CREDIT POLICY FOR AN INVENTORY MODEL OF A DETERIORATING ITEM HAVING VARIABLE DEMAND CONSIDERING DEFAULT RISK

Rituparna Mondal^a, Prasenjit Pramanik ^{b,*}, Ranjan Kumar Jana ^{a}, Manas Kumar Maiti ^{c}

^{a} Applied Mathematics and Humanities Department (AMHD), Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat 395007, India.

^{b} Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore, Paschim-Medinipur, West Bengal 721102, India.

^{c} Department of Mathematics, Mahishadal Raj College, Mahishadal, Purba-Medinipur, West Bengal 721628, India.

ABSTRACT. In this study, a supplier-retailer-customer supply chain has been proposed for a deteriorating item with expiration time and dynamic deterioration rate. Here, the supplier adopt full credit policy for the retailer to enhance the retailer's order volume. This facility influences the retailer to provide some partial credit opportunity to the customers to boost the demand. For this credit policy, the retailer always faces a risk due to defaulters, which is termed as default credit risk. The default credit risk is considered in more realistic manner, which depends on the customers' partial credit period and credit amount. The market demand is influenced by customers' credit amount, customers' credit period and retail price of the item. Optimal decision is searched by maximizing the average profit of the system. For the search process, an artificial bee colony algorithm is implemented, tested and used. Illustration of the model is done with some hypethetical examples.

Key words : Expiration time; Dynamic deterioration; Credit policy; Credit amount; Default credit risk.

1. **Introduction**

In any supply chain, among different parameters, market demand is the key factor as revenue of the system fully comes from the sell revenue of the item. This phenomenon influences both the supplier and retailer to adopt different promotional activities to improve their sales. Trade credit is one of the important strategies to improve the demand of each player involved in the system. To increase the retailer's order size, the supplier offers a credit period to the retailer. This opportunity promotes the retailer to adopt some credit policy for his/her customers. Pioneering work in this direction was made by Goyal[1] incorporating credit policy. Afterwards, there are several research works done incorporating full credit policy[2, 3, 4, 5, 6, 7] and partial credit policy[8, 9, 10, 11, 12, 13].

Though credit policy is a good promotional tool for the enhancement of the market demand of any item, any credit policy involves credit risk. In fact, in any business transaction under trade credit policy, there are some business bonding between the supplier and the retailer, but there is no such bonding between the retailer and the customers. In reality customers are floating in nature and so some of the customers normally do not follow business ethics and move to another shop without paying the credit amount. For that, researchers developed their models[8, 12] considering partial credit policy in retailer-customers level to decrease the default credit risk. However, partial credit policy is an excellent attempt to decrease the credit risk, but still it is very critical to draw the optimal decision for the decision maker(DM) as the number of defaulters is not known. To overcome this difficulty, Pramanik *et al.*[14] considered a percentage of total customers as defaulters. But in reality, it is impossible to predict the amount of defaulters at the beginning. So it is more appropriate to consider the default credit risk as a function of customers' credit period and credit amount as the number of defaulters increases/decreases with customers credit amount and credit period proportionally. According to the authors best knowledge, there is only one article[15], considered the default risk as a function of customers' credit period. But number of defaulters may vary with the customers' credit amount also. So default risk must be a function of credit amount and the

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duration of credit period. Though it is a real life phenomenon, till now no researcher developed their model incorporating it.

In 1963, Ghare and Schrader[16] first developed an EOQ model on deteriorating items. Afterwards, in different circumstances, researchers developed their models for deteriorating items assuming constant deterioration rate[2, 17], time dependent deterioration rate[8, 18]. Recently, Pramanik et al.[19] gives a new direction for modelling the inventory models of deteriorating items introducing the interest paid situation for the deteriorated units. As far as authors knowledge go, till now none has introduced any supply chain model on deteriorating items under trade credit incorporating default credit risk, which depends on credit amount and credit period.

During last two decades, soft computing algorithms are successfully applied to solve inventory control problems[10, 11, 17, 20, 21, 22]. Among different soft computing algorithms, recently Artificial Bee colony (ABC) algorithm draws more attention due to its accuracy of searching the optimal solution with significantly large decimal places. Recently Pramanik and Maiti[23] uses this approach to solve an inventory control problem. Due to this reason here ABC approach is followed to draw marketing decision.

Removing the above mentioned shortcomings, the following new concepts are introduced in the present investigation.

- Two level trade credit policy with default customers is considered.
- Here, number of default customers depend not only on the customers' credit period but also on the credit amount which have not been considered till now.
- Very few have investigated the inventory policy of deteriorating items with default customers. Here, time dependent deteriorating items is considered.
- ABC algorithm has been modified suitably and applied to the investigated problem.
- Contradictions between the number of defaulters and credit amount as well as credit period are pointed out and appropriate managerial decisions are derived.

In a nutshell, a supply chain model has been formulated for a deteriorating item under two level partial credit policies. Due to credit opportunity of the customers, there are some defaulters, who are not come back to repay the credit amount. The default credit risk depends on customers' credit amount and credit period. Market demand is influenced by the customers' credit amount, customers' credit period and retail price of the item. To find the optimal decision for the DM, an ABC algorithm is implemented, tested and used. For the real life implementation of the model, it is justified with some hypothetical data.

2. Notations and Assumptions

Throughout this paper, the following notations are used and assumptions are made.

2.1. **Notations.**

    (i) $I(t)$, retailer's inventory level at any instant t.

    (ii) Q, retailer's order quantity.

    (iii) A, per order ordering cost for the retailer in \$.

    (iv) h, unit time inventory holding cost in \$.

    (v) p, unit purchase cost in \$.

    (vi) β, mark-up of selling price.

    (vii) s, unit selling price in \$, where $s = \beta p$.

    (viii) T , retailer's replenishment time(a decision variable).

    (ix) $\theta(t)$, dynamic deterioration rate, depends on time t, where $0 \leq \theta(t) < 1$.

    (x) E_p, expiration period of the product.

    (xi) I_e, rate of interest offered by the bank per \$ per unit time.

    (xii) I_p, rate of payable interest per \$ per unit time to the bank.

    (xiii) Z, per unit time profit of the retailer in \$.

2.2. **Assumptions.**

    (i) A supplier renders a full credit period (M) to its retailer to increase the order amount. This opportunity encourages the retailer to adopt a credit policy for the customers. Due to credit risk, the retailer provides a credit duration (N) to the customers on a part $(\alpha(0 < \alpha < 1))$ of the procured amount and the rest amount is to be paid by the customer at the time of purchase.

At the end of his/her credit duration, the retailer has to pay interest to the supplier on the outstanding amount at a rate I_p and the retailer can earn interest on the sales revenue at a rate I_e in the grace period. Under the circumstance, the retailer pays the total outstanding amount to the supplier just after the end of grace period by a bank-loan with the same rate of payable interest I_p and continue to repay the bank loan as soon as units are sold.

(ii) Usually, the customers pay their credit amount just after the grace period, otherwise, the customer is treated as defaulter, i.e., no credit amount should be refunded from the default customer.

(iii) All the units of the item should be sold within the expiration period otherwise the units will be fully deteriorated and the deteriorated units are neither repaired nor sold. The dynamic deterioration rate, $\theta(t)$, is expiration time dependent and is of the form:

$$\theta(t) = \frac{1}{1 + E_p - t}, 0 \le t \le T \le E_p \tag{1}$$

Equation (1) implies that the retailer's replenishment time should not exceed the expiration period of the item.

(iv) In a business transaction, default credit risk depends on the credit amount and the credit period, i.e., in the presence of large(short) credit period the credit risk is very high(low) and also in the presence of large (small) credit amount the risk is significantly high(low). Moreover the credit period and the credit amount both simultaneously influence the credit risk. Due to this reason, in this study, the default risk of the retailer, $F(\alpha, N), (0 \le F(\alpha, N) < 1)$ is assumed as a function of the credit amount(α) and the credit period(N) and is of the form:

$$F(\alpha, N) = 1 - e^{-a_1 \alpha N} \tag{2}$$

(v) In reality, the market demand is influenced by different factors such as selling price, credit policy, etc. In this study, the demand of the item is influenced by the credit policy and the retail selling price. Also it is well known that the effect of selling price on demand is always inversely proportional to some extent. Here the market demand is designed in such a manner that in the absence of credit policy, the demand is influenced by the selling price only and vice versa. In the presence of the credit policy, the demand increases with the duration of credit period and the credit amount at a decreasing rate. D is the per unit time market demand, which is a function of retailers' credit amount, credit period and selling price and is in the form:

$$D = \frac{a + b(1 - e^{-c\alpha^r N^q})}{s^\gamma} \tag{3}$$

where a, b, c, c, r, q and γ are the positive real numbers chosen form the expert's opinion for the best fit of the demand.

3. **Mathematical formulation and analysis of the model with defaulters (Model-1)**

According to the assumptions, the retailer begins each cycle by ordering Q units of the item; replenishment is instantaneous and the inventory depleted gradually due to the market demand and deterioration. Thus at any time, t, the inventory level, $I(t)$, can be mathematically represented as:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D; 0 \le t \le T. \tag{4}$$

with boundary conditions $I(0) = Q$ and $I(T) = 0$.

Solving Equation (4) we have

$$I(t) = D(1 + E_p - t)\ln\left(\frac{1 + E_p - t}{1 + E_p - T}\right) \qquad \text{for } 0 \le t \le T \tag{5}$$

$$\text{Thus, } Q = I(0) = D(1 + E_p)\ln\left(\frac{1 + E_p}{1 + E_p - T}\right) \tag{6}$$

The following costs and revenues govern the retailer's average profit:

$$\text{(i) Sales revenue, } SR \quad = \quad \frac{1}{T}\int_0^T (1-\alpha)Dsdt + \frac{1}{T}\int_0^T D\alpha s[1-F(\alpha,N)]dt = Ds[(1-\alpha)+\alpha e^{-a\alpha N}] \tag{7}$$

$$\text{(ii) Ordering cost, } OC \quad = \quad \frac{A}{T} \tag{8}$$

$$\text{(iii) Purchase cost, } PC \quad = \quad \frac{Qp}{T} = \frac{p}{T}I(0) = \frac{pD(1+E_p)}{T}\ln\left(\frac{1+E_p}{1+E_p-T}\right) \tag{9}$$

$$\text{(iv) Holding Cost, } HC \quad = \quad \frac{h}{T}\int_0^T I(t)dt = \frac{hD}{T}\left[\frac{(1+E_p)^2}{2}\ln\left(\frac{1+E_p}{1+E_p-T}\right)+\frac{T^2}{4}-\frac{(1+E_p)T}{2}\right] \tag{10}$$

(v) Payable and earned interest, as calculated below.

Determination of interest amounts (payable and earned):

To determine the average payable interest and average earned interest, the following two situations are studied :

Case 1 ($N < M$): This situation is mostly practised by the retailers to decrease paid interest to the bank.

Case 2 ($N \geq M$): In some cases it is observed that customers purchase the units during the whole month and paid the whole amount after getting their salaries. To capture those customers the retailer some time provides such amount of delay period though the supplier's credit duration may less than the retailer's credit duration.

Case 1: $N < M$

According to the assumption, in this case, customers' credit duration is smaller than the retailer's credit duration. Here three scenarios may arise-

- Scenario-1.1 : $0 < T + N \leq M$
- Scenario-1.2 : $T \leq M \leq T + N$
- Scenario-1.3 : $M \leq T$

Scenario-1.1 : $0 < T + N \leq M$

In this scenario, the retailer receives all the sales revenue from the customers before the grace period offered by the supplier. So the retailer has not to pay any interest.

$$\text{Payable interest, } IP = 0 \tag{11}$$

The retailer can earn interest from the collateral deposit of the customers for the units sold in $(0,T]$ and from the credit payment of the customers for the units sold in $(0,T]$. Those are presented mathematically in Equation (13) and Equation (14) respectively.

Thus in this scenario, retailer's total earned interest is in the form:

$$\text{Interest earned, } IE = \frac{1}{T}\left[IE_1 + IE_2\right], \text{ where} \tag{12}$$

$$IE_1 \quad = \quad I_e\int_0^T (1-\alpha)sD(M-t)dt = I_e(1-\alpha)sD\left[MT-\frac{T^2}{2}\right] \tag{13}$$

$$IE_2 \quad = \quad I_e\int_0^T \alpha sD(M-N-t)[1-F(\alpha,N)]dt = I_e\alpha sDTe^{-a\alpha N}\left(M-N-\frac{T}{2}\right) \tag{14}$$

Also the Scenario-1.1 is presented graphically in the Figure-1.

Scenario-1.2 : $T \leq M \leq T + N$

In this scenario since $M \leq T + N$, so the settlement at bank is made at $t = T + N$, i.e., when all revenues are collected from the customers. The retailer has to pay interests- (i) due to the customers' credit for the units sold in $(M - N, T]$ as the credit payment of the customers should be made after the time $t = (M - N) + N$, i.e., $t = M$, which is the grace period of the retailer. This phenomenon is formulated mathematically in Equation (16).

(ii) Also the retailer bound to pay interest for the units deteriorated in $(0, T]$ as the cost of those units should have to pay the supplier at the time $t = M$, which governs the Equation (17).

(iii) Retailer also have to pay interest on the credit amount of the defaulters for the sold units during $(0, M - N]$ as well as $(M - N, T]$, those are respectively presented mathematically in Equation (18) and Equation (19).

Thus in this scenario, the retailer has to pay per unit time interest, IP as

$$IP = \frac{1}{T}[IP_1 + IP_2 + IP_3 + IP_4], \text{ where} \tag{15}$$

$$IP_1 = I_p \int_{M-N}^{T} \alpha p D(t + N - M)dt = I_p \frac{\alpha p D}{2}(T + N - M)^2 \tag{16}$$

$$IP_2 = I_p p(T + N - M) \int_{0}^{T} \theta(t)I(t)dt = I_p p D(T + N - M)\left[(1 + E_p)\ln\left(\frac{1 + E_p}{1 + E_p - T}\right) - T\right] \tag{17}$$

$$IP_3 = I_p \int_{0}^{M-N} \alpha p D(T + N - M)F(\alpha, N)dt = I_p \alpha p D F(\alpha, N)(T - M + N)(M - N) \tag{18}$$

$$IP_4 = I_p \int_{M-N}^{T} \alpha p D F(\alpha, N)[(T + N) - (t + N)]dt = \frac{I_p}{2}\alpha p D F(\alpha, N)(T - M + N)^2 \tag{19}$$

Similar to Scenario-1.1, in this scenario, the retailer can able to earn interest on the customers' collateral deposit of the customers for the units sold in $(0, T]$ and for the customers' credit payment for the units sold in $(0, M - N]$, those are presented mathematically in Equation (21) and Equation (22) respectively. Thus the per unit time earned interest of the retailer, IE is

$$IE = \frac{1}{T}[IE_1 + IE_2], \text{ where} \tag{20}$$

$$IE_1 = I_e \int_{0}^{T}(1 - \alpha)sD(M - t)dt = I_e(1 - \alpha)sD\left[MT - \frac{T^2}{2}\right] \tag{21}$$

$$IE_2 = I_e \int_{0}^{M-N} \alpha s D(M - N - t)[1 - F(\alpha, N)]dt = I_e \alpha s D e^{-a\alpha N}\frac{(M - N)^2}{2} \tag{22}$$

The above stated situations for the Scenario-1.2 are pictorially presented for better understanding in Figure-2.

Scenario-1.3 : $M \leq T$

In this scenario, the retailer have to pay per unit time interest, IP as

$$IP = \frac{1}{T}[IP_1 + IP_2 + IP_3 + IP_4 + IP_5 + IP_6 + IP_7] \tag{23}$$

$$IP_1 = I_p p \int_M^T I(t)dt = I_p pD \left[\frac{(1+E_p-M)^2}{4}\left\{2\ln\left(\frac{1+E_p-M}{1+E_p-T}\right) - 1\right\} + \frac{(1+E_p-T)^2}{4}\right] \tag{24}$$

$$IP_2 = I_p \int_{M-N}^M \alpha pD(t+N-M)dt = I_p \alpha pD\frac{N^2}{2} \tag{25}$$

$$IP_3 = I_p p \int_M^T \alpha DNdt = I_p \alpha pDN(T-M) \tag{26}$$

$$IP_4 = I_p \int_0^{M-N} \alpha pDF(\alpha, N)(T+N-M)dt = I_p \alpha pDF(\alpha, N)(T+N-M)(M-N) \tag{27}$$

$$IP_5 = I_p \int_{M-N}^T \alpha pDF(\alpha, N)[(T+N)-(t+N)]dt = I_p \alpha pDF(\alpha, N)\frac{(T+N-M)^2}{2} \tag{28}$$

$$IP_6 = I_p P(T+N-M)\int_0^M \theta(t)I(t)dt$$
$$= I_p pD(T+N-M)\left[(1+E_p)\ln\left(\frac{1+E_p}{1+E_p-M}\right) - M\left\{1 - \ln\left(\frac{1+E_p-M}{1+E_p-T}\right)\right\}\right] \tag{29}$$

$$IP_7 = I_p P\int_M^T \theta(t)I(t)(T+N-t)dt$$
$$= I_p pD\left[\left\{\frac{M^2}{2} - \frac{(1+E_p)^2}{2} + (T+N)(1+E_p-M)\right\}\right.$$
$$\left.\ln\left(\frac{1+E_p-M}{1+E_p-T}\right) + \frac{T^2-M^2}{4} - (T+N)(T-M) + \frac{(1+E_p)(T-M)}{2}\right] \tag{30}$$

On the other hand, the retailer has an opportunity to earn interest on the collateral deposit of the customers and on the credit payment of the customers. Thus the per unit time interest, earned by the retailer is

$$IE = \frac{1}{T}[IE_1 + IE_2] \tag{31}$$

$$IE_1 = I_e \int_0^M (1-\alpha)sD(M-t)dt = I_e(1-\alpha)sD\frac{M^2}{2} \tag{32}$$

$$IE_2 = I_e \int_0^{M-N} \alpha sD(M-N-t)[1-F(\alpha, N)]dt = I_e \alpha sDe^{-a\alpha N}\frac{(M-N)^2}{2} \tag{33}$$

Therefore, the average profit of the retailer for Case-1 : $N < M$ is given by

$$Z_1 = \begin{cases} Z_{11} & \text{if} & 0 < T+N \leq M \\ Z_{12} & \text{if} & T \leq M \leq T+N \\ Z_{13} & \text{if} & M \leq T \end{cases} \quad \text{where} \tag{34}$$

$$Z_{1j} = SR - OC - PC - HC + IE - IP \text{ for j=1, 2 and 3, where} \tag{35}$$

SR, OC, PC, HC are in Equation (7), Equation (8), Equation (9), Equation (10) respectively and for j=1,2,3; IP and IE are respectively given by Equation (11),Equation (12); Equation (15),Equation (20); Equation (23),Equation (31).

Case 2: $N \geq M$

In this case to determine the payable and earn interest of the retailer we have to study the following three scenarios:

- Scenario-2.1 : $N \geq M \geq T$
- Scenario-2.2 : $N \geq T \geq M$
- Scenario-2.3 : $T \geq N \geq M$

Scenario-2.1 : $N \geq M \geq T$

In this scenario, per unit time payable interest of the retailer is

$$IP = \frac{1}{T}[IP_1 + IP_2 + IP_3], \text{ where} \tag{36}$$

$$IP_1 = I_p \int_0^T \alpha p D(t + N - M) = I_p \alpha p D \left[\frac{T^2}{2} + (N - M)T \right] \tag{37}$$

$$IP_2 = I_p p(T + N - M) \int_0^T \theta(t)I(t)dt = I_p p D(T + N - M) \left[(1 + E_p) \ln \left(\frac{1 + E_p}{1 + E_p - T} \right) - T \right] \tag{38}$$

$$IP_3 = I_p p \int_0^T \alpha D F(\alpha, N)[(T + N) - (t + N)]dt = I_p \alpha p D F(\alpha, N) \frac{T^2}{2} \tag{39}$$

On the other hand, the retailer earn interest only on the customers' collateral deposit. Thus the per unit time earn interest of the retailer is

$$IE = \frac{1}{T} IE_1, \text{ where} \tag{40}$$

$$IE_1 = I_e \int_0^T (1 - \alpha)sD(M - t)dt = I_e(1 - \alpha)sD \left[MT - \frac{T^2}{2} \right] \tag{41}$$

Scenario-2.2 : $N \geq T \geq M$

In this scenario, the retailer have to pay per unit time payable interest as

$$IP = \frac{1}{T}[IP_1 + IP_2 + IP_3 + IP_4 + IP_5 + IP_6], \text{ where} \tag{42}$$

$$IP_1 = I_p p \int_M^T I(t)dt = I_p pD \left[\frac{(1+E_p-M)^2}{4} \left\{ 2\ln\left(\frac{1+E_p-M}{1+E_p-T}\right) - 1 \right\} + \frac{(1+E_p-T)^2}{4} \right] \tag{43}$$

$$IP_2 = I_p \int_0^M \alpha pD(t+N-M])dt = I_P \alpha pD \frac{M(2N-M)}{2} \tag{44}$$

$$IP_3 = I_p \int_M^T \alpha pDNdt = I_p \alpha pDN(T-M) \tag{45}$$

$$IP_4 = I_p pD \int_0^T \alpha F(\alpha,N)[(T+N)-(t+N)]dt = I_p pD\alpha F(\alpha,N)\frac{T^2}{2} \tag{46}$$

$$IP_5 = I_p p(T+N-M) \int_0^M \theta(t)I(t)dt$$

$$= I_p pD(T+N-M) \left[(1+E_p)\ln\left(\frac{1+E_p}{1+E_p-M}\right) - M\left\{ 1 - \ln\left(\frac{1+E_p-M}{1+E_p-T}\right) \right\} \right] \tag{47}$$

$$IP_6 = I_p p \int_M^T \theta(t)I(T)(T+N-t)dt$$

$$= I_p pD \left[\left\{ \frac{M^2}{2} - \frac{(1+E_p)^2}{2} + (T+N)(1+E_p-M) \right\} \ln\left(\frac{1+E_p-M}{1+E_p-T}\right) \right.$$

$$\left. + \frac{T^2-M^2}{4} - (T+N)(T-M) + \frac{(1+E_p)(T-M)}{2} \right] \tag{48}$$

Also in this scenario, the retailer can earn interest on the customers' collateral deposit for the sold units during $(0, M]$, which is written mathematically in Equation (50). Thus the retailer's per unit time earn interest is

$$IE = \frac{1}{T}IE_1, \text{ where} \tag{49}$$

$$IE_1 = I_e \int_0^M (1-\alpha)sD(M-t)dt = I_e(1-\alpha)sD\frac{M^2}{2} \tag{50}$$

Scenario-2.3 : $T \geq N \geq M$

In this scenario, the retailer have to pay interest per unit time is

$$IP = \frac{1}{T}[IP_1 + IP_2 + IP_3 + IP_4 + IP_5 + IP_6], \text{ where} \tag{51}$$

$$IP_1 = I_p p \int_M^T I(t)dt = I_p pD \left[\frac{(1 + E_p - M)^2}{4} \left\{ 2\ln\left(\frac{1 + E_p - M}{1 + E_p - T}\right) - 1 \right\} + \frac{(1 + E_p - T)^2}{4} \right] \tag{52}$$

$$IP_2 = I_p \int_0^M \alpha pD(t + N - M)dt = I_P \alpha pD \frac{M(2N - M)}{2} \tag{53}$$

$$IP_3 = I_p \int_M^T \alpha pDNdt = I_p \alpha pDN(T - M) \tag{54}$$

$$IP_4 = I_p pD \int_0^T \alpha F(\alpha, N)\left[(T + N) - (t + N)\right]dt = I_p pD\alpha F(\alpha, N)\frac{T^2}{2} \tag{55}$$

$$IP_5 = I_p p(T + N - M) \int_0^M \theta(t)I(t)dt$$

$$= I_p pD(T + N - M)\left[(1 + E_p)\ln\left(\frac{1 + E_p}{1 + E_p - M}\right) - M\left\{1 - \ln\left(\frac{1 + E_p - M}{1 + E_p - T}\right)\right\}\right] \tag{56}$$

$$IP_6 = I_p p \int_M^T \theta(t)I(T)(T + N - t)dt$$

$$= I_p pD\left[\left\{\frac{M^2}{2} - \frac{(1 + E_p)^2}{2} + (T + N)(1 + E_p - M)\right\}\ln\left(\frac{1 + E_p - M}{1 + E_p - T}\right)\right.$$

$$\left. + \frac{T^2 - M^2}{4} - (T + N)(T - M) + \frac{(1 + E_p)(T - M)}{2}\right] \tag{57}$$

Also in this scenario, the retailer can earn interest for the customers' collateral deposit for the units sold during $(0, M]$, which is presented in Equation (59). Thus the retailer's per unit time earn interest is

$$IE = \frac{1}{T}IE_1, \text{ where} \tag{58}$$

$$IE_1 = I_e \int_0^M (1 - \alpha)sD(M - t)dt = I_e(1 - \alpha)sD\frac{M^2}{2} \tag{59}$$

Therefore, the per unit time profit of the retailer for Case-2 : $N \geq M$ is given by

$$Z_2 = \begin{cases} Z_{21} & \text{if} & T \leq M \leq N \\ Z_{22} & \text{if} & M \leq T \leq N \\ Z_{23} & \text{if} & M \leq N \leq T \end{cases} \text{ where} \tag{60}$$

$$Z_{2j}(T) = SR - OC - PC - HC + IE - IP \text{ for j=1, 2 and 3, where} \tag{61}$$

SR, OC, PC, HC are in Equation (7), Equation (8), Equation (9), Equation (10) respectively and for j=1,2,3; IP and IE are respectively given by Equation (36),Equation (40); Equation (42),Equation (49); Equation (51),Equation (58).

Hence the problem is to

$$\begin{cases} \text{Determine } T, \alpha, N \text{ and } s \text{ to} \\ \text{Maximize } Z = \begin{cases} Z_1 & \text{when} & N \leq M \\ Z_2 & \text{when} & N \geq M \end{cases} \end{cases} \tag{62}$$

where $Z_i, i = 1, 2$ is defined in equation (34) and equation (60) respectively.

Proposed Model without defaulters (Model-2)

In this case, it is assumed that all customers are honest and repay their dues to the retailer in due time. Results for this model are obtained putting $a_1 = 0$ in the expressions of Model-1.

4. **Solution Methodology**

The above discussed models are optimized with the help of a soft computing technique, Artificial Bee Colony(ABC) algorithm. The detail discussion of the algorithm is presented below.

Artificial bee colony algorithm(ABC): Imitating the food search technique of honey bees, the search algorithm ABC was introduced by Karaboga[24] for optimisation problems having only continuous decision variables. After that the performance of the algorithm is improved by the modification of different functions by several researchers [25, 26, 27]. In this paper, the ABC algorithm is implemented, tested and used for the continuous optimization problem. For the detail mechanism of ABC algorithm, reader may go through[19, 23].

Implementation and Parameter setting : The ABC algorithm is implemented using C++ programming language. For the implementation of the ABC algorithm, the stooping criterion is chosen as the maximum number of iteration, $maxit$ with the population size $n = 60$ and the limit value is set to $lim = Dim \times n$[24], where Dim is the dimension of the problem.

Experiments and Efficiency : To investigate the efficiency and consistency of the ABC algorithm, it is tested for 10 benchmark test functions, which are listed in Table-5 in Appendix-A. All the test functions are optimized (minimized) for 30 dimension problems. The obtained results are presented in Table-6 in Appendix-A. From the results it is evident that the implemented ABC algorithm is efficient to draw the decisions for any continuous optimization problems.

5. **Numerical Illustration**

To observe the real life implication of the proposed model, here six numerical examples are taken into account. The parametric values of different parameters are given in the corresponding example.

Example-1 : $A = 10$, $h = 0.05$, $p = 0.5$, $I_p = 0.1$, $I_e = 0.08$, $M = 1.3$, $E_p = 1$, $q = 0.34$, $a = 100$, $b = 200$, $c = 4.$, $r = 0.7$, $\gamma = 2.5$, $a_1 = 0.2$

Example-2 : $A = 10$, $h = 0.05$, $p = 0.65$, $I_p = 0.15$, $I_e = 0.05$, $M = 1$, $E_p = 1$, $q = 0.5$, $a = 110$, $b = 220$, $c = 4.2$, $r = 0.7$, $\gamma = 2.$, $a_1 = 0.25$

Example-3 : $A = 8$, $h = 0.03$, $p = 0.65$, $I_p = 0.1$, $I_e = 0.05$, $M = 0.35$, $E_p = 1$, $q = 0.25$, $a = 130$, $b = 180$, $c = 4.3$, $r = 0.65$, $\gamma = 2.5$, $a_1 = 0.35$

Example-4 : $A = 10$, $h = 0.05$, $p = 0.5$, $I_p = 0.1$, $I_e = 0.07$, $M = 1$, $E_p = 1$, $q = 0.54$, $a = 100$, $b = 200$, $c = 4.$, $r = 0.7$, $\gamma = 2.5$, $a_1 = 0.2$

Example-5 : $A = 10$, $h = 0.07$, $p = 0.65$, $I_p = 0.1$, $I_e = 0.05$, $M = 0.35$, $E_p = 1$, $q = 0.75$, $a = 100$, $b = 200$, $c = 4.3$, $r = 0.4$, $\gamma = 2.5$, $a_1 = 0.3$

Example-6 : $A = 15$, $h = 0.03$, $p = 0.75$, $I_p = 0.15$, $I_e = 0.05$, $M = 0.35$, $E_p = 1$, $q = 0.51$, $a = 100$, $b = 200$, $c = 5.0$, $r = 0.5$, $\gamma = 2.$, $a_1 = 0.25$

For these examples, results are obtained using ABC algorithm and are presented in Table-1.

From the results of Table-1, it is seen that the results of the different examples are belongs to different scenarios of the model. So from these outcomes, one can conclude that the proposed model is applicable for any retail shop and also the DM may able to draw the optimal decision in any circumstances. Also

from Table-1, a DM can able to realise the actual loss due to the defaulters.

Parametric study on α**:** To establish the relation between α and N in a business transaction, here a parametric study is performed with respect to α with the different parametric values as in Example-1. The computed results are tabulated in Table-2.

From Table-2, it is observed that if the customer's initial payment is higher(lower), the retailer wish to offer larger(smaller) credit period to the customers. This phenomenon basically occurs due to the credit risk; as higher credit period/credit amount indicates higher credit risk. So the retailer wishes to offer less credit amount with higher credit period or higher credit amount for lesser credit period. This phenomenon agrees with any real-life business transaction. Also the above stated phenomenon is presented graphically in Figure-3.

Parametric study on N **for a fixed** α**:** To observe the effect of customer's credit period on a fixed credit amount in a business transaction, here a parametric study is performed with respect to N with the Example-1. The obtained results are presented in Table-3.

From Table-3, it is seen that initially the system profit (Z) increases with the increase of customer's credit period (N), but after a certain value of N, profit decreases with the increase of N. This phenomenon basically happens due to two reasons: first is that if the customers' credit period increases then the demand increases and also the amount of payable interest increase. If the profit due to the increased demand dominates the loss due to the interest charge then profit increases and if the loss due to the interest charges dominate the gain due to the increased demand then profit decreases. Second is that higher value of credit period calls higher credit risk, in other words the number of defaulters increases with the increase of N, on the other hand increases of N indicates the increases of demand. Thus a particular value of N provides the optimal profit of any inventory control system. The above stated phenomenon also represented graphically in Figure-4.

Parametric study on α **for a fixed** N**:** To study the effect of customer's collateral deposit on a business transaction for a fixed credit period, here a parametric study is performed with respect to α with the Example-1. The obtained results are presented in Table-4.

In any business transaction, if the initial payment of the customers increases then the total profit of the retailer should be increases but higher value of initial payment indicates the lower demand as well as lower profit. So an appropriate value of customer's collateral deposit gives the optimal profit of the system. These real life observations are agree with the results of Table-4. Also from Table-4, it is to be noted that if the value of collateral deposit is 38% of the total purchase amount, then the system provides the optimal profit and after this value of α, α increases but the profit decreases. This phenomenon is presented pictorially in Figure-5.

6. Conclusion

In this paper, for the first time a three layer supply chain of a deteriorating item is considered under credit policy and incorporating the effect of unfaithfulness of customers. Here a supplier offers a credit period to its retailer to enhance his/her order quantity and the retailer also offers a credit period to his/her customers to increase the base demand of the items. For the credit opportunity offered by the retailer, the retailer always takes a credit risk to sell the items. The default credit risk is considered in more realistic manner, which depends on the customers' credit period and credit amount. Demand is influenced by the customers' credit amount, customers' credit period and selling price of the item. To determine the marketing decisions here a soft computing algorithm ABC is implemented and tested. The main findings of this research work are summarised below:

- In each experiment it is observed that customers' credit period is positive, i.e., in presence of defaulters also the customers' credit period has a positive effect in the return of the supply chain.
- A particular value of credit period gives maximum return of the supply chain.
- A particular value of collateral deposit gives maximum return of the supply chain.
- Defaulters are mainly influenced by the credit amount and credit period. Here, for the first time a generalised function of defaulters is considered and analysed.

- The collateral deposit$(1 - \alpha)$ increases with credit period(N) and vice versa, which agrees with reality in presence of unfaithful customers.

Acknowledgement

Appendix A

Table-5. List of test functions

Function name	Function	Search Range
Sphere	$F_1(x) = \sum_{i=1}^{n} x_i^2$	$[-100, 100]$
Elliptic	$F_2(x) = \sum_{i=1}^{n} (10^6)^{(i-1)/(n-1)} x_i^2$	$[-100, 100]$
Schwefel 2.22	$F_3(x) = \sum_{i=1}^{n} |x_i| + \prod_{i=1}^{n} |x_i|$	$[-10, 10]$
Step	$F_4(x) = \sum_{i=1}^{n} (x_i + 0.5)^2$	$[-100, 100]$
Quatric WN	$F_5(x) = \sum_{i=1}^{n} i x_i^4 + random[0,1)$	$[-1.28, 1.28]$
Rosenbrock	$F_6(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i)^2 + (x_i - 1)^2]$	$[-10, 10]$
Rastrigin	$F_7(x) = \sum_{i=1}^{n} [x_i^2 - 10cos(2\pi x_i) + 10]$	$[-5.12, 5.12]$
Griewank	$F_8(x) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos(\frac{x_i}{\sqrt{i}}) + 1$	$[-600, 600]$
Schwefel 2.26	$F_9(x) = 418.98288727243369 \times n - \sum_{i=1}^{n} x_i sin(\sqrt{|x_i|})$	$[-500, 500]$
Penalized 1	$F_{10}(x) = \frac{\pi}{n} \{ sin^2(\pi y_1) + \sum_{i=1}^{n-1} (y_i - 1)^2 [1 + sin^2(\pi y_{i+1})] + (y_n + 1)^2 \}$ $+ \sum_{i=1}^{n} u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{1}{4}(x_i + 1)$ and $u_{x_i,a,k,m} = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a \leq x_i \geq a \\ k(-x_i - a)^m & x_i < -a \end{cases}$	$[-50, 50]$

Table-6. Obtained results of test functions using ABC

Function name	ABC	
	Mean	Std.Dev.
Sphere	5.10E-16	8.40E-17
Elliptic	4.79E-16	9.88E-17
Schwefel 2.22	1.28E-15	1.44E-16
Step	0	0
Quatric WN	4.86E-02	1.49E-02
Rosenbrock	4.32E-02	4.71E-02
Rastrigin	0	0
Griewank	7.62E-11	4.18E-10
Schwefel 2.26	1.09E-12	9.06E-13
Penalized 1	5.08E-16	5.15E-17

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1   **Caption of Tables**
2   Table-1. Results of the models
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13
14   **Tables**

TABLE 1. Results of the models

Example	Model-1					Model-2					Loss due to
	T	α	N	s	Z(in $)	T	α	N	s	Z(in $)	defaulters(in $)
1	0.3478	0.8167	0.1766	0.88	108.66	0.3325	1.0000	0.2591	0.87	120.88	12.21
2	0.4899	0.4041	0.7361	1.54	75.82	0.4821	1.0000	0.5157	1.50	87.26	11.44
3	0.4146	0.7198	0.1228	1.25	61.67	0.3500	1.0000	0.13139	1.21	70.27	8.60
4	0.3594	0.2525	1.5037	0.93	94.86	0.3528	1.0000	0.4395	0.91	106.76	11.90
5	0.3956	0.0810	2.8357	1.27	57.17	0.3500	0.7285	1.6212	1.22	60.50	3.33
6	0.6358	0.2943	0.6358	1.59	30.86	0.5714	0.8014	0.5714	1.55	34.51	3.65

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Table-2
Relation between α and N

α	N	T	Q	Z
0.13	2.9197	0.4675	61.0695	71.87
0.14	2.7040	0.4697	61.8738	72.36
0.15	2.5116	0.4716	62.6275	72.79
0.20	1.8126	0.4788	65.4934	74.26
0.30	1.0848	0.4858	68.9006	75.51
0.40	0.7458	0.4898	70.6471	75.82
0.50	0.5471	0.4948	71.5968	75.67
0.60	0.4423	0.5120	73.2352	75.18
0.70	0.3413	0.5136	72.9311	74.57
0.80	0.2722	0.5152	72.6607	74.00

Table-3
Parametric study on N for fixed α

N	T	Q	Z
0.60	0.4944	71.3267	75.49
0.71	0.4906	70.8554	75.81
0.72	0.4904	70.8089	75.82
0.73	0.4899	70.7500	75.82
0.74	0.4899	70.6959	75.82
0.75	0.4896	70.6231	75.82
0.76	0.4894	70.5524	75.81
0.77	0.4893	70.5196	75.80
0.78	0.4890	70.4330	75.79
0.80	0.4886	70.3136	75.76
0.90	0.4878	69.5919	75.47

Table-4
Parametric study on α for fixed N

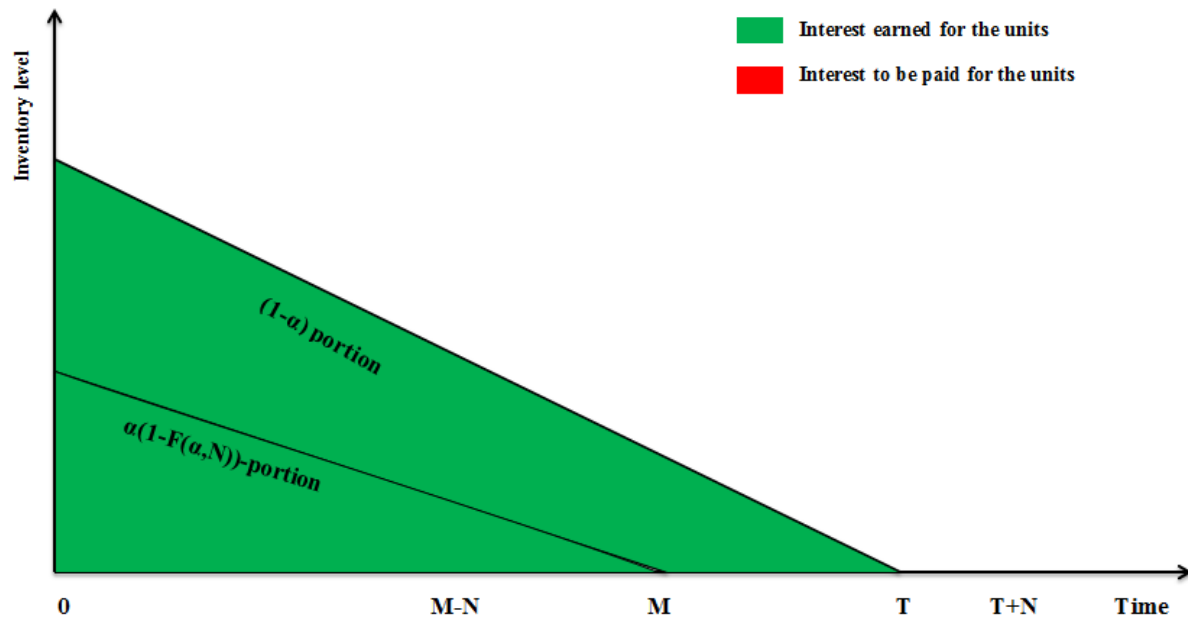| α	T	Q	Z
0.36	0.4893	70.4007	75.73
0.37	0.4889	70.3818	75.78
0.38	0.4888	70.3728	75.81
0.39	0.4887	70.3723	75.80
0.40	0.4886	70.3319	75.78
0.41	0.4887	70.2799	75.73
0.42	0.4889	70.2198	75.66
0.43	0.4888	70.1578	75.56
0.50	0.4909	69.4418	74.38

Figures

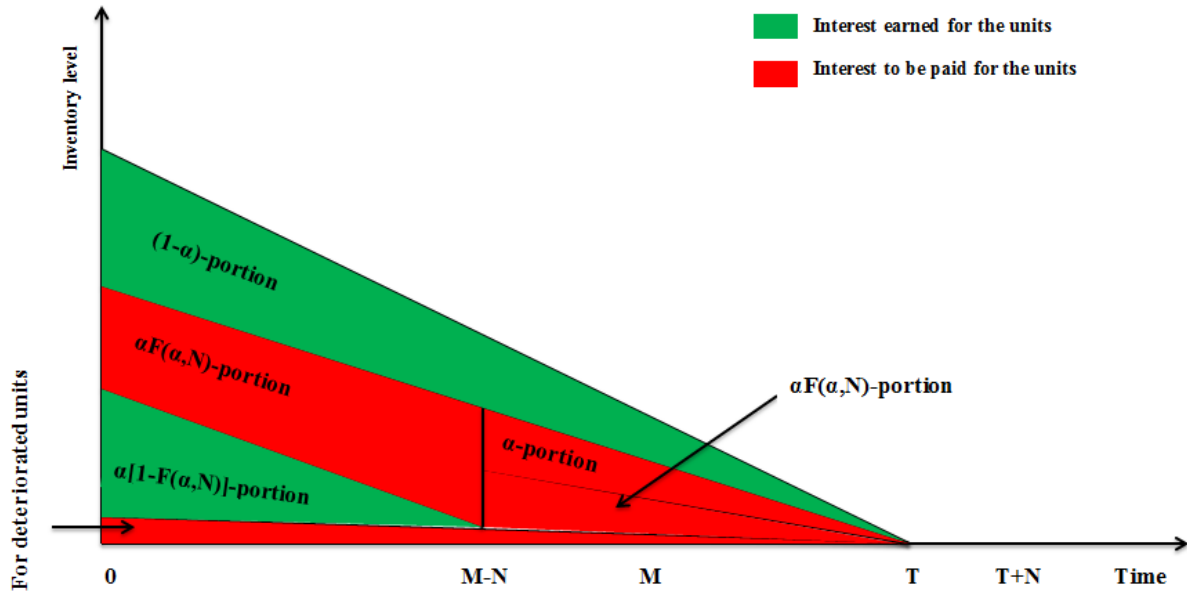FIGURE 1. Pictorial representation Scenario-1.1

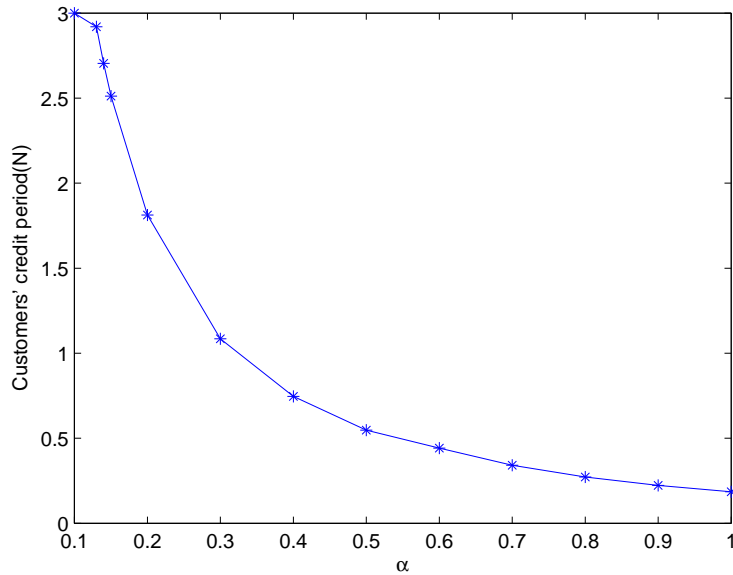FIGURE 2. Pictorial representation Scenario-1.2

FIGURE 3. Pictorial representation between α and N

Authors Biography

Rituparna Mondal completed M.Sc in mathematics from the department of Mathematics, Indian Institute of Technology Madras (IIT Madras), Tamil Nadu, India in 2015. Since 2019, she is working as an institute research scholar at Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat, India. Her research area includes operation research, Fractional calculus.
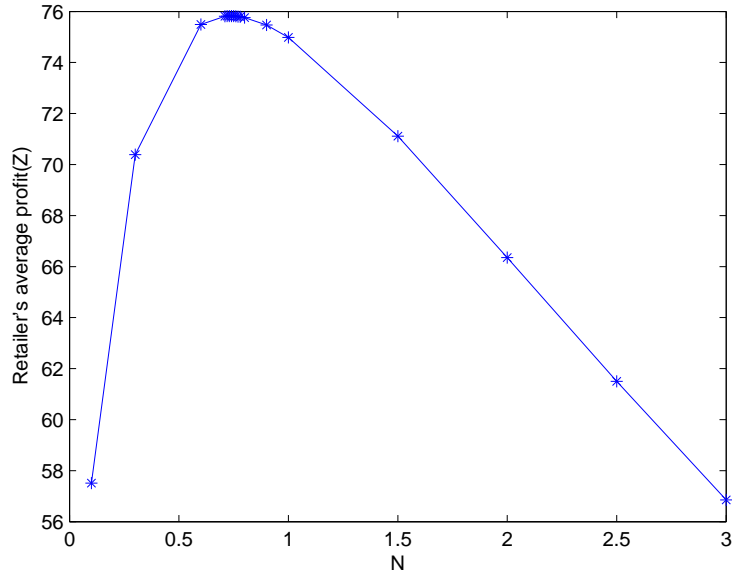
Figure 4. Effect of N on Z
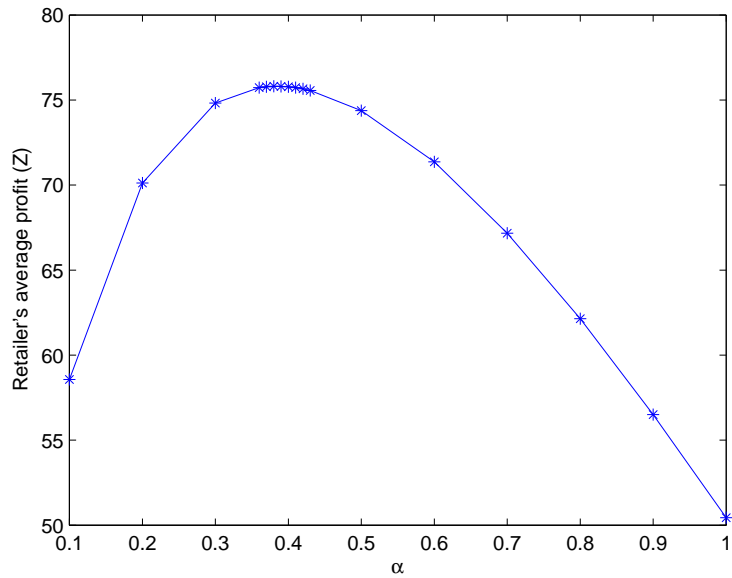
Figure 5. Effect of α on Z

Prasenjit Pramanik has received his M.Sc. in Applied Mathematics from Vidyasagar University, India; M.Tech in Computer Science from I.I.T. Kharagpur, India and awarded Ph.D. in Applied Mathematics from Vidyasagar University, India. His fields of research includes Inventory Control System, Supply Chain Management, Fuzzy Optimization, Evolutionary Computation, etc. Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore, West Bengal, 721102, India. E-mail: pramanik.prasenjit72@gmail.com, Tel: +919851411811.

Dr. Ranjan Kumar Jana is working as Assistant Professor in Mathematics at SVNIT, Surat since 07.12.2006. Prior to his joining SVNIT, he also worked as Lecturer at Nirma University, Ahmedabad, Research Fellow at Space Applications Centre (ISRO), Ahmedabad (2004-06) and Visva-Bharati Santinikean (2003-04). Dr. Jana has been awarded Indo-US Research Fellowship for the year 2013 by IUSSTF, New Delhi, for post-doctoral studies at University of Illinois at Urbana-Champaign, USA and worked with Prof. Bruce C. Berndt. He has been awarded Research Associate ship by Institute of Mathematical Sciences, Chennai from 01.06.2015 to 31.05.2018. At SVNIT, his group is working on Special Functions, Integral Transform, Operations Research and Data Assimilation. He has published 52 research papers in national / international journal of repute and conference proceedings, reviewed 54 papers. He has guided 06 students for PhD, 08 students are currently working towards PhD degree. 27 students completed their Master dissertation, 15 students completed their Summer / Winter Internship under his able guidance. He has completed 04 research projects each sponsored by DST, CSIR, SVNIT and SERB. Currently, 01 project is ongoing, which is sponsored by ISRO. Dr. Jana has organized 14 national / international conferences, 54 Seminars Symposium, 12 Workshops, 11 Short Term Training Programme (STTP), 02 Lecture Series, 22 Expert Lectures at SVNIT. He has also visited different universities e.g. George Washington University, Washington DC, University of Illinois at Urbana-Champaign, University of Wisconsin at Madison, Texas Tech University, University of Wisconsin at Eau Claire in USA; Rajamangala University of Technology at Thailand and University of Porto at Portugal for academic purpose. He has delivered 94 invited lectures at different Universities /Institution. He is member of American Mathematical Society, Society for Industrial and Applied Mathematics and life member of Indian Mathematical Society, Ramanujan Mathematical Society, Calcutta Mathematical Society, Society for Special Functions and Applications, Operational Research Society of India etc.

Manas Kumar Maiti is an Associate Professor of Mathematics in Mahishadal Raj College. He received his M.Sc. and M.Phil. in Applied Mathematics from Calcutta University, India, and M.Tech. in Computer Science from I.I.T. Kharagpur, India and awarded Ph.D. in Applied Mathematics from Vidyasagar University, India. His research interest includes Evolutionary Algorithms, Operations Research, Fuzzy Mathematics, Rough Set, etc. Department of Mathematics, Mahishadal Raj College, Mahishadal, Purba Medinipur, West Bengal, 721628, India. E-mail: manasmaiti@yahoo.co.in, Tel: +919434611765.