Optimization with Genetic Algorithm (GA): Planar Mechanism Synthesis
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ABSTRACT

Dimensional synthesis of mechanisms to trace given points is an important issue in mechanism and machine science. Having no exact solution makes this issue an optimization problem. This study offers an optimization approach for the dimensional synthesis of planar mechanisms. Four-bar mechanisms having one Degree of Freedom (DOF) are chosen as the configurations. The proposed method is implemented by establishing the objective functions with specified constraints and searching for the results by using an optimization algorithm. Genetic Algorithm (GA) in Optimization Toolbox-Matlab® is selected as a solver. Different types of four-bar mechanisms like crank-rocker and double-crank including different target points are performed. Mechanisms are depicted by resulted parameters and a Matlab® script prepared plays their animations. As a result, it is proved that the mechanisms whose dimensional properties are obtained by the GA solver have a good tracing capability for the desired paths. This study has the property of being a design guide. Its application is not limited to four bar mechanism. Planar mechanisms with different configurations can be easily synthesized by using this technique.

KEYWORDS - Mechanism synthesis, Optimization, Genetic algorithm (GA), Four-bar mechanism, Path generation.

1 INTRODUCTION

Dimensional synthesis is defined as the determination of kinematic dimensions of a mechanism to perform required motion (link lengths, offsets, etc.) Graphical and analytical methods are available. Optimization methods are applied in recent years. Different algorithms are used. Problems are defined as; motion generation, path generation, function generation. Generally, a multidisciplinary design optimization procedure is applied to synthesize mechanisms. Some input data which the designer needs is given in the algorithm. These are Degree of Freedom (DOF), range of all geometric parameters, inputs to the mechanism, outputs from the mechanism, and the required kinematic characteristics of the mechanism [1].

Many simple machines use a four-bar linkage; a windshield wiper, a rock crusher, an oil well, a door closer, or a high lift mechanism can be given as examples, 1 DOF. Four-bar mechanisms are
simple but practically important mechanisms. The interest is taken on a four-bar linkage where some studies are summarized here. R.C. Blackett (2001) has presented a study on the optimal synthesis of planar five-bar linkages [1]. J. A. Cabrera et al. (2002) have studied the optimal synthesis of mechanisms by using GA [2]. Zohoor and Nia (2005) have suggested a Genetic Algorithm (GA) based optimization for the synthesis of planar and spatial mechanisms [3]. Geletu (2007) has studied optimization problems using Matlab Optimization Toolbox [4]. N. N. Zadeh et al. (2009) have introduced hybrid multi-objective GA for pareto optimum synthesis of four-bar mechanisms. Two objective functions; tracking error and transmission angle (μ) are minimized simultaneously [5]. S.K. Archaryya and M. Mandal (2009) have performed a study on the synthesis of the four-bar mechanism using GA, Particle Swarm Optimization (PSO), and Differential Evolution (DE). A comparative study is carried on three evolutionary algorithms [6]. H. Erdogan (2011) has performed a thesis on synthesis on planar mechanisms by applying evolutionary algorithms, GA [7]. S.S. Shete and S.A. Kulkarni (2015) have performed a study on the dimensional synthesis with GA analysing three problems. Straight-line generation problem through six desired points without prescribed timing [8]. S. Sleesongsom and S. Bureerat (2018) have presented optimum path generation for synthesizing four-bar linkage. Seven metaheuristic techniques are applied, and compared. Optimum parameters of a four-bar linkage are found [9]. P. Chaphet et al. (2019) are studied four-bar as the high-lift mechanism. Teaching learning-based optimization (TLBO) is applied. [10]. Pavlovic et al. (2019) offered a study on the optimal synthesis of manipulator drive mechanisms in hydraulic excavators [11]. A Sardashi et al. (2022) presented a methodology for path generation synthesis of the four bar mechanism by introducing the Geometrical Similarity Error Function (GSEF) [12]. Y.H.Kang et al. (2022) performed a comparative study on the optimum synthesis of path generation. Five metaheuristic optimization algorithms; two swarm based (PSO, HPSO) and three evolutionary based (DE-gr, EPSDE and L-EPSD) applied to four bar mechanism [13].

The purpose of this study is to apply an evolutionary method for the synthesis of four-bar mechanisms and present a design guide to be used in linkage mechanisms. The type of the four-bar mechanism can be changed in an easy way with minor modifications done in the functions. Path generation cases with and without prescribed timing are given in the manuscript. Minimization of structural error is performed by considering the size and geometric constraints; Grashof theorem and the transmission angle for example. This study is organized as follows. Section 1 gives an introduction. Section 2 presents definition and main principles of Genetic Algorithm (GA). Section 3 includes kinematics equations, objective and constraint functions. Section 4 discusses
implementation to the four-bar mechanism with case studies. Finally, conclusions are drawn in Section 5.

2 GENETIC ALGORITHM (GA)

GA is proposed by Holland in 1975 for solving different optimization problems. GA is popular in solving complicated nonlinear problems in engineering. It is inspired by natural selection in the evolution of living organisms. The principles of genetics and natural selection are fundamentals of this technique. GA uses the analogy to chromosome encoding and natural selection [14]. Nobakhti (2010) has performed a study on natural-based optimization [15]. Khan and Bajpai (2013) have also given implementation of GA for different fields of mechanical engineering & other problems in the real world with advantages and limitations [16]. Bhoskar et al. (2015) have presented a review on GA applications ranging from scheduling problems to manufacturing and material science also [17]. Y. M. Al-Smadi et al. (2022) investigated sensitivity of four bar coupler motion by analysing position error applying GA for four-bar mechanism. The hood mechanism is chosen the Plymouth Satellite automobile as case study [18]. During optimization, an engineering issue is defined by considering the boundaries of the problem. This can be accepted as a constraint for the solution finding minima and maxima of functions. The problem must be formulated correctly.

3 SYNTHESIS OF MECHANISMS BY OPTIMIZATION

Synthesis of a mechanism means the generation of solutions for a particular type of problem in linkage design. It can be utilized as; type synthesis, number synthesis, and dimensional synthesis [19]. Dimensional synthesis is focused on in this study to determine the lengths of links necessary to get the desired motions.

The mechanism to be synthesized is a four-bar mechanism given in Figure 1.

3.1 Closed Loop Equation of the Mechanism

The first thing to do is to write a loop equation for the mechanism,

$$\vec{r}_2 + \vec{r}_3 - \vec{r}_4 - \vec{r}_1 = 0$$

Complex number notation is substituted and Equation 1 is rewritten as;

$$r_2 e^{i\theta_2} + r_3 e^{i\theta_3} - r_4 e^{i\theta_4} - r_1 e^{i\theta_1} = 0$$

Real and imaginary parts are separated as given in Equation 3,

$$r_2 \cos \theta_2 + r_3 \cos \theta_3 - r_4 \cos \theta_4 - r_1 = 0$$  \hspace{1cm} (3a)

$$r_2 \sin \theta_2 + r_3 \sin \theta_3 - r_4 \sin \theta_4 = 0$$  \hspace{1cm} (3b)

Angles $\theta_2$ and $\theta_4$ are solved for input angle $\theta_2$ from Freudenstein's equation;

$$F_1 \cos \theta_4 - F_2 \cos \theta_2 + F_3 = \cos(\theta_2 - \theta_4)$$
where the angular displacement of third and fourth links are found as given in Equation 5.

\[
\theta_{a,2} = 2 \tan^{-1}\left(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right) (5a)
\]

\[
\theta_{a,2} = 2 \tan^{-1}\left(\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right) (5b)
\]

where

\[
A = \cos \theta_2 - F_1 - F_2 \cos \theta_2 + F_3 \quad B = -2 \sin \theta_2 \quad C = F_1 - (F_2 + 1) \cos \theta_2 + F_3
\]

\[
D = \cos \theta_2 - F_4 + F_4 \cos \theta_4 + F_5 \quad E = -2 \sin \theta_2 \quad F = F_1 + (F_4 - 1) \cos \theta_2 + F_3
\]

Position of the coupler C, in the reference frame AX'Y' is

\[
C_{x, i} = r_2 \cos \theta_2 + r_{x, i} \cos \theta_3 - r_{y, i} \sin \theta_3
\]

\[
C_{y, i} = r_2 \sin \theta_2 + r_{x, i} \sin \theta_3 - r_{y, i} \cos \theta_3
\]

or written on world coordinate system OXY

\[
\begin{bmatrix}
C_x \\
C_y
\end{bmatrix} = \begin{bmatrix}
\cos \theta_0 & -\sin \theta_0 \\
\sin \theta_0 & \cos \theta_0
\end{bmatrix} \begin{bmatrix}
C_{x, i} \\
C_{y, i}
\end{bmatrix} + \begin{bmatrix}
X_0 \\
Y_0
\end{bmatrix}
\]

3.2 Objective Function

The objective function is computing the position as the sum of the squares of the Euclidian distances between each \( C_d^i \) and \( C^i \). It is given in Equation 8.

\[
f_{obj} = \sum_{i=1}^{N} \left[ (C_{x, d} - C_{x, i})^2 + (C_{y, d} - C_{y, i})^2 \right] (8)
\]

\( C_d^i \) is a group of target point defined by designer that should be traced by the coupler of the mechanism. \( C^i \) is a group of traced point obtained in Equation 7. \( N \) denotes the number of points to be synthesized.

3.3 Constraint Function

Two criteria are taken into account in defining the constraint function. The first one is Grashof condition and the second one is the transmission angle.

Grashof theorem says that a four-bar mechanism has at least one revolving link if

\[
s + l \leq p + q
\]
where s and l are shortest and longest, p and q are the intermediate links of the mechanisms, successively. Two options providing the condition given above are mechanisms called crank-rocker and double-crank as shown in Figure 2 (a, b), respectively.

- The mechanism is called a crank-rocker if the shortest link is a side link. The shortest link is the crank in the mechanism.
- The mechanism is called a double-crank if the shortest link is the frame.

Transmission angle is an important criterion in designing linkages. The force and the motion must be effectively transferred between them to obtain smooth link movements with respect to each other. It is denoted by $\mu$, illustrated in Figure 3.

It is clear that the optimum value of the transmission angle is $90^0$. In practice, it has been found out that if the maximum deviation of the transmission angle from $90^0$ is more than $40^0$ or $50^0$, the mechanism will lock [20]. It is given in Equation 9.

$$
\mu = \cos^{-1} \left( \frac{r_2^2 - r_1^2 - r_4^2 + r_3^2 + 2r_1r_3 \cos \theta_2}{2r_3r_4} \right)
$$

4 CASE STUDIES

This section presents a group of results obtained when applying the functions derived in the previous section. Optimization Toolbox-Matlab® providing a numeric computing environment is used in this section [21-22].

4.1 Case I_ Path Generation without Prescribed Timing

There are four coupler points required to find out an optimal solution. These points are designed to trace a vertical straight line by changing Y coordinate only. The problem is then defined by;

(i) The variables of the mechanism;

$$V = [r_1, r_2, r_3, r_4, r_c, r_y, x_0, y_0, \theta_0, \theta_2]$$

where $i=1,2,...,N$ and $N=4$

(ii) Target points are chosen as:

$$[C_x^{\text{cd}}, C_y^{\text{cd}}] = [(20,30), (20,35), (20,40), (20,45)]$$

(iii) Limits:

$$r_1, r_2, r_3, r_4 \in [5, 60] \quad r_c, r_y, x_0, y_0 \in [-60, 60] \quad \theta_0, \theta_1^i, \theta_2^1, \theta_2^3, \theta_4^i \in [0, 2\pi]$$

Parameters of GA are given in Table 1.

Crank-rocker mechanism variables obtained after synthesis are given below;
\[ \begin{align*}
r_1 &= 59.636 \quad r_2 = 24.941 \quad r_3 = 53.618 \quad r_4 = 37.089 \quad r_{cx} = -17.126 \quad r_{cy} = 8.173 \\
x_0 &= 5.593 \quad y_0 = 9.28 \quad \theta_0 = 0.205 \quad \theta_1 = 0 \quad \theta_2 = 0.151 \quad \theta_3 = 0.339 \quad \theta_4 = 0.543
\end{align*} \]

Coupler points tracing desired target points are given in Figure 4(a-d), successively.

Desired and traced points in x and y directions and transmission angle obtained are given in Table 2.

Double-crank mechanism variables obtained after synthesis are given below;
\[ \begin{align*}
r_1 &= 10.501 \quad r_2 = 48.57 \quad r_3 = 43.381 \quad r_4 = 40.431 \quad r_{cx} = -1.189 \quad r_{cy} = 33.716 \\
x_0 &= -9.287 \quad y_0 = 27.733 \quad \theta_0 = 0.655 \quad \theta_1 = 0.177 \quad \theta_2 = 0.343 \quad \theta_3 = 0.502 \quad \theta_4 = 0.649 \\
f_{obj} &= 0.807
\end{align*} \]

Coupler points tracing desired target points are given in Figure 5(a-d), successively.

Desired and traced points in x and y directions and transmission angles are given in Table 3.

4.2 Case II_Path Generation with Prescribed Timing

There are four coupler points required to find out an optimal solution. They are chosen to get an optimal solution for tracing an arc. The problem is then defined by;

(i) The variables of the mechanism;
\[ V = [r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, x_0, y_0, \theta_0] \]

(ii) Target points are chosen as:
\[ \left[ C^i_{sd}, C^j_{sd} \right] = [(0,0),(1.9098,5.8779),(6.9098,9.5106),(13.09,9.5106)] \]
\[ \left[ \theta^i_2 \right] = \left[ \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3} \right] \]

(iii) Limits:
\[ r_1, r_2, r_3, r_4 \in [5,60] \quad r_{cx}, r_{cy}, x_0, y_0 \in [-60,60] \]

Parameters of GA are given in Table 1.

Crank-rocker mechanism variables obtained after synthesis are given below;
\[ \begin{align*}
r_1 &= 60 \quad r_2 = 20.267 \quad r_3 = 35.465 \quad r_4 = 59.71 \quad r_{cx} = 56.865 \quad r_{cy} = 3.414 \quad x_0 = -0.091 \quad y_0 = 44.654 \quad \theta_0 = 0 \\
f_{obj} &= 2.15
\end{align*} \]

Coupler points tracing desired target points are given in Figure 6 (a-d), successively.

Desired and traced points in x and y directions and transmission angles are given in Table 4.

5 CONCLUSIONS

This paper has presented a study for the synthesis of planar mechanisms by a Genetic algorithm solver; one DOF four-bar mechanism is used as a simple example. The algorithm is developed only for well-known Grashof type four-bar mechanisms. Different case studies with
crank-rocker and double-crank mechanisms having straight-line and arc-shaped target points are presented by using Matlab® Optimization Toolbox. The study is enriched with case studies including path generations with and without prescribed timing. The position error between desired and actual values of the coupler point of the mechanism is optimized by using objective and constraint functions. This technique presents the utility of being basic and straightforward. It also shows that using GA in the dimensional synthesis of mechanisms is a good alternative solution to analytical and graphical solutions.

The proposed method gives users an option to increase and decrease the number of precision points so easily without changing the mathematical expressions. Adapting this technique to other planar mechanisms is simply possible. This adaptation may also include different objective functions except for position error minimization. The parameters of the options in the optimization toolbox can be studied to get better results.

REFERENCES
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NOMENCLATURE

Degree of Freedom DOF
Genetic Algorithm GA
Group of target point $C_j$
Group of traced point $C_i$
Transmission angle $\mu$

Figure 1. Four-Bar Mechanism
Figure 2. Crank-Rocker and Double-Crank Mechanisms
Figure 3. Transmission Angle, $\mu$
Figure 4. Tracing Desired Points with Crank-Rocker Mechanism
Figure 5. Tracing Desired Points with Double-Crank Mechanism
Figure 6. Tracing Arc Shaped Path with Crank-Rocker Mechanism

Table 1. Parameters of Genetic Algorithm
Table 2. Desired & Traced Points and Transmission Angles (µ) for Crank-Rocker Mechanism
Table 3. Desired & Traced Points and Transmission Angles (µ) for Double-Crank Mechanism
Table 4. Desired & Traced Points and Transmission Angles (µ) for Crank-Rocker Mechanism

Figure 1. Four-Bar Mechanism

Figure 2. Crank-Rocker and Double-Crank Mechanisms
Figure 3. Transmission Angle, μ

Figure 4. Tracing Desired Points with Crank-Rocker Mechanism
Figure 5. Tracing Desired Points with Double-Crank Mechanism
Figure 6. Tracing Arc Shaped Path with Crank-Rocker Mechanism

Table 1. Parameters of Genetic Algorithm

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<th>Values</th>
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<tr>
<td>Population size</td>
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<td>Crossover possibility</td>
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<tr>
<td>Mutation possibility</td>
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<td>Selection type</td>
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Table 2. Desired & Traced Points and Transmission Angles ($\mu$) for Crank-Rocker Mechanism

<table>
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<th>X</th>
<th>Y</th>
<th>Transmission Angle $\mu$</th>
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<tr>
<td>20</td>
<td>19.31</td>
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Table 3. Desired & Traced Points and Transmission Angles ($\mu$) for Double-Crank Mechanism

<table>
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<th>Transmission Angle $\mu$</th>
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</tr>
<tr>
<td>$\theta_2$</td>
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<td>Y</td>
</tr>
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<td>---------</td>
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<tr>
<td>Desired</td>
<td>Traced</td>
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<tr>
<td>$2\pi/3$</td>
<td>13.09</td>
<td>13.02</td>
</tr>
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Table 4. Desired & Traced Points and Transmission Angles ($\mu$) for Crank-Rocker Mechanism

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