structure and its constraints of deformation and stress introduced in the design regulations. One of the first researchers who studied the discrete optimization of steel frames for the first time was Camp et al. [34], who optimized steel frame structures using the ant colony optimization algorithm.

Also, Kaveh and Shojaee utilized ACO algorithm to optimize several applicable problems [35]. Ghatte [36] developed a Hybrid Firefly and Biogeography-Based Optimization Algorithms (FA-BBO) for the optimal design of steel frames based on flashing patterns and biogeography-based optimization (BBO). Kaveh et al. [37] proposed the Advanced Charged System Search algorithm (ACSS) for the optimal design of steel structures. They used the idea of opposition-based learning to improve the performance and enhance the capabilities of the standard CSS algorithm.

The standard AISA algorithm has a high ability in solving optimization problems due to the lack of control parameters and the ability to balance between two opposing components of exploration and exploitation. Therefore, the Modified Adolescent Identity Search Algorithm (MAISA) is used to optimize skeletal structures. Identity is a combination of different behaviors, thoughts, abilities, and beliefs that a person acquires during adolescence. During this period, a person hesitates about many issues around him and sometimes disagrees with others and even his parents to achieve his goals and desires. In fact, in this period, a person wants to move from childhood dependence to independence and have more independence in the family and school environment and decide for himself. Mathematical simulation of how adolescents' personalities are formed during this process has led to the emergence of this algorithm.

In Section 2, the optimal design process of frame structures will be discussed, in Section 3. the formulation of the MAISA algorithm is extended. In Section 4. The efficiency of the proposed method is evaluated by numerical examples and in Section 5. discusses and concludes the performance of the proposed algorithm.

2. Problem statement

Recently, one of the main goals in structural optimization is minimizing the weight of the structure's elements. Metaheuristic algorithms are a great tool to fulfill this goal, in which the first population is randomly generated, and then by making a balance between exploration and exploitation, the population is promoted and at the end of each iteration, the obtained solutions are compared with the constraints of the problem, which are mainly displacements, stresses, and position of nodes in civil engineering problems. In order to qualitatively express the search conditions of the population in the feasible space, the search space is displayed in two dimensions and three dimensions in Fig. (1b) to Fig. (1g) show how adolescents search in this space. In Fig. (1b), the red and green dots represent the initial population produced and the position of the best person is shown in green. In each iteration, adolescents search and explore this space and with each iteration, the adolescents get closer to the optimal solution, and finally, all people converge to the optimal point (Fig. (1g)).

A structural optimization problem can be formulated as follows:

$$\begin{aligned} &Minimize \quad f(A) \\ &Subject\ to \quad \xi^k(A) \le 0 \qquad k = 1,2,...,N_c \\ &\qquad A = \{a_1, a_2, ..., a_j, ..., a_n\} \hat{I} R^d \end{aligned} \tag{1}$$

where $f(A)$ is the objective function, A is the vector of variables (we will see in the next sections $A = a^i$) and ξ^k represents the constrains of the problem, where N_c is the number of constraints. In structural optimization problems, the main goal is usually to minimize the weight of the structure by considering design constraints. Design variables are selected as the cross sections of elements, which are usually selected from discrete categories. Therefore, the optimization problem is defined by using the following equation.

$$Minimize \quad f(A) = W(A) = \sum_{j=1}^{Ne} \rho_j S_j L_j$$

$$Subject\ to \quad \xi_s^d = \frac{\delta_s}{\delta_u} - 1 \le 0 \quad s = 1,2,...,N_n$$

$$\xi_j^\sigma = \begin{cases} (\frac{P_u}{\phi_c P_n}) + \frac{8}{9}(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}) - 1 & for \ \frac{P_u}{\phi P_n} \ge 0.2 \\ (\frac{P_u}{2\phi_c P_n}) + (\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}) - 1 & for \ \frac{P_u}{\phi P_n} < 0.2 \end{cases} \quad j = 1,2,...,N_e \tag{2}$$

$$S_j \in D_j = \{S_1, S_2, ..., S_t\}$$

in which *W(A)* is the structure weight, S_j, ρ_j and L_j are cross-section, material density and length of the j_{th} elements, respectively. ξ_s^d and ξ_j^σ are constraints conditions for steel frames. ξ_s^d is the displacement violation and ξ_j^σ is stress violation based on LRFD requirements in AISC specification (Equation H1-1) [38]. δ_s and δ_u denote the displacement of the s_{th} story and allowable displacement equal to (story height)/300. P_u is the required axial force, M_u represents the required bending moment, P_n denotes the nominal axial capacity, and M_n is the nominal bending capacity. ϕ_c and ϕ_b are resistance factors (ϕ_c =0.9 for tension and 0.85 for compression, and ϕ_b =0.9 for flexure). N_e represents the number of members and N_n denotes the number of stories. D_j is the available profile list, and t is the total number of profiles.

The penalty method is a systematic method for controlling and limiting the error due to violation of the constraint, which is formed using the Riley-Ritz method. The penalized weight is calculated based on the penalty function as follows:

$$\lambda(A) = W(A)[1 + \varepsilon_1 \eta]^{\varepsilon_2} \tag{3}$$

where $\lambda(A)$ is the penalized weight, η is the constraint violation function, and ε_1 and ε_2 denote the coefficients of the penalty function. The violation of the constraint is defined as:

$$\eta = \sum_{s}^{N_n} \xi_s^d + \sum_{j}^{N_e} \xi_j^\sigma \tag{4}$$

and the penalty value ξ is equal to:

$$\xi = \begin{cases} 0 & if \quad \xi \le 0 \\ \xi & if \quad \xi > 0 \end{cases} \tag{5}$$

3. Standard Adolescent Identity Search Algorithm (AISA)

The Adolescent Identity Search Algorithm (AISA) [39] was developed in 2020 by Bogar and Beyhan for electronic optimization problems. This algorithm is a population-based method that was inspired by how adolescent identity is formed. Adolescence is a specific period of life that is different from other stages of growing up in terms of its characteristics and conditions. One of these important characteristics is the adolescent's desire to gain independence. The adolescent tends to display his abilities and forces and prove his power and ability to those around him. He wants to act alone and not rely on those around him as much as he can, especially his parents. One of the causes of defiance, stubbornness, and aggression by adolescents is seeking independence. Independence provides the initial basis for his entry into society and prepares him for the acceptance of social responsibilities. However, its incorrect orientation will be accompanied by problems for him and those around him. During this period, the adolescent is no longer considered a child and has not yet fully matured. He is on the border between childhood and adulthood and faces the pressures and expectations of both periods. During this period, the adolescent becomes very worried and anxious, so he cannot organize the various aspects of his personality in a properly coordinated and acceptable way. Adolescence is one of the most important and sensitive periods of human life because in this period, adolescents, to find their identity, take a step towards forming a self-reliant personality. There are many questions in the teenager's mind that if he cannot find the right answer to them, he may find a shaky and possibly dual personality. Identity is a combination of different behaviors, thoughts, abilities, and beliefs that a person acquires during adolescence. Family environment and how parents interact with adolescents in this period play an important role in the development and formation of the adolescent's personality. Regardless of norms and structures, peer groups also play an important role in adolescent development because adolescents spend most of their time at school, on the playground, etc. with this group. Experiences gained in the peer group may have a positive or negative effect on adolescents, which has an important role in self-awareness, self-esteem, and the ability to make decisions and take responsibility in an environment where adults do not exist. Modeling the process of adolescent identity formation is a complex and costly task because many factors are influential in this regard. Briefly, the adolescent identity search algorithm can be defined as follows:

The adolescent's identity is shaped by imitation. This imitation originates from a pattern that probably has certain personality traits in the adolescent. These characteristics that formed the adolescent's identity are obtained by observing and arguing behaviors and social feedback. Sometimes adolescents may lose their social status due to destructive factors and undesirable environmental conditions. These states are modeled mathematically as follows [39]:

$$a^i_{new} = \begin{cases} case\ \ 1: & a^i - r_1(a^i - a^*), & r_4 \le 1/3 \\ case\ \ 2: & a^i - r_2(a^p - a^{rm}), & 1/3 < r_4 \le 2/3 \\ case\ \ 3: & a^i - r_3(a^i - a^q), & 2/3 < r_4 \end{cases} \tag{6}$$

where a^i represents the identities of person *i*, and other parameters are defined in next section in detail.

The AISA algorithm does not take into account the fact that adolescents generally desire to break norms and break social rules. Therefore, the probability that his personality traits decline is less than other presented rules. This is why the AISA algorithm considers the selection of each of the three cases equally (Equation (6)). To modify this issue, some changes have been made to the formulation of the problem, which is described in detail.

4. Modified Adolescent Identity Search Algorithm formulation (MAISA)

This algorithm works similarly to adolescent identity development. As the adolescent takes the people who behave better within the norms of society as a model, this algorithm also finds the most optimal solution among the multitude of answers. If no adolescent achieves the desired values, the optimization problem would be considered unsolvable, and this is a possible situation of not being able to solve the social problems of the society [39]. In the proposed method, by modifying the balance between the two components of exploration and exploitation using a control parameter, the balance between global search (exploration) and local search (exploitation) is increased, which reduces the cost of analysis.

The MAISA method can be summarized in five main steps, which are as follows:

Step 1: In this method, similar to other optimization methods, uniform random population production is used (Equation (7)) to create the initial population, which is quantified by using the objective function.

$$a_j^i = L_j + r(0,1)_j \times (U_j - L_j), \quad i = 1,2,...,N\,;\, j = 1,2,...,n \tag{7}$$

where, a^i_j represents the j_{th} characteristic of the person *i*. *N* represents the number of people, *n* represents the number of design variables, and *L* and *U* represent the lower and upper bounds of the design variable, respectively. r(0,1) is a random number between 0 and 1.

Step 2: In this step, new personalities are formed and if the new features are better than the previous ones, they will be replaced in the next steps. Although each person's personality is affected by numerous factors, in this algorithm it is assumed that each person randomly chooses one of the three important states mentioned in the previous sections. Selecting any of these three cases requires obtaining a control parameter that is proportional to the number of current iteration to max iteration, starting with small values and increasing non-linearly as the number of iteration increases. Higher *CP* values increase the local search capability and convergence rate, while lower *CP* values increase the global search capability and decrease the convergence rate.

$$CP = \sin\left(\frac{\pi}{2} \times \frac{iter + 0.25Max_{iter}}{2Max_{iter}}\right) \times Ep \tag{8}$$

in this relation, *Ep* is a control parameter that determines the maximum value of *CP* and its value is experimentally considered to be 0.5.

Case 1: If a random number called r_4 is greater than *CP*, the new properties of the person are generated according to Equation (9):

$$a_{new}^i = a^i - r_1(a^i - a^*) \tag{9}$$

in the above relation, r_1 is a random number in the range [0,1] and a^* is the best identity that exists among individuals, which is defined as follows:

$$a_j^* = a_j^{m^j}, \quad m^j = \arg_l \min\left\{\hat{f}_j^{\,l} \,\middle|\, l = 1,2,...,N\right\}, \quad \forall j \tag{10}$$

where $\hat{f}_j^{\,l}$ is the j_{th} characteristic of the person *l* that exists in the $\hat{F}$ matrix. To create this matrix, the following procedure is performed:

The matrix of identities (*H*) is normalized and becomes the $\hat{H}$ matrix.

$$\hat{H} = \begin{bmatrix} \hat{a}_1^1 & \hat{a}_2^1 & \cdots & \hat{a}_n^1 \\ \hat{a}_1^2 & \hat{a}_2^2 & \cdots & \hat{a}_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{a}_1^N & \hat{a}_2^N & \cdots & \hat{a}_n^N \end{bmatrix}_{N \times n} \tag{11}$$

Using Chebyshev polynomials, the Γ regressor matrix is defined as follows:

$$\Gamma = \begin{bmatrix} T_1(\hat{a}_1^1) & \cdots & T_k(\hat{a}_1^1) & \cdots & \cdots & T_1(\hat{a}_n^1) & \cdots & T_k(\hat{a}_n^1) \\ T_1(\hat{a}_1^2) & \cdots & T_k(\hat{a}_1^2) & \cdots & \cdots & T_1(\hat{a}_n^2) & \cdots & T_k(\hat{a}_n^2) \\ \vdots & \ddots & \vdots & \ddots & \ddots & \vdots & \ddots & \vdots \\ T_1(\hat{a}_1^N) & \cdots & T_k(\hat{a}_1^N) & \cdots & \cdots & T_1(\hat{a}_n^N) & \cdots & T_k(\hat{a}_n^N) \end{bmatrix}_{N\times(n\times k)} \tag{12}$$

Then, the weight vector can be calculated by Equation (13).

$$\begin{aligned} \hat{\mathbf{\Omega}} &= (\Gamma^T\Gamma)^{-1}\Gamma^T \boldsymbol{f} \\ \hat{\mathbf{\Omega}} &= [\omega_1^1 \quad \cdots \quad \omega_k^1 \quad \cdots \quad \cdots \quad \omega_1^n \quad \cdots \quad \omega_k^n]^T_{1\times(n\times k)} \\ &= [\boldsymbol{\omega}^1 \quad \cdots \quad \cdots \quad \boldsymbol{\omega}^n]^T \end{aligned} \tag{13}$$

Finally, the relative values of the fitness function are obtained through Equation (14) as follows:

$$\hat{f}_j^i = \gamma_j^i \boldsymbol{\omega}^i \tag{14}$$

$$\hat{\boldsymbol{F}} = \begin{bmatrix} \hat{f}_1^1 & \hat{f}_2^1 & \cdots & \hat{f}_n^1 \\ \hat{f}_1^2 & \hat{f}_2^2 & \cdots & \hat{f}_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{f}_1^N & \hat{f}_2^N & \cdots & \hat{f}_n^N \end{bmatrix}_{N\times n} \tag{15}$$

in which, γ_j^i is the sub-regressor vector of the i_{th} person and the j_{th} identity.

Case 2: If the random number r_4 is in the interval $0.5CP < r_4 \le CP$, the intended person will model the best person and try to make his / her character look like the best person. The new identities of the individual are produced according to Equation (16):

$$a_{new}^i = a^i - r_2(a^p - a^{rm}) \tag{16}$$

where, a^{rm} is the attribute of the best person and a^p is the attribute of the p_{th} person (provided that $p \neq rm$).

Case 3: Sometimes, the adolescent suffers from behavioral and moral deviations due to destructive factors, which causes his personality to decline. In algorithm, this case occurs when r_4 is less than $0.5CP$ and the new identity is generated using Equation (17):

$$a_{new}^i = a^i - r_3(a^i - a^q) \tag{17}$$

in which, a^q is a vector of negative properties which is defined as follows:

$$a^q = a^u[1 \quad 1 \quad \cdots \quad 1]^T_{1\times n} \tag{18}$$

in this relation, a^u is a negative identity that is randomly selected.

In summary, one of the following three cases occurs for each person:

$$a_{new}^i = \begin{cases} case\ 1: & a^i - r_1(a^i - a^*), \quad r_4 > CP \\ case\ 2: & a^i - r_2(a^p - a^{rm}), \quad 0.5CP < r_4 \le CP \\ case\ 3: & a^i - r_3(a^i - a^q), \quad 0.5CP \ge r_4 \end{cases} \tag{19}$$

Step 3: If a person violates the feasible search space, at this stage, by generating a new random feature, the person is returned to the allowed environment.

Step 4: In this step, the quality of the new character created in the second step is compared with the previous values of each person and if it is better, it will replace them.

Step 5: In the last step, if the termination criterion rule is established, the algorithm will end and the best person will be introduced as the answer. In the MAISA algorithm, similar to most existing algorithms, the maximum number of iterations of the algorithm is selected as the final criterion.

The flowchart shown in Fig. 2 summarizes how the MAISA algorithm works.

5. Practical examples

In the following section, several benchmark steel frames are examined. For each of these examples, 20 independent runs were performed. It should be noted that in 1-bay, 8-story steel frame, 3-bay, 15-story steel frame, and 3-bay, 24-story steel frame, the values of ε_1 and ε_2 are considered to be 1 and 2, respectively. In the following section, the results obtained are compared with other similar methods in tables to better compare the results obtained from different algorithms. The details of each of the cases are discussed in more detail in the next parts.

5.1. 1-bay, 8-story steel frame

The 8-storey steel frame is under lateral load as shown in Fig. 3. The modulus of elasticity of steel is 200 Gpa and its specific gravity is considered to be 76.8 kN/m^3. The members of this 2D frame are divided into 8 groups, the sections of which are selected from 267 W-shaped rolled sections according to AISC. The only constraint is the displacement of the roof floor, which is limited to 5.08 cm.

By comparing the results obtained from solving the frame of the 1-bay, 8-story frame, the MAISA algorithm is in the first place in the comparison. This algorithm has reduced the structure weight by almost 1.92% compared to the AISA algorithm (Table 1). Due to the lack of results of mean weight and standard deviation of other algorithms, it is not possible to make an accurate comparison in these cases, but the AISA algorithm has performed better than MAISA by more than 1.34% in the average weights. Fig. 4 shows the optimal response of each run, which indicates their slight dispersion. Fig. 5 is a diagram of the convergence history of the MAISA and AISA method.

5.2 3-bay, 15-story steel frame

The 3-bay, 15-story frame is considered shown in Fig. 6 is considered here. The frame consists of 105 members, whose sections are selected from all 267 rolled W-shaped sections. The frame is divided into 11 groups, including one group for the beams, 5 groups for the side columns, and 5 groups for the middle columns. In this example, the resistance and displacement constraints are considered based on the requirements of AISC-LRFD, and the maximum lateral displacement of the last floor is limited to 23.5 cm. The unbraced length for each beam is considered to be one-fifth the length of the span length and it is assumed that the columns are non-braced along their length. The properties of the materials are equal to E = 200 GPa and Fy = 248.2 MPa.

The MAISA algorithm has the best performance compared to the other methods listed in Table 2. This algorithm has reduced the weight of the structure by almost 0.53% compared to the TLBO method and reduced the weight of the structure by 0.97% compared to AISA, which is in fourth place in terms of the best answer. Also, by examining the average of the answers obtained from 20 independent runs, the MAISA algorithm obtained about 0.41% lower average than the AISA algorithm which is in second place in this comparison. The standard deviation is an indicator that measures how the data is scattered in a collection by its average. A low standard deviation indicates the stability of the answers of an algorithm. Therefore, smaller standard deviation values of the proposed algorithm compared to other ones indicate that the answers obtained from this algorithm are more stable (Fig. 7).

Fig. 8 shows the optimal answer for each run. As mentioned, the proposed method has high stability. Therefore, a low scatter of points proportional to the mean line and low standard deviation is not far from expectation. By comparing the convergence diagram in fig. 9, it can be seen that the MAISA algorithm has reached the optimal solution faster. Drift is defined as the ratio of the difference between the above and below displacements of a floor to the height of that floor. Because excessive displacement of the structure causes the destruction of non-structural components and also disturbs the peace of its inhabitants, the regulations limit its amount. Fig. 10 shows the drift of the optimal answer to the allowable drift per floor, most of which belongs to the 7th floor with value of 0.0037. The stress ratios of the members obtained from the best answers are given in Fig. 11. The highest and lowest stress ratios occurred in members 4 and 101, respectively, with the values of 1.00 and 0.16.

5.3 3-bay, 24-story steel frame

In the third example, a steel frame with 3 bays and 24 stories, as shown in Fig. 12, is examined. This frame consists of 168 members and 20 groups, with four groups for the beams and 16 groups for the columns. The side columns are separated from the middle columns. In this example, the cross-section of the columns is selected from 37 W-14 sections and the cross-section of the beams is selected from all W-shaped sections. Similar to the 15-story frame, the effective length factors of the members are calculated as $K_x \geq 0$ for a sway-permitted frame and the out-of-plane effective length factor is specified as $K_y = 1.0$. The resistance and displacement constraints are based on AISC-LRFD requirements. The Material properties are equal to E = 29732 ksi and $F_y = 33.4$ ksi.

The MAISA algorithm has the lowest weight compared to the other methods in Table 3. Also, by examining the average of the answers obtained from 20 independent runs, the MAISA algorithm obtained about 1.29% lower average than the AISA algorithm. By comparing the standard deviations in Fig. 13, the better performance of the proposed method can be seen. Fig. 14 shows the results of each of the 20 runs. This graph shows the small scatter of optimal answers to the mean weight ratio, which somehow shows the standard deviation. The low number of obtained answers in this example indicates the high ability of this method to solve optimization problems with a higher number of variables. Also, the MAISA algorithm has converged faster than the AISA method, which can be seen in Fig. 15. Fig. 16 shows the drift of each floor, where the highest value belongs to the 13rd and 16rd floors and it’s equal to 0.00587. Fig. 17 shows the stress ratio for each of the 168 frame members. Due to the grouping of members, the stress ratio scatter is seen in this graph. The highest stress ratio is in element 14 with a ratio of 0.94 and the lowest stress ratio is in element 167 with a value of 0.06.

6. Discussion and conclusion

In this study, a method to improve the performance of the adolescent identity search algorithm was presented. The main purpose of this article was to balance the local search with the global search. To improve the way particles are searched in the global space or around themselves, a control parameter was introduced according to which, as the number of iterations increases, the particles move from the global space search to the local one. To evaluate the effectiveness of the proposed method, three practical problems were solved. The results of the optimal designs obtained from this algorithm showed that this method can be used as an ideal algorithm in the optimal design of structures. As shown in the graphs, the stories' drift values were close to the maximum allowable values and the member stress ratios had values close to one, which demonstrates the considerable ability of the proposed method for solving optimization problems. This method was compared with the AISA algorithm and some similar metaheuristic algorithms. The results showed that, although both AISA and MAISA methods had a good performance in optimizing the weight of structures the MAISA showed better performance. Also, by examining the obtained average weights, it was concluded that the MAISA algorithm has high reliability because the obtained average weight was less than the average weight of other compared methods. For investigating the stability of the obtained results, the standard deviation of these methods was compared. Lower standard deviation results meant that the answers of this algorithm compared to the other ones are more stable. Despite the positive aspects mentioned, it is suggested to increase the convergence rate in future works by adding new ideas to the algorithm to reduce the computational cost.

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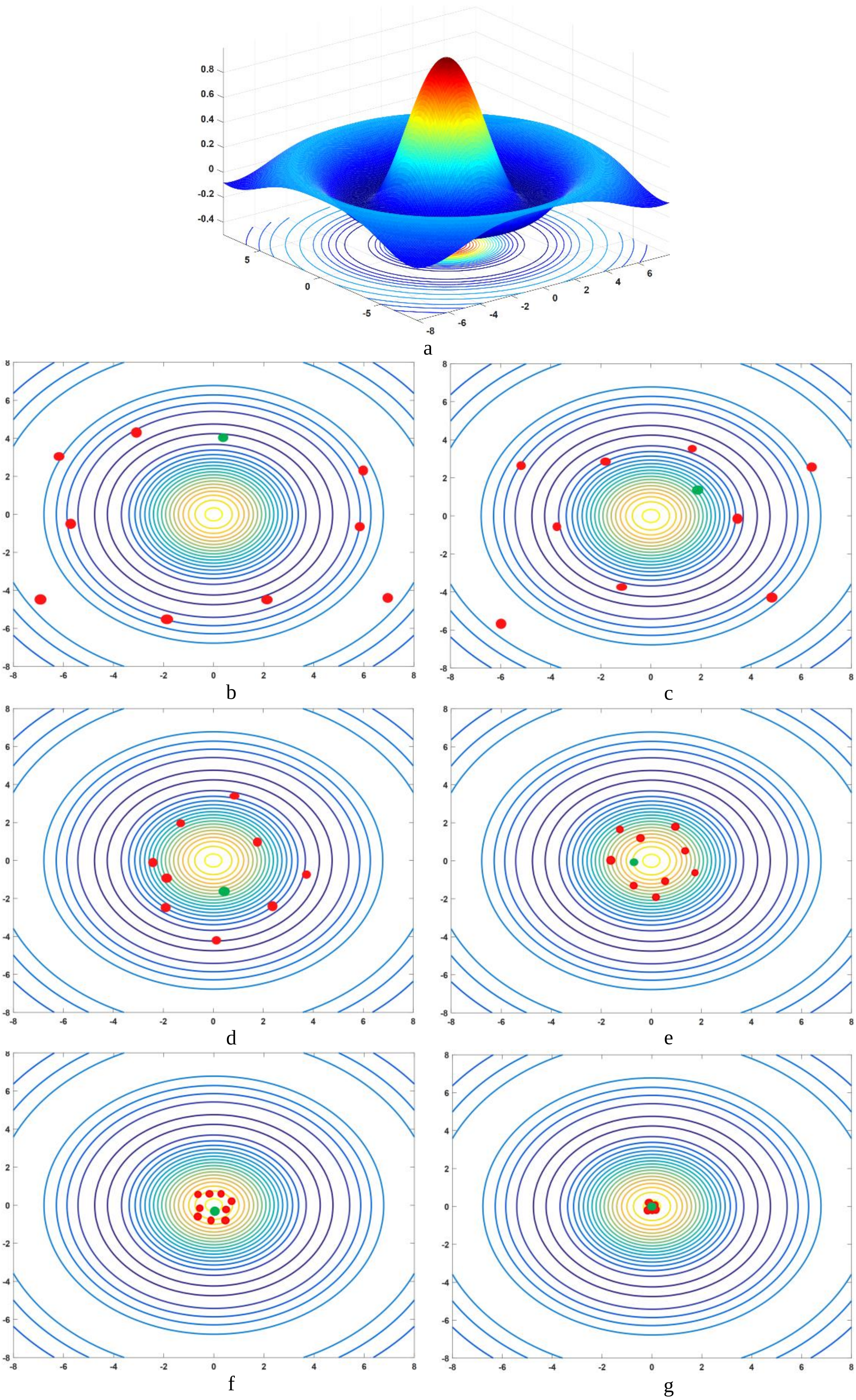

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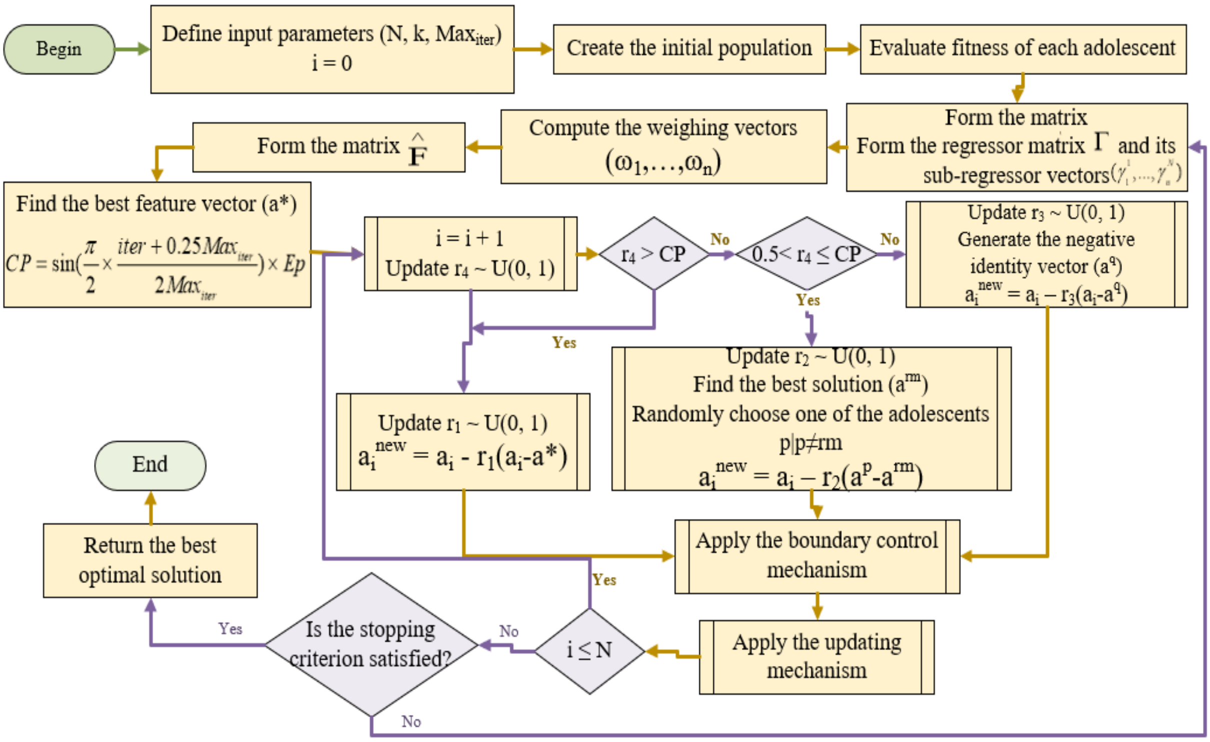

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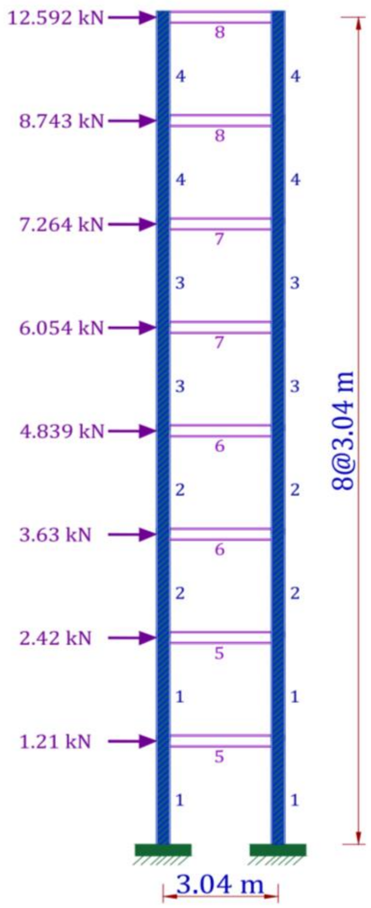

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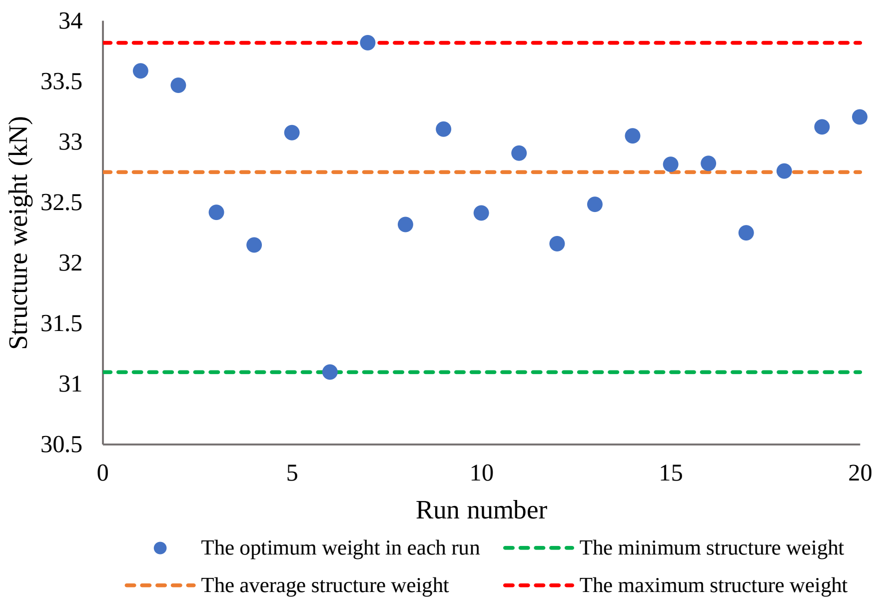

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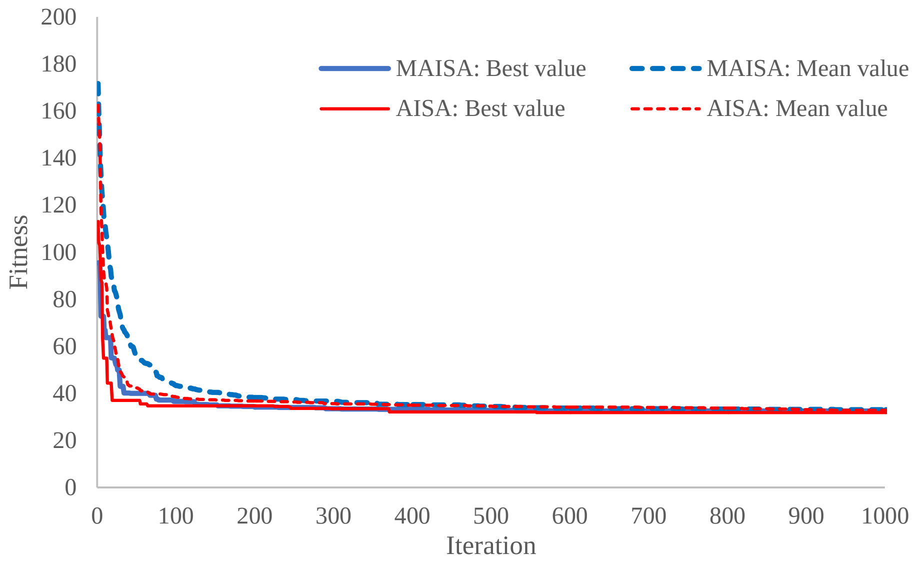

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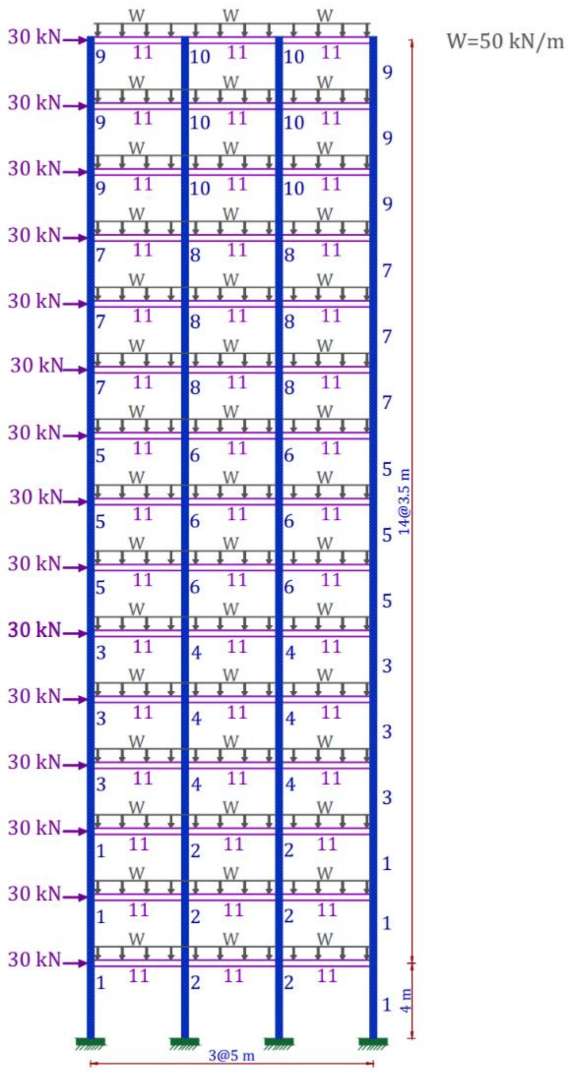

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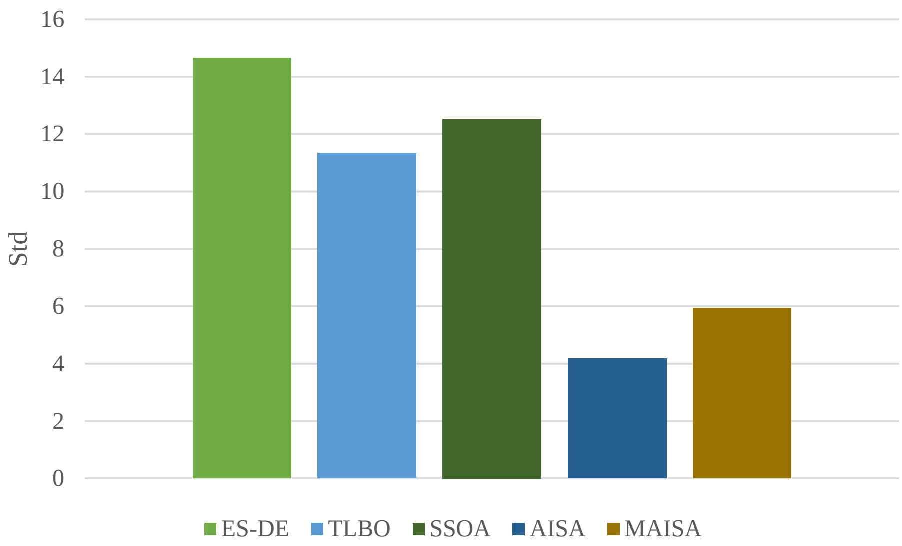

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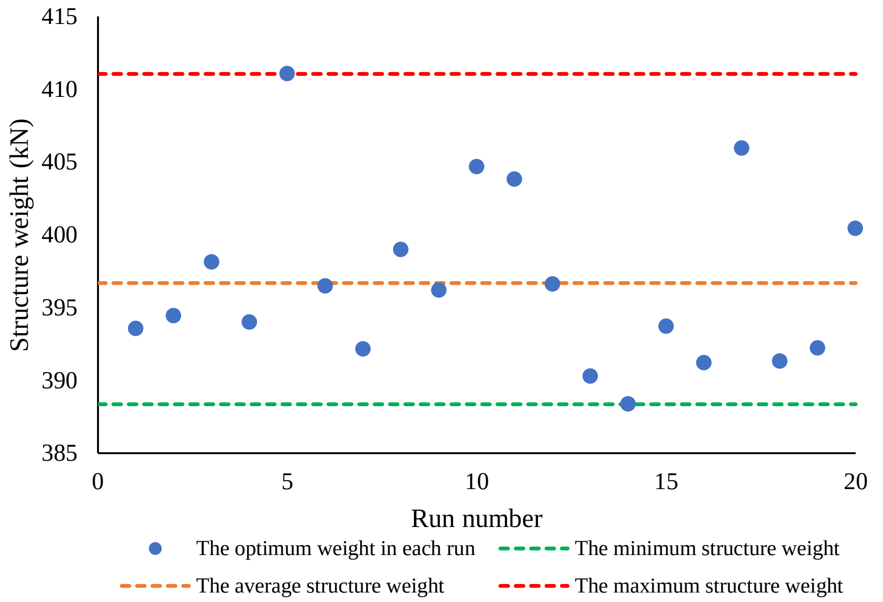

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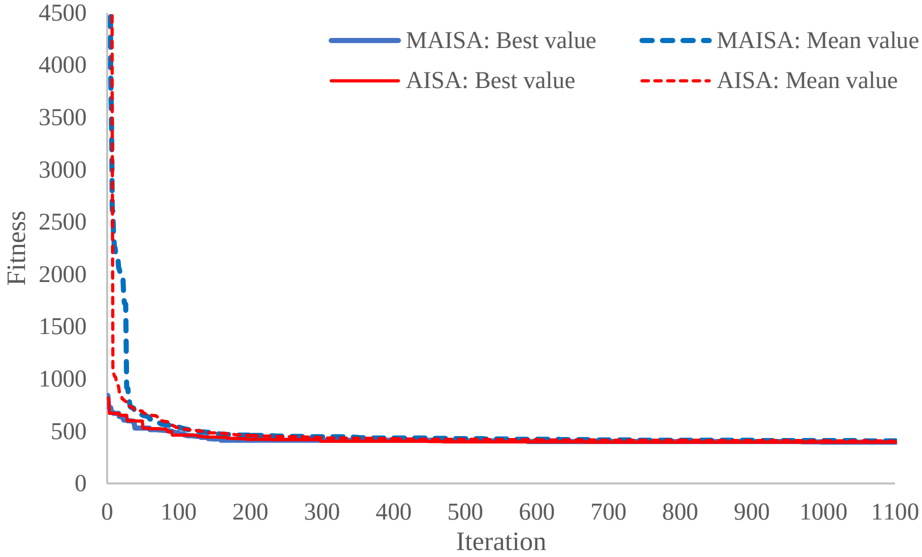

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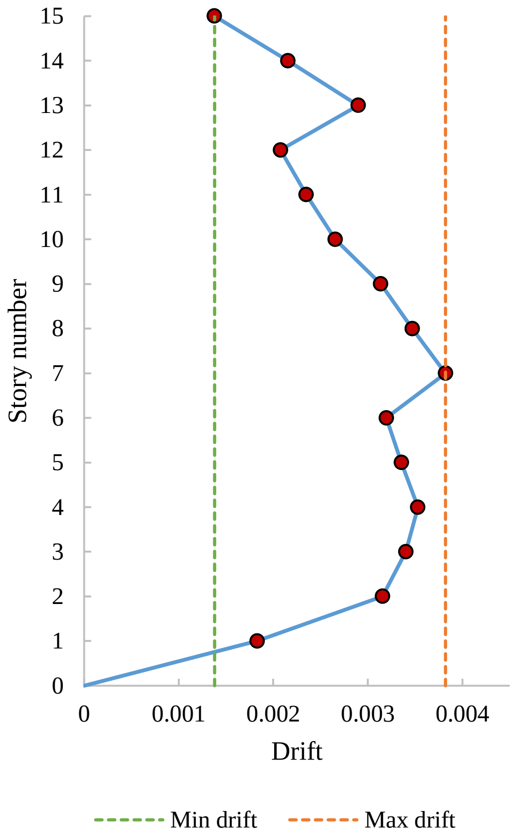

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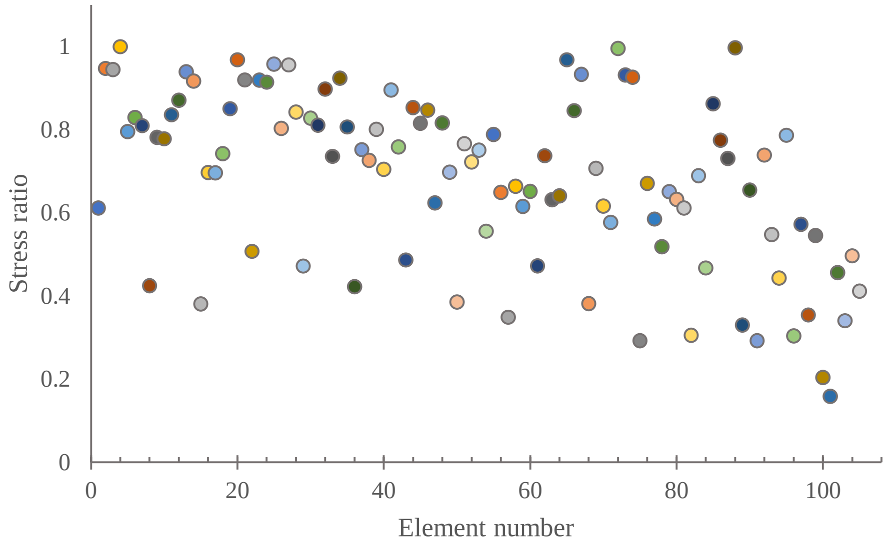

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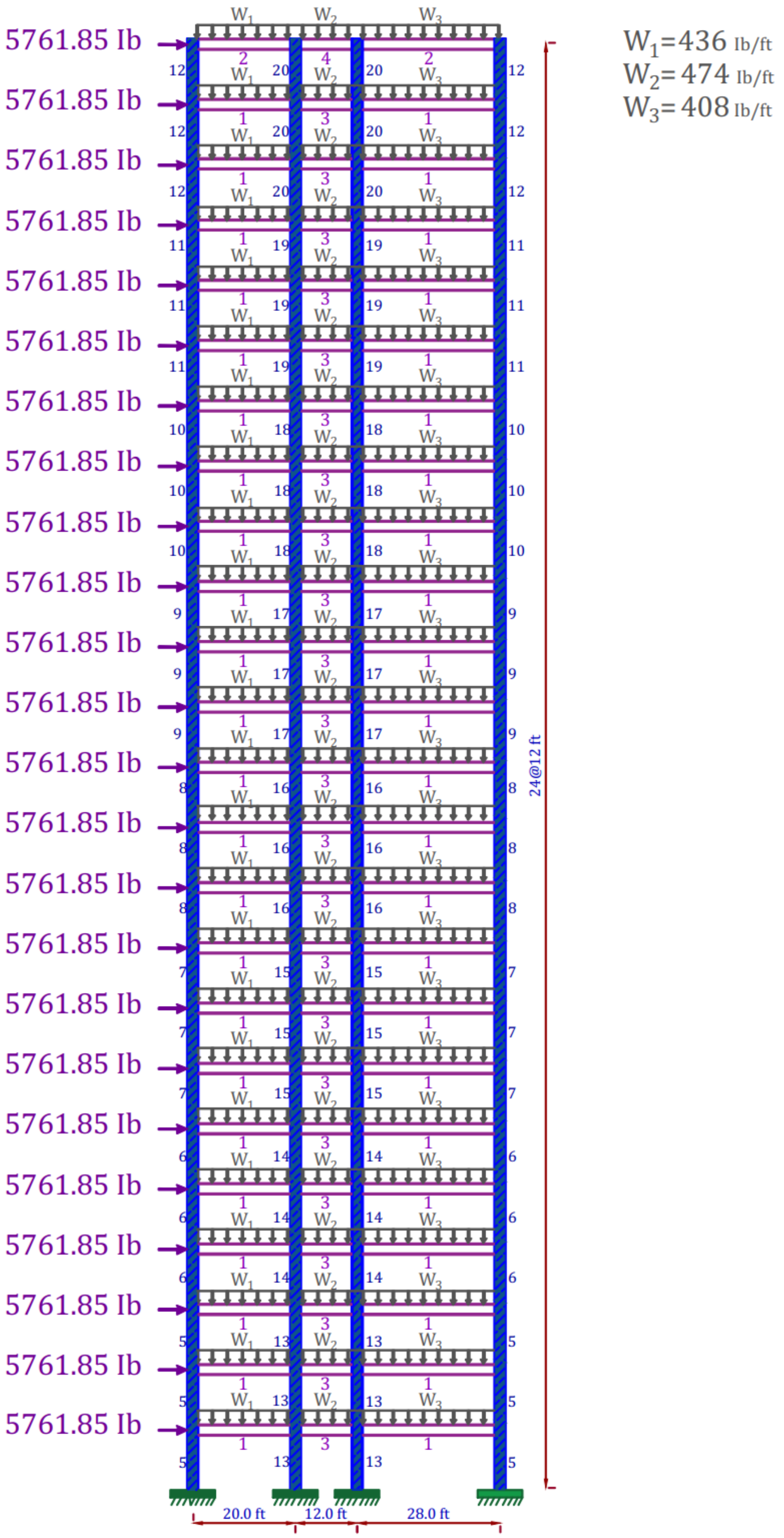

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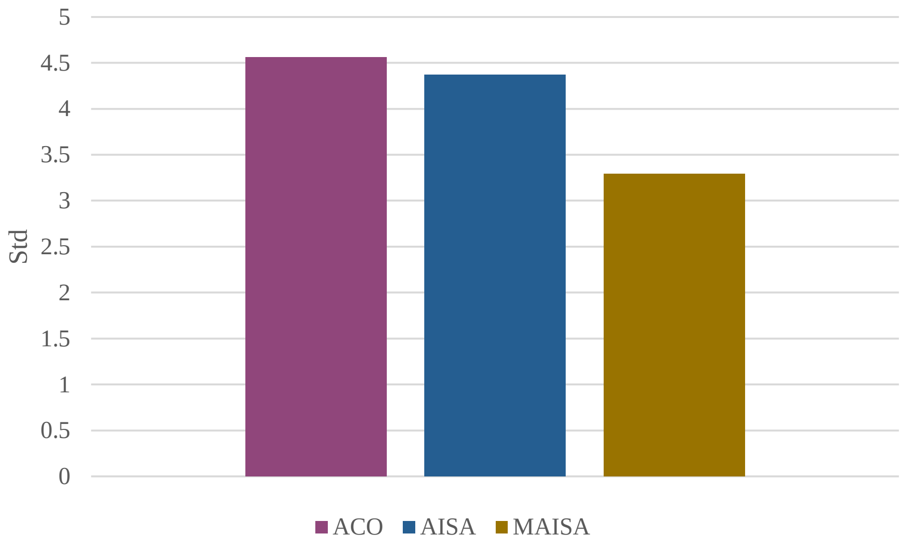

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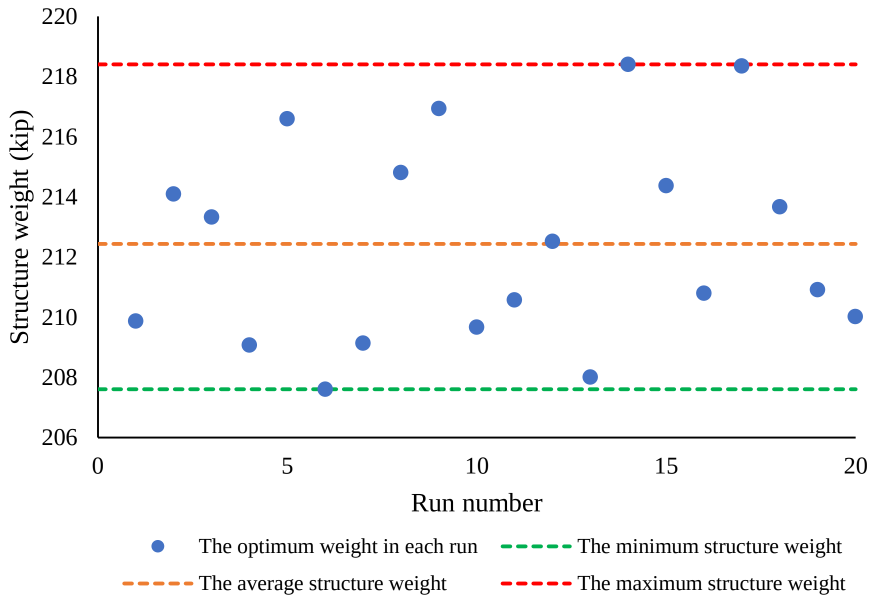

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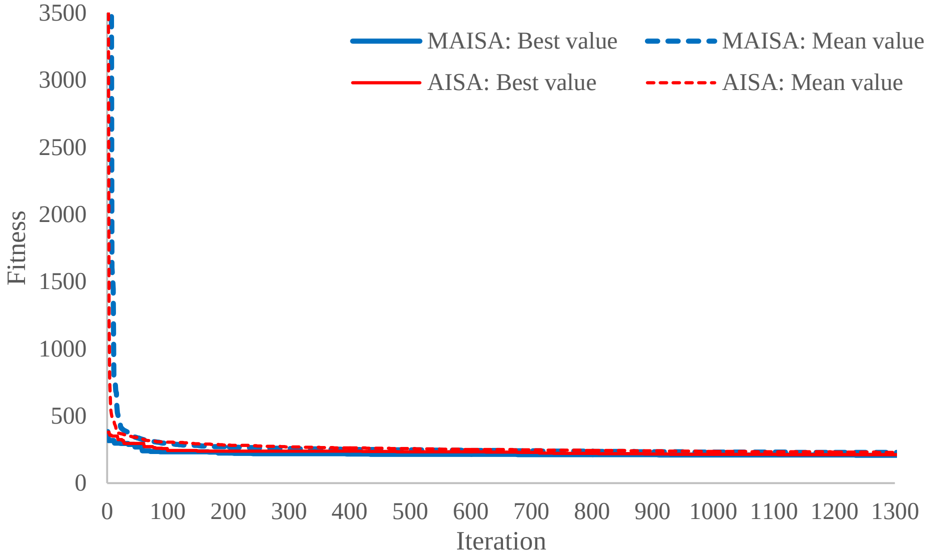

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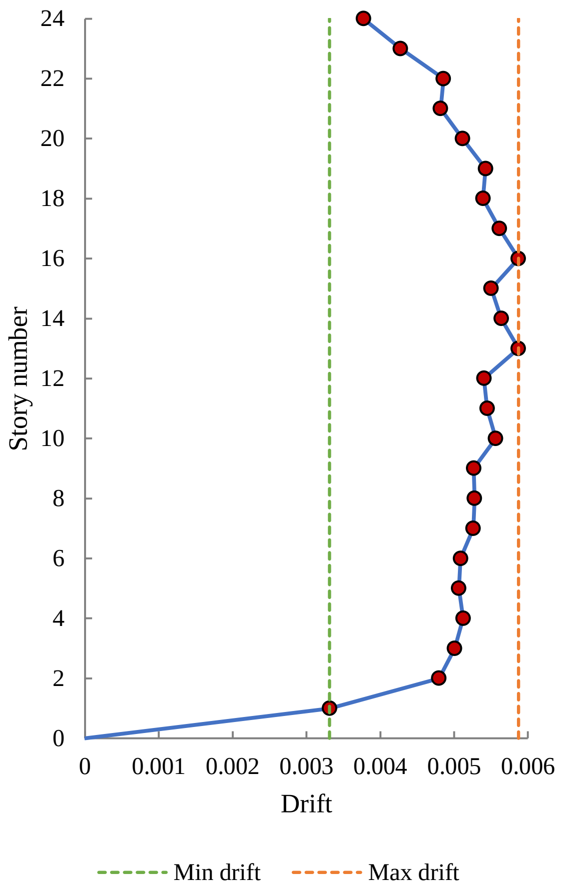

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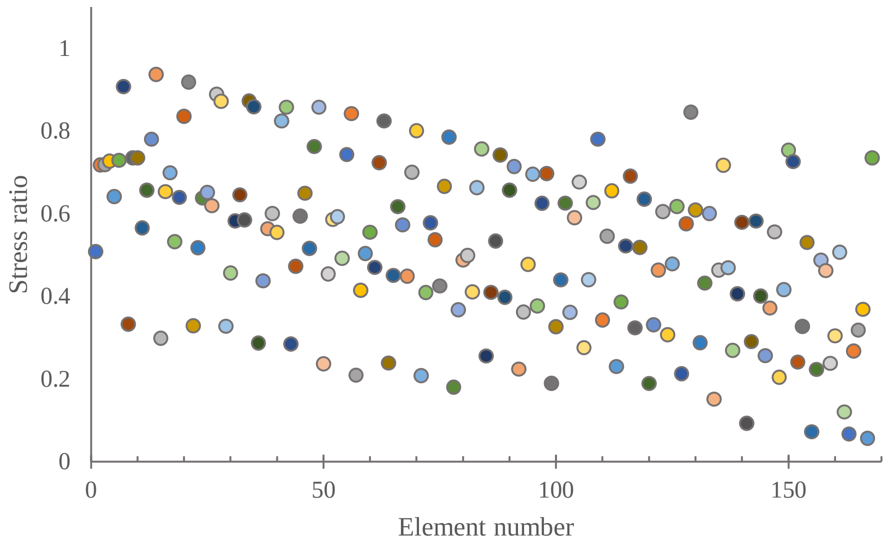

Fig. 34: The stress ratio of each member in the optimal solution

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Table 1: Performance comparison for the 1-bay, 8- story frame.

Group No.	EBA [40]	PSOPC [41]	PSOPC+ACO [41]	ES-DE [42]	Present work	
					AISA	MAISA
1	W18×35	W18×35	W18×35	W18×40	W18×35	W18×40
2	W16×31	W14×26	W16×31	W18×35	W14×34	W18×35
3	W16×31	W16×26	W14×22	W14×22	W14×34	W14×22
4	W12×14	W14×26	W12×16	W12×14	W12×16	W12×14
5	W18×35	W24×62	W21×48	W18×46	W16×26	W18×35
6	W18×35	W18×35	W18×40	W18×35	W16×45	W18×35
7	W18×35	W16×31	W16×31	W18×35	W18×35	W18×35
8	W16×31	W12×30	W16×36	W12×19	W10×33	W14×22
W_{Best} (kN)	31.86	34.21	32.29	31.77	31.70	**31.09**
$W_{Avg.}$ (kN)	N/A	N/A	N/A	33.65	**32.30**	32.74
Std	N/A	N/A	N/A	2.32	**0.29**	0.62

Table 2: Performance comparison for the 3-bay, 15-story frame.

Group No.	ES-DE [42]	TLBO [43]	EWOA [44]	SSOA [45]	Present work	
					AISA	MAISA
1	W18×106	W12×96	W14×99	W14×90	W21×111	W24×104
2	W36×150	W27×161	W27×161	W36×170	W18×143	W24×146
3	W12×79	W27×84	W27×84	W27×84	W12×79	W18×76
4	W27×114	W24×104	W24×104	W27×114	W14×109	W26×114
5	W30×90	W10×68	W21×68	W24×68	W10×77	W12×72
6	W10×88	W30×90	W18×86	W18×86	W24×104	W14×90
7	W18×71	W8×48	W21×48	W21×48	W8×67	W18×60
8	W18×65	W24×68	W14×68	W14×68	W18×86	W18×60
9	W8×28	W8×28	W8×31	W8×31	W8×28	W5×19
10	W12×40	W10×39	W10×45	W10×39	W12×40	W14×43
11	W21×48	W21×50	W21×44	W21×44	W21×44	W21×44
W_{Best} (kN)	415.06	390.42	392.00	393.23	392.19	**388.37**
$W_{Avg.}$ (kN)	438.26	423.67	403.99	407.89	398.31	**396.68**
Std	14.65	11.35	N/A	12.52	**4.17**	5.94

Table 3: Performance comparison for the 3-bay 24-story frame.

Group No.	ACO [34]	HS [46]	CBO [47]	ES-DE [42]	Present work	
					AISA	MAISA
1	W30×90	W30×90	W27×102	W30×90	W30×90	W30×90
2	W8×18	W10×22	W8×18	W21×55	W24×62	W24×55
3	W24×55	W18×40	W24×55	W24×48	W24×55	W21×44
4	W8×21	W12×16	W6×8.5	W10×45	W24×162	W18×158
5	W14×145	W14×176	W14×132	W14×145	W14×159	W14×176
6	W14×132	W14×145	W14×120	W14×109	W14×159	W14×120
7	W14×132	W14×176	W14×145	W14×99	W14×90	W14×132
8	W14×132	W14×132	W14×82	W14×145	W14×109	W14×99
9	W14×68	W14×132	W14×61	W14×109	W14×48	W14×61
10	W14×53	W14×109	W14×43	W14×48	W14×53	W14×48
11	W14×43	W14×109	W14×38	W14×38	W14×34	W14×43
12	W14×43	W14×82	W14×22	W14×30	W14×34	W14×30
13	W14×145	W14×82	W14×99	W14×99	W14×90	W14×90
14	W14×145	W14×61	W14×109	W14×132	W14×90	W14×120
15	W14×120	W14×74	W14×82	W14×109	W14×109	W14×90
16	W14×90	W14×48	W14×90	W14×68	W14×68	W14×82
17	W14×90	W14×34	W14×74	W14×68	W14×82	W14×74
18	W14×61	W14×30	W14×61	W14×68	W14×48	W14×53
19	W14×30	W14×22	W14×30	W14×61	W14×34	W14×38
20	W14×26	W14×22	W14×22	W14×22	W14×26	W14×22
W_{Best} (kip)	220.47	214.86	215.87	212.39	207.79	**207.60**
$W_{Avg.}$ (kip)	229.56	222.62	225.07	N/A	215.21	**212.43**
Std	4.56	N/A	N/A	N/A	4.37	**3.29**