Numerical simulation of melting heat transfer towards stagnation point region over a permeable shrinking surface

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Abstract:
The aim of deploying hybrid nanofluids is to optimize the thermal transport characteristics of the model under consideration. Hybrid nanofluids incorporate composite nanoparticles, which enhance thermal conductivity. Here, Silver (Ag) and Graphene oxide (Go) are used as nanoparticles with kerosene oil as a base fluid. The impact of Ohmic heating, viscous dissipation, and thermal radiation are taken to model the problem of steady flow over a stretching/shrinking geometry. The model equations are tackled by a built-in scheme, bvp4c in Matlab. Moreover, the existence of dual solutions is found for a given range of pertinent parameters. The impact of the melting heat transfer parameter is heeded on the coefficient of skin friction and Nusselt number for both hybrid nanofluid and nanoparticles. A comparison is established with the pre-existing results, which is in good agreement. It is noted that the values of the coefficient of friction drag for the upper branch decrease for a particular range of shrinking parameter; however, for the lower branch opposite trend is observed. The magnetic force decreases the flow field and energy distribution for the stable branch; however, enhances for lower branch.

Keywords: Numerical solution; Melting heat transfer; Thermal radiation; Ohmic heating; Viscous dissipation.

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Nomenclature:

\( u, v \) velocity components \( [\text{m/s}] \)
\( \nu \) kinematic viscosity \( [\text{m}^2 \text{s}^{-1}] \)
\( k_f \) thermal conductivity \( [\text{kgmK}^{-1} \text{s}^{-1}] \)
\( \rho_f \) density \( [\text{kgm}^{-3}] \)
\( \mu_f \) dynamic viscosity \( [\text{kgm}^{-1} \text{s}^{-1}] \)
\( \alpha_f \) thermal diffusivity \( [\text{m}^2 \text{s}^{-1}] \)
\( (c_p)_f \) specific heat \( [\text{m}^2 \text{s}^{-2} \text{K}^{-1}] \)
\( \phi_1 \) Ag volume fraction
\( \phi_2 \) Go volume fraction
\( T \) fluid temperature \( [\text{K}] \)
\( T_o \) ambient temperature \( [\text{K}] \)
\( M \) magnetic parameter
\( \Pr \) Prandtl number
\( \chi \) stretching/shrinking parameter
\( m_e \) Melting parameter
\( \theta_w \) temperature ratio parameter
\( Ec \) Eckert number
\( R_d \) thermal radiation

For hybrid nanofluid

\( k_{hnf} \) thermal conductivity \( [\text{kgmK}^{-1} \text{s}^{-3}] \)
\( \alpha_{hnf} \) thermal diffusivity \( [\text{m}^2 \text{s}^{-1}] \)
\( (c_p)_{hnf} \) specific heat \( [\text{m}^2 \text{s}^{-2} \text{K}^{-1}] \)
\( \mu_{hnf} \) dynamic viscosity \( [\text{kgm}^{-1} \text{s}^{-1}] \)
\( \nu_{hnf} \) kinematic viscosity \( [\text{m}^2 \text{s}^{-1}] \)
\( \rho_{hnf} \) density \( [\text{kgm}^{-3}] \)

For nanofluid

\( k_{nf} \) thermal conductivity \( [\text{kgmK}^{-1} \text{s}^{-3}] \)
\( \alpha_{nf} \) thermal diffusivity \( [\text{m}^2 \text{s}^{-1}] \)
\( (c_p)_{nf} \) specific heat \( [\text{m}^2 \text{s}^{-2} \text{K}^{-1}] \)
\( \mu_{nf} \) dynamic viscosity \( [\text{kgm}^{-1} \text{s}^{-1}] \)
\( \nu_{nf} \) kinematic viscosity \( [\text{m}^2 \text{s}^{-1}] \)
\( \rho_{nf} \) density \( [\text{kgm}^{-3}] \)

Introduction

The recent discovery of hybrid nanocomposites has piqued curiosity about their possible applications in a variety of fields. Hybrid nanofluids combine metallic, polymeric, or non-metallic nanoparticles with a base fluid to boost heat transmission. They are known for their great electrical and thermal conductivity. Their exceptional dynamic strengths and specific heat are due to their nanostructure and particle bonds. These features are expected to aid various fields of industry and technology, including telecommunications, ophthalmology, thermoplastic elastomers, biotechnology, and on and on. Many research groups have been motivated by Sumio Iijima's discovery of carbon nanotubes in 1991 [1]. Huang et al. [2] studied the pressure drop and energy transport enhancement properties of hybrid nanofluid. By taking multiple diameters of water confined in single-walled carbon nanotubes, Liu et al. [3] investigated the flow transport and its structural properties. Madhesh and Kalaiselvam [4] experimentally noted the influence of forced convection on hybrid nanofluids through heat exchangers. Toghraie et al. [5] also conducted experimental verification of nanoparticle concentration on energy enhancement for ZnO-TiO_2 /EG. The hybrid nanofluid model with free convection within porous media was presented by Tlili et al. [6]. They applied SEM and Gauss-Seidel method to obtain the desired outcomes. Yang et al.
[7] did thermal transport investigation for Casson nanofluid flow. Tang et al. [8] gave a comparison of interfacial properties on crude oil-water with rheological attributes of polymeric nanofluids. Further recent studies related to hybrid nanofluid are presented in Refs. [9–16].

Melting heat transfer is a complicated and significant field of thermo-physics that is linked to phase-transition issues in production such as, electromagnetic crucible systems, metallic processing, glass treatment, polymer synthesis, and laser ablation. In thermally driven flows, melting heat transfer simulation is complicated because the fluid dynamics must be combined with a moving interface. Such issues are commonly referred to as the "Stefan shifting boundary" problem. The melting process and heat transfer rate are the two most important elements to consider when calculating the melting heat transfer process. To address melting heat transfer issues effectively and precisely as moving boundary problems, strong computational techniques are necessary. An alternative approach is to model the melting effect as a boundary condition, which avoids the need to explicitly simulate the moving interface. This method works with boundary layer flow models as well, but it is less accurate. Robert [17] first reported the melting behaviour of an ice slab in a hot air stream. Yacob et al. [18] scrutinized the heat transfer in a micropolar fluid boundary layer stagnation point flow across a stretching/contracting sheet with a melting effect. Bachok et al. [19] explored the characteristics of melting heat transport in the viscous fluid flow toward a stretching surface. Hayat et al. [20] scrutinized the flow of Maxwell fluid towards a stretched surface with a melting phenomenon. The melting process in a magnetized flow over a moving surface with heat radiation was explored by Das [21]. The related similar studies can be found in Refs. [22–27].

The goal of this effort is to reveal the properties of melting heat transfer in the flow of hybrid nanofluid flow across a stretching/shrinking surface in the stagnation point region. Here, kerosene oil is assumed as the base fluid, while silver (Ag) and Graphene oxide (Go) are the nanoparticles. The results for various physical parameters are obtained and presented graphically and in tables. Additionally, the limiting case comparison results are based on previously published data.

**Formulation of the problem**

We assume a boundary layer flow of hybrid nanofluid over a shrinking surface, with melting heat transfer. It is assumed that the external flow velocity \( u_e(x) = ax \), where the stretching/shrinking velocity is \( u_w(x) = bx \). The coordinates system is selected in such a manner that, the direction of the flow is along the \( x-axis \) and the \( y-axis \) is perpendicular to it. The magnetic force with strength \( B_o \) is imposed in a vertical direction. Moreover, non-linear thermal radiation with Ohmic heating and viscous dissipation is taken into account, to describe the thermal transport of a hybrid nanofluid. The melting surface temperature is assumed to be \( T_m \), whereas the free-stream temperature is \( T_\infty \), where \( T_\infty > T_m \). The flow structure of our considered problem is depicted in
Fig. 1.
Using the assumption above, we design the flow problem as follows

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} + \frac{\sigma_{nf} B^2_o}{\rho_{nf}} (u_e - u), \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{(\rho c_p)_{nf}} \left\{ k_{nf} + \frac{16\sigma^* T^3}{3\kappa^*} \right\} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho c_p)_{nf}} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_{nf} B^2_o u^2}{(\rho c_p)_{nf}}, \tag{3}
\]

the boundary conditions are

\[
u = u_w, \quad T = T_m, \quad k_{nf} \left( \frac{\kappa}{\kappa_f} \right)_{y=0} = \rho_{nf} \left\{ L + c_{s(T_m-T_a)} \right\} v(x,y) \sin \theta, \quad \text{at} \quad y = 0 \tag{4}
\]

\[
u \to u_e \text{ and } T \to T_o, \quad \text{as} \quad y \to \infty
\]

Letting

\[u = axf^\eta(\eta), \quad v = -\sqrt{av_f} f(\eta), \quad \theta = \frac{T - T_m}{T_o - T_m}, \quad \eta = \frac{a}{\sqrt{a}} y, \tag{5}\]

which satisfies equation (1) and converts the motion equation as

\[rac{1}{\rho_{nf} / \rho_f} \left\{ \frac{\mu_{nf}}{\mu_f} f'' + ff'' - f'^2 + \frac{\sigma_{nf}}{\sigma_f} \right\} M (1 - f') + 1 = 0, \tag{6}
\]

\[
\frac{1}{Pr} \left( \frac{1}{\rho c_p} \right)_{nf} \left( \frac{1}{\rho c_p} \right)_{f} \left\{ \left( k_{nf} + \frac{2}{3} R_d \left( \frac{\theta_{w} - 1}{\theta_{w}} \right)^3 \left( \theta' - 1 \right) \right)' + f \theta' \right\} \\
+ \frac{E_c}{Pr} \left( \frac{1}{\rho c_p} \right)_{nf} \left( \frac{1}{\rho c_p} \right)_{f} \left\{ \frac{\rho_{nf}}{\rho_f} f'' + \frac{\sigma_{nf}}{\sigma_f} Mf'' \right\} = 0, \tag{7}
\]

with boundary condition's

\[f' = \chi, \quad \theta = 0, \quad Me \frac{k_{nf}}{k_f} \theta' + Pr \frac{\rho_{nf}}{\rho_f} f = 0 \quad \text{at} \quad y = 0, \tag{8}
\]

\[f' \to 1, \quad \theta \to 1 \quad \text{as} \quad \eta \to \infty
\]

the involved physical parameters are defined as \(\chi = \frac{k_{nf}}{k_f}\) the stretching/shrinking parameter, \(M = \frac{a B_o^2}{\mu_f}\) is the magnetic field parameter, \(Me = \frac{c_{s(T_m-T_a)}}{L + c_{s(T_m-T_a)}}\) is the melting heat transfer parameter and \(Pr = \frac{v_f}{k_f}\) is the Prandtl number, \(R_d = \frac{4\sigma T^3}{k_f} \kappa^*\) thermal radiation, \(\theta_{w} = \frac{T_w}{T_o} > 1\) the temperature ratio parameter, \(E_c \frac{u^2}{c_p(T_m-T_a)}\) the Eckert number.
**Physical quantities**

The expressions for physical quantities are expressed as

\[ C_f = \frac{\tau_w}{\rho_f u_w^2}, \quad \text{and} \quad Nu = \frac{\chi q_w}{k_f (T - T_m)} \]  \hspace{1cm} (9)

where \( \tau_w \) and \( q_w \) symbolize the surface heat, and mass fluxes, which are defined as

\[ \tau_w = \mu_{inf} \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad \text{and} \quad q_w = -\left( k_{inf} + \frac{16 \sigma^*}{3k^*} T^3 \right) \left( \frac{\partial T}{\partial y} \right)_{y=0} \]  \hspace{1cm} (10)

the dimensionless form of equation (10) is

\[ \text{Re}^{1/2} C_f = f''(0), \quad \text{and} \quad \text{Re}^{1/2} Nu = -\left( \frac{k_{inf}}{k_f} + \frac{4}{3} R_d \left\{ \left( \theta^* \theta(0) \right) + \frac{1}{2} \theta'(0) \right\} \right) \theta'(0) \]  \hspace{1cm} (11)

in which \( \text{Re} \left( = \frac{\rho u^2}{\mu} \right) \) denoted the local Reynolds number.

**Results and discussion**

With the help of the shooting method, the governing equations (6) and (7) as well as the boundary conditions (8) are numerically solved. The dimensionless fluid velocity and temperature distribution, as well as the skin friction coefficient and heat transfer rate, are depicted graphically in figures (2) to (12) as a result of changing various physical parameters. Throughout the analyses, the numerical significances are assigned to grasp the physical purpose of the problem, that is \( M = 0.1, \quad m_v = 0.5, \quad \chi = -2.0, \quad R_d = 1.0, \quad \theta_w = 1.1 \) and \( Ec = 0.1 \). In particular, the numerical values of \( \text{Re}^{1/2} C_f \) and \( \text{Re}^{1/2} Nu \) against the shrinking parameter \( \chi \) with \( M = (0.1, 0.3, 0.5) \) are depicted in **Fig 2(a) and (b)**. From these illustrated upshots, we concluded that the necessary values changed from the left to the right when the magnetic parameter varied from 0.1 to 0.5, corresponding to \( \chi_c = -2.8412 \) to \( M \approx 4.3573 \). These burification values make it clear that both branches of the solutions exist in the range \( \chi_c < \chi \), whereas no solution is found outside of turning points, or when \( \chi < \chi_c \), and only one branch is discovered when \( \chi = \chi_c \). **Fig 3(a) and (b)** shows the influences of \( m_v \) against the shrinking parameter \( \chi \) on \( \text{Re}^{1/2} C_f \), and \( \text{Re}^{1/2} Nu \). These conclusions show that improving the melting parameter force drops the skin friction coefficient and heat transfer rate (in an absolute sense). **Fig. 4** shows how the volume fractions of nanoparticles affect the heat transfer rate. Physically, due to an increase in the kinetic energy of the system, the heat transfer rate is inclined. This increase in kinetic energy improves the heat transfer rate in upper branch solution. The consequence of the Eckert number.
Ec on the $Re_x^{\frac{1}{2}} Nu_x$ is depicted in Fig. 5. Here, it is noted that the suspense in a boundary layer separation is unaffected by boosting the Eckert number. Due to this, the dual solutions are exclusively valid up to the exact necessary value for all Ec. The profile $f'(\eta)$ for distinct values of Magnetic field parameter $M$ and shrinking parameter $\chi$ are shown in Figs. 6 and 7, respectively. The increment in these constraints improved the velocity of the fluid in the upper solution branch while it declines in the lower solution branch. Fig. 8 illustrates the influence of nanoparticles volume fraction on the fluid velocity. Physically, the conduct is an outcome of intermolecular oscillations of nanoparticles, which improves mass diffusion and reduces the flow field. Fig. 9 demonstrates how the temperature curves are affected by the melting parameter $m_c$. Further, it is noted that when the melting parameter augments, the first solution decays, whereas the second solution expands. According to Fig. 10, the temperature distribution becomes more even for higher Eckert number Ec. Higher values of Ec physically retain more heat energy in the fluid. Therefore, the friction forces ultimately improve the temperature profile. In Fig. 11 there has been an increasing trend in the thermal distribution for the radiation parameter $dR$. The dominance of heat radiation over conduction is therefore indicated by higher values of $dR$. As a result, rising values show that the system is absorbing more radiative heat energy, which raises the value of $\theta(\eta)$. Temperature distribution in the existence of melting heat exhibits an increasing behaviour for both the upper and lower branch solutions as the temperature ratio parameter $\theta_n$ is increased, as shown in Fig. 12.

Thermophysical properties of hybrid nanofluid and nanofluid are [29]:

$$\rho_{nf} = (1-\phi_f)\left((1-\phi)\rho_f + \phi\rho_m\right) + \phi_2\rho_{m_2}$$

$$\left(\rho c_v\right)_{nf} = (1-\phi_f)\left((1-\phi)\left(\rho c_v\right)_f + \left(\rho c_v\right)_m\phi\right) + \phi_2\left(\rho c_v\right)_{m_2}$$

$$\frac{k_{nf}}{\kappa_f} = \frac{k_{m} + 2\kappa_{m} - 2(k_f - k_n)\phi}{k_{m} + 2\kappa_{m} + (k_f - k_n)\phi}$$

where

$$\frac{k_{nf}}{\kappa_f} = \frac{k_{m} + 2\kappa_{m} - 2(k_f - k_n)\phi}{k_{m} + 2\kappa_{m} + (k_f - k_n)\phi}$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{\frac{1}{2}}}(1-\phi_f)^{\frac{1}{2}} \cdot \frac{\sigma_{nf}}{\sigma_f} = \frac{\sigma_{m} + 2\sigma_{nf} - 2(\sigma_f - \sigma_m)\phi}{\sigma_{m} + 2\sigma_{nf} + (\sigma_f - \sigma_m)\phi}$$

and

$$\rho_{nf} = (1-\phi_f)\rho_f + \phi_1\rho_m$$

$$\left(\rho c_v\right)_{nf} = (1-\phi_f)\left(\rho c_v\right)_f + \phi_1\left(\rho c_v\right)_m$$

$$\frac{k_{nf}}{\kappa_f} = \frac{k_{m} + 2\kappa_{m} - 2(k_f - k_n)\phi}{k_{m} + 2\kappa_{m} + (k_f - k_n)\phi}$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{\frac{1}{2}}} \cdot \frac{\sigma_{nf}}{\sigma_f} = \frac{\sigma_{m} + 2\sigma_{nf} - 2(\sigma_f - \sigma_m)\phi}{\sigma_{m} + 2\sigma_{nf} + (\sigma_f - \sigma_m)\phi}$$

Table 1 shows the thermo-physical properties of base fluid the nanoparticles.
Tabular comparison
A comparison of $f'(0)$ for the shrinking case ($\chi < 0$) is shown in Table. 2, which demonstrate a positive correlation with past research.

Concluding remarks
This study investigates the stagnation point flow of a hybrid nanofluid over a permeable shrinking surface. Using similarity transformations, the boundary layer governing equations are converted to nonlinear ordinary differential equations, which are then solved in Matlab using bvp4c. The influence of several dimensionless constraints on fluid velocity and temperature distribution is scrutinized. The numerical results of friction drag, and heat transfer rates are discussed concerning various parameters. The key findings of the study are

- An increase in the melting parameter speeds up the separation of the boundary layer and decreases the velocity and temperature profiles marginally.
- In the presence of a magnetic parameter, the fluid velocity is reduced due to produced Lorentz force.
- Larger radiative and temperature ratio parameter causes a significant increase in the thermal state and related boundary layer thickness.
- For the thermal radiation parameter, the temperature field of hybrid nano liquid is significantly improved.
- An increase in Eckert number and temperature ratio parameter enhanced the temperature of the fluid.
References


Fig. 1: Geometry of the problem.
Figs. 2(a-b): Plot of $M$ via $\text{Re}^{1/2} C_f$ and $\text{Re}^{-1} Nu_x$.
Figs. 3(a-b): Plot of $m_e$ via $\text{Re}^{1/2} C_f$ and $\text{Re}^{1/2} Nu_x$.

Fig. 4: Plot of $\phi_2$ via $\text{Re}^{1/2} Nu_x$. 
Fig. 5: Plot of $Ec$ via $Re^{-\frac{1}{2}}Nu_x$.
Fig. 6: Plot of $M$ via $f'(\eta)$.
Fig. 7: Plot of $\chi$ via $f'(\eta)$. 

$\chi = -2, -1.8, -1.6$
Fig. 8: Plot of $\phi_2$ via $f'(\eta)$.
Fig. 9: Plot of $m_c$ via $\theta(\eta)$. 
Fig. 10: Plot of $Ec$ via $\theta(\eta)$. 
Fig. 11: Plot of $R_d$ via $\theta(\eta)$. 
Fig. 12: Plot of $\theta_w$ via $\theta(\eta)$.

Table 1: Thermo-physical properties [29]:

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>$c_p$ (J/kg K)</th>
<th>$k$ (W/m K)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$\sigma$ (S/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag</td>
<td>235</td>
<td>429</td>
<td>10500</td>
<td>$63 \times 10^{-6}$</td>
</tr>
<tr>
<td>Go</td>
<td>717</td>
<td>5000</td>
<td>1800</td>
<td>$6.30 \times 10^7$</td>
</tr>
<tr>
<td>Kerosene oil</td>
<td>2090</td>
<td>0.145</td>
<td>783</td>
<td>$21 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 2: The numerical values of $f''(0)$ for shrinking parameter $\chi$ when $m_r = 0$.

<table>
<thead>
<tr>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>F. S.</td>
<td>S. S.</td>
<td>F. S.</td>
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<tr>
<td>-1.0</td>
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<td>0.1167022</td>
<td>1.082431</td>
</tr>
<tr>
<td>-1.2</td>
<td>0.9324739</td>
<td>0.2336497</td>
<td>0.932743</td>
</tr>
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</table>
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