Computational analysis on radiative non-Newtonian Carreau nanofluid flow in a microchannel under the magnetic properties

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Abstract: Microfluidic technology and Micro Electromechanical Systems (MEMS) have received much attention in science and engineering fields over the last few years. MEMS can be found in many areas like heat exchangers, chemical separation devices, bio-chemical analysis and micro pumps. Keeping these facts in mind, the prime purpose of the current paper is to present the flow of Carreau nanofluids through the micro-channel with the electro-osmosis, Joule heating and chemical reactions. The effect of external magnetic field is also considered into account. For the formulation of the problem, the Cartesian coordinate system is considered. The perturbed solutions have been presented by making use of regular perturbation method. The graphical results also prepared corresponding to numerous values of fluid flow phenomenon like velocity, temperature, solutal nano-particle concentration, Sherwood number and Nusselt number with different fluid variables. It is concluded from our analysis that; velocity decrement is identified with respect to the enhancing the magnetic parameter (Hartmann number). The Schmidt number, Radiation term, Prandtl number and chemical reaction term increase the solutal nano-particle concentration. The outcomes of the

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Newtonian liquid model can be obtained from our scrutiny. The present scrutiny has many applications in engineering sciences such as electromagnetic micro pumps and nano-mechanics.

**Keywords:** Electro-osmosis flow, Regular perturbation method, Thermal radiation, Nanofluid dynamics, Helmholtz-Smoluchowski velocity, Slip boundary conditions.

**Nomenclature:**

\( B_0 \): Magnetic field strength [T]

\( x \): Axial direction of flow [m]

\( y \): Transverse direction of flow [m]

\( \bar{q} \): Velocity vector [m s\(^{-1}\)]

\( \rho_{ef} \): Effective density of nanoliquid \([\text{kg m}^3]\)

\( P \): Pressure [Pa]

\( \sigma_{ef} \): Effective electrical conductivity \([\text{S m}^{-1}]\)

\( (c_p)_f \): Effective heat capacity of the nanofluid \([\text{J kg}^{-1} \text{K}^{-1}]\)

\( t \): Time [s]

\( k_{ef} \): Effective thermal conductivity \([\text{W m}^{-1} \text{K}^{-1}]\)

\( T \): Temperature [K]

\( C \): Concentration of the liquid \([\text{mol m}^3]\)

\( D_{ic} \): Temperature diffusivity coefficient \([\text{m}^2 \text{s}^{-1}]\)

\( D_{ct} \): Mass diffusivity coefficient \([\text{m}^2 \text{s}^{-1}]\)

\( E_x \): Applied electrical field \([\text{V m}^{-1}]\)
$D_s$: Solutal diffusivity [m$^2$s$^{-1}$]

$k$: Chemical reaction parameter [mol m$^{-2}$ s$^{-1}$]

$T_0$: Ambient temperature [-]

$C_0$: Ambient concentration [-]

$u$: Velocity component [m s$^{-1}$]

$g$: Acceleration due to gravity [m s$^{-2}$]

$\beta_c$: Concentration expansion coefficient [K$^{-1}$]

$\beta_t$: Thermal expansion coefficient [K$^{-1}$]

$\eta_\infty$: Viscosity of liquid at infinite shear rate [kg m$^{-1}$ s$^{-1}$]

$\eta_0$: Viscosity of liquid at zero shear rate [kg m$^{-1}$ s$^{-1}$]

$\Gamma$: Material constant [-]

$a$: Non-Newtonian liquid constant [-]

$\phi$: Electric potential term [V]

$\rho_e$: Electrical charge density [C m$^{-3}$]

$n$: Power law index [-]

$\varepsilon_{ef}$: Medium permittivity [F m$^{-1}$]

$n$: Ions bulk concentration in the electrolyte [m$^{-3}$]

$M$: Hartmann number [-]

$\kappa$: Electro osmosis term [-]

$U_{HS}$: Helmholtz-Smoluchowski velocity [m s$^{-1}$]

$G_n$: Thermal Grashof number [-]
\( G_{rc} \): Solutal Grashof number [-]

\( Pr \): Prandtl number [-]

\( Rn \): Radiation parameter [-]

\( q_r \): Radiative heat flux [W m\(^{-2}\)]

\( S \): Joule heating parameter [-]

\( N_{\alpha} \): Dufour thermo-diffusive parameter [-]

\( N_{\alpha} \): Soret diffuso-thermo parameter [-]

\( \alpha \): Dimensionless chemical reaction term [-]

\( Sc \): Schmidt number [-]

\( \xi_1 \): Temperature slip term [-]

\( \xi_2 \): Concentration slip term [-]

\( \beta \): Velocity slip parameter [-]

\( Sh \): Sherwood number [-]

\( Nu \): Nusselt number [-]

1. Introduction

During past few years, the study of nanofluids has attracted continuous consideration of several researchers because of its various engineering applications. Nanotechnology became the most important and interesting in diverse fields in biology, chemistry, engineering and physics [1-5]. It shows an incredible endeavor towards giving us plenteous pervasion which will change the direction of technological improvements in wide scope of utilizations. Because of augmentation of nanofluids in thermal conductivity, potential advantages and having several applications in biomedical engineering such as cancer treatment using thermal
therapy, power generation, heat exchanger, transportation, microelectronics, ventilation, air-conditioning and atomic framework cooling. The nanofluids plays an important role in nanotechnology. It has also many industrial applications such as production of plastic, expulsion, liquefy turning, hot moving, glass fiber creation, wire drawing and elastic plates. Salahuddin et al. [6] have investigated Carreau nanofluids flow influenced by a stretching cylinder utilizing Keller box technique with the slip effects. Mabood et al. [7] have scrutinized the impacts of magnetohydrodynamics and thick dispersal on the laminar boundary layer stream of nanoliquid through a nonlinear extending sheet. Waqas et al. [8] have discussed the flow of a Carreau nanoliquid via exponentially convected stretchable area and this model has computed by Range-Kutta-Fehlberg technique. Hatami and Jing [9] have presented the differential transformation method to study viscous nanofluid flow. Din et al. [10] have inspected the impacts of mass and heat exchange for the flow of a nanofluids between two plates utilizing homotopy analysis technique. Hayat et al. [11] have scrutinized the steady laminar boundary layer flow of a Carreau nanoliquid via a stretching sheet and also discussed the impacts of brownian movement and thermophoresis. Acharya et al. [12] have presented the squeezing flow of nanoliquids Cu-H2O and Copper-kerosene among two plates using differential transformation technique. Kumar et al. [13] have presented heat transfer mechanism and application of nanofluid micro-channel using heat convection techniques. Irfan et al. [14] have built up a scientific connection for unsteady 3-D constrained Carreau nano-liquid convective flow through an extended surface. Rana and Bhargava [15] have presented laminar boundary flow of a nanoliquid via a nonlinear stretching sheet and the resulting nonlinear governing equations have been solved by variational finite element technique. Rohni et al. [16] have contemplated the time dependent flow through a ceaselessly contracting area with wall mass suction into H2O based nanofluids. Sheikholeslami and Ganji [17] have scrutinized about the nanofluids stream and heat transfer through the plates at equi-
distant utilising differential transformation technique. Sheikholeslami et al. [18] have introduced the shaky or unsteady flow of a nanofluids squeezing among two plates utilizing adomian decomposition scheme. Sheikholeslami et al. [19] have scrutinized the free convection of nanofluids using lattice Boltzmann technique. Ali et al. [20] have utilized bvp4c MATLAB scheme to investigate the Cross nanofluid flow in the process of melting. Shah et al. [21] have presented the solutions of Kellor box and bvp4c for the Cross nanofluid flow with chemical process on a melting surface. Ayub et al. [22] have presented bvp4c solutions to study the radiative 3D Cross nano-liquid motion with buoyancy opposing/assisting effects. Haider et al. [23] have obtained the numerical solutions for the movement of second grade nanofluid via a stretching surface using bvp4c scheme of MATLAB. Ayub et al. [24] have utilized bvp4c and Kellor box methods to discuss the cross nanofluid flow which is described for blood’s melting heat transfer.

In the areas of technology and science, the MHD flow of the non-Newtonian liquids has assembled a lot of attention. This attention due to the many applications in innovation and science, for example, structure for cooling of atomic reactors, blood stream estimation procedures, turbo apparatus development of heat exchangers, establishment of atomic quickening agents, and so on. Internal flows of MHD fluid in ducts and channels have received special attention. As the investigators are aware of the fact that magnetic field has the power to induce current in a progressive conductive liquid and in turn produces the forces on liquid with varying magnetic field itself. This entire basic idea is behind the introduction of MHD. When an electrically permitting fluid flows through a magnetic field the interaction between the electromagnetic field and hydrodynamics produces magneto-hydrodynamics. Impact of magnetic field on the nanofluids with various geometries has been examined by many investigators. Haq et al. [25] have presented the stagnation point flow of a nanoliquid with magneto-hydrodynamics and thermal radiation effects. Ellahi and Riaz [26] have
examined the impact of magneto-hydrodynamics through the pipe flow of a third-grade liquid with variable viscosity. Ramesh and Tushar [27] have presented the fundamental flows of MHD Carreau liquid introducing slip boundary conditions with radiation parameters and Joule heating utilizing regular perturbation method. The flow and heat transport of MHD Go-$H_2O$ nanofluids among two flat plates has been investigated through Dogonchi et al. [28] in the existence of thermal radiation utilizing Duan-Rach Approach. Pushpa et al. [29] scrutinized heat transfer of copper-water nanoliquid in a cylindrical annulus with baffle. Khan et al. [30] have presented the MHD squeeze flow of electrically conducting liquid among two parallel disks with suction / injection surface and hybrid nanofluid. Kandasamy et al. [31] have contemplated a nano-particle shapes through the pressed MHD nanofluids stream of ethylene glycol, water and motor oil through a permeable sensor area within the sight of thermal radiation. Nadeem and Akbar [32] have scrutinized the Peristaltic transfer of Newtonian MHD liquid with variable viscosity in a symmetric channel under the impact of heat transport using adomian decomposition method. Nadeem and Akram [33] have investigated the impacts of partial slip over the peristaltic stream of a magneto-hydrodynamic Newtonian liquid in non-symmetric channel. Warke et al. [34] have presented the MHD flow of micropolar liquid induced by a heated stretching sheet and nonlinear radiation. Srinivas and Kothandapani [35] have researched the impacts of mass and heat transport on the peristaltic transfer in a permeable space with consistent walls. Hayat and Hina [36] have presented the impacts of mass and heat transport on the MHD peristaltic flow into a planar channel through compliant walls. Some more recent and important studies in the direction of MHD nanoliquid flows in diverse directions can be seen in [37-51] and the references therein.

The MEMS and microfluidic technology possess many applications in different branches in engineering and science like lab-on-a-chip system in favour of drug conveyance, heat
exchangers, micro pumps, chemical separation devices, biomedical diagnostics and biochemical analysis. Heat transport and fluid flow inside micro-channels are involved in all the above-mentioned devices and instruments [52]. One of the significant difficulties in micro-scale transport phenomena is to have a reliable flow incitation. The most commonly utilized flow activation mechanism in micro devices is by creating a pressure gradient utilizing a pumping device. Such procedures are cumbersome, utilize moving parts to generate flow, and requires frequent upkeep. During the previous decade, utilization of electro-kinetics as a stream impelling system in micro-devices is getting increasingly famous. Stream incitation in micro-channels because of a remotely applied electric field has discovered striking applications in various micro-fluidics devices and systems. Keeping all these applications in mind, many researchers have focussed their research in micro-channel flows with electric fields. Sridhar and Ramesh [53] have presented the analytical investigation on the electro-magneto-hydrodynamic (EMHD) radiative Jeffrey nano-liquid flow in an asymmetric peristaltic mechanism. Munawar and Saleem [54] have discussed the motion of Williamson hybrid nano-liquid through the ciliated walls with EMHD effects. Ijaz et al. [55] have provided Mathematica solutions for the EMHD nano-bio-fluid flow in a peristaltic curved plate. Mandal et al. [56] have discussed the propulsion of super imposed liquids in the strait confinements in the presence of EMHD influences. Ghorbani et al. [57] have the finite difference solutions for the effect of EMHD on the motion of Carreau-Yasuda liquid through the rectangular micro-channel. Murtaza et al. [58] have used the Laplace transform technique to discuss the flow situation of EMHD Maxwell nanofluid in a channel. Noreen et al. [59] have discussed the EMHD nanofluid motion through asymmetric peristaltic plates with various zeta potentials. Mahapatra et al. [60] have analyzed the motion of electro-osmotic viscoelastic liquid over the surfaces of high zeta potentials.
Enliven through these findings, the present work deals with a novel model for simulating the flow of solar magneto-hydrodynamic Carreau-nanofluid due to electro-osmosis which can be considered as generalization of the viscous fluid model. The working liquid is a magnetized Carreau-nanofluid which includes a base liquid containing suspended magnetic nanoparticles. The curiosity of current work is the consolidation of magneto-hydrodynamics and nanofluids dynamics to design a hybrid solar pump system model. The problem is first modelled and then the perturbation solutions are calculated for the resultant system of equations. The graphical results are prepared for velocity, solutal nano-particle concentration, temperature, Nusselt number and Sherwood number. This model can assist with examining the fluid dynamics problems administered by electro-osmosis mechanism. The structure of this article as : The next section describes the modelling of the problem along through the solutions. Section 3 provides the numerical outcomes and discussion, and the conclusions of the study is given in Section 4.

2. Modeling

Consider an incompressible flow of a Carreau-nano-liquid in the micro-channel with distance \(2h\). The Cartesian system \((x, y)\) is engaged to study the present problem. In this system, the \(x\)–axis is in the way of fluid movement and the \(y\)–axis is taken vertical to it with its origin at the micro-channel center. In transverse way of the motion, liquid is conceived to be electrically conducting with an applied magnetic field of magnitude \(B_0\). Fluid particles are confined between two walls. The upper and lower walls have been kept up at the steady temperature \(T_0\) and \(C_0\) is the nano-particle volume fraction for these walls. Electro-osmosis flow has also been taken into consideration and it gives bulk motion of ionized fluid within the sight of fixed charged surface. Schematic delineation of electro-osmotic stream along horizontal micro-channel is depicted through Figure 1. From this representation, it has been
observed that the surface of channel walls gets attached with net-negative charges due to which –ve ions get repulsed away from wall and +ve ions get pulled in towards the wall, shaping an EDL (Electric Double Layer) near the channel wall. At the point where the EDL collaborates via the applied electric field, there generates the electro-osmotic flow. The +vely charged ions of EDL layer are magnetized near cathode and repulsed via the anode resulting in net transport of ionized fluid towards electric field.

The equations of continuity, motion, energy, and solutal-nano-particle concentration for the Carreau-nanofluids can be put in the form [27, 61]:

\[ \nabla \cdot \bar{q} = 0, \]  
\[ \rho_d \left( \frac{\partial \bar{q}}{\partial t} + (\bar{q} \cdot \nabla \bar{q}) \right) = -\nabla P + \nabla \cdot \bar{S} + \bar{J} \times \bar{B} + \bar{f}_g + \rho_e E_x, \]  
\[ \left( \rho c_p \right)_d \left( \frac{\partial \bar{q}}{\partial t} + (\bar{q} \cdot \nabla T) \right) = k_d \nabla^2 T + D_c \nabla^2 C + \nabla \cdot q_x + \sigma_{ef} E_x^2, \]  
\[ \frac{\partial C}{\partial t} + (\bar{q} \cdot \nabla C) = D_s \nabla^2 C + D_d \nabla^2 T - k (C - C_0), \]

where the velocity vector is \( \bar{q} = (u, 0, 0) \), \( \rho_d \) represents the effective density of nanofluids, \( P \) is the pressure, \( \bar{J} \) denotes the electric current density, \( \bar{B} \) represents the total magnetic field, \( \bar{f}_g = \rho_g g \left( \beta_r (T - T_0) + \beta_c (C - C_0) \right) \), \( \rho_e \) denotes electrical charge density, \( E_x \) denotes applied electrical field, \( c_p \) denotes specific heat at constant pressure, \( \left( \rho c_p \right)_d \) represents the heat capacity of the nanofluids, \( T \) denotes the temperature, \( t \) is the time, \( k_{ef} \) denotes the effective thermal conductivity, \( \sigma_{ef} \) denotes the electrical conductivity, \( C \) is the solute concentration, and \( \bar{S} \) is the extra stress tensor, which is given by [11, 27]
\[ \bar{S} = \left( \eta_{\infty} + (\eta_0 - \eta_{\infty}) (1 + \Gamma \bar{\gamma})^{w} \right)^{\frac{w-1}{a}} \gamma_{ij}, \]  

(5)

where

\[ \bar{\gamma} = \sqrt{\frac{1}{2} \sum_i \sum_j \bar{\gamma}_{ij} \bar{\gamma}_{ji}} = \sqrt{\frac{1}{2} \Pi}, \]  

(6)

here \( \Pi \) stands for second invariant strain tensor, \( \eta_{\infty} \) denotes viscosity of fluid at infinite shear rate, \( \eta_0 \) denotes viscosity of fluid at zero shear rate, \( \Gamma \) and \( a \) stand for non-Newtonian liquid quantities, \( w \) denotes dimensionless power law index and \( \bar{\gamma} \) stands for shear rate. The limiting cases can be captured for shear thickening and shear thinning impacts when \( w < 1 \) and \( w > 1 \) respectively. In the ongoing investigation, \( \mu_{\infty} = 0 \) is considered. Besides, for \( a = 2 \) this model provides the Carreau fluid model, and for \( \Gamma = 0 \) or \( w = 1 \) it is a Newtonian fluid model.

According to notable Poisson-Boltzmann equation for micro-channel, the electric potential \( \phi \) is defined as [61]:

\[ \nabla^2 \phi = -\frac{\rho_e}{\varepsilon_{ef}}, \]  

(7)

here \( \rho_e = e \mathbf{z}(\bar{n}_+ - \bar{n}_-) \) represents electrical charge density, \( \bar{n}_+ \) and \( \bar{n}_- \), respectively, denote positive and negative ions containing bulk concentration, and \( \varepsilon_{ef} \) denotes medium permittivity.

Nernst-Planck condition is characterized to ascertain the potential dispersion and portrays the charge number density for every specie as:

\[ \frac{\partial \bar{n}_z}{\partial t} + (\bar{q}, \nabla) \bar{n}_z = D \nabla^2 \bar{n}_z \pm \frac{D e}{k_B T} \left( \nabla \cdot (\bar{n}_z \nabla \phi) \right), \]  

(8)
where $D$ denotes to chemical species diffusivity. Here, Einstein formula for the species mobility is considered for both the species with equal ionic diffusion coefficients. It is appropriate to stabilize the conservation equations, in this manner non-dimensional parameters have been introduced as:

\[ \bar{x} = \frac{x}{h}, \bar{y} = \frac{y}{h}, \bar{u} = \frac{u}{U}, M = \sqrt{\frac{\sigma_\text{fluid}}{\eta_0} B_0 h}, \bar{T} = \frac{T - T_0}{T_0}, \bar{C} = \frac{C - C_0}{C_0}, \gamma = \frac{\Gamma U}{h} \]

\[ G_n = \frac{\rho_c g h \beta T_0}{\eta_0 U}, G_{rc} = \frac{\rho_c g h \beta c T_0}{\eta_0 U}, \kappa = \frac{2n_0}{h}, \bar{n} = \frac{n}{n_0}, \xi_1 = \frac{L_1}{h}, \xi_2 = \frac{L_2}{h}, \]

\[ U_{HS} = -\frac{E_c \varepsilon_\text{fluid} \zeta}{\eta_0 U}, S = -\frac{\sigma_\text{fluid} E^2 h^2}{k_c T_0}, N_c = \frac{D_c C_0}{k_c T_0}, N_{ci} = \frac{D_{ci} T_0}{D_c C_0}, \beta = \frac{L}{h}. \quad (9) \]

Here $M$ is the Hartmann number, $u$ denotes the dimensionless axial component, $\bar{T}$ represents the non-dimensional temperature, $\bar{p}$ denotes the non-dimensional pressure, $\bar{C}$ is the non-dimensional solute nanoparticle concentration, $G_{rc}$ denotes the solutal Grashof number, $G_n$ is the thermal Grashof number, $n$ is the dimensionless ions bulk concentration in the electrolyte, $U_{HS}$ denotes velocity of the Helmholtz-Smoluchowski, $S$ is the Joule heating parameter, $N_c$ represents the Dufour thermo-diffusive parameter, $N_{ci}$ is parameter of the Soret diffuso-thermo and $\phi$ represents dimensionless electric potential term. In steady state, the condition of the Poisson-Boltzmann is diminished to:

\[ \frac{d^2 \phi}{dy^2} = -\kappa^2 \left( \frac{n_i - n}{2} \right), \quad (10) \]

where $\kappa$ stands for the electro-osmotic term. The ionic dissemination can be achieved through Nernst Planck equation, which is defined as:

\[ \frac{d^2 n_i}{dy^2} \pm \frac{d}{dy} \left( n_i \frac{d \phi}{dy} \right) = 0, \quad (11) \]
exposed to the conditions, at $\phi = 0$, $n_z = 1$ and $\frac{\partial n_z}{\partial y} = 0$ at $\frac{\partial \phi}{\partial t} = 0$. Utilizing these, obtained a much-anticipated Boltzmann dispersion for the ions, and it yields as follows:

$$n_z = \varepsilon^\phi. \quad (12)$$

By substituting Eq. (12) in Eq. (10), obtained the Poisson-Boltzmann model for the electrical potential dissemination in the electrolyte, and it yields:

$$\frac{d^2 \phi}{dy^2} = \kappa^2 \sinh (\phi). \quad (13)$$

Further, under the estimate of low-zeta potential eq.(13) is linearized, it tends to be disentangled as:

$$\frac{d^2 \phi}{dy^2} = \kappa^2 \phi. \quad (14)$$

The Eq. (14) can be simplified under the conditions $\frac{\partial \phi}{\partial t} = 0$ and $\phi|_{y=1} = 1$, whereas potential function solution can be composed as:

$$\phi = \frac{\cosh (\kappa y)}{\cosh \kappa}. \quad (15)$$

Using the above-mentioned assumptions along with the dimensionless quantities, the subsequent governing differential equations (after dropping the bars) can be composed as:

$$\frac{d^2 u}{dy^2} + \frac{3}{2} (w-1) \gamma^2 \left( \frac{du}{dy} \right)^2 \left( \frac{d^2 u}{dy^2} \right) - M^2 u + \kappa^2 U_{HS} \frac{\cosh (\kappa y)}{\cosh \kappa} + G_n T + G_r C = 0, \quad (16)$$

$$\left(1 + Rn Pr\right) \frac{d^2 T}{dy^2} + N_\infty \frac{d^2 C}{dy^2} + S = 0, \quad (17)$$

$$\frac{d^2 C}{dy^2} + N_\infty \frac{d^2 T}{dy^2} - \alpha Sc C = 0, \quad (18)$$
with the non-dimensional form of slip boundary condition as

\[
   u - \beta \left( \frac{du}{dy} + \frac{1}{2} (w-1) y^2 \left( \frac{du}{dy} \right)^3 \right) = U_1, \quad T - \xi_1 \frac{dT}{dy} = 0, \quad C - \xi_2 \frac{dC}{dy} = 0 \quad \text{at} \quad y = -1, \quad (19)
\]

\[
   u + \beta \left( \frac{du}{dy} + \frac{1}{2} (w-1) y^2 \left( \frac{du}{dy} \right)^3 \right) = U_2, \quad T + \xi_1 \frac{dT}{dy} = 0, \quad C + \xi_2 \frac{dC}{dy} = 0 \quad \text{at} \quad y = 1. \quad (20)
\]

In general, it is considered \((U_1 = 0 \text{ and } U_2 = 0)\). But it is extended the current findings for diverse values of \(U_1\) and \(U_2\) (means, the upper and lower plates are moving in various direction with constant velocities) graphically. Using the eqs. (16)-(18) with the assistance of boundary conditions (19)-(20), the expressions for solutal nano-particle concentration, temperature and velocity distributions are obtained as

\[
   C = C_1 e^{my} + C_2 e^{-my} - \frac{R}{m^2}, \quad (21)
\]

\[
   T = B_1 e^{my} + B_2 e^{-my} + B_3 y^2 + C_3 y + C_4, \quad (22)
\]

\[
   u = C_5 e^{My} + C_6 e^{-My} + B_1 \left( e^{My} + e^{-My} \right) + B_5 e^{my} + B_6 e^{-my} + B_7 y^2 + B_8 y + B_9 + \gamma^2 \left( C_7 e^{My} + C_8 e^{-My} \right) + B_1 y e^{-ym} + B_1 y e^{ym} + B_6 e^{-ym} + B_6 e^{ym} + B_7 e^{My} + B_7 e^{-My} + B_8 e^{My} + B_8 e^{-My} + B_9 e^{My} + B_9 e^{-My} + B_{10} e^{M(m-y)} + B_{10} e^{(m+y)} + B_{11} e^{(m-y)M} + B_{11} e^{(y-M)} + B_{12} e^{(m-y)M} + B_{12} e^{(y-M)} + B_{13} e^{(m-y)M} + B_{13} e^{(y-M)} + B_{14} e^{(m-y)M} + B_{14} e^{(y-M)} + B_{15} e^{(m-y)M} + B_{15} e^{(y-M)} + B_{16} e^{(m-y)M} + B_{16} e^{(y-M)} + B_{17} e^{(m-y)M} + B_{17} e^{(y-M)} + B_{18} e^{(m-y)M} + B_{18} e^{(y-M)} + B_{19} e^{(m-y)M} + B_{19} e^{(y-M)} + B_{20} e^{(m-y)M} + B_{20} e^{(y-M)} + B_{21} e^{(m-y)M} + B_{21} e^{(y-M)} + B_{22} e^{(m-y)M} + B_{22} e^{(y-M)} + B_{23} e^{(m-y)M} + B_{23} e^{(y-M)} + B_{24} e^{(m-y)M} + B_{24} e^{(y-M)} + B_{25} e^{(m-y)M} + B_{25} e^{(y-M)} + B_{26} e^{(m-y)M} + B_{26} e^{(y-M)} + B_{27} e^{(m-y)M} + B_{27} e^{(y-M)} + B_{28} e^{(m-y)M} + B_{28} e^{(y-M)} + B_{29} e^{(m-y)M} + B_{29} e^{(y-M)} + B_{30} e^{(m-y)M} + B_{30} e^{(y-M)} + B_{31} e^{(m-y)M} + B_{31} e^{(y-M)} + B_{32} e^{(m-y)M} + B_{32} e^{(y-M)} + B_{33} e^{(m-y)M} + B_{33} e^{(y-M)} + B_{34} e^{(m-y)M} + B_{34} e^{(y-M)} + B_{35} e^{(m-y)M} + B_{35} e^{(y-M)} + B_{36} e^{(m-y)M} + B_{36} e^{(y-M)} + B_{37} e^{(m-y)M} + B_{37} e^{(y-M)} + B_{38} e^{(m-y)M} + B_{38} e^{(y-M)} + B_{39} e^{(m-y)M} + B_{39} e^{(y-M)} + B_{40} e^{(m-y)M} + B_{40} e^{(y-M)} + B_{41} e^{(m-y)M} + B_{41} e^{(y-M)} + B_{42} e^{(m-y)M} + B_{42} e^{(y-M)} + B_{43} e^{(m-y)M} + B_{43} e^{(y-M)} + B_{44} e^{(m-y)M} + B_{44} e^{(y-M)} + B_{45} e^{(m-y)M} + B_{45} e^{(y-M)} + B_{46} e^{(m-y)M} + B_{46} e^{(y-M)} + B_{47} e^{(m-y)M} + B_{47} e^{(y-M)} + B_{48} e^{(m-y)M} + B_{48} e^{(y-M)} + B_{49} e^{(m-y)M} + B_{49} e^{(y-M)} + B_{50} e^{(m-y)M} + B_{50} e^{(y-M)} + B_{51} e^{(m-y)M} + B_{51} e^{(y-M)} + B_{52} e^{(m-y)M} + B_{52} e^{(y-M)} + B_{53} e^{(m-y)M} + B_{53} e^{(y-M)} + B_{54} e^{(m-y)M} + B_{54} e^{(y-M)} + B_{55} e^{(m-y)M} + B_{55} e^{(y-M)} + B_{56} e^{(m-y)M} + B_{56} e^{(y-M)} + B_{57} e^{(m-y)M} + B_{57} e^{(y-M)} + B_{58} e^{(m-y)M} + B_{58} e^{(y-M)} + B_{59} e^{(m-y)M} + B_{59} e^{(y-M)} + B_{60} e^{(m-y)M} + B_{60} e^{(y-M)} + B_{61} e^{(m-y)M} + B_{61} e^{(y-M)} + B_{62} e^{(m-y)M} + B_{62} e^{(y-M)} + B_{63} e^{(m-y)M} + B_{63} e^{(y-M)} + B_{64} e^{(m-y)M} + B_{64} e^{(y-M)} + B_{65} e^{(m-y)M} + B_{65} e^{(y-M)} + B_{66} e^{(m-y)M} + B_{66} e^{(y-M)} + B_{67} e^{(m-y)M} + B_{67} e^{(y-M)} + B_{68} e^{(m-y)M} + B_{68} e^{(y-M)} + B_{69} e^{(m-y)M} + B_{69} e^{(y-M)} + B_{70} e^{(m-y)M} + B_{70} e^{(y-M)} + B_{71} e^{(m-y)M} + B_{71} e^{(y-M)} + B_{72} e^{(m-y)M} + B_{72} e^{(y-M)} + B_{73} e^{(m-y)M} + B_{73} e^{(y-M)} + B_{74} \),

where \(C_i \)'s \((i = 1, 2, \ldots, 8)\) and \(B_j \)'s \((j = 1, 2, \ldots, 74)\) are basic algebraic calculations computed by Mathematica programming software.
3. Results and conversation

Here the effect of numerous physical parameters through the temperature $T$, velocity $U$, solutal nanoparticle concentration $C$, Sherwood number $Sh$ and $Nu$ have been considered. It is analysed from these Figs that velocity, temperature, and solute $NPs$ concentration profiles are almost parabolic in nature.

Figure 2(a) is sketched to see the alteration of chemical reaction term $\alpha$ via the velocity dissemination. It is seen that the velocity is getting diminished via enhancing estimations of chemical reaction parameter when Joule heating parameter is positive. This is because of the chemical reaction in this system brings about use of the chemical, and hence outcomes in velocity portray abatement. But without Joule heating parameter and with negative values, the velocity increases as the value of chemical reaction parameter increases. Figure 2(b) is sketched to visualize the conduct of velocity dissemination for numerous values of $\beta$. The velocity enhancement with rising values of velocity slip term in all instances of Joule heating parameter is observed from Fig. 2(b). It is physically reasonable on the grounds that, while the slipping of liquid takes place at the boundary, liquid velocity is not equivalent to the boundary at that point. Also, as more liquid slips at the boundary, less in its velocity and is influenced through the boundary movement. From Fig. 2(c) the velocity lessens via enhancing estimations of $M$ in every case of the three referenced Joule heating parameters. This is due to Lorentz force generated through the application of constant magnetic field which offers resistance opposing the fluid motion and hence decreasing the flow. Figure 2(d) tends to visualize the conduct of velocity dissemination for numerous values of electro-
osmosis term $\kappa$ in three different situations of Joule heating parameters. It is found from the Fig. 2(d) that the velocity increments with augmenting estimations of electro-osmosis parameter. Figure 2(e) is depicted to see the behavior of motion of the superior plate via the velocity distribution. Increase in velocity in all the cases (such as when the movement of the upper plate in the opposite direction of the flow, both the plates are fixed and the movement of top plate with the constant velocity) is observed. In all these cases, it is additionally noticed that the higher velocities are detected on account of the upper plate moving with constant velocity. It is genuinely supported that, when the plate is moving, the layers which are near to the plate gets affected, with this explanation the higher velocities are noticed. The similar results can be seen with movement of lower plate (see Figure 2(f)).

Figure 3(a) is sketched to analyse variations of $\alpha$ on the temperature dissemination. It is found that, the temperature diminishes through enhancing $\alpha$ when Joule heating term is positive, and the temperature increases on account of Joule heating parameter is negative. Figure 3(b) illustrates the variation of Soret diffuso-thermo parameter with respect to the temperature, and from this figure, the temperature improves via boosting estimations of Soret diffuso-thermo term on the account of positive estimation of Joule heating term, whereas in the event of negative estimation of Joule heating term, the pattern is reversed. Figure 3(c) is committed to see the variations of $Pr$ on the temperature profile. It is seen from this plot that with rising estimations of $Pr$, there is an ascent in temperature for negative Joule heating term, the pattern is inverted on the account of positive Joule heating term. Since an expansion in Prandtl number offers ascend to more fragile thermal diffusivity and more slender boundary layer thickness. Bigger Prandtl number have lower thermal diffusivity and lower $Pr$ have higher thermal diffusivity. For lesser $Pr$, the higher temperature dissemination is noticed. The similar behavior is observed in Figures 3(d)-(e) with enhancing $Rn$ and $Sc$. It is due to an augment in $Rn$ prompts decline in the boundary layer thickness, and upgrades the
heat transport rate on dissolving area within the sight of chemical impact. Plot 3(f) is set up to see the nature of temperature slip term. It is seen that the temperature increments with rising values of temperature slip parameter when Joule heating parameter is positive, and the temperature decreases on account of Joule heating parameter is negative.

Figure 4(a) has been plotted to study the impact of $\alpha$ over the solutal nanoparticle concentration. From this figure, the solutal nanoparticle concentration is a reducing function of $\alpha$ for negative estimation of Joule warming parameter because of this explanation that chemical reaction brings about utilisation of the chemical species, so concentration profile diminishes and ascending function for positive estimation of Joule heating parameter. Figure 4(b) depicts that with rising $N_n$, solutal nanoparticle concentration profile keeps increasing for negative Joule heating term and lessens for positive Joule heating term. Figure 4(c) established to see the variations of different values of $Pr$ on the solutal nanoparticle concentration distribution. It is depicted from Fig.4(c) that; a rise of $Pr$ leads to reduce in the solutal nanoparticle concentration profile for $S = -2$ and when $S = 2$, the trend is opposite. Figure 4(d) is plotted to observe the variations of $Rn$ via the solutal nanoparticle concentration distribution. Also noticed that, when Joule heating parameter is positive, the solutal-nano-particle concentration enhances with boosting values of radiation parameter and when Joule heating parameter is negative, the solutal nanoparticle concentration diminishes with boosting values of radiation parameter. A similar behavior is followed with the increase of Schmidt number $Sc$ (see Figure 4(e)). Truly, increasing estimations of $Sc$ relate to high rate of viscous dissemination which causes the solutal-nano-particle concentration of a fluid to increment. The impact of concentration slip parameter through the solutal $NP$ concentration is described in Fig. 4(f). It is obvious from this figure that, the concentration augments with enhancing values of concentration slip parameter for negative values of Joule heating term.
and the pattern is over turned for positive value of Joule heating term. In all the cases, no variation is noticed for $S = 0$.

Figure 5(a)-(e) is sketched to visualize the features between radiation parameter $Rn$ and Nusselt number $Nu$. From Figure 5(a)-(c), the $Nu$ augments with a rise of $\alpha$, $Pr$ and $Sc$ for negative values of Joule heating term, the contrary pattern is pursued for positive Joule heating term. This is on the grounds that a development in the radiation term $Rn$ prompts decline in the boundary layer thickness and upgrades the heat transport rate on melting area within the sight of chemical impact. Figure 5(d)-(e) is plotted to contemplate the impacts of $N_{ct}$ and $N_{nc}$ on $Nu$. It is noticed that, $Nu$ diminishes via boosting values of $N_{ct}$ and $N_{nc}$ for $S = -2$ and the case is inverted for $S = 2$. To study the effects between radiation term $Rn$ and Sherwood number $Sh$, Figure 6(a)-(e) has been prepared. It is illustrated from Fig. 6(a)-(c) that, through augment values of $\alpha$, Prandtl number and $Sc$, there is an ascent in Sherwood number for the non-negative Joule heating term, the pattern is reversed for non-positive Joule heating term. Figure 6(d)-(e) are graphed to visualise the conduct of $N_{ct}$ and $N_{nc}$ on the $Sh$. It is depicted that, $Sh$ rises via increasing values of $N_{ct}$ and $N_{nc}$ for negative values of Joule heating term and Sherwood number decreases for positive values of Joule heating term. Without Joule heating term, no change in $Nu$ and $Sh$ is detected. It is clearly mentioned from the figure 7 that, the reported outcomes are in close agreement via the existing results of Ramesh and Tushar [27] with the limiting cases of the current analysis.

4. Conclusions

The current study is a worthwhile attempt to undertake the study the transport of Carreau-nanofluid in the infinite parallel micro-channels. The impacts of radiation, magnetic field, chemical reaction and Helmholtz-Smoluchowski velocity have been considered into account.
The solutions for the temperature, velocity and solutal nanoparticle concentration have been introduced utilizing regular perturbation technique considering Carreau-liquid parameter as perturbation parameter. The numerical simulation is presented to see the nature of the flow quantities.

The significant perceptions of research undertaken are summed up in the following form:

- The velocity is a rising function of chemical reaction term and slip velocity parameter for the case of negative Joule heating term, and the pattern is inverted for positive Joule heating term.
- There is a decrement in velocity with enhancement of Hartmann number in all the cases of Joule heating term.
- The temperature diminishes with increment of $\alpha$, $Pr$, $Rn$, $Sc$ with positive Joule heating term.
- The solutal nano-particle concentration increments via increment of $\alpha$, $Pr$, $Rn$ and $Sc$ for the positive Joule heating term.
- The very similar observations are noticed in $Nu$ and $Sh$ through all the cases of Joule heating parameter.
- The Newtonian liquid model outcomes can be caught by setting $\gamma = 0$ and $w = 1$.

The findings of the current mathematical scrutiny will the benchmark for simulating the more generalized model in three-dimensions for the nanofluid/hybrid nanofluid flow in various directions with thermophysical properties.

**Competing Interests:**

There is no conflict of interests.
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Fig. 1. Physical sketch of the electro-osmosis modulated flow of a Carreau-nano-liquid in a micro-channel.
Fig. 2. Profiles of velocity for various involved fluid parameters.
Fig. 3. Temperature distributions via various involved fluid terms.
Fig. 4. Solutal nanoparticle concentration via various involved liquid parameters.
Fig. 5. $Nu$ distributions via various involved fluid terms.
(b) \( Sc = 1, 1.5, 2 \)

(c) \( Pr = 1, 2, 3 \)
Fig. 6. $Sh$ distributions for diverse involved fluid parameters.
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