Assessment of hedging strategy on supply chain performance with a single supplier and two retailers

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Abstract

The previous studies have shown that the application of option contracts affects the coordination of the supply chain. Though, based on authors' research there appears to be no survey conducted to measure the effect of hedging on supply chain from quantitative viewpoint. Generally, it is assumed that the product price is held fixed in the hedging; however, the competitors or the partners might sell the product cheaper. This condition restricts the hedger's opportunity to benefit. In this study we examine if the application of hedging through option contracts improves the performance of the total supply chain. To illustrate the answer, the supply chain consisting of one supplier and two retailers are considered. Regarding hedging, eight scenarios are created. The results indicate that the total supply chain profit is at the maximum benefit- among all possible scenarios- when hedging is completed properly. The research provides new insights that how hedging can maximize total supply chain profit, although it is possible that each individual member's profit may not be maximized.

Keywords: Supply Chain performance, Hedging, Option Contracts, demand.

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1. Introduction

Previously, the interaction between the members of the supply chain was competitive rather than cooperative. Every member was first concerned about optimizing its goals without concerning others in the same chain. However, the companies realized if organizations manage the total performance of the supply chain, they will have an improved value and strategic superiority in their market [1]. As a result, a company should not compete with the other members in the same supply chain. The real competition is between supply chains, not firms. Accordingly, the managers of the supply chain would cooperate in deciding. Customers want to receive high-quality goods and services at reasonable prices in the shortest time. But different types of risk affect the performance of companies and supply chains [2, 3, 4, 5].

For managing risk, financial derivatives offer an effective mechanism that facilitates price fluctuations. This mechanism is hedging. Every company would use the hedging strategy to decrease the effects of price fluctuations risk. But, there is no guarantee that a firm's outcome with hedging will be better than its outcome without hedging. Because in a supply chain, companies find themselves exposed not only to their price risks but also to the price risks of their partners. However, the financial theories do not examine the hedging effects on the total supply chain [6, 7, 8, 9, 10, 11].

There are few works related to hedging interactions among firms [12]. Models in risk management usually analyse the hedging decision of a firm in isolation condition [13]. The motivation of this study is to quantify the impact of the retailers' hedging on the performance of the supply chain. The novelty of this research is to investigate the effects of retailers' different hedging decisions on the supply chain members and the total supply chain. It wants to find the best decision for the total supply chain. The present paper aims to provide new insights for supply chain managers. As a result, they would have a better perception of the consequence of their hedging or not-hedging decisions.

The remainder of this paper is organized as follows: We reviewed the related literature in section 2, the problem description in section 3, the proposed model in section 4, the numerical analysis and the results in section 5, the proposed managerial insights, and some topics for future research in section 6.

2. Literature review

Recently, a plethora of research investigated the exclusive use of hedging in the financial markets. However, a few studies have investigated the hedging impacts on the supply chain. In the following lines, we will only review the literature relevant to this investigation.

Mian [14] provides empirical evidence on the determinants of corporate hedging decisions. He finds that large firms are more likely to hedge. Petersen and Thiagarajan [15], Mello and Ruckes [13] and Liu and Wang [16] survey the impact of financial hedging and production strategies on the interaction between firms.

Brown [17], Nain [11], Adam et al [18] and Haushalter et al. [19] find a firm's hedging choice depends on the hedging choices of its competitors, the number of firms in the industry, the elasticity of demand, and the convexity of production costs. Caldentey and Haugh [20] study the performance of a stylized supply chain. They consider three types of contracts. For each contract, they compare the decentralized competitive solution with the solution obtained by the central planner.

Liu and Parlour [21] characterize optimal hedging strategies in a competition framework. Also, Loss [22], Adam and Nain [23], and Fas and Senel [24] analyze the hedging strategies of firms that are competitors with each other. Turcic et al. [10] identify conditions under which the risk of the supply chain breakdown and its impacts on the firms' operations will cause the supply chain members to hedge their input costs.

Wang and Yao [25] studied a production planning with risk hedging, under partial information and budget constraint. Caldentey and Haugh [26] study the performance of a supply chain where multiple budget-constrained retailers and a single producer compete in the Cournot-Stackelberg game with a hedging strategy. Brusset and Bertrand [27] present an approach to reduce sales volatility using weather index-based financial instruments. Kouvelis et al [28] study hedging cash-flow risks in a supply chain where firms invest internal funds to improve production efficiencies.

Kouvelis et al [12] offer a conceptual framework for understanding supply chain cash hedging strategies. Awudu et al. [29] develop an optimization model considering maximization of the supply chain profit with hedging. Chen and Bolandifar [30] model a retailer procuring goods through index-based price contracts from two commodity processors.

Wang et al [31] survey supply chain coordination and supply chain members' decisions with a customer return policy. Firouzi and Vahdatmanesh [32] show how construction industry can take advantage of well-developed financial derivatives. Merkert and Swidan [33] re-examine risk management theories in the airline context and investigates whether financial hedging is an effective strategy for enhancing operational profitability.

Yu et al [34] propose a new coordination mechanism of contract farming supply chain. Sadefo Kamdem and Moumouni [35] investigate the different hedging strategies. Yan et al [36] survey the price risk of fresh agriculture products, and solve it with hedging. Xing et al [37] study the risk management strategies of two manufacturers. Wang et al [38] study the optimal financial hedging strategy and sustainable development in four supply chain structures. Xu and Duan [39] propose the hedging function of the futures market to reduce price risk. Wang et al [40] survey risk hedging in the online procurement problem.

In recent years, many researches have been accomplished in the field of financial hedging. However, based on authors' knowledge there is no research referring hedging with call options in the supply chain. The main purpose of the present paper is to address the question of "what are the effects of using hedging with call options contracts on the total supply chain profit?" Our objective is to illustrate how financial tools affect the supply chain operations.

3. Problem description

We consider a one-period two-echelon supply chain consisting of one supplier and two retailers. The supplier is responsible for the product sale to retailers and they are responsible to satisfy the end customers' demand. The product price is independent and identically distributed ($i.i.d.$) from normal distribution with mean μ and variance σ^2. We assume that only one type product is passed on the retailers by the supplier. Figure 1 illustrates flow of product in the supply chain.

[Figure 1]

For managing end customers' demands, the retailers can use the call option contracts for long hedging. The shortage is not allowed and the supplier can buy additional units from an emergency source with a higher price than itself. Option contracts are signed between supplier and each retailer before the primary period. The retailers can exercise their call option in primary period (see Figure 2).

[Figure 2]

At primary period (Figure 2), we face two conditions. The exercise price is bigger than the spot price or vice versa. For each event, four other conditions occur. The first and second retailers decide to hedge or not. Therefore, we have eight scenarios. Table 1 shows them. w_2

is a spot price. If the exercise price is less than w_2, we denote it by w_1 and otherwise, we indicate it by $w_1^{'}$ $\left(w_1 < w_2 < w_1^{'}\right)$.

[Table 1]

We consider the supplier and the retailers to be risk neutral. As a result, they choose to maximize their own expected profit respectively [41]. We consider supplier as a leader in Stackelberg game. Two retailers are followers, they themselves decide about hedging or not.

We want to know different hedging scenarios by retailers, what are the effects on the supply chain members' profit and the total supply chain profit?

4. Mathematical model

In this section, we calculate the profit function of the supply chain for each scenario and compare them with each other. Then, we find the scenario in which total supply chain has a maximum profit.

4.1 Notations

To develop the model, notations are summarized as follows:

- **Sets**

$i \in \{1,2,\ldots,8\}$: Scenario number

$j \in \{r_1, r_2, s, TSC\}$: r_1, r_2, s and TSC represent the first retailer, the second retailer, the supplier and the total supply chain respectively.

- **Decision Variables**

y_{1i} : The first retailer's order quantity under scenario i

y_{2i} : The second retailer's order quantity under scenario i

q_i : The supplier's capacity under scenario i

- **Parameters**

c : The supplier's production cost per unit

e : The emergency purchasing cost per unit

c_0 : The option price per unit

m : The retailers' fixed percentage profit margin $(0 < m < 1)$

p_1 : The retail price per unit under the hedging strategy when $w_1 < w_2$

$p_1^{'}$: The retail price per unit under the hedging strategy when $w_1^{'} > w_2$

p_2 : The retail price per unit under the optimal order quantity strategy

γ : The degree of substitutability between retailers $(0 \le \gamma \le 1)$

ν : The salvage value per unit $(\nu > 0)$

d_1^i : The end customers' demand to first retailer under scenario i

d_2^i : The end customers' demand to second retailer under scenario i

$F(x_1)$: The cumulated distribution function of end customers demand to the first retailer

$F(x_2)$: The cumulated distribution function of end customers demand to the second retailer

$f(x_1)$: The probability distribution function of end customers demand to the first retailer

$f(x_2)$: The probability distribution function of end customers demand to the second retailer

π_j^i : Expected profit under scenario i , j

$S\left(y_{1i}\right)$: The first retailer's expected sales

$S\left(y_{2i}\right)$: The second retailer's expected sales

$S\left(q_i\right)$: The supplier's expected sales

$I\left(q_i\right)$: The supplier's expected leftover inventory

$H\left(y_{1i}+y_{2i},q_i\right)$: The supplier's expected order quantity to the emergency source

a: Intercept of the retailers' demand curve- the end customers' choke price

b: Slope of the retailers' demand curve $\left(b>0\right)$

μ: The mean of the product price

σ^2: The variance of the product price

σ: The standard deviation of the product price

$\mu_{d_1^i}$: The mean of customers' demand to the first retailer under scenario i

$\mu_{d_2^i}$: The mean of customers' demand to the second retailer under scenario i

$\sigma_{d_1^i}^2$: The variance of customers' demand to the first retailer under scenario i

$\sigma_{d_2^i}^2$: The variance of customers' demand to the second retailer under scenario i

$\sigma_{d_1^i}$: The standard deviation of customers' demand to the first retailer under scenario i

$\sigma_{d_2^i}$: The standard deviation of customers' demand to the second retailer under scenario i

$\left(w_1<w_2<w_1^{'}<e;\ w_1<w_2<w_1^{'}<p_1,p_2;v,c<e\right)$.

4.2 The relation between the spot price and the retail price

For hedging strategy, the retail price at the beginning of sale period is as formula (1) or formula (2).

$$\boldsymbol{p}_1 = \left(1+\boldsymbol{m}\right)\boldsymbol{w}_1 \tag{1}$$

$$\boldsymbol{p}_1^{'} = \left(1+\boldsymbol{m}\right)\boldsymbol{w}_1^{'} \tag{2}$$

For optimal order quantity strategy, the retail price at the beginning of sale period is as formula (3).

$$\boldsymbol{p}_2 = \left(1+\boldsymbol{m}\right)\boldsymbol{w}_2 \tag{3}$$

Also, $\boldsymbol{p}_1$, $\boldsymbol{p}_1^{'}$ and $\boldsymbol{p}_2$ are independent.

4.3 The expected sales, the expected leftover inventory and the expected order quantity to the emergency source

The supplier's expected sales and the expected leftover inventory will be as formula (4) and formula (5), respectively.

$$S\left(q_i\right) = min\left(y_{1i} + y_{2i}, q_i\right) = q_i - \int_0^{q_i}\left(F\left(x_1\right) + F\left(x_2\right)\right)dx \tag{4}$$

$$I\left(q_i\right) = E\left(q_i - min\left(y_{1i} + y_{2i}, d_1^i + d_2^i\right)\right)^+ = q_i - S\left(q_i\right) \tag{5}$$

The first retailer's expected sales and the second retailer's expected sales will be as formula (6) and formula (7), respectively.

$$S\left(y_{1i}\right) = \left[\min\left(d_1^i, y_{1i}\right)\right] = y_{1i} - \int_0^{y_{1i}} F\left(x_1\right)dx \tag{6}$$

$$S\left(y_{2i}\right) = \left[\min\left(d_2^i, y_{2i}\right)\right] = y_{2i} - \int_0^{y_{2i}} F\left(x_2\right)dx \tag{7}$$

The expected order quantity to the emergency source is formula (8).

$$H\left(y_{1i}+y_{2i},q_i\right)=E\left[min\left(d_1^i+d_2^i,y_{1i}+y_{2i}\right)-q_i\right]^+=S\left(y_{1i}\right)+S\left(y_{2i}\right)-S\left(q_i\right) \quad (8)$$

4.4 The end customers' reaction to the product prices

By considering the exercise price and the spot price, each retailer may decide to hedge or not. Thus, one of the retailers might sell his product cheaper than another. As a result, some end customers will buy their products from the retailer at a cheap price. Therefore:

- When the first retailer sells his product cheaper than the second retailer, some end customers of the second retailer (γ) will buy their products from the first retailer as well. See Figure 3.

[Figure 3]

- When the second retailer sells his product cheaper than the first retailer, some customers of the first retailer (γ) will buy their products from the second retailer as well. See Figure 4.

[Figure 4]

In this paper, the end customers' demand of each retailer is considered a function of the first retailer's product price and the second retailer's product price.

4.5 The supplier's optimal capacity and two retailers' optimal order quantity

First, we calculate the Nash equilibrium for two retailers under different scenarios. Then, by substituting calculated variable with the supplier's expected profit function and taking the first and the second derivatives with respect to q_i, we will obtain supplier's optimal capacity. Second, for each scenario, we calculate the profit function of the total supply chain and with substituting the first retailer's optimal order quantity, the second retailer's optimal order quantity and the supplier's optimal capacity in the profit function of the total supply chain, we find the scenario in which the total supply chain has a maximum profit.

- **Scenario 1**

The end customers' demand of the first retailer and the end customers' demand of the second retailer are as formula (9) and formula (10) respectively.

$$\boldsymbol{d}_1^1 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}}\boldsymbol{p}_1 \tag{9}$$

$$\boldsymbol{d}_2^1 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}}\boldsymbol{p}_1 \tag{10}$$

Expected profit function for the first retailer, the second retailer and the supplier are as formula (11), formula (12) and formula (13) respectively.

$$\pi_{r_1}^1 = p_1 S\left(y_{11}\right) - c_0 y_{11} - w_1 y_{11} \tag{11}$$

$$\pi_{r_2}^1 = p_1 S\left(y_{21}\right) - c_0 y_{21} - w_1 y_{21} \tag{12}$$

$$\begin{aligned}\pi_s^1 &= w_1 S\left(y_{11}\right) + w_1 S\left(y_{21}\right) + c_0\left(y_{11} + y_{21}\right) + \\ &\nu I\left(q_1\right) - eH\left(y_{11} + y_{21}, q_1\right) - cq_1 \\ &= w_1\left(S\left(y_{11}\right) + S\left(y_{21}\right)\right) + c_0\left(y_{11} + y_{21}\right) + \left(\nu - c\right)q_1 \\ &+ \left(e - \nu\right)S\left(q_1\right) - e\left(S\left(y_{11}\right) + S\left(y_{21}\right)\right)\end{aligned} \tag{13}$$

Proposition 1. We solve formula (11), formula (12), and formula (13) for y_{11}, y_{21} and q_1 respectively.

$$\left(y_{11}\right)^* = \mu_{d_1^1} + \sqrt{2\pi}\sigma_{d_1^1}\left(\frac{mw_1 - c_0}{\left(1+m\right)w_1} - \frac{1}{2}\right) \tag{14}$$

$$\left(y_{21}\right)^* = \mu_{d_2^1} + \sqrt{2\pi}\sigma_{d_2^1}\left(\frac{mw_1 - c_0}{\left(1+m\right)w_1} - \frac{1}{2}\right) \tag{15}$$

$$\left(q_1\right)^* = \mu_{d_1^1+d_2^1} + \sqrt{2\pi}\sigma_{d_1^1+d_2^1}\left(\frac{1}{2} - \frac{v-c}{e-v}\right) \tag{16}$$

Proof. See Appendix 1.

- **Scenario 2**

The end customers' demand of the first retailer and the end customers' demand of the second retailer are as formula (17) and formula (18) respectively [42].

$$\boldsymbol{d}_1^2 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}}\boldsymbol{p}_1 + \frac{\boldsymbol{\gamma}}{\boldsymbol{b}}\boldsymbol{p}_2 \tag{17}$$

$$\boldsymbol{d}_2^2 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}}\boldsymbol{p}_2 - \frac{\boldsymbol{\gamma}}{\boldsymbol{b}}\boldsymbol{p}_2 \tag{18}$$

Expected profit function for the first retailer, the second retailer and the supplier are as formula (19), formula (20), and formula (21) respectively.

$$\pi_{r_1}^2 = p_1 S\left(y_{12}\right) - c_0 y_{12} - w_1 y_{12} = p_1 S\left(y_{12}\right) - c_0 y_{12} - w_1 y_{12} \tag{19}$$

$$\pi_{r_2}^2 = p_2 S\left(y_{22}\right) - w_2 y_{22} = p_2 S\left(y_{22}\right) - w_2 y_{22} \tag{20}$$

$$\begin{aligned} \pi_s^2 &= w_1 S\left(y_{12}\right) + w_2 y_{22} + c_0 y_{12} - c q_2 \\ &+ vI\left(q_2\right) - eH\left(y_{12} + y_{22}, q_2\right) \\ &= w_1 S\left(y_{12}\right) + w_2 y_{22} + c_0 y_{12} + \left(v-c\right) q_2 \\ &+ \left(e-v\right) S\left(q_2\right) - e\left(S\left(y_{12}\right) + S\left(y_{22}\right)\right) \end{aligned} \tag{21}$$

Proposition 2. We solve formula (19), formula (20), and formula (21) for y_{12}, y_{22} and q_2 respectively.

$$\left(y_{12}\right)^{*}=\mu_{d_1^2}+\sqrt{2\pi}\sigma_{d_1^2}\left(\frac{mw_1-c_0}{\left(1+m\right)w_1}-\frac{1}{2}\right) \tag{22}$$

$$\left(y_{22}\right)^{*}=\mu_{d_2^2}+\sqrt{2\pi}\sigma_{d_2^2}\left(\frac{m}{1+m}-\frac{1}{2}\right) \tag{23}$$

$$\left(q_{2}\right)^{*}=\mu_{d_1^2+d_2^2}+\sqrt{2\pi}\sigma_{d_1^2+d_2^2}\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right) \tag{24}$$

Proof. See Appendix 2.

- **Scenario 3**

The end customers' demand of the first retailer and the end customers' demand of the second retailer are as formula (25) and formula (26) respectively.

$$\boldsymbol{d}_1^3=\frac{\boldsymbol{a}}{\boldsymbol{b}}-\frac{1}{\boldsymbol{b}}\boldsymbol{p}_2-\frac{\boldsymbol{\gamma}}{\boldsymbol{b}}\boldsymbol{p}_2 \tag{25}$$

$$\boldsymbol{d}_2^3=\frac{\boldsymbol{a}}{\boldsymbol{b}}-\frac{1}{\boldsymbol{b}}\boldsymbol{p}_1+\frac{\boldsymbol{\gamma}}{\boldsymbol{b}}\boldsymbol{p}_2 \tag{26}$$

Expected profit function for the first retailer, the second retailer and the supplier are as formula (27), formula (28), and formula (29) respectively.

$$\pi_{r_1}^3=p_2S\left(y_{13}\right)-w_2y_{13}=p_2S\left(y_{13}\right)-w_2y_{13} \tag{27}$$

$$\pi_{r_2}^3=p_1S\left(y_{23}\right)-c_0y_{23}-w_1y_{23}=p_1S\left(y_{23}\right)-c_0y_{23}-w_1y_{23} \tag{28}$$

$$\pi_{s}^3=w_2y_{13}+w_1S\left(y_{23}\right)+c_0y_{23}$$

$$-cq_3+vI\left(q_3\right)-eH\left(y_{13}+y_{23},q_3\right) \tag{29}$$

$$=w_2y_{13}+w_1S\left(y_{23}\right)+c_0y_{23}+\left(\nu-c\right)q_3$$

$$+(e-v)S(q_3)-e\left(S(y_{13})+S(y_{23})\right)$$

Proposition 3. We solve formula (27), formula (28), and formula (29) for y_{13}, y_{23} and q_3 respectively.

$$(y_{13})^* = \mu_{d_1^3} + \sqrt{2\pi}\sigma_{d_1^3}\left(\frac{m}{1+m}-\frac{1}{2}\right) \tag{30}$$

$$(y_{23})^* = \mu_{d_2^3} + \sqrt{2\pi}\sigma_{d_2^3}\left(\frac{mw_1-c_0}{(1+m)w_1}-\frac{1}{2}\right) \tag{31}$$

$$(q_3)^* = \mu_{d_1^3+d_2^3} + \sqrt{2\pi}\sigma_{d_1^3+d_2^3}\left(\frac{1}{2}-\frac{v-c}{e-v}\right) \tag{32}$$

Proof. See Appendix 3.

- **Scenario 4**

The end customers' demand of the first retailer and the end customers' demand of the second retailer are as formula (33) and formula (34) respectively.

$$\boldsymbol{d}_1^4 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}}\boldsymbol{p}_2 \tag{33}$$

$$\boldsymbol{d}_2^4 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}}\boldsymbol{p}_2 \tag{34}$$

Expected profit function for the first retailer, the second retailer and the supplier are as formula (35), formula (36) and formula (37) respectively.

$$\pi_{r_1}^4 = p_2S(y_{14}) - w_2y_{14} = p_2S(y_{14}) - w_2y_{14} \tag{35}$$

$$\pi_{r_2}^4 = p_2S(y_{24}) - w_2y_{24} = p_2S(y_{24}) - w_2y_{24} \tag{36}$$

$$\pi_s^4 = w_2y_{14} + w_2y_{24} - cq_4 \tag{37}$$

$+vI\left(q_4\right)-eH\left(y_{14}+y_{24},q_4\right)$

$=w_2\left(y_{14}+y_{24}\right)+\left(v-c\right)q_4$

$+\left(e-v\right)S\left(q_4\right)-e\left(S\left(y_{14}\right)+S\left(y_{24}\right)\right)$

Proposition 4. We solve formula (35), formula (36) and formula (37) for y_{14}, y_{24} and q_4 respectively.

$$\left(y_{14}\right)^* = \mu_{d_1^4} + \sqrt{2\pi}\sigma_{d_1^4}\left(\frac{m}{1+m}-\frac{1}{2}\right) \tag{38}$$

$$\left(y_{24}\right)^* = \mu_{d_2^4} + \sqrt{2\pi}\sigma_{d_2^4}\left(\frac{m}{1+m}-\frac{1}{2}\right) \tag{39}$$

$$\left(q_4\right)^* = \mu_{d_1^4+d_2^4} + \sqrt{2\pi}\sigma_{d_1^4+d_2^4}\left(\frac{1}{2}-\frac{v-c}{e-v}\right) \tag{40}$$

Proof. See Appendix 4.

- **Scenario 5**

The end customers' demand of the first retailer and the end customers' demand of the second retailer are as formula (41) and formula (42) respectively.

$$\boldsymbol{d}_1^5 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}}\boldsymbol{p}_1^{'} \tag{41}$$

$$\boldsymbol{d}_2^5 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}}\boldsymbol{p}_1^{'} \tag{42}$$

Expected profit function for the first retailer, the second retailer and the supplier are as formula (43), formula (44), and formula (45) respectively.

$$\pi_{r_1}^5 = p_1^{'}S\left(y_{15}\right)-c_0y_{15}-w_1^{'}y_{15} = p_1^{'}S\left(y_{15}\right)-c_0y_{15}-w_1^{'}y_{15} \tag{43}$$

$$\pi_{r_2}^5 = p_1^{'}S\left(y_{25}\right)-c_0y_{25}-w_1^{'}y_{25} = p_1^{'}S\left(y_{25}\right)-c_0y_{25}-w_1^{'}y_{25} \tag{44}$$

$$\pi_s^5 = w_1^{'}S\left(y_{15}\right)+w_1^{'}S\left(y_{25}\right)+c_0y_{15} \tag{45}$$

$$+c_0 y_{25} - cq_5 + vI(q_5) - eH(y_{15} + y_{25}, q_5)$$

$$= w_1^{'}\left(S(y_{15}) + S(y_{25})\right) + c_0(y_{15} + y_{25})$$

$$+(\nu - c)q_5 + (e-\nu)S(q_5) - e\left(S(y_{15}) + S(y_{25})\right)$$

Proposition 5. We solve formula (43), formula (44) and formula (45) for y_{15}, y_{25}, and q_5 respectively.

$$(y_{15})^* = \mu_{d_1^5} + \sqrt{2\pi}\sigma_{d_1^5}\left(\frac{mw_1^{'} - c_0}{(1+m)w_1^{'}} - \frac{1}{2}\right) \tag{46}$$

$$(y_{25})^* = \mu_{d_2^5} + \sqrt{2\pi}\sigma_{d_2^5}\left(\frac{mw_1^{'} - c_0}{(1+m)w_1^{'}} - \frac{1}{2}\right) \tag{47}$$

$$(q_5)^* = \mu_{d_1^5 + d_2^5} + \sqrt{2\pi}\sigma_{d_1^5 + d_2^5}\left(\frac{1}{2} - \frac{\nu - c}{e - \nu}\right) \tag{48}$$

Proof. See Appendix 5.

- **Scenario 6**

The end customers' demand of the first retailer and the end customers' demand of the second retailer are as formula (49) and formula (50) respectively.

$$\boldsymbol{d}_1^6 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}}\boldsymbol{p}_1^{'} - \frac{\boldsymbol{\gamma}}{\boldsymbol{b}}\boldsymbol{p}_1^{'} \tag{49}$$

$$\boldsymbol{d}_2^6 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}}\boldsymbol{p}_2 + \frac{\boldsymbol{\gamma}}{\boldsymbol{b}}\boldsymbol{p}_1^{'} \tag{50}$$

Expected profit function for the first retailer, the second retailer and the supplier are as formula (51), formula (52), and formula (53) respectively.

$$\pi_{r_1}^6 = p_1^{'}S(y_{16}) - c_0 y_{16} - w_1^{'}y_{16} = p_1^{'}S(y_{16}) - c_0 y_{16} - w_1^{'}y_{16} \tag{51}$$

$$\pi_{r_2}^6 = p_2 S\left(y_{26}\right) - w_2 y_{26} = p_2 S\left(y_{26}\right) - w_2 y_{26} \tag{52}$$

$$\begin{aligned}\pi_s^6 &= w_1^{'} S\left(y_{16}\right) + w_2 y_{26} + c_0 y_{16} - c q_6 \\ &+ vI\left(q_6\right) - eH\left(y_{16} + y_{26}, q_6\right) \\ &= w_1^{'} S\left(y_{16}\right) + w_2 y_{26} + c_0 y_{16} + \left(v - c\right) q_6 \\ &+ \left(e - v\right) S\left(q_6\right) - e\left(S\left(y_{16}\right) + S\left(y_{26}\right)\right)\end{aligned} \tag{53}$$

Proposition 6. We solve formula (51), formula (52), and formula (53) for y_{16}, y_{26} and q_6 respectively.

$$\left(y_{16}\right)^* = \mu_{d_1^6} + \sqrt{2\pi}\sigma_{d_1^6}\left(\frac{m w_1^{'} - c_0}{\left(1+m\right) w_1^{'}} - \frac{1}{2}\right) \tag{54}$$

$$\left(y_{26}\right)^* = \mu_{d_2^6} + \sqrt{2\pi}\sigma_{d_2^6}\left(\frac{m}{1+m} - \frac{1}{2}\right) \tag{55}$$

$$\left(q_6\right)^* = \mu_{d_1^6 + d_2^6} + \sqrt{2\pi}\sigma_{d_1^6 + d_2^6}\left(\frac{1}{2} - \frac{v - c}{e - v}\right) \tag{56}$$

Proof. See Appendix 6.

- **Scenario 7**

The end customers' demand of the first retailer and the end customers' demand of the second retailer are as formula (57) and formula (58) respectively.

$$\boldsymbol{d}_1^7 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}} \boldsymbol{p}_2 + \frac{\boldsymbol{\gamma}}{\boldsymbol{b}} \boldsymbol{p}_1^{'} \tag{57}$$

$$\boldsymbol{d}_2^7 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}} \boldsymbol{p}_1^{'} - \frac{\boldsymbol{\gamma}}{\boldsymbol{b}} \boldsymbol{p}_1^{'} \tag{58}$$

Expected profit function for the first retailer, the second retailer and the supplier are as formula (59), formula (60), and formula (61) respectively.

$$\pi_{r_1}^7 = p_2 S(y_{17}) - w_2 y_{17} = p_2 S(y_{17}) - w_2 y_{17} \tag{59}$$

$$\pi_{r_2}^7 = p_1^{'} S(y_{27}) - c_0 y_{27} - w_1^{'} y_{27} = p_1^{'} S(y_{27}) - c_0 y_{27} - w_1^{'} y_{27} \tag{60}$$

$$\begin{aligned} \pi_s^7 &= w_2 y_{17} + w_1^{'} S(y_{27}) + c_0 y_{27} \\ &-cq_7 + vI(q_7) - eH(y_{17} + y_{27}, q_7) \\ &= w_2 y_{17} + w_1^{'} S(y_{27}) + c_0 y_{27} + (v - c) q_7 \\ &+(e - v) S(q_7) - e(S(y_{17}) + S(y_{27})) \end{aligned} \tag{61}$$

Proposition 7. We solve formula (59), formula (60), and formula (61) for y_{17}, y_{27} and q_7 respectively.

$$(y_{17})^* = \mu_{d_1^7} + \sqrt{2\pi}\sigma_{d_1^7}\left(\frac{m}{1+m} - \frac{1}{2}\right) \tag{62}$$

$$(y_{27})^* = \mu_{d_2^7} + \sqrt{2\pi}\sigma_{d_2^7}\left(\frac{mw_1^{'} - c_0}{(1+m)w_1^{'}} - \frac{1}{2}\right) \tag{63}$$

$$(q_7)^* = \mu_{d_1^7 + d_2^7} + \sqrt{2\pi}\sigma_{d_1^7 + d_2^7}\left(\frac{1}{2} - \frac{v - c}{e - v}\right) \tag{64}$$

Proof. See Appendix 7.

- **Scenario 8**

The end customers' demand of the first retailer and the end customers' demand of the second retailer are as formula (65) and formula (66) respectively.

$$\boldsymbol{d}_1^8 = \frac{\boldsymbol{a}}{\boldsymbol{b}} - \frac{1}{\boldsymbol{b}}\boldsymbol{p}_2 \tag{65}$$

$$d_2^8 = \frac{a}{b} - \frac{1}{b} p_2 \tag{66}$$

Expected profit function for the first retailer, the second retailer, and the supplier are as formula (67), formula (68) and formula (69) respectively.

$$\pi_{r_1}^{8} = p_2 S\left(y_{18}\right) - w_2 y_{18} = p_2 S\left(y_{18}\right) - w_2 y_{18} \tag{67}$$

$$\pi_{r_2}^{8} = p_2 S\left(y_{28}\right) - w_2 y_{28} = p_2 S\left(y_{28}\right) - w_2 y_{28} \tag{68}$$

$$\begin{aligned} \pi_s^8 &= w_2 y_{18} + w_2 y_{28} - c q_8 \\ &+ v I\left(q_8\right) - e H\left(y_{18} + y_{28}, q_8\right) \\ &= w_2\left(y_{18} + y_{28}\right) + \left(v - c\right) q_8 \\ &+ \left(e - v\right) S\left(q_8\right) - e\left(S\left(y_{18}\right) + S\left(y_{28}\right)\right) \end{aligned} \tag{69}$$

Proposition 8. We solve formula (67), formula (68) and formula (69) for y_{18}, y_{28} and q_8 respectively.

$$\left(y_{18}\right)^* = \mu_{d_1^8} + \sqrt{2\pi}\sigma_{d_1^8}\left(\frac{m}{1+m} - \frac{1}{2}\right) \tag{70}$$

$$\left(y_{28}\right)^* = \mu_{d_2^8} + \sqrt{2\pi}\sigma_{d_2^8}\left(\frac{m}{1+m} - \frac{1}{2}\right) \tag{71}$$

$$\left(q_8\right)^* = \mu_{d_1^8 + d_2^8} + \sqrt{2\pi}\sigma_{d_1^8 + d_2^8}\left(\frac{1}{2} - \frac{v-c}{e-v}\right) \tag{72}$$

Proof. See Appendix 8.

4.6 Profit functions of total supply chain

The profit of the total supply chain is obtained the sum of the members' profits in the supply chain. The profit function of the total supply chain can be written as formula (73).

$$\pi_{TSC}^{i} = \pi_{r_1}^{i} + \pi_{r_2}^{i} + \pi_s^{i} \tag{73}$$

For scenario 1 to scenario 8, the profit function of total supply chain has been written as formula (74), formula (75), formula (76), formula (77), formula (78), formula (79), formula (80), and formula (81) respectively.

$$\pi_{TSC}^{1}=\left(p_1+w_1-e\right)\left[S\left(y_{11}\right)+S\left(y_{21}\right)\right]-w_1\left(y_{11}+y_{21}\right)+\left(v-c\right)q_1+\left(e-v\right)S\left(q_1\right) \tag{74}$$

$$\pi_{TSC}^{2}=\left(p_1+w_1-e\right)S\left(y_{12}\right)+\left(p_2-e\right)S\left(y_{22}\right)-w_1y_{12}+\left(v-c\right)q_2+\left(e-v\right)S\left(q_2\right) \tag{75}$$

$$\pi_{TSC}^{3}=\left(p_2-e\right)S\left(y_{13}\right)+\left(p_1+w_1-e\right)S\left(y_{23}\right)-w_1y_{23}+\left(v-c\right)q_3+\left(e-v\right)S\left(q_3\right) \tag{76}$$

$$\pi_{TSC}^{4}=\left(p_2-e\right)\left[S\left(y_{14}\right)+S\left(y_{24}\right)\right]+\left(v-c\right)q_4+\left(e-v\right)S\left(q_4\right) \tag{77}$$

$$\pi_{TSC}^{5}=\left(p_1^{'}+w_1^{'}-e\right)\left[S\left(y_{15}\right)+S\left(y_{25}\right)\right]-\left(y_{15}+y_{25}\right)w_1^{'}+\left(v-c\right)q_5+\left(e-v\right)S\left(q_5\right) \tag{78}$$

$$\pi_{TSC}^{6}=\left(p_1^{'}+w_1^{'}-e\right)S\left(y_{16}\right)+\left(p_2-e\right)S\left(y_{26}\right)-w_1^{'}y_{16}+\left(v-c\right)q_6+\left(e-v\right)S\left(q_6\right) \tag{79}$$

$$\pi_{TSC}^{7}=\left(p_2-e\right)S\left(y_{17}\right)+\left(p_1^{'}+w_1^{'}-e\right)S\left(y_{27}\right)-w_1^{'}y_{27}+\left(v-c\right)q_7+\left(e-v\right)S\left(q_7\right) \tag{80}$$

$$\pi_{TSC}^{8}=\left(p_2-e\right)\left[S\left(y_{18}\right)+S\left(y_{28}\right)\right]+\left(v-c\right)q_8+\left(e-v\right)S\left(q_8\right) \tag{81}$$

Theorem 1. The profit of the supply chain in scenario 1 is greater than other scenarios if we have $a<\mu$, $p_2<w_1+p_1<e$, and $2w_1<w_2$.

Proof. See Appendix 9.

According to Theorem 1, for the supply chain profit in the first scenario to be greater than the supply chain profit in other scenarios, we must consider five parameters and their relations to analyze: intercept of the retailers' demand curve (a), the mean of the product price (μ), the exercise price (w_1), the spot price (w_2), the emergency purchasing cost (e).

The intercept of the retailers' demand curve (a) is also called the choke price. It is the lowest price at which the end customers' demand is equal to zero. As mentioned by Theorem 1, the mean of the product price (μ) should be higher than the choke price. Also, the retailers' inference of purchase price of goods through the hedging strategy and the optimal order quantity strategy, and the relationship of these prices with each other is effective in select the optimal strategy. If at the beginning of the sale period, p_2 is less than $w_1 + p_1$, and $w_1 + p_1$ is less than e, also $2w_1 < w_2$, and the first retailer and the second retailer accept to select hedging strategy, the performance of the supply chain will be best.

5. Numerical Analysis

This section has two parts:

- First, in section 5.1, we execute the models through a numerical example for each scenario and then we compare them to each other.
- Second, in section 5.2, we show the impact of changing the value of parameters on the profit function retailer 1, retailer 2, the supplier, and the supply chain for the first scenario. We survey the effect of changing σ, μ, b, and w_1 on the values of objective functions of the first scenario in subsection 5.2.1, subsection 5.2.2, subsection 5.2.3, and subsection 5.2.4, respectively.

We consider $m = 0.1$, $a = 3$, $b = 5$, $\gamma = 0.3$, $c_0 = 3$, $w_1 = 10$, $w_1^{'} = 15$, $w_2 = 30$, $c = 1$, $v = 7$, $e = 90$, $\mu = 5$, $\sigma = 0.5$. It should be mentioned that the conditions of Theory 1 are established for these values. Also, MATLAB is used for numerical analysis.

5.1 Comparison of the objective functions values of scenarios

Table 2 shows the value of objective functions for each scenario.

[Table 2]

Also, Figure 5 represents the profit function retailer 1, retailer 2, the supplier, and the supply chain for all scenarios. Vertical axis shows the value of objective functions. The horizontal axis represents retailer 1, retailer 2, supplier, and the supply chain.

[Figure 5]

According to Table 2 and Figure 5, under scenario 1, the profit of total supply chain will obtain the maximum benefit compared with other scenarios. However, for the first scenario, the retailers' profit is not the highest compared with other scenarios. But the supplier's profit is higher than in other scenarios.

Based on Table 2 and Figure 5, and by comparing profit amount for retailer 1 and retailer 2 in scenario 2 and scenario 3, it shows that if retailer 1 or retailer 2 does not hedge but another does, it is possible to have less or more profit than another. Also, this situation applies for scenario 6 and scenario 7. Additionally, retailer 1 in the second scenario or retailer 2 in the third scenario has the highest profit compared to others scenarios. The first retailer in scenario 2 or the second retailer in scenario 3 has an accurate price forecast, and it hedges proper. But the decision of retailer 2 under second scenario and retailer 1 under third scenario is not suitable, and it affects the profit of the supply chain. As a result, the profit of the supply chain under the second scenario and the profit of the supply chain under the third scenario are lower than the profit of the supply chain under the first scenario.

Under scenario 5, the profits of the first retailer and the second retailer are the lowest. Because regarding the spot price and the exercise price, they choose the worst strategy. But after the first scenario, the supplier's profit is highest under scenario 5.

Under scenario 8, the retailers do not hedge. The exercise price is more than the spot price ($w_1^{'} > w_2$) and the retailers' decision is the best. But the profit of the supplier is the lowest.

The results show that sharing the hedging decision among the members of the supply chain and correct prediction of the spot price at the time of exercising the contract are the approaches that maximize the total supply chain profit. Although it is possible that the profit of each member is not maximized. We can see these conditions in the first scenario.

5.2 The effect of parameters changing on the values of objective functions of the first scenario

In preceding section, we are going to present the analysis of sensitivity model for its key parameters.

5.2.1 The standard deviation of the product price (σ)

Figure 6 illustrates how $\pi_{r_1}^1, \pi_{r_2}^1, \pi_s^1$, and π_{TSC}^1 change for various values of σ. We consider $\sigma \in \{0.3, 0.5, 0.7\}$. When we increase $\sigma = 0.3$ to $\sigma = 0.7$, this means that the price changes are much. In this situation, the profits of retailer 1, retailer 2, the supplier, and the supply chain raise. When we increase the standard deviation of the price, the rate of increase supplier's profit is much higher than the rate of increase retailers' profits. In this condition, the result shows that the hedging of both retailers is still the best strategy to have maximum supply chain performance.

[Figure 6]

5.2.2 The mean of the product price (μ)

Figure 7 illustrates how $\pi_{r_1}^1, \pi_{r_2}^1, \pi_s^1$, and π_{TSC}^1 change for various values of μ. We consider $\mu \in \{4, 5, 6\}$. By increasing μ, the profits of retailer 1, retailer 2, the supplier, and the supply chain raise. But in comparison with the standard deviation, the rates of profits rising are lower.

[Figure 7]

5.2.3 Slop of the retailers' demand curve (b)

Figure 8 illustrates how $\pi_{r_1}^1, \pi_{r_2}^1, \pi_s^1$, and π_{TSC}^1 change for various values of b. We consider $b \in \{4,5,6\}$. Because d_1^1 and d_2^1 are correlated with the slope of demand curve, negatively. With increasing b, the number of end customers declines. When slop of the retailers' demand curve increases from 4 to 6, the values of expected profit functions of the retailers, the supplier, and supply chain decrease. But the total supply chain profit is higher than the profits of retailer 1, retailer 2, and the supplier.

[Figure 8]

5.2.4 The exercise price (w_1)

Figure 9 illustrates how $\pi_{r_1}^1, \pi_{r_2}^1, \pi_s^1$, and π_{TSC}^1 change for various values of w_1. We consider $w_1 \in \{5,10,15\}$. When the exercise price increases, the profits of the retailers rise. But the supplier profit and the supply chain profit decrease. However, the supply chain profit is more than the profit of each member of chain.

[Figure 9]

6. Conclusions

The present study has been designed to determine the effect of hedging in the two echelon supply chain. It is important to realize that hedging can relatively decrease or increase the individual company's profit. However, based on authors' information, there has been no study accomplished to address what will happen for the profit of total supply chain. We have adopted a quantitative approach to answer the aforementioned question. We found out that hedging may not guarantee to lead the profit for each individual company, but it leads the benefit the total supply chain. In another word, sharing the hedging decision among the members of the supply chain and precise prediction of the spot price at the time of exercising the contract are the approaches that maximize the total supply chain profit. We are looking for total optimization, rather than the local optimization. Obviously, by negotiation members can share the benefits and satisfy the one which got less advantage to compare with others. The results provide some useful managerial insights on the implementation of these strategies:

1. Retailers' ordering strategy affects good's price. Each retailer that can sell the product cheaper to the end customers will have more customers (See the Figure 1, Figure 2, and Figure 3). As a result, the retailers' ordering strategy will have impacts on the number of end customers and the retailers' profit or loss.
2. The partner's behavior affects the other retailer's profit/loss directly. We are looking at the big picture of hedging which views all members in the supply chain rather than local optimization. The decision to hedge without considering the other members' decision does not necessarily mean choosing the best strategy. As a result, companies are affected by their proceedings and their partners' operational and financial actions.
3. The supply chain coordination could be achieved with hedging strategy. The retailers send their orders to the supplier with flexibility because the call option contracts are concluded before the start of the period. Besides, since ordering process occurs before the start of the period, the supplier can plan better.
4. For scenario 1, the total supply chain performance is the highest, but the supply chain members' profit is not maximum. In such a situation, the supply chain members can negotiate with each other, and the member who earns the most profit gives a part of its profit to the other members. So that other members are motivated to use the suitable strategy.
5. It should be considered that increasing the number of supply chain members and their decision to hedging will complicate the analysis of the supply chain performance.

Further research regarding the role of hedging in the supply chain would be interesting. In future studies, we can consider a supply chain with several suppliers that they compete with each other, then see the effect of hedging on each member and the supply chain performance. In this study, we do not use price forecast methods. Also, we plan to combine price forecasting methods with the decision to use hedging strategies. Additionally, we suggest that short hedge can be examined for the supplier in the supply chain so that one will be concerned with the selling price of his/her product. The impacts of this on the supplier and the supply chain can be investigated. Also, hedging with option contracts can also compared to other contracts will be beneficial too. Moreover, these calculations can be performed with three-echelon supply chains and in the future it is recommended to check out different supply chain configurations.

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Figure and Table captions:

Figure 1. Flow of product in supply chain
Figure 2. Time line for contracting
Figure 3. The proportion of end customers of the second retailer who buy from the first retailer
Figure 4. The proportion of end customers of the first retailer who buy from the second retailer
Figure 5. Profit function values for retailer 1, retailer 2, supplier and total supply chain under various scenarios
Figure 6. Profit function values for retailer 1, retailer 2, supplier and total supply chain for first scenario when σ changes
Figure 7. Profit function values for retailer 1, retailer 2, supplier and total supply chain for first scenario when μ changes
Figure 8. Profit function values for retailer 1, retailer 2, supplier and total supply chain for first scenario when b changes
Figure 9. Profit function values for retailer 1, retailer 2, supplier and total supply chain for first scenario when w_1 changes
Table 1. Description of scenarios
Table 2. The value of objective functions for each scenario

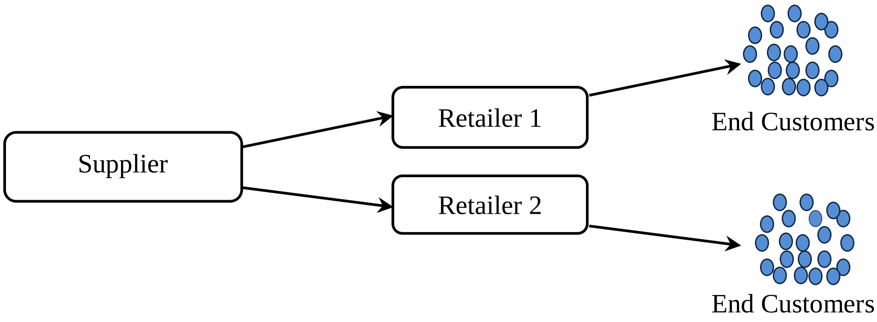

Figure 1

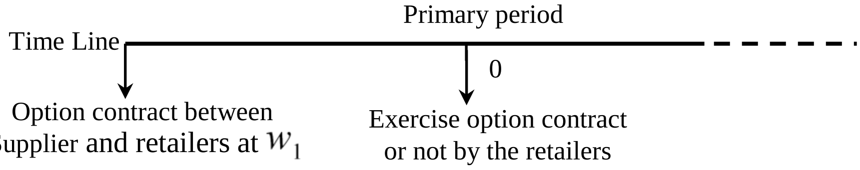

Figure 2

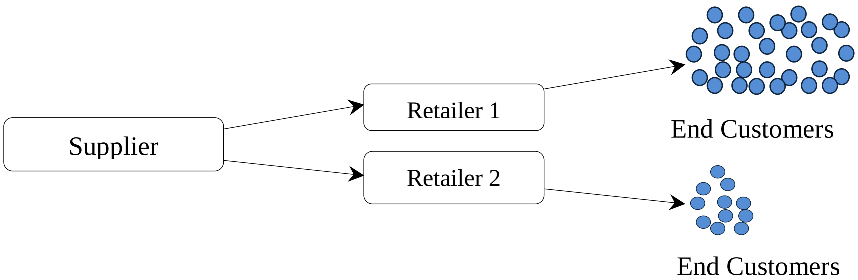

Figure 3

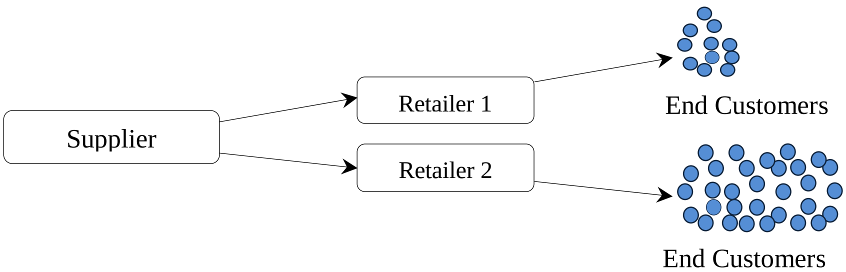

Figure 4

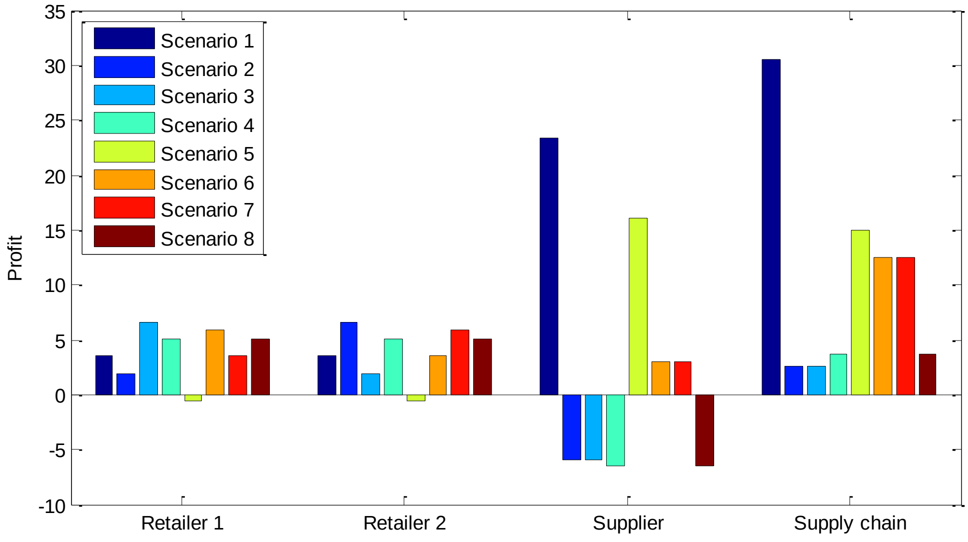

Figure 5

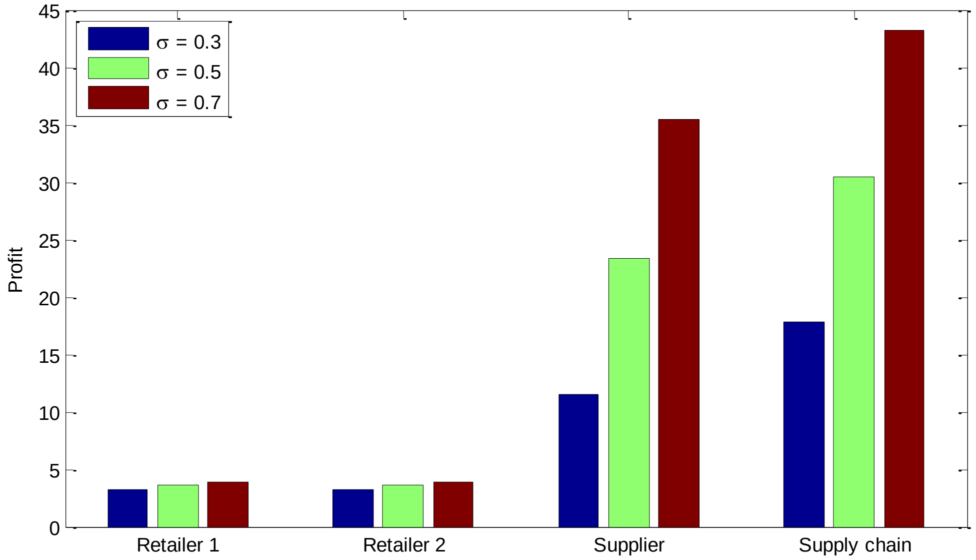

Figure 6

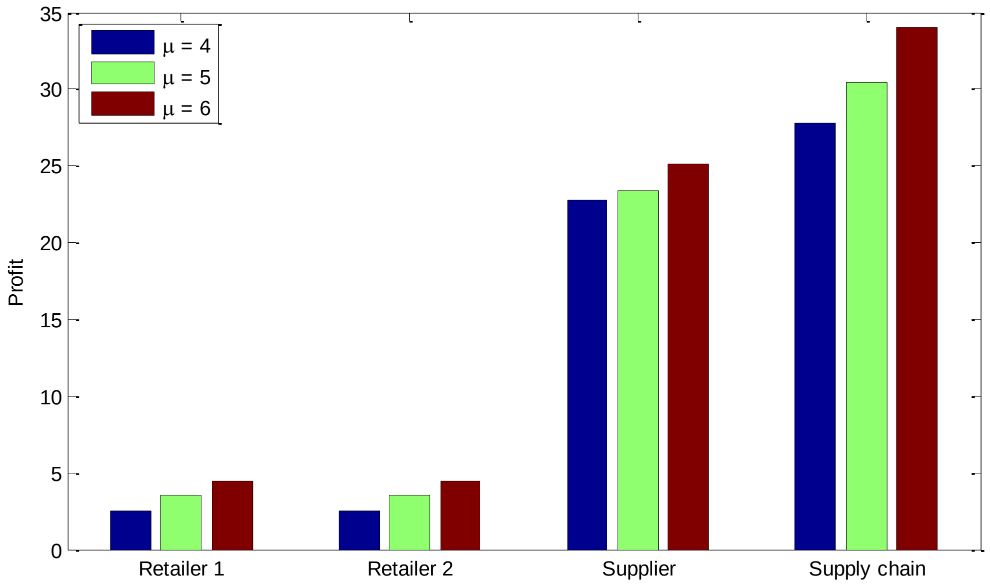

Figure 7

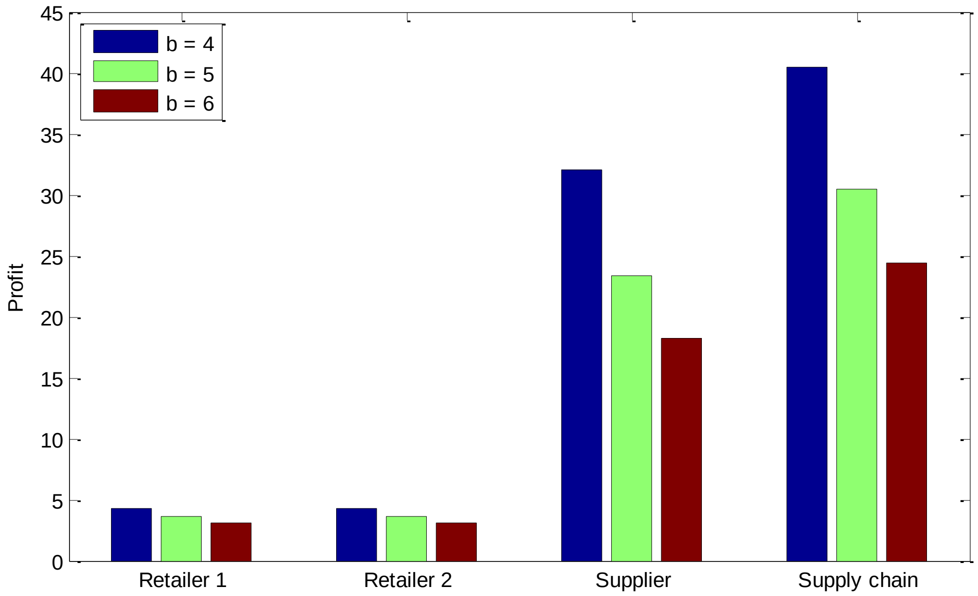

Figure 8

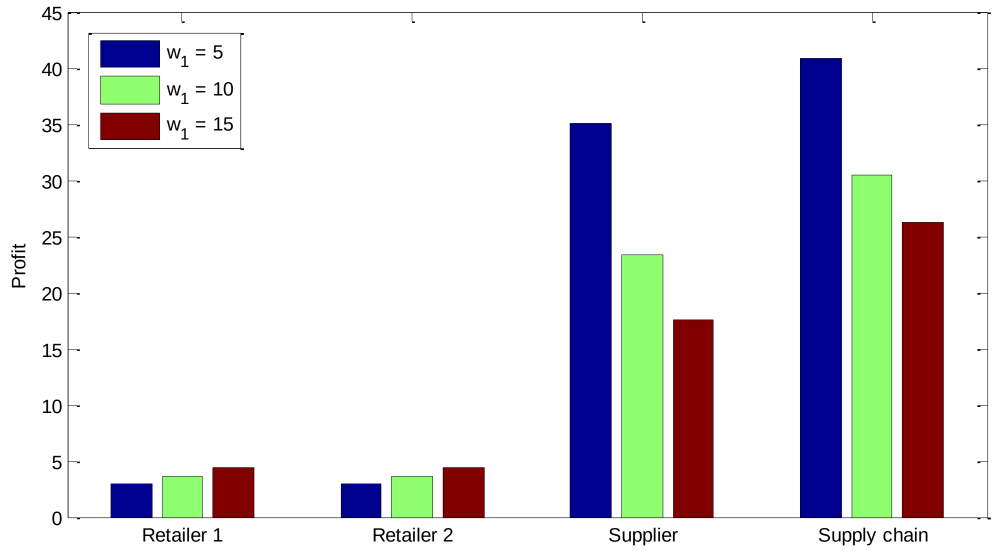

Figure 9

Table 1.

scenario	Description	
1	$w_1 < w_2$	Both retailers hedge.
2		Retailer 1 hedges and Retailer 2 does not hedge.
3		Retailer 1 does not hedge and Retailer 2 hedges.
4		None of the retailers hedge.
5	$w_1^{'} > w_2$	Both retailers hedge.
6		Retailer 1 hedges and Retailer 2 does not hedge.
7		Retailer 1 does not hedge and Retailer 2 hedges.
8		None of the retailers hedge.

Table 2.

Scenario	1	2	3	4	5	6	7	8
Retailer 1	3.5625	1.9147	6.5956	5.1156	-0.5792	5.9207	3.5841	5.1156
Retailer 2	3.5625	6.5956	1.9147	5.1156	-0.5792	3.5841	5.9207	5.1156
Supplier	23.3594	-5.9041	-5.9041	-6.5433	16.0999	3.0455	3.0455	-6.5433
Total Supply chain	30.4843	2.6062	2.6062	3.6880	14.9414	12.5503	12.5503	3.6880

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Appendix 1

Regarding formula (6), formula (7) and taking first derivative with respect to y_{1i} and y_{2i} , we have:

$$\frac{\partial S\left(y_{1i}\right)}{\partial y_{1i}}=1-F\left(y_{1i}\right) \tag{A.1}$$

$$\frac{\partial S\left(y_{2i}\right)}{\partial y_{2i}}=1-F\left(y_{2i}\right) \tag{A.2}$$

Regarding formula (A.1), formula (A.2) and taking the second derivative with respect to y_{1i} and y_{2i} , we have:

$$\frac{\partial^2 S\left(y_{1i}\right)}{\partial\left(y_{1i}\right)^2}=-f\left(y_{1i}\right) \tag{A.3}$$

$$\frac{\partial^2 S\left(y_{2i}\right)}{\partial\left(y_{2i}\right)^2}=-f\left(y_{2i}\right) \tag{A.4}$$

Regarding formula (11), formula (A.1) and taking first derivative with respect to y_{11} , we have:

$$\frac{\partial \pi_{r_1}^1}{\partial y_{11}}=p_1\frac{\partial S\left(y_{11}\right)}{\partial y_{11}}-c_0-w_1=p_1\left(1-F\left(y_{11}\right)\right)-c_0-w_1 \tag{A.5}$$

Regarding formula (A.5), formula (A.3) and taking second derivative with respect to y_{11} , we have:

$$\frac{\partial^2 \pi_{r_1}^1}{\partial y_{11}^2}=-p_1 f\left(y_{11}\right)<0 \tag{A.6}$$

$\pi_{r_1}^{1}$ is concave in y_{11}. The first retailer's optimal order quantity is given by:

$$y_{11} = \frac{\partial \pi_{r_1}^{1}}{\partial y_{11}} = 0 \tag{A.7}$$

$$y_{11} = F^{-1}\left(1 - \frac{c_0 + w_1}{p_1}\right) \tag{A.8}$$

We consider to $x \sim N\left(\mu, \sigma^2\right)$; we will have:

$$F^{-1}(x) = \mu + \left(\sqrt{2}\sigma\right) erf^{-1}(2x - 1) \tag{A.9}$$

$$erf^{-1}(x) = \sqrt{\pi}\left(\frac{1}{2}x + \frac{1}{24}\pi x^3 + \frac{7}{960}\pi^2 x^5 + \dots\right) \tag{A.10}$$

$$erf^{-1}(x) \sim \frac{\sqrt{\pi}}{2}x \tag{A.11}$$

$$erf^{-1}(2x - 1) = \frac{\sqrt{\pi}}{2}(2x - 1) \tag{A.12}$$

By substituting formula (A.12) into formula (A.9), we have:

$$F^{-1}(x) = \mu + \frac{\sqrt{2\pi}\sigma}{2}(2x - 1) \tag{A.13}$$

By considering formula (1), formula (A.8), and formula (A.13), we have:

$$\left(y_{11}\right)^{*} = \mu_{d_1^1} + \sqrt{2\pi}\sigma_{d_1^1}\left(\frac{mw_1 - c_0}{(1+m)w_1} - \frac{1}{2}\right) \tag{A.14}$$

Regarding formula (12) and formula (A.2) taking first derivative with respect to y_{21}, we have:

$$\frac{\partial \pi_{r_2}^{1}}{\partial y_{21}} = p_1 \frac{\partial S\left(y_{21}\right)}{\partial y_{21}} - c_0 - w_1 = p_1\left(1 - F\left(y_{21}\right)\right) - c_0 - w_1 \tag{A.15}$$

Regarding formula (A.15), formula (A.4), and taking second derivative with respect to y_{21}, we have:

$$\frac{\partial^2 \pi_{r_2}^{1}}{\partial y_{21}^{2}} = -p_1 f\left(y_{21}\right) < 0 \tag{A.16}$$

$\pi_{r_2}^{1}$ is concave in y_{21}. The second retailer's optimal order quantity is given by:

$$y_{21} = \frac{\partial \pi_{r_2}^{1}}{\partial y_{21}} = 0 \tag{A.17}$$

$$y_{21} = F^{-1}\left(1 - \frac{c_0 + w_1}{p_1}\right) \tag{A.18}$$

By considering formula (1), formula (A.13), and formula (A.18), we have:

$$\left(y_{21}\right)^{*}=\mu_{d_2^1}+\sqrt{2\pi}\sigma_{d_2^1}\left(\frac{c_0+w_1}{(1+m)w_1}-\frac{1}{2}\right) \tag{A.19}$$

By considering $x_1 \sim N\left(\mu_1,\sigma_1^2\right)$ and $x_2 \sim N\left(\mu_2,\sigma_2^2\right)$, we have:

$$F(x)=F\left(x_1\right)+F\left(x_2\right) \tag{A.20}$$

$$x \sim N\left(\mu_1+\mu_2,\sigma_1^2+\sigma_2^2\right) \tag{A.21}$$

Regarding formula (4), formula (A.20), and formula (A.21), we have:

$$\frac{\partial S\left(q_i\right)}{\partial q_i}=1-F\left(q_i\right) \tag{A.22}$$

Regarding formula (A.22) and taking the second derivative with respect to q_i , we have:

$$\frac{\partial^2 S\left(q_i\right)}{\partial\left(q_i\right)^2}=-f\left(q_i\right) \tag{A.23}$$

Regarding formula (13), formula (A.22), and taking first derivative with respect to q_1, we have:

$$\frac{\partial\pi_s^1}{\partial q_1}=(v-c)+(e-v)\frac{\partial S\left(q_1\right)}{\partial q_1}=(v-c)+(e-v)\left(1-F\left(q_1\right)\right) \tag{A.24}$$

Regarding formula (A.24), formula (A.23), and taking second derivative with respect to q_1, we have:

$$\frac{\partial^2\pi_s^1}{\partial q_1^2}=-(e-v)f\left(q_1\right)<0 \tag{A.25}$$

π_s^1 is concave in q_1. The supplier’s optimal capacity is given by:

$$q_1=\frac{\partial\pi_s^1}{\partial q_1}=0 \tag{A.26}$$

$$q_1=F^{-1}\left(1-\frac{v-c}{e-v}\right) \tag{A.27}$$

By considering formula (A.13) and formula (A.18), we have:

$$\left(q_1\right)^{*}=\mu_{d_1^1+d_2^1}+\sqrt{2\pi}\sigma_{d_1^1+d_2^1}\left(\frac{1}{2}-\frac{v-c}{e-v}\right) \tag{A.28}$$

Appendix 2

Regarding formula (19), formula (A.1), and taking first derivative with respect to y_{12}, we have:

$$\frac{\partial\pi_{r_1}^2}{\partial y_{12}}=p_1\left(1-F\left(y_{12}\right)\right)-c_0-w_1 \tag{B.1}$$

Regarding formula (B.1), formula (A.3), and taking second derivative with respect to y_{12}, we have:

$$\frac{\partial^2 \pi_{r_1}^2}{\partial y_{12}^2} = -p_1 f\left(y_{12}\right) < 0 \tag{B.2}$$

$\pi_{r_1}^2$ is concave in y_{12}. The first retailer's optimal order quantity is given by:

$$y_{12} = \frac{\partial \pi_{r_1}^2}{\partial y_{11}} = 0 \tag{B.3}$$

$$y_{12} = F^{-1}\left(1 - \frac{\left(c_0 + w_1\right)}{p_1}\right) \tag{B.4}$$

By considering formula (1), formula (A.13), and formula (B.4), we have:

$$\left(y_{12}\right)^* = \mu_{d_1^2} + \sqrt{2\pi}\sigma_{d_1^2}\left(\frac{mw_1 - c_0}{(1+m)w_1} - \frac{1}{2}\right) \tag{B.5}$$

Regarding formula (20), formula (A.2), and taking first derivative with respect to y_{22}, we obtain formula (B.6).

$$\frac{\partial \pi_{r_2}^2}{\partial y_{22}} = p_2\left(1 - F\left(y_{22}\right)\right) - w_2 \tag{B.6}$$

Regarding formula (B.6), formula (A.4), and taking second derivative with respect to y_{22}, we have:

$$\frac{\partial^2 \pi_{r_2}^2}{\partial y_{22}^2} = -p_2 f\left(y_{22}\right) < 0 \tag{B.7}$$

$\pi_{r_2}^2$ is concave in y_{22}. The second retailer's optimal order quantity is given by:

$$y_{22} = \frac{\partial \pi_{r_2}^2}{\partial y_{22}} = 0 \tag{B.8}$$

$$y_{22} = F^{-1}\left(1 - \frac{w_2}{p_2}\right) \tag{B.9}$$

By considering formula (A.13) and formula (B.9), we have:

$$\left(y_{22}\right)^* = \mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2}\left(\frac{m}{1+m} - \frac{1}{2}\right) \tag{B.10}$$

Regarding formula (21), formula (A.22), and taking first derivative with respect to q_2, we obtain formula (B.11).

$$\frac{\partial \pi_s^2}{\partial q_2} = \left(v - c\right) + \left(e - v\right)\frac{\partial S\left(q_2\right)}{\partial q_2} = \left(v - c\right) + \left(e - v\right)\left(1 - F\left(q_2\right)\right) \tag{B.11}$$

Regarding formula (B.11), formula (A.23), and taking second derivative with respect to q_2, we have:

$$\frac{\partial^2 \pi_s^2}{\partial q_2^2} = -\left(e - v\right) f\left(q_2\right) < 0 \tag{B.12}$$

π_s^2 is concave in q_2. The supplier's optimal capacity is given by:

$$q_2 = \frac{\partial \pi_s^2}{\partial q_2} = 0 \tag{B.13}$$

$$q_2 = F^{-1}\left(1 - \frac{v-c}{e-v}\right) \tag{B.14}$$

By considering formula (A.13) and formula (B.14), we have:

$$\left(q_2\right)^* = \mu_{d_1^2+d_2^2} + \sqrt{2\pi}\sigma_{d_1^2+d_2^2}\left(\frac{1}{2} - \frac{v-c}{e-v}\right) \tag{B.15}$$

Appendix 3

Regarding formula (27), formula (A.1) and taking first derivative with respect to y_{13}, we have:

$$\frac{\partial \pi_{r_1}^3}{\partial y_{13}} = p_2\left(1 - F\left(y_{13}\right)\right) - w_2 \tag{C.1}$$

Regarding formula (C.1), formula (A.3) and taking second derivative with respect to y_{13}, we have:

$$\frac{\partial^2 \pi_{r_1}^3}{\partial y_{13}^2} = -p_2 f\left(y_{13}\right) < 0 \tag{C.2}$$

$\pi_{r_1}^3$ is concave in y_{13}. The first retailer’s optimal order quantity is given by:

$$y_{13} = \frac{\partial \pi_{r_1}^3}{\partial y_{13}} = 0 \tag{C.3}$$

$$y_{13} = F^{-1}\left(\frac{p_2 - w_2}{p_2}\right) \tag{C.4}$$

By considering formula (3), formula (A.13) and formula (C.4), we have:

$$\left(y_{13}\right)^* = \mu_{d_1^3} + \sqrt{2\pi}\sigma_{d_1^3}\left(\frac{m}{1+m} - \frac{1}{2}\right) \tag{C.5}$$

Regarding formula (28), formula (A.2) and taking first derivative with respect to y_{23}, we have:

$$\frac{\partial \pi_{r_2}^3}{\partial y_{23}} = p_1\left(1 - F\left(y_{23}\right)\right) - c_0 - w_1 \tag{C.6}$$

Regarding formula (C.6), formula (A.4) and taking second derivative with respect to y_{23}, we have:

$$\frac{\partial^2 \pi_{r_2}^3}{\partial y_{23}^2} = -p_1 f\left(y_{23}\right) < 0 \tag{C.7}$$

$\pi_{r_2}^3$ is concave in y_{23}. The second retailer’s optimal order quantity is given by:

$$y_{23} = \frac{\partial \pi_{r_2}^3}{\partial y_{23}} = 0 \tag{C.8}$$

$$y_{23} = F^{-1}\left(1 - \frac{c_0 + w_1}{p_1}\right) \tag{C.9}$$

By considering formula (1), formula (A.13) and formula (C.9), we have:

$$\left(y_{23}\right)^* = \mu_{d_2^3} + \sqrt{2\pi}\sigma_{d_2^3}\left(\frac{mw_1 - c_0}{(1+m)w_1} - \frac{1}{2}\right) \tag{C.10}$$

Regarding formula (29), formula (A.22) and taking first derivative with respect to q_3, we have:

$$\frac{\partial \pi_s^3}{\partial q_3} = (\nu - c) + (e - \nu)\frac{\partial S(q_3)}{\partial q_3} = (\nu - c) + (e - \nu)\left(1 - F(q_3)\right) \tag{C.11}$$

Regarding formula (C.11), formula (A.23) and taking second derivative with respect to q_3, we have:

$$\frac{\partial^2 \pi_s^3}{\partial q_3^2} = -(e - \nu) f\left(q_3\right) < 0 \tag{C.12}$$

π_s^3 is concave in q_3. The supplier's optimal capacity is given by:

$$q_3 = \frac{\partial \pi_s^3}{\partial q_3} = 0 \tag{C.13}$$

$$q_3 = F^{-1}\left(1 - \frac{\nu - c}{e - \nu}\right) \tag{C.14}$$

By considering formula (A.13) and formula (C.14), we have:

$$\left(q_3\right)^* = \mu_{d_1^3 + d_2^3} + \sqrt{2\pi}\sigma_{d_1^3 + d_2^3}\left(\frac{1}{2} - \frac{\nu - c}{e - \nu}\right) \tag{C.15}$$

Appendix 4

Regarding formula (35), formula (A.1) and taking first derivative with respect to y_{14}, we have:

$$\frac{\partial \pi_{r_1}^4}{\partial y_{14}} = p_2\left(1 - F\left(y_{14}\right)\right) - w_2 \tag{D.1}$$

Regarding formula (D.1), formula (A.3) and taking second derivative with respect to y_{14}, we have:

$$\frac{\partial^2 \pi_{r_1}^4}{\partial y_{14}^2} = -p_2 f\left(y_{14}\right) < 0 \tag{D.2}$$

$\pi_{r_1}^4$ is concave in y_{14}. The first retailer's optimal order quantity is given by:

$$y_{14} = \frac{\partial \pi_{r_1}^4}{\partial y_{14}} = 0 \tag{D.3}$$

$$y_{14} = F^{-1}\left(1 - \frac{w_2}{p_2}\right) \tag{D.4}$$

By considering formula (3), formula (A.13) and formula (D.4), we have:

$$\left(y_{14}\right)^{*}=\mu_{d_1^4}+\sqrt{2\pi}\sigma_{d_1^4}\left(\frac{m}{1+m}-\frac{1}{2}\right) \tag{D.5}$$

Regarding formula (36), formula (A.2) and taking first derivative with respect to y_{24}, we have:

$$\frac{\partial \pi_{r_2}^4}{\partial y_{24}}=p_2\left(1-F\left(y_{24}\right)\right)-w_2 \tag{D.6}$$

Regarding formula (D.6), formula (A.4) and taking second derivative with respect to y_{24}, we have:

$$\frac{\partial^2 \pi_{r_2}^4}{\partial y_{24}^2}=-p_2 f\left(y_{24}\right)<0 \tag{D.7}$$

$\pi_{r_2}^4$ is concave in y_{24}. The second retailer's optimal order quantity is given by:

$$y_{24}=\frac{\partial \pi_{r_2}^4}{\partial y_{24}}=0 \tag{D.8}$$

$$y_{24}=F^{-1}\left(1-\frac{w_2}{p_2}\right) \tag{D.9}$$

By considering formula (3), formula (A.13) and formula (D.9), we have:

$$\left(y_{24}\right)^{*}=\mu_{d_2^4}+\sqrt{2\pi}\sigma_{d_2^4}\left(\frac{m}{1+m}-\frac{1}{2}\right) \tag{D.10}$$

Regarding formula (37), formula (A.22) and taking first derivative with respect to q_4, we have:

$$\frac{\partial \pi_s^4}{\partial q_4}=\left(\nu-c\right)+\left(e-\nu\right)\frac{\partial S\left(q_4\right)}{\partial q_4}=\left(\nu-c\right)+\left(e-\nu\right)\left(1-F\left(q_4\right)\right) \tag{D.11}$$

Regarding formula (D.11), formula (A.23) and taking second derivative with respect to q_2, we have:

$$\frac{\partial^2 \pi_s^4}{\partial q_4^2}=-\left(e-\nu\right)f\left(q_4\right)<0 \tag{D.12}$$

π_s^4 is concave in q_4. The supplier's optimal capacity is given by:

$$q_4=\frac{\partial \pi_s^4}{\partial q_4}=0 \tag{D.13}$$

$$q_4=F^{-1}\left(1-\frac{\nu-c}{e-\nu}\right) \tag{D.14}$$

By considering formula (A.13) and formula (D.14), we have:

$$\left(q_4\right)^{*}=\mu_{d_1^4+d_2^4}+\sqrt{2\pi}\sigma_{d_1^4+d_2^4}\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right) \tag{D.15}$$

Appendix 5

Regarding formula (43), formula (A.1) and taking first derivative with respect to y_{15}, we have:

$$\frac{\partial \pi_{r_1}^5}{\partial y_{15}} = p_1^{'}\left(1 - F\left(y_{15}\right)\right) - c_0 - w_1^{'} \tag{E.1}$$

Regarding formula (E.1), formula (A.3) and taking second derivative with respect to y_{15}, we have:

$$\frac{\partial^2 \pi_{r_1}^5}{\partial y_{15}^2} = -p_1^{'} f\left(y_{15}\right) < 0 \tag{E.2}$$

$\pi_{r_1}^5$ is concave in y_{15}. The first retailer's optimal order quantity is given by:

$$y_{15} = \frac{\partial \pi_{r_1}^5}{\partial y_{15}} = 0 \tag{E.3}$$

$$y_{15} = F^{-1}\left(1 - \frac{c_o + w_1^{'}}{p_1^{'}}\right) \tag{E.4}$$

By considering formula (2), formula (A.13) and formula (E.4), we have:

$$\left(y_{15}\right)^* = \mu_{d_1^5} + \sqrt{2\pi}\sigma_{d_1^5}\left(\frac{mw_1^{'} - c_0}{\left(1+m\right)w_1^{'}} - \frac{1}{2}\right) \tag{E.5}$$

Regarding formula (44), formula (A.2) and taking first derivative with respect to y_{25}, we have:

$$\frac{\partial \pi_{r_2}^5}{\partial y_{25}} = p_1^{'}\left(1 - F\left(y_{25}\right)\right) - c_0 - w_1^{'} \tag{E.6}$$

Regarding formula (E.6), formula (A.4) and taking second derivative with respect to y_{25}, we have:

$$\frac{\partial^2 \pi_{r_2}^5}{\partial y_{25}^2} = -p_1^{'} f\left(y_{25}\right) < 0 \tag{E.7}$$

$\pi_{r_2}^5$ is concave in y_{25}. The second retailer's optimal order quantity is given by:

$$y_{25} = \frac{\partial \pi_{r_2}^5}{\partial y_{25}} = 0 \tag{E.8}$$

$$y_{25} = F^{-1}\left(1 - \frac{c_o + w_1^{'}}{p_1^{'}}\right) \tag{E.9}$$

By considering formula (2), formula (A.13) and formula (E.9), we have:

$$\left(y_{25}\right)^* = \mu_{d_2^5} + \sqrt{2\pi}\sigma_{d_2^5}\left(\frac{mw_1^{'} - c_0}{\left(1+m\right)w_1^{'}} - \frac{1}{2}\right) \tag{E.10}$$

Regarding formula (37), formula (A.22) and taking first derivative with respect to q_5, we have:

$$\frac{\partial \pi_s^5}{\partial q_5} = (\nu - c) + (e - \nu)\frac{\partial S(q_5)}{\partial q_5} = (\nu - c) + (e - \nu)(1 - F(q_5)) \quad \text{(E.11)}$$

Regarding formula (E.11), formula (A.23) and taking second derivative with respect to q_5, we have:

$$\frac{\partial^2 \pi_s^5}{\partial q_5^2} = -(e - \nu) f(q_5) < 0 \quad \text{(E.12)}$$

π_s^5 is concave in q_5. The supplier's optimal capacity is given by:

$$q_5 = \frac{\partial \pi_s^5}{\partial q_5} = 0 \quad \text{(E.13)}$$

$$q_5 = F^{-1}\left(1 - \frac{\nu - c}{e - \nu}\right) \quad \text{(E.14)}$$

By considering formula (A.13) and formula (E.14), we have:

$$(q_5)^* = \mu_{d_1^5 + d_2^5} + \sqrt{2\pi}\sigma_{d_1^5 + d_2^5}\left(\frac{1}{2} - \frac{\nu - c}{e - \nu}\right) \quad \text{(D.15)}$$

Appendix 6

Regarding formula (51), formula (A.1) and taking first derivative with respect to y_{16}, we have:

$$\frac{\partial \pi_{r_1}^6}{\partial y_{16}} = p_1^{'}(1 - F(y_{16})) - c_0 - w_1^{'} \quad \text{(F.1)}$$

Regarding formula (F.1), formula (A.3) and taking second derivative with respect to y_{16}, we have:

$$\frac{\partial^2 \pi_{r_1}^6}{\partial y_{16}^2} = -p_1^{'} f(y_{16}) < 0 \quad \text{(F.2)}$$

$\pi_{r_1}^6$ is concave in y_{16}. The first retailer's optimal order quantity is given by:

$$y_{16} = \frac{\partial \pi_{r_1}^6}{\partial y_{16}} = 0 \quad \text{(F.3)}$$

$$y_{16} = F^{-1}\left(1 - \frac{(c_o + w_1^{'})}{p_1^{'}}\right) \quad \text{(F.4)}$$

By considering formula (2), formula (A.13) and formula (F.4), we have:

$$(y_{16})^* = \mu_{d_1^6} + \sqrt{2\pi}\sigma_{d_1^6}\left(\frac{mw_1^{'} - c_0}{(1+m)w_1^{'}} - \frac{1}{2}\right) \quad \text{(F.5)}$$

Regarding formula (52), formula (A.2) and taking first derivative with respect to y_{26}, we have:

$$\frac{\partial \pi_{r_2}^6}{\partial y_{26}} = p_2(1 - F(y_{26})) - w_2 \quad \text{(F.6)}$$

Regarding formula (F.6), formula (A.4) and taking second derivative with respect to y_{26}, we have:

$$\frac{\partial^2 \pi_{r_2}^6}{\partial y_{26}^2} = -p_2 f\left(y_{26}\right) < 0 \tag{F.7}$$

$\pi_{r_2}^6$ is concave in y_{26}. The second retailer's optimal order quantity is given by:

$$y_{26} = \frac{\partial \pi_{r_2}^6}{\partial y_{26}} = 0 \tag{F.8}$$

$$y_{26} = F^{-1}\left(1 - \frac{w_2}{p_2}\right) \tag{F.9}$$

By considering formula (3), formula (A.13) and formula (F.9), we have:

$$\left(y_{26}\right)^* = \mu_{d_2^6} + \sqrt{2\pi}\sigma_{d_2^6}\left(\frac{m}{1+m} - \frac{1}{2}\right) \tag{F.10}$$

Regarding formula (53), formula (A.22) and taking first derivative with respect to q_6, we have:

$$\frac{\partial \pi_s^6}{\partial q_6} = (\nu - c) + (e - \nu)\frac{\partial S\left(q_6\right)}{\partial q_6} = (\nu - c) + (e - \nu)\left(1 - F\left(q_6\right)\right) \tag{F.11}$$

Regarding formula (F.11), formula (A.23) and taking second derivative with respect to q_6, we have:

$$\frac{\partial^2 \pi_s^6}{\partial q_6^2} = -(e - \nu) f\left(q_6\right) < 0 \tag{F.12}$$

π_s^6 is concave in q_6. The supplier's optimal capacity is given by:

$$q_6 = \frac{\partial \pi_s^6}{\partial q_6} = 0 \tag{F.13}$$

$$q_6 = F^{-1}\left(1 - \frac{\nu - c}{e - \nu}\right) \tag{F.14}$$

By considering formula (A.13) and formula (F.14), we have:

$$\left(q_6\right)^* = \mu_{d_1^6 + d_2^6} + \sqrt{2\pi}\sigma_{d_1^6 + d_2^6}\left(\frac{1}{2} - \frac{\nu - c}{e - \nu}\right) \tag{F.15}$$

Appendix 7

Regarding formula (59), formula (A.1) and taking first derivative with respect to y_{17}, we have:

$$\frac{\partial \pi_{r_1}^7}{\partial y_{17}} = p_2\left(1 - F\left(y_{17}\right)\right) - w_2 \tag{G.1}$$

Regarding formula (G.1), formula (A.3) and taking second derivative with respect to y_{17}, we have:

$$\frac{\partial^2 \pi_{r_1}^7}{\partial y_{17}^2} = -p_2 f\left(y_{17}\right) < 0 \quad \text{(G.2)}$$

$\pi_{r_1}^7$ is concave in y_{17}. The first retailer's optimal order quantity is given by:

$$y_{17} = \frac{\partial \pi_{r_1}^7}{\partial y_{17}} = 0 \quad \text{(G.3)}$$

$$y_{17} = F^{-1}\left(1 - \frac{w_2}{p_2}\right) \quad \text{(G.4)}$$

By considering formula (3), formula (A.13) and formula (G.4), we have:

$$\left(y_{17}\right)^* = \mu_{d_1^7} + \sqrt{2\pi}\sigma_{d_1^7}\left(\frac{m}{1+m} - \frac{1}{2}\right) \quad \text{(G.5)}$$

Regarding formula (60), formula (A.2) and taking first derivative with respect to y_{27}, we have:

$$\frac{\partial \pi_{r_2}^7}{\partial y_{27}} = p_1^{'}\left(1 - F\left(y_{27}\right)\right) - c_o - w_1^{'} \quad \text{(G.6)}$$

Regarding formula (G.6), formula (A.4) and taking second derivative with respect to y_{27}, we have:

$$\frac{\partial^2 \pi_{r_2}^7}{\partial y_{27}^2} = -p_1^{'} f\left(y_{27}\right) < 0 \quad \text{(G.7)}$$

$\pi_{r_2}^7$ is concave in y_{27}. The second retailer's optimal order quantity is given by:

$$y_{27} = \frac{\partial \pi_{r_2}^7}{\partial y_{27}} = 0 \quad \text{(G.8)}$$

$$y_{27} = F^{-1}\left(1 - \frac{c_o + w_1^{'}}{p_1^{'}}\right) \quad \text{(G.9)}$$

By considering formula (2), formula (A.13) and formula (G.9), we have:

$$\left(y_{27}\right)^* = \mu_{d_2^7} + \sqrt{2\pi}\sigma_{d_2^7}\left(\frac{mw_1^{'} - c_0}{\left(1+m\right)w_1^{'}} - \frac{1}{2}\right) \quad \text{(G.10)}$$

Regarding formula (61), formula (A.22) and taking first derivative with respect to q_7, we have:

$$\frac{\partial \pi_s^7}{\partial q_7} = \left(v - c\right) + \left(e - v\right)\frac{\partial S\left(q_7\right)}{\partial q_7} = \left(v - c\right) + \left(e - v\right)\left(1 - F\left(q_7\right)\right) \quad \text{(G.11)}$$

Regarding formula (G.11), formula (A.23) and taking second derivative with respect to q_7, we have:

$$\frac{\partial^2 \pi_s^7}{\partial q_7^2} = -\left(e - v\right) f\left(q_7\right) < 0 \quad \text{(G.12)}$$

π_s^7 is concave in q_7. The supplier's optimal capacity is given by:

$$q_7 = \frac{\partial \pi_s^7}{\partial q_7} = 0 \quad \text{(G.13)}$$

$$q_7 = F^{-1}\left(1 - \frac{v-c}{e-v}\right) \quad \text{(G.14)}$$

By considering formula (A.13) and formula (G.14), we have:

$$\left(q_7\right)^* = \mu_{d_1^7+d_2^7} + \sqrt{2\pi}\sigma_{d_1^7+d_2^7}\left(\frac{1}{2} - \frac{v-c}{e-v}\right) \quad \text{(G.15)}$$

Appendix 8

Regarding formula (67), formula (A.1) and taking first derivative with respect to y_{18}, we have:

$$\frac{\partial \pi_{r_1}^8}{\partial y_{18}} = p_2\left(1 - F\left(y_{18}\right)\right) - w_2 \quad \text{(H.1)}$$

Regarding formula (H.1), formula (A.3) and taking second derivative with respect to y_{18}, we have:

$$\frac{\partial^2 \pi_{r_1}^8}{\partial y_{18}^2} = -p_2 f\left(y_{18}\right) < 0 \quad \text{(H.2)}$$

$\pi_{r_1}^8$ is concave in y_{18}. The first retailer's optimal order quantity is given by:

$$y_{18} = \frac{\partial \pi_{r_1}^8}{\partial y_{18}} = 0 \quad \text{(H.3)}$$

$$y_{18} = F^{-1}\left(1 - \frac{w_2}{p_2}\right) \quad \text{(H.4)}$$

By considering formula (3), formula (A.13) and formula (H.4), we have:

$$\left(y_{18}\right)^* = \mu_{d_1^8} + \sqrt{2\pi}\sigma_{d_1^8}\left(\frac{m}{1+m} - \frac{1}{2}\right) \quad \text{(H.5)}$$

Regarding formula (68), formula (A.2) and taking first derivative with respect to y_{28}, we have:

$$\frac{\partial \pi_{r_2}^8}{\partial y_{28}} = p_2\left(1 - F\left(y_{28}\right)\right) - w_2 \quad \text{(H.6)}$$

Regarding formula (H.6), formula (A.4) and taking second derivative with respect to y_{28}, we have:

$$\frac{\partial^2 \pi_{r_2}^8}{\partial y_{28}^2} = -p_2 f\left(y_{28}\right) < 0 \quad \text{(H.7)}$$

$\pi_{r_2}^8$ is concave in y_{28}. The second retailer's optimal order quantity is given by:

$$y_{28} = \frac{\partial \pi_{r_2}^8}{\partial y_{28}} = 0 \quad \text{(H.8)}$$

$$y_{28} = F^{-1}\left(1 - \frac{w_2}{p_2}\right) \quad \text{(H.9)}$$

By considering formula (3), formula (A.13) and formula (H.9), we have:

$$\left(y_{28}\right)^* = \mu_{d_2^8} + \sqrt{2\pi}\sigma_{d_2^8}\left(\frac{m}{1+m} - \frac{1}{2}\right) \quad \text{(H.10)}$$

Regarding formula (69), formula (A.22) and taking first derivative with respect to q_8, we have:

$$\frac{\partial \pi_s^8}{\partial q_8} = \left(\nu - c\right) + \left(e - \nu\right)\frac{\partial S\left(q_8\right)}{\partial q_8} = \left(\nu - c\right) + \left(e - \nu\right)\left(1 - F\left(q_8\right)\right) \quad \text{(H.11)}$$

Regarding formula (H.11), formula (A.23) and taking second derivative with respect to q_8, we have:

$$\frac{\partial^2 \pi_s^2}{\partial q_8^2} = -\left(e - \nu\right) f\left(q_8\right) < 0 \quad \text{(H.12)}$$

π_s^8 is concave in q_8. The supplier’s optimal capacity is given by:

$$q_8 = \frac{\partial \pi_s^8}{\partial q_8} = 0 \quad \text{(H.13)}$$

$$q_8 = F^{-1}\left(1 - \frac{\nu - c}{e - \nu}\right) \quad \text{(H.14)}$$

By considering formula (A.13) and formula (H.14), we have:

$$\left(q_8\right)^* = \mu_{d_1^8 + d_2^8} + \sqrt{2\pi}\sigma_{d_1^8 + d_2^8}\left(\frac{1}{2} - \frac{\nu - c}{e - \nu}\right) \quad \text{(H.15)}$$

Appendix 9

The mean of the end customers’ demand will be as formula (I.1), formula (I.2), and formula (I.3).

$$\mu_{d_1^1} = \mu_{d_1^4} = \mu_{d_1^5} = \mu_{d_1^8} = \mu_{d_2^1} = \mu_{d_2^4} = \mu_{d_2^5} = \mu_{d_2^8} = \frac{a - \mu}{b} \quad \text{(I.1)}$$

$$\mu_{d_1^2} = \mu_{d_1^7} = \mu_{d_2^3} = \mu_{d_2^6} = \frac{a + \left(\gamma - 1\right)\mu}{b} \quad \text{(I.2)}$$

$$\mu_{d_2^2} = \mu_{d_1^6} = \mu_{d_1^3} = \mu_{d_2^7} = \frac{a - \left(\gamma + 1\right)\mu}{b} \quad \text{(I.3)}$$

Regarding formula (I.1), formula (I.2) and formula (I.3), we have:

$$\mu_{d_1^1 + d_2^1} = \mu_{d_1^2 + d_2^2} = \mu_{d_1^3 + d_2^3} = \mu_{d_1^4 + d_2^4} = \mu_{d_1^5 + d_2^5} = \mu_{d_1^6 + d_2^6} = \mu_{d_1^7 + d_2^7} = \mu_{d_1^8 + d_2^8} = \frac{2\left(a - \mu\right)}{b} \quad \text{(I.4)}$$

The variance of the end customers’ demand will be as formula (I.5) and formula (I.6).

$$\sigma^2_{d_1^1} = \sigma^2_{d_1^4} = \sigma^2_{d_1^5} = \sigma^2_{d_1^8} = \sigma^2_{d_2^1} = \sigma^2_{d_2^4} = \sigma^2_{d_2^5} = \sigma^2_{d_2^8} = \left(\frac{\sigma}{b}\right)^2 \quad \text{(I.5)}$$

$$\sigma^2_{d_1^2}=\sigma^2_{d_2^2}=\sigma^2_{d_1^3}=\sigma^2_{d_2^3}=\sigma^2_{d_1^6}=\sigma^2_{d_2^6}=\sigma^2_{d_1^7}=\sigma^2_{d_2^7}=\left(\frac{1+\gamma^2}{b^2}\right)\sigma^2 \tag{I.6}$$

We know formula (I.7) and formula (I.8) as follows:

$$var(x+y)=var(x)+var(y)+2cov(x,y) \tag{I.7}$$

$$cov(a+bx,a+cx)=(bc)var(x) \tag{I.8}$$

Regarding formula (I.5), formula (I.6), formula (I.7), formula (I.8), we have:

$$\sigma^2_{d_1^1+d_2^1}=\sigma^2_{d_1^4+d_2^4}=\sigma^2_{d_1^5+d_2^5}=\sigma^2_{d_1^8+d_2^8}=\left(\frac{2\sigma}{b}\right)^2 \tag{I.9}$$

$$\sigma^2_{d_1^2+d_2^2}=\sigma^2_{d_1^3+d_2^3}=\sigma^2_{d_1^6+d_2^6}=\sigma^2_{d_1^7+d_2^7}=\left(\frac{1-\gamma^2}{b^2}\right)\sigma^2 \tag{I.10}$$

We obtain the decision variables as follows:

$$(y_{11})^*=\left[\frac{a-\mu}{b}\right]+\sqrt{2\pi}\,\frac{\sigma}{b}\left(\frac{mw_1-c_0}{(1+m)w_1}-\frac{1}{2}\right) \tag{I.11}$$

$$(y_{21})^*=\left[\frac{a-\mu}{b}\right]+\sqrt{2\pi}\,\frac{\sigma}{b}\left(\frac{mw_1-c_0}{(1+m)w_1}-\frac{1}{2}\right) \tag{I.12}$$

$$(q_1)^*=\frac{2(a-\mu)}{b}+2\sqrt{2\pi}\left[\frac{\sigma}{b}\right]\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right) \tag{I.13}$$

$$(y_{12})^*=\left[\frac{a+(\gamma-1)\mu}{b}\right]+\sqrt{2\pi}\left[\frac{\sqrt{1+\gamma^2}}{b}\right]\left(\frac{mw_1-c_0}{(1+m)w_1}-\frac{1}{2}\right)\sigma \tag{I.14}$$

$$(y_{22})^*=\left[\frac{a-(\gamma+1)\mu}{b}\right]+\sqrt{2\pi}\left[\frac{\sqrt{1+\gamma^2}}{b}\right]\left(\frac{m}{1+m}-\frac{1}{2}\right)\sigma \tag{I.15}$$

$$(q_2)^*=\left[\frac{2(a-\mu)}{b}\right]+\sqrt{2\pi}\left[\frac{\sqrt{1-\gamma^2}}{b}\right]\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)\sigma \tag{I.16}$$

$$(y_{13})^*=\left[\frac{a-(\gamma+1)\mu}{b}\right]+\sqrt{2\pi}\left[\frac{\sqrt{1+\gamma^2}}{b}\right]\left(\frac{m}{1+m}-\frac{1}{2}\right)\sigma \tag{I.17}$$

$$(y_{23})^*=\left[\frac{a+(\gamma-1)\mu}{b}\right]+\sqrt{2\pi}\left[\frac{\sqrt{1+\gamma^2}}{b}\right]\left(\frac{mw_1-c_0}{(1+m)w_1}-\frac{1}{2}\right)\sigma \tag{I.18}$$

$$(q_3)^*=\left[\frac{2(a-\mu)}{b}\right]+\sqrt{2\pi}\left[\frac{\sqrt{1-\gamma^2}}{b}\right]\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)\sigma \tag{I.19}$$

$$\left(y_{14}\right)^{*}=\left[\frac{a-\mu}{b}\right]+\sqrt{2\pi}\left(\frac{\sigma}{b}\right)\left(\frac{m}{1+m}-\frac{1}{2}\right) \tag{I.20}$$

$$\left(y_{24}\right)^{*}=\left[\frac{a-\mu}{b}\right]+\sqrt{2\pi}\left(\frac{\sigma}{b}\right)\left(\frac{m}{1+m}-\frac{1}{2}\right) \tag{I.21}$$

$$\left(q_{4}\right)^{*}=\frac{2\left(a-\mu\right)}{b}+2\sqrt{2\pi}\,\frac{\sigma}{b}\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right) \tag{I.22}$$

$$\left(y_{15}\right)^{*}=\left[\frac{a-\mu}{b}\right]+\sqrt{2\pi}\left(\frac{\sigma}{b}\right)\left(\frac{mw_{1}^{'}-c_{0}}{\left(1+m\right)w_{1}^{'}}-\frac{1}{2}\right) \tag{I.23}$$

$$\left(y_{25}\right)^{*}=\left[\frac{a-\mu}{b}\right]+\sqrt{2\pi}\left(\frac{\sigma}{b}\right)\left(\frac{mw_{1}^{'}-c_{0}}{\left(1+m\right)w_{1}^{'}}-\frac{1}{2}\right) \tag{I.24}$$

$$\left(q_{5}\right)^{*}=\frac{2\left(a-\mu\right)}{b}+2\sqrt{2\pi}\,\frac{\sigma}{b}\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right) \tag{I.25}$$

$$\left(y_{16}\right)^{*}=\left[\frac{a-\left(\gamma+1\right)\mu}{b}\right]+\sqrt{2\pi}\left[\frac{\sqrt{1+\gamma^{2}}}{b}\right]\left(\frac{mw_{1}^{'}-c_{0}}{\left(1+m\right)w_{1}^{'}}-\frac{1}{2}\right)\sigma \tag{I.26}$$

$$\left(y_{26}\right)^{*}=\left[\frac{a+\left(\gamma-1\right)\mu}{b}\right]+\sqrt{2\pi}\left[\frac{\sqrt{1+\gamma^{2}}}{b}\right]\left(\frac{m}{1+m}-\frac{1}{2}\right)\sigma \tag{I.27}$$

$$\left(q_{6}\right)^{*}=\left[\frac{2\left(a-\mu\right)}{b}\right]+\sqrt{2\pi}\left[\frac{\sqrt{1-\gamma^{2}}}{b}\right]\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)\sigma \tag{I.28}$$

$$\left(y_{17}\right)^{*}=\left[\frac{a+\left(\gamma-1\right)\mu}{b}\right]+\sqrt{2\pi}\left[\frac{\sqrt{1+\gamma^{2}}}{b}\right]\left(\frac{m}{1+m}-\frac{1}{2}\right)\sigma \tag{I.29}$$

$$\left(y_{27}\right)^{*}=\left[\frac{a-\left(\gamma+1\right)\mu}{b}\right]+\sqrt{2\pi}\left[\frac{\sqrt{1+\gamma^{2}}}{b}\right]\left(\frac{mw_{1}^{'}-c_{0}}{\left(1+m\right)w_{1}^{'}}-\frac{1}{2}\right)\sigma \tag{I.30}$$

$$\left(q_{7}\right)^{*}=\left[\frac{2\left(a-\mu\right)}{b}\right]+\sqrt{2\pi}\left[\frac{\sqrt{1-\gamma^{2}}}{b}\right]\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)\sigma \tag{I.31}$$

$$\left(y_{18}\right)^{*}=\left[\frac{a-\mu}{b}\right]+\sqrt{2\pi}\left(\frac{\sigma}{b}\right)\left(\frac{m}{1+m}-\frac{1}{2}\right) \tag{I.32}$$

$$\left(y_{28}\right)^* = \left[\frac{a-\mu}{b}\right] + \sqrt{2\pi}\left(\frac{\sigma}{b}\right)\left(\frac{m}{1+m} - \frac{1}{2}\right) \tag{I.33}$$

$$\left(q_8\right)^* = \frac{2\left(a-\mu\right)}{b} + 2\sqrt{2\pi}\left(\frac{\sigma}{b}\right)\left(\frac{1}{2} - \frac{\nu - c}{e-\nu}\right) \tag{I.34}$$

We know formula (I.35) and formula (I.36) as follows:

$$F\left(x\right) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} exp\left(-\frac{u^2}{2}\right)du = \frac{1}{2}\left[1 + erf\left(\frac{x}{\sqrt{2}}\right)\right] \tag{I.35}$$

$$erf\left(x\right) = \frac{2}{\sqrt{\pi}}\left(x - \frac{x^3}{3} + \frac{x^5}{5.2!} - \frac{x^7}{7.3!} + \ldots\right) \tag{I.36}$$

$$erf\left(x\right) \simeq \frac{2x}{\sqrt{\pi}} \tag{I.37}$$

We have the supply chain profit functions as follows:

$$\begin{aligned}
\pi_{TSC}^1 &= \left(mw_1 + \nu - 2c\right)\left[\frac{a-\mu}{b}\right] + \sqrt{2\pi}\frac{\sigma}{b}\left(mw_1 - e\right)\left(\frac{mw_1 - c_0}{\left(1+m\right)w_1} - \frac{1}{2}\right) \\
&- \frac{1}{\sqrt{2\pi}}\left(-2\nu + 2w_1 + mw_1 + e\right)\left[\frac{a-\mu}{b}\right]^2 \\
&- \sqrt{2\pi}\left(\left(2+m\right)w_1 - e\right)\left(\frac{\sigma}{b}\right)^2\left(\frac{mw_1 - c_0}{\left(1+m\right)w_1} - \frac{1}{2}\right)^2 \\
&- 2\left(\left(2+m\right)w_1 - e\right)\left(\frac{\sigma}{b}\right)\left[\frac{a-\mu}{b}\right]\left(\frac{mw_1 - c_0}{\left(1+m\right)w_1} - \frac{1}{2}\right) \\
&+ \left[\sqrt{2\pi}\left(\nu - 2c + e\right) - 4\left(e-\nu\right)\left[\frac{a-\mu}{b}\right]\right]\left[\frac{\sigma}{b}\right]\left(\frac{1}{2} - \frac{\nu - c}{e-\nu}\right) \\
&- 2\sqrt{2\pi}\left(e-\nu\right)\left(\frac{\sigma}{b}\right)^2\left(\frac{1}{2} - \frac{\nu - c}{e-\nu}\right)^2
\end{aligned} \tag{I.38}$$

$$\begin{aligned}
\pi_{TSC}^2 = \pi_{TSC}^3 &= \frac{1}{2}\left(mw_1 - e\right)\left[\frac{a + \left(\gamma - 1\right)\mu}{b}\right] \\
&+ \left[\frac{\sqrt{2\pi}}{2}\left(mw_1 - e\right) - \left(\left(2+m\right)w_1 - e\right)\left[\frac{a + \left(\gamma - 1\right)\mu}{b}\right]\right]\left[\frac{\sqrt{1+\gamma^2}}{b}\right]\left(\frac{mw_1 - c_0}{\left(1+m\right)w_1} - \frac{1}{2}\right)\sigma \\
&- \frac{1}{2\sqrt{2\pi}}\left(\left(2+m\right)w_1 - e\right)\left[\frac{a + \left(\gamma - 1\right)\mu}{b}\right]^2 \\
&- \frac{\sqrt{2\pi}\left(1+\gamma^2\right)}{2b^2}\left(\left(2+m\right)w_1 - e\right)\left(\frac{mw_1 - c_0}{\left(1+m\right)w_1} - \frac{1}{2}\right)^2\sigma^2 \\
&+ \frac{1}{2}\left(\left(1+m\right)w_2 - e\right)\left[\frac{a - \left(\gamma + 1\right)\mu}{b}\right]
\end{aligned} \tag{I.39}$$

$$+\left[\frac{\sqrt{2\pi}}{2}-\left[\frac{a-(\gamma+1)\mu}{b}\right]\right]\left((1+m)w_2-e\right)\left[\frac{\sqrt{1+\gamma^2}}{b}\right]\left(\frac{m}{1+m}-\frac{1}{2}\right)\sigma$$

$$-\frac{1}{2\sqrt{2\pi}}\left((1+m)w_2-e\right)\left[\frac{a-(\gamma+1)\mu}{b}\right]^2$$

$$-\frac{\sqrt{2\pi}\left(1+\gamma^2\right)}{2b^2}\left((1+m)w_2-e\right)\left(\frac{m}{1+m}-\frac{1}{2}\right)^2\sigma^2$$

$$+\left(\nu-2c+e\right)\left[\frac{a-\mu}{b}\right]$$

$$+\left[\frac{\sqrt{2\pi}}{2}(\nu-2c+e)-(e-\nu)\left[\frac{2(a-\mu)}{b}\right]\right]\left[\frac{\sqrt{1-\gamma^2}}{b}\right]\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)\sigma$$

$$-\frac{2}{\sqrt{2\pi}}(e-\nu)\left[\frac{a-\mu}{b}\right]^2-\frac{\sqrt{2\pi}\left(1-\gamma^2\right)}{2b^2}(e-\nu)\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)^2\sigma^2$$

$$\pi_{TSC}^4=\left((1+m)w_2+\nu-2c\right)\left[\frac{a-\mu}{b}\right]$$

$$+\left((1+m)w_2-e\right)\left[\sqrt{2\pi}-2\left[\frac{a-\mu}{b}\right]\right]\left(\frac{\sigma}{b}\right)\left(\frac{m}{1+m}-\frac{1}{2}\right)$$

$$-\frac{1}{\sqrt{2\pi}}\left((1+m)w_2+e-2\nu\right)\left[\frac{a-\mu}{b}\right]^2$$

$$-\sqrt{2\pi}\left((1+m)w_2-e\right)\left(\frac{\sigma}{b}\right)^2\left(\frac{m}{1+m}-\frac{1}{2}\right)^2 \qquad \text{(I.40)}$$

$$+\left[\sqrt{2\pi}(\nu-2c+e)-4(e-\nu)\left[\frac{a-\mu}{b}\right]\right]\frac{\sigma}{b}\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)$$

$$-2\sqrt{2\pi}(e-\nu)\left(\frac{\sigma}{b}\right)^2\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)^2$$

$$\pi_{TSC}^5=\left(mw_1^{'}+\nu-2c\right)\left[\frac{a-\mu}{b}\right]$$

$$+\sqrt{2\pi}\frac{\sigma}{b}\left(mw_1^{'}-e\right)\left(\frac{mw_1^{'}-c_0}{(1+m)w_1^{'}}-\frac{1}{2}\right)$$

$$-\frac{1}{\sqrt{2\pi}}\left(-2\nu+2w_1^{'}+mw_1^{'}+e\right)\left[\frac{a-\mu}{b}\right]^2 \qquad \text{(I.41)}$$

$$-\sqrt{2\pi}\left((2+m)w_1^{'}-e\right)\left(\frac{\sigma}{b}\right)^2\left(\frac{mw_1^{'}-c_0}{(1+m)w_1^{'}}-\frac{1}{2}\right)^2$$

$$
-2\left(\left(2+m\right)w_1^{'}-e\right)\left(\frac{\sigma}{b}\right)\left[\frac{a-\mu}{b}\right]\left(\frac{mw_1^{'}-c_0}{\left(1+m\right)w_1^{'}}-\frac{1}{2}\right)
$$

$$
+\sqrt{2\pi}\left(\nu-2c+e\right)\left[\frac{\sigma}{b}\right]\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)-2\sqrt{2\pi}\left(e-\nu\right)\left(\frac{\sigma}{b}\right)^2\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)^2
$$

$$
-4\left(e-\nu\right)\left[\frac{a-\mu}{b}\right]\left[\frac{\sigma}{b}\right]\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)
$$

$$
\pi_{TSC}^{6}=\pi_{TSC}^{7}=\frac{1}{2}\left(mw_1^{'}-e\right)\left[\frac{a+\left(\gamma-1\right)\mu}{b}\right]
$$

$$
+\left[\frac{\sqrt{2\pi}}{2}\left(mw_1^{'}-e\right)-\left(\left(2+m\right)w_1^{'}-e\right)\left[\frac{a+\left(\gamma-1\right)\mu}{b}\right]\right]\left[\frac{\sqrt{1+\gamma^2}}{b}\right]\left(\frac{mw_1^{'}-c_0}{\left(1+m\right)w_1^{'}}-\frac{1}{2}\right)\sigma
$$

$$
-\frac{1}{2\sqrt{2\pi}}\left(\left(2+m\right)w_1^{'}-e\right)\left[\frac{a+\left(\gamma-1\right)\mu}{b}\right]^2
$$

$$
-\frac{\sqrt{2\pi}\left(1+\gamma^2\right)}{2b^2}\left(\left(2+m\right)w_1^{'}-e\right)\left(\frac{mw_1^{'}-c_0}{\left(1+m\right)w_1^{'}}-\frac{1}{2}\right)^2\sigma^2
$$

$$
+\frac{1}{2}\left(\left(1+m\right)w_2-e\right)\left[\frac{a-\left(\gamma+1\right)\mu}{b}\right]
$$

$$
+\left[\frac{\sqrt{2\pi}}{2}-\left[\frac{a-\left(\gamma+1\right)\mu}{b}\right]\right]\left(\left(1+m\right)w_2-e\right)\left[\frac{\sqrt{1+\gamma^2}}{b}\right]\left(\frac{m}{1+m}-\frac{1}{2}\right)\sigma \qquad \text{(I.42)}
$$

$$
-\frac{1}{2\sqrt{2\pi}}\left(\left(1+m\right)w_2-e\right)\left[\frac{a-\left(\gamma+1\right)\mu}{b}\right]^2
$$

$$
-\frac{\sqrt{2\pi}\left(1+\gamma^2\right)}{2b^2}\left(\left(1+m\right)w_2-e\right)\left(\frac{m}{1+m}-\frac{1}{2}\right)^2\sigma^2+\left(\nu-2c+e\right)\left[\frac{a-\mu}{b}\right]
$$

$$
+\left[\frac{\sqrt{2\pi}}{2}\left(\nu-2c+e\right)-\left(e-\nu\right)\left[\frac{2\left(a-\mu\right)}{b}\right]\right]\left[\frac{\sqrt{1-\gamma^2}}{b}\right]\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)\sigma
$$

$$
-\frac{2}{\sqrt{2\pi}}\left(e-\nu\right)\left[\frac{a-\mu}{b}\right]^2-\frac{\sqrt{2\pi}\left(1-\gamma^2\right)}{2b^2}\left(e-\nu\right)\left(\frac{1}{2}-\frac{\nu-c}{e-\nu}\right)^2\sigma^2
$$

$$
\pi_{TSC}^{8}=\left(\left(1+m\right)w_2+\nu-2c\right)\left[\frac{a-\mu}{b}\right]
$$

$$
+\left(\left(1+m\right)w_2-e\right)\left[\sqrt{2\pi}-2\left[\frac{a-\mu}{b}\right]\right]\left(\frac{\sigma}{b}\right)\left(\frac{m}{1+m}-\frac{1}{2}\right) \qquad \text{(I.43)}
$$

$$
-\frac{1}{\sqrt{2\pi}}\left(\left(1+m\right)w_2+e-2\nu\right)\left[\frac{a-\mu}{b}\right]^2
$$

$$-\sqrt{2\pi}\left((1+m)w_2-e\right)\left(\frac{\sigma}{b}\right)^2\left(\frac{m}{1+m}-\frac{1}{2}\right)^2$$
$$+\left[\sqrt{2\pi}(v-2c+e)-4(e-v)\left[\frac{a-\mu}{b}\right]\right]\frac{\sigma}{b}\left(\frac{1}{2}-\frac{v-c}{e-v}\right)$$
$$-2\sqrt{2\pi}(e-v)\left(\frac{\sigma}{b}\right)^2\left(\frac{1}{2}-\frac{v-c}{e-v}\right)^2$$

We compare formula (I.38), formula (I.39), formula (I.40), formula (I.41), and formula (I.42) to obtain the scenario in which the supply chain has a maximum profit.

- If $a<\mu$ and $w_1+p_1<e$, then π_{TSC}^1 is bigger than π_{TSC}^5.
- If $a<\mu, w_1+p_1<e$, $p_2<e$ and $p_2-p_1>w_1$ then π_{TSC}^1 is bigger than π_{TSC}^4 and π_{TSC}^8.
- If $a<(1-\gamma)\mu$ then $\pi_{TSC}^2=\pi_{TSC}^3$ is bigger than $\pi_{TSC}^6=\pi_{TSC}^7$.
- If $a<\mu, w_1+p_1<e$, $p_2<e$, and $2w_1<w_2$ then π_{TSC}^1 is bigger than π_{TSC}^2 and π_{TSC}^3.

Taking into account the total conditions, if we have $a<\mu$, $w_1+p_1<e$, $p_2<e$, $p_2-p_1>w_1$, and $2w_1<w_2$, the supply chain profit in scenario 1 is greater than in the other scenarios.