Influence of Curvature Dependent Channel walls on MHD Peristaltic Flow of Viscous Fluid with Hall Currents and Joule Dissipation

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Abstract

The prime motive of this study is to assess the behavior of curvature-dependent channel boundaries on the MHD peristaltic flow of viscous liquid with heat transportation effects via a curved channel. The analysis is reported by considering the Hall currents, Joule and viscous dissipations effects. Further, no-slip momentum and thermal conditions are incorporated. Mathematical model is subjected to the implication of large wavelength and weaker magnetic Reynolds number schemes. Galilean transformation is used to convert the problem from laboratory frame to wave frame. The solution of the system of equations is executed by employing the numerical method ND Solve (Built-in command in Mathematica). The volume and mean flow rates are computed and examined. Graphical illustrations are provided to evaluate the nature of distinct constraints on velocity, temperature, and rate of heat transportation. Results show that the curvature constraint has significant influences on the mechanical and thermal features of the flow. The damping effects of the magnetic field improve the temperature and rate of heat transportation at the boundary. Moreover, a reduction in thermal transportation rate is noticed for the increasing curvature constraint values.
**Keywords:** Hall current; Viscous fluid; Peristaltic flow; Joule dissipation; Curvature dependent channel walls

**Nomenclature**

<table>
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<tr>
<th>2a channel width</th>
<th>$F$ dimensionless flow rate in wave frame</th>
<th>$m$ Hall parameter</th>
<th>$q^*$ volume flow rate in moving frame</th>
<th>$(u,v)$ velocity components in wave frame</th>
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<tr>
<td>$b$ dimensionless wave amplitude</td>
<td>$J (Am^{-2})$ current density</td>
<td>$n_e$ free electron density</td>
<td>$R^*$ radius</td>
<td>$Br$ Brinkman number</td>
</tr>
<tr>
<td>$B_0 (Am^{-1})$ strength of magnetic field</td>
<td>$K \left(\frac{W}{mk}\right)$ thermal conductivity</td>
<td>$\bar{P}(\bar{R},\bar{X},\bar{t})$ pressure in stationary frame</td>
<td>$s (ms^{-1})$ wave speed</td>
<td>$\bar{W}(\bar{X},\bar{t})$ peristaltic wall</td>
</tr>
<tr>
<td>$c_p (Jkg^{-1}k^{-1})$ Specific heat</td>
<td>$k$ curvature parameter</td>
<td>$p(r,x)$ pressure in wave frame</td>
<td>$\bar{t}$ time</td>
<td>$w(x)$ dimensionless peristaltic wall</td>
</tr>
<tr>
<td>$e$ charge on electron</td>
<td>$M$ Hartmann number</td>
<td>$Q^* (m^3s^{-1})$ Volume flow rate in fixed frame</td>
<td>$(\bar{U},\bar{V})$ velocity components in fixed frame</td>
<td>$(\bar{R},\bar{X})$ curvilinear coordinate</td>
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**Greek letters**

<table>
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<tr>
<th>$\lambda$ wavelength</th>
<th>$\psi$ stream function</th>
<th>$\sigma$ electrical conductivity</th>
<th>$P$ fluid density</th>
<th>$\theta$ dimensionless temperature</th>
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</thead>
<tbody>
<tr>
<td>$\eta$ dimensionless flow rate in fixed frame</td>
<td>$\mu$ dynamic viscosity</td>
<td>$\rho$ fluid density</td>
<td>$\kappa$ thermal conductivity</td>
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1. **Introduction**
The continuous transport of body fluid through a distensible tube because of the change in pressure along the tube walls is known as peristaltic pumping. The peristaltic behavior of fluids has a potential role in distinct physiological processes. The physiological processes used the mechanism of peristalsis for the transport of fluid in biological systems. These involve the passage of food bolus via the esophagus, lymph motion in lymphatic vessels, discharge of waste material from the body, working of the digestive system, spermatic transport in the male reproductive system, the blood circulation in blood vessels, ovum motion in fallopian tubes and digestive system functioning. Besides this, the mechanism of peristalsis also has importance in designing biomedical instruments like heart-lung dialysis machine, blood pumps, etc. The process of peristalsis is also used to prohibit the direct linkage of the liquid with pump components during the flow of corrosive fluids or purified fluids and as in finger/roller pumps. Further, this process is utilized in the transport of toxic fluids (in the nuclear industries, this mechanism is exploited in the transportation of toxic liquids). This phenomenon of peristalsis has achieved the vital attention of researchers across the globe. Latham [1] first initiated the concept of peristalsis through an experimental study. Onwards, the mechanism of peristalsis has investigated by several researchers with different approaches. The viscous fluid flow in a thick-walled viscoelastic pipe given specific flow and pressure conditions was analyzed by Whirlow and Rouleau [2]. Shapiro [3] studied the pumping of retrograde diffusion in the peristaltic waves. Shapiro et al. [4] introduced the lubrication theory model for peristaltic flow by considering the negligible effect of fluid inertia and wave number. Ali et al. [5] reported the thermal transportation aspects in viscid fluid flowing through a curved channel having wavy walls. Hayat et al. [6] addressed the magnetized flow of third-order peristaltic fluid through a channel under slip and variable viscosity aspects. Numerical computations for the evaluation of micropolar
peristaltic fluid motion in a curved channel is reported by Ali et al. [7]. Vajravelu et al. [8] elucidated the peristaltic transportation of Williamson liquid in an asymmetric channel by adopting the peristalsis wave train on the walls with distinct phases and amplitudes. The mathematical formulation for the peristaltic pumping of liquids mixtures through circular cylinders is executed by Mekheimer et al. [9]. Tripathi et al. [10] constructed a mathematical model to analyze the peristaltic flow of non-Newtonian physiological liquid via an asymmetric channel with a porous medium. Makinde and Reddy [11] disclosed the nature of magneto peristaltic movement of Casson fluid fills the porous space. Hayat et al. [12] addressed the entropy production phenomenon in Sisko nano liquid flow that fills the non-Darcy’s porous space. The role of Brownian movement, Coriolis forces, and thermophoresis factor on the peristaltic movement of viscous liquid is studied by Mabood et al. [13].

Peristalsis with MHD plays a vital role in biomedical sciences. These effects are used in MRI (magnetic resonance imaging) to detect and cure tumor and in magneto therapy. MHD peristaltic transport is important because of its vital applications in biomedical science, few examples are MHD compressors, Magnetic Resonance Image (MRI) utilizes to detect and cure tumors, ulceration and urinary tract problems. Further, peristalsis through heat transportation is utilized in the process such as hemodialysis, oxygenation, hyperthermia etc. The magnetic peristaltic transportation of Carreau fluid with the transfer of heat through convection is executed by Hayat et al. [14]. MHD peristaltic flow of Carreau-Yasuda (C-Y) fluid via a curved geometry under the consideration of slip condition has been inspected by Abbasi et al. [15]. Rashidi et al. [16] examined the peristaltic movement of Casson blood fluid under heat mass transportation features. Asha and Sunitha [17] discussed the Joule heating and magnetic force influences Eyring-Powell blood fluid flow under suspension of nanoparticles passing through the non-
uniform channel. Endoscope influences on peristaltic transportation of couple stress liquid between co-axial tubes is addressed by Ramesh and Devakar [18]. Selimefendigil et al. [19] enlightened the magnetic field and convection aspects of nanofluid in a vertical cavity with a stretchy fin that is connected to its outer wall. Hayat et al. [20] elucidated the nanoparticles behavior in irreversible flow of peristaltic fluid under second-order slip regime. Iqbal et al. [21] addressed the energy transportation of C-Y nanoliquid flow with radial magnetic force. Analysis of mass transportation in magnetized Casson fluid flowing over a starching/shrinking sheet has been reported by Mahabaleshwar et al. [22]. Shah et al. [23] employed the shooting technique to analyze the magnetized entropy generation in stagnation-point flowing through an extendable Riga boundary with porous media. The impact of heat transfer on peristaltic Newtonian fluid across a porous conduit is investigated by Kiran et al. [24]. A theoretical analysis of thermal ignition in Oldroyd 8-constant liquid via a channel is introduced by Salawu et al. [25]. Shamshuddin et al. [26] reported the influence of transverse magnetic field and Hall current influences in non-Newtonian liquid flowing over an exponentially stretching sheet. Akbar et al. [27] addressed the Hall current influences in nanofluid flow having distinct nanoparticles through a straight channel having pulsating walls. Tripathi et al. [28] and [29] demonstrated the electroosmotic peristalsis of micro polar nanofluids with double diffusion and electro kinetically driven peristaltic movement of hybrid nanomaterials. Ranjit et al. [30] scrutinized the two-layered electroosmotic non-Newtonian fluid flowing in an asymmetric channel with velocity and thermal slip conditions. Selimefendigil and Chamkha [31] utilized the FEM approach for mixed convected magneto hybrid nanofluid through a triangular shaped cavity. Abdal et al. [32] reported the thermal transportation influence in MHD bio convective Maxwell nanofluid over a stretchable sheet with radiative heat flux, specific temperature and thermal gradient conditions.
Entropy production in magnetic stagnation point flow of tangent-hyperbolic nanomaterial with mass and thermal transportation is evaluated by Zhao et al. [33]. Hashmi et al. [34] investigated the simultaneous impact of magnetic dipole and mixed convection in reactive transportation of non-Newtonian material flowing over an extendable disk. Nazeer et al. [35] presented a comprehensive study on multi-phase Newtonian and non-Newtonian fluids over an inclined channel. Nisar et al. [36] reported the influence of Robin’s condition and wall features on magnetized peristaltic motion of Eyring-Powell nano liquid through radiative channel. Rao and Shamshuddin [37] investigates the time-dependent Hall current aspects in radiated chemically reacting liquid flow. Saba et al. [38] executed the simultaneous influences of magnetic and electric fields on peristaltic flow of viscous fluid via curved channel. Peristaltically driven magnetized nanofluids under electric field via straight and curved geometries through different assumptions being studied by Akram et al. [39-41]. As, the curved geometries and curvature are ambiguously encountered in physiological process and biological mechanism/studies. Curved surfaces are found in biological tissues. Curvature of tissues increased surface area required for nutrition absorption in the colon or gas exchange in the lungs. Many processes, including cell mobility, stem cell formation, and disease progression, can be influenced by curvature of the cells. Tissues in our body are unable to see, they are aware of their environment and their shapes [42]. Many cellular processes, as well as tissue morphogenesis, pathology, and repair, are guided by physical cues in their environments, according to recent studies. The nature of local geometries as an extracellular cue is one aspect that is gaining impetus. Understanding how geometries influences the tissue and cell function is critical for developing artificial scaffolds and better understanding of tissue remodeling and growth. Surface curvature is the most basic description of local geometry, and an increasing body of evidence shows that it has an impact on
the spatiotemporal establishment of tissues and cells [43]. The curvature and wall thickness influences on wall stress in patient-specific electromechanical model of the left atrium has been studied by Augustin [44]. The pulsating flow of viscous fluid flowing over a curved channel with thermal transport effects is discussed by Saba et al. [45].

Several investigations on peristalsis through different geometries under the different physical effects have been found in literature. But not enough study has been found in literature to investigate the peristaltic movement of fluid through a curved channel in which curvature engages in the modeling of channel walls. Motivated from the above studies, here the purpose is to introduce a mathematical model which involves the effects of curvature dependent channel boundaries on the peristaltic motion of viscous fluid via a curved channel. Further, the thermal transport, viscous dissipation, Joule dissipation and Hall effects are also considered. It is noteworthy that effects of curvature-dependent channel walls have not received due attention in above mentioned studies and it provides a opportunity for further research in this directions particularly for nano and hybrid nanofluids.

2. Statement of problem

MHD peristalsis of viscous fluid via a curved channel with channel width "2a" bent in a circle centered at "O" and radius "R*" has been considered. The curvilinear coordinate system (\(\tilde{R}, \tilde{X}\)) is taken in such a manner that \(\tilde{R}\)-axis is chosen along radial direction and \(\tilde{X}\)-axis is considered normal to it. The sinusoidal waves imposed on the channel boundaries are accountable for fluid flow generation within a channel (see Fig. 1). Both walls are kept at temperature"\(T_0\)." Mathematically, the channel walls of the present flow problem are defined as [45]:

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\[ \pm W(\bar{X},\bar{t}) = \pm a \pm d \cos \left( \frac{2\pi}{\lambda} (\bar{X} - s \bar{t}) \right) + R^*. \] (1)

Here, \( \bar{t}, s, d, \lambda \) and \( R^* \) represent the time, speed of the sinusoidal waves, amplitude, wavelength, and radius of the channel, whereas \( +W \) and \( -W \) characterize the above and the lower boundaries of the channel.

The applied magnetic field is defined as:

\[ B = (0,0,B_0). \] (2)

Mathematically, the Ohm’s law in the existence of Hall effects is given as [38]:

\[ J = \sigma \left( E + V \times B - \frac{(J \times B)}{en_e} \right). \] (3)

Thus, the Lorentz force in the light of Eqs. (2) and (3) is defined as:

\[ F = \left( \frac{\sigma B_0^2}{1+m^2} (-U + m\bar{V}), -\frac{\sigma B_0^2}{1+m^2} (\bar{V} + m\bar{U}), 0 \right). \] (4)

where \( \sigma, n_e, e \) and \( m \) denote the electrical conductivity, free electron density, the charge on electron and Hall parameter, respectively. The expressions which govern the fluid movement through the curved channel in laboratory frame are defined as [5] and [45]:

\[ (R^* + \bar{R}) \frac{\partial \bar{U}}{\partial \bar{R}} + \bar{U} + R^* \frac{\partial \bar{V}}{\partial \bar{X}} = 0, \] (5)

\[ \rho \left( \frac{\partial \bar{U}}{\partial \bar{t}} + \bar{U} \frac{\partial \bar{U}}{\partial \bar{R}} + (R^* \bar{V} \frac{\partial \bar{U}}{\partial \bar{X}} - \bar{V}^2) \frac{1}{\bar{R} + R^*} \right) = -\frac{\partial \bar{P}}{\partial \bar{R}} + \frac{1}{(\bar{R} + R^*)} \frac{\partial}{\partial \bar{R}} \left( S_{kk}(\bar{R} + R^*) \right) \] (6)

\[ + \frac{R^*}{(\bar{R} + R^*)} \frac{\partial}{\partial \bar{X}} S_{k\bar{k}} - \frac{S_{k\bar{k}}}{(\bar{R} + R^*)} + \sigma \frac{B_0^2}{1+m^2} \left[ -\bar{U} + m\bar{V} \right]. \] (7)
\[
\rho \left[ \frac{\partial \tilde{V}}{\partial t} + \tilde{U} \frac{\partial \tilde{V}}{\partial R} + \left( R^* \frac{\partial \tilde{V}}{\partial x} + \tilde{U} \right) \frac{\tilde{V}}{R + \tilde{R}} \right] = - \frac{R^*}{R^* + \tilde{R}} \frac{\partial \tilde{P}}{\partial x} + \frac{1}{(R^* + \tilde{R})^2} \frac{\partial}{\partial R} \left( (R^* + \tilde{R})^2 S_{\tilde{R} \tilde{R}} \right) \\
+ \frac{R^*}{(R^* + \tilde{R}) \tilde{R}} \frac{\partial}{\partial x} S_{\tilde{R} \tilde{x}} - \frac{\sigma B_0^2}{1 + m^2} (\tilde{V} + m \tilde{U}),
\]

\[
\rho C_p \left[ \frac{\partial \tilde{V}}{\partial t} + \tilde{V} \cdot \nabla \right] T = KV^2 + S.L + \frac{1}{\sigma} J.J.
\]

Here \( \tilde{P}(\tilde{R}, \tilde{x}, \tilde{t}), \rho, C_p, B_0, m, \) and \( K \) symbolize the fluid’s pressure, density, specific heat at constant pressure, strength of applied magnetic field, Hall parameter and thermal conductivity respectively.

Further,

\[
S_{\tilde{R} \tilde{R}} = 2\mu \frac{\partial}{\partial \tilde{R}} \tilde{U},
\]

\[
S_{\tilde{R} \tilde{x}} = \mu \left( \frac{\partial \tilde{V}}{\partial \tilde{R}} + \frac{1}{(R + \tilde{R})} \left( R^* \frac{\partial \tilde{U}}{\partial \tilde{x}} - \tilde{V} \right) \right),
\]

\[
S_{\tilde{x} \tilde{x}} = \mu \left( \frac{1}{R^* + \tilde{R}} \left( 2R^* \frac{\partial \tilde{V}}{\partial \tilde{x}} + 2 \tilde{U} \right) \right),
\]

\[
\nabla = \frac{\partial}{\partial \tilde{R}} + \frac{1}{R^* + \tilde{R}} \frac{\partial}{\partial \tilde{x}},
\]

\[
\nabla^2 = \frac{\partial^2}{\partial \tilde{R}^2} + \frac{1}{R^* + \tilde{R}} \frac{\partial^2}{\partial \tilde{R}^2} + \frac{1}{(R^* + \tilde{R})^2} \frac{\partial^2}{\partial \tilde{x}^2},
\]

\[
tr(S.L) = \mu \left( 2 \left( \frac{\partial \tilde{U}}{\partial \tilde{R}} \right)^2 + \frac{\partial \tilde{V}}{\partial \tilde{R}} \left( R^* \frac{\partial \tilde{U}}{\partial \tilde{x}} - \tilde{V} \right) \right)^2 + \frac{2}{(R^* + \tilde{R})^2} \left( R^* \frac{\partial \tilde{V}}{\partial \tilde{x}} + \tilde{U} \right)^2,
\]
Following transformation has been considered to convert the above expressions from laboratory to the wave frame [10].

$$\tilde{R} = \tilde{r}, \tilde{X} = \tilde{x} + s \tilde{t}, \tilde{P}(\tilde{R}, \tilde{X}, \tilde{t}) = \tilde{p}(\tilde{r}, \tilde{x}), \tilde{V}(\tilde{R}, \tilde{X}, \tilde{t}) = \tilde{v}(\tilde{r}, \tilde{x}) + s, \tilde{U}(\tilde{R}, \tilde{X}, \tilde{t}) = \tilde{u}(\tilde{r}, \tilde{x}).$$  \hspace{1cm} (10)

Eqs. (5)-(8) take the following form after employing Eqs. (9) and (10).

$$\left( \frac{\tilde{r} + R^*}{\tilde{r}} \right) \frac{\partial \tilde{u}}{\partial \tilde{r}} + \tilde{u} + R^* \frac{\partial}{\partial \tilde{x}} (\tilde{v} + s) = 0,$$  \hspace{1cm} (11)

$$\rho \left[ \frac{\partial \tilde{v}}{\partial \tilde{r}} + \frac{1}{\tilde{r} + R^*} \left( R^* \frac{\partial \tilde{v}}{\partial \tilde{x}} - (\tilde{v} + s) \tilde{u} \right) \right] = -\frac{R^*}{\tilde{r} + R^*} \frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{\mu}{(\tilde{r} + R^*)} \frac{\partial}{\partial \tilde{r}} \left( \frac{2(\tilde{r} + R^*)}{\tilde{r}} \frac{\partial \tilde{u}}{\partial \tilde{r}} \right)$$

$$+ \frac{R^* \mu}{(\tilde{r} + R^*)} \frac{\partial}{\partial \tilde{r}} \left( \frac{\partial (\tilde{v} + s)}{\partial \tilde{r}} + \frac{1}{(\tilde{r} + R^*)} \left( R^* \frac{\partial \tilde{u}}{\partial \tilde{x}} - (\tilde{v} + s) \right) \right)$$

$$- \frac{\mu}{(\tilde{r} + R^*)^2} \left( 2R^* \frac{\partial (\tilde{v} + s)}{\partial \tilde{x}} + 2\tilde{u} \right) + \sigma \frac{B^2_0}{1 + m^2} [\tilde{u} + m(\tilde{v} + s)],$$  \hspace{1cm} (12)

$$\rho C_p \left[ \frac{\partial \tilde{V}}{\partial \tilde{r}} + \frac{R^*}{\tilde{r} + R^*} (\tilde{v} + s) \frac{\partial \tilde{V}}{\partial \tilde{x}} \right] = K \left[ \frac{\partial^2 \tilde{T}}{\partial \tilde{r}^2} + \frac{1}{R^* + \tilde{r}} \frac{\partial \tilde{T}}{\partial \tilde{x}} + \left( \frac{R^*}{R^* + \tilde{r}} \right)^2 \frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} \right]$$

$$+ \mu \left( 2 \left( \frac{\partial \tilde{u}}{\partial \tilde{r}} \right)^2 + \frac{\partial}{\partial \tilde{r}} (\tilde{v} + s) + \frac{R^*}{R^* + \tilde{r}} \left( \frac{\partial \tilde{u}}{\partial \tilde{x}} \right) - (\tilde{v} + s)^2 \right) + 2 \left( \frac{R^*}{R^* + \tilde{r}} \left( \frac{\partial (\tilde{v} + s)}{\partial \tilde{x}} + \tilde{u} \right) \right)^2 + \frac{\sigma B^2_0}{1 + m^2} (\tilde{v} + s)^2.$$  \hspace{1cm} (14)
Dimensionless variables [44]:

\[ w = \pm \frac{W}{a}, \, \text{Re} = \frac{\rho_{sa}}{\mu_0}, \, \gamma = \frac{\bar{r}}{a}, \, \delta = \frac{a}{\lambda}, \, p = \frac{a^2 \bar{p}}{s \lambda \mu_0}, \, m = \frac{\sigma B_0}{\epsilon n_e}, \]

\[ k = \frac{R^*}{a}, \quad \theta = \frac{T - T_0}{T_0}, \quad x = \frac{\bar{x}}{\lambda}, \quad u = \frac{\bar{u}}{s}, \quad Br = \frac{\mu_0 s^2}{kT_0}, \quad v = \frac{\bar{v}}{s}. \]  

\[ b = \frac{d}{a}, \quad M^2 = \frac{\sigma B_0^2 a}{\mu_0}, \quad u = \frac{\partial \psi}{\partial x} \frac{k \delta}{k + y}, \quad v = -\frac{\partial \psi}{\partial y}. \]

After using above mentioned non-dimensional variables, negligible Reynolds number and wave number assumptions, Eq. (11) is satisfied identically and equations (12)-(14) become:

\[ \frac{\partial p}{\partial x} = -\frac{1}{k(k + y)} \frac{\partial}{\partial y} (k + y)^2 \left( \frac{1}{k + y} \left( 1 - \frac{\partial \psi}{\partial y} \right) + \frac{\partial^2 \psi}{\partial y^2} \right) \]

\[ - (k + y) \frac{M^2}{k(1 + m^2)} \left( 1 - \frac{\partial \psi}{\partial y} \right), \]  

\[ \frac{\partial p}{\partial y} = 0, \]  

\[ \frac{\partial^2 \theta}{\partial y^2} + \frac{1}{y + k} \frac{\partial \theta}{\partial y} + Br \left( \frac{\partial^2 \psi}{\partial y^2} + \frac{1}{k + y} \left( 1 - \frac{\partial \psi}{\partial y} \right)^2 \right) + Br \left( \frac{M^2}{1 + m^2} \right) \left( 1 - \frac{\partial \psi}{\partial y} \right)^2 = 0. \]  

Cross differentiation of equations (16) and (17) provides the following Eq.

\[ \frac{\partial}{\partial y} \left( -\frac{1}{k(k + y)} \frac{\partial}{\partial y} \left( \frac{1}{y + k} \left( 1 - \frac{\partial \psi}{\partial y} \right) + \frac{\partial^2 \psi}{\partial y^2} \right) (y + k)^2 \right) \]

\[ - \frac{\partial}{\partial y} \left( \frac{M^2 (k + y)}{k(1 + m^2)} \left( 1 - \frac{\partial \psi}{\partial y} \right) \right) = 0. \]
Here, the non-dimensional quantities $\theta, m, M, k, b, \psi, p, Br$ and $Re$ characterize the temperature, Hall parameter, Hartman number, curvature parameter, wave amplitude, stream function, pressure, Brinkman, and Reynolds number, respectively.

In the fixed and wave frames, the volume flow rates are given as:

$$Q^* = \int_{-w}^{+w} \tilde{V}(\tilde{R}, \tilde{X}, \tilde{r})d\tilde{R}, \quad q^* = \int_{-w}^{+w} \tilde{v}(\tilde{r}, \tilde{x})d\tilde{r}. \quad (20)$$

The flow rates in wave and laboratory frames are linked as follows:

$$Q^* = q^* + 2sw(x).$$

Hence, the mean flow rate can be written as [44]:

$$Q^{**} = q^* + 2as + 2sR^*. \quad (21)$$

Dimensionless pattern of flow rates in wave and laboratory frames is given below:

$$\eta = F + 2 + 2k. \quad (22)$$

The dimensionless pattern of peristaltic walls and boundary conditions is described as:

$$\pm w(x) = \pm 1 \pm b \cos(2\pi x) + k. \quad (23)$$

$$\psi[-w] = \frac{F}{2}, \quad \psi[w] = -\frac{F}{2}, \quad \frac{\partial \psi}{\partial y}[\pm w] = 1, \quad \theta[\pm w] = 0. \quad (24)$$

3. Solution methodology
Equations (18) and (19) with boundary conditions (24) have been solved numerically with the aid of the Built-in numerical solver NDSo\lve in Mathematica 11. The step size for these computations is chosen as 0.01 for variations in $x$ and $y$. To test numerical results, comparison of a sub-case of present study and analytical results obtained from an existing study [5] is presented via tables 1 and 2, and both are found in good agreement.

4. Results and discussion

This section has been prepared to evaluate the numerical outcomes through graphs. The effect of involved sundry parameters on fluid’s velocity, temperature and thermal transport rate at the channel wall are plotted as well as physically interpreted.

4.1. Velocity profile

Figs. 2-5 depict the changes in axial velocity "$V(y)$" for the various involved sundry parameters such as the dimensionless flow rate "$\eta$", Hartmann number "$M$", Hall constraint "$m$" and curvature parameter "$k$". These Figs. also indicate that the velocity has parabolic curves with the highest velocity existing near the middle of channel and these curves satisfying the boundary conditions. Fig. 2 expresses that $\eta = 7.5$ is the critical flow rate value and it is also the minimum flow rate value to continue the fluid flow within a curved channel having curvature dependent channel boundaries. It is observed that the velocity graphs are concave up and concave down with highest and lowest value of velocity arising close to the middle of curved channel for $\eta > 7.5$ and $\eta < 7.5$ respectively. Further, an increase and a decline in the velocity are noted with an increment in "$\eta$" above and below the critical value, respectively. Fig. 3 demonstrates a decrease in fluid’s velocity closest to the channel center for rising "$M$". However, a reverse impact is observed near channel walls. It is because the Lorentz force (i.e., resistive force) enhanced by enhancing the strength of applied magnetic field. The axial velocity variation for
distinct "m" values is shown in Fig. 4. It is discovered that axial velocity is directly proportional to "m", as it is evident from the Hall effect formula that the Joule heating effect decreases for larger values of "m" and thus the damping effects of Lorentz force also decreases. The axial velocity retards for the higher values of "k". The central axis moved forward for increasing values of "k". This is because of the existence of "k" in the mathematical modeling of channel walls. Further, the velocity curves show a concave up behavior for \( \eta > 7.5 \) and \( k > 4 \) this indicates that the value of \( \eta \) should be greater than its critical value to maintain channel flow for large values of curvature parameter (see Fig. 5).

4.2. Temperature profile and thermal transport rate

The temperature profile "\( \theta \)" with respect to "\( y \)" for distinct values of sundry parameters is plotted in the Figs. 6-10. It is noted that the "\( \theta \)" graphs are also parabolic form with highest temperature occurring close to the middle of curved geometry. Figs. 6a and 6b reveal that for "\( \eta > 7.5 \)" the rise in temperature profile is accountable with rise in "\( \eta \)". Whereas for "\( \eta < 7.5 \)" decreasing behavior of temperature is visualized for enlarging "\( \eta \)". Fig. 7 reports an augmentation in temperature with an enhancement in "\( M \)". This increase in temperature profile is because of resisting nature of magnetic force (physically, due to the Joule heating effects (i.e., heat dissipation improved)). Fig. 8 defines a reduction in fluid’s temperature with a rise in "m". This is because that an increase in "m" lowers the fluid’s thermal conductivity and thus reduced the resistive effects of Lorentz force. Also found that for smaller values of "m", the temperature profile tends to maintain symmetry near the central axis. Fig. 9 indicates that "\( k = 4 \)" is the critical number of the curvature constraint. For "\( k > 4 \)" , temperature of fluid enhances with an enhancement in curvature constraint. A reverse trend is observed in the temperature profile of
fluid below the critical value of "k" with an increment in "k". A rise in the values of "Br" causes an enhancement in the viscous dissipation effects and thus enhances the fluid temperature as depicted in Fig. 10.

The nature of involved flow constraints on $-\theta'(w)$ has been executed in the Figs. 11-15. Figure 11 and figure 12 show that $-\theta'[w]$ is proportional to "M" and "Br" respectively. Fig. 13 indicates a decrement in the thermal transport rate for growing values of "m". Fig. 14 shows that below the critical value of flow rate the heat transfer rate rises with a reduction in "\eta" but above the flow rate critical value, the heat transmission rate enhanced with an increment in "\eta". Figs. 15a and 15b reveals that when "k > 4" heat transfer rate is proportional to "k" while a decrease in thermal transfer rate is found with a rise in "k" respectively.

Comparison between numerical results of a limiting case of present study with exact solutions obtained by Ali et al. [5] for temperature and velocity profiles for different values of the involved parameters at the center of the channel is given through Tables 1 and 2. It noted that the findings of this investigation are consistent with earlier research.

5. Conclusions

The effect of curvature dependent channel boundaries on the peristaltic movement of viscous liquid through curved channel with Hall, Joule and viscous dissipation effects is investigated in this study. The following key points are derived from above discussion.

- Above and below the critical value flow rate an increment in the axial velocity is achieved against the augmented flow rate.
• A rise in Hartmann number causes a drop in velocity profile. Whereas an enhancement in axial velocity is found for the greater Hall constraint values.

• The velocity profile demonstrates that \( k = 4 \) is the critical number of curvature constraint. It is noted that below and above the critical number of curvature constraint, the fluid velocity reduces with an increment in the curvature parameter.

• Below the critical value of flow rate, a decrease in \( \eta \) resulted an augmentation in temperature profile. However, above the critical value of flow rate fluid temperature enhances with a rise in \( \eta \).

• Below the critical value of curvature parameter, a reduction in temperature profile is found with an enhancement in \( k \). Whereas a reverse behavior is noted above the critical value of curvature parameter.

• Thermal transport rate rises with a rise in \( M \) and \( Br \).

• A fall in thermal transmission rate is obtained against higher \( m \) values.

References


[26] Shamshuddin, M.D., Salawu, S.O., Ogunseye, H.A. and Mabood, F. “Dissipative power-law fluid flow using spectral quasi linearization method over an exponentially stretchable surface...


Figure Captions

Fig. 1. Geometry of the problem.

Fig. 2. Velocity curves for $\eta$ when $m = 2, d = 0.3, M = 2, Br = 2, k = 4$ and $x = 0$.

Fig. 3. Velocity curves for $M$ when $m = 2, \eta = 8, d = 0.3, Br = 2, k = 4$ and $x = 0$.

Fig. 4. Velocity curves for $m$ when $\eta = 8, d = 0.3, M = 2, Br = 2, k = 4$ and $x = 0$.

Fig. 5. Velocity curves for $k$ when $\eta = 7.5, d = 0.3, m = 2, Br = 2, M = 2$ and $x = 0$.

Fig. 6a. Temperature curves for $\eta$ when $m = 2, d = 0.3, M = 2, Br = 2, k = 4$ and $x = 0$.

Fig. 6b. Temperature curves for $\eta$ when $m = 2, d = 0.3, Br = 2, M = 2, k = 4$ and $x = 0$.

Fig. 7. Temperature curves for $M$ when $m = 2, d = 0.3, \eta = 8, Br = 2, k = 4$ and $x = 0$.

Fig. 8. Temperature curves for $m$ when $\eta = 8, d = 0.3, M = 2, Br = 2, k = 4$ and $x = 0$.

Fig. 9. Temperature curves for $k$ when $m = 2, d = 0.3, M = 2, Br = 2, \eta = 7.5$ and $x = 0$.

Fig. 10. Temperature curves for $Br$ when $m = 2, d = 0.3, M = 2, k = 4, \eta = 8$ and $x = 0$.

Fig. 11. Thermal transport rate at the boundary for $M$ when $m = 2, d = 0.3, \eta = 8, Br = 2, k = 4$ and $x = 0$.

Fig. 12. Thermal transport rate at the boundary for $Br$ when $m = 2, d = 0.3, M = 2, \eta = 8, k = 4$ and $x = 0$.

Fig. 13. Thermal transport rate at the boundary for $m$ when $M = 2, d = 0.3, \eta = 8, Br = 2, k = 4$ and $x = 0$. 

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Fig. 14. Thermal transport rate at the boundary for $\eta$ when $m = 2, d = 0.3, M = 2, Br = 2, k = 4$ and $x = 0$.

Fig. 15a. Thermal transport rate at the boundary for $k$ when $m = 2, d = 0.3, \eta = 8, Br = 2, M = 2$ and $x = 0$.

Fig. 15b. Thermal transport rate at the boundary for $k$ when $m = 2, d = 0.3, \eta = 8, Br = 2, M = 4$ and $x = 0$.

Table Captions

**Table 1.** Comparison between exact [10] and numerical results when $x = 0, k = 3, d = 0.6, \eta = 2, Br = 2$ and $y = 0$ (center of channel).

**Table 2.** Comparison between exact and numeric solutions for various values of curvature parameter.
Fig. 1. Geometry of the problem.

Fig. 2. Velocity curves for $\eta$ when $m = 2, d = 0.3, M = 2, Br = 2, k = 4$ and $x = 0$. 
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Fig. 15a. Thermal transport rate at the boundary for \( k \) when \( m = 2, d = 0.3, \eta = 8, Br = 2, M = 2 \) and \( x = 0 \).

![Fig. 15a](image-url)

Fig. 15b. Thermal transport rate at the boundary for \( k \) when \( m = 2, d = 0.3, \eta = 8, Br = 2, M = 4 \) and \( x = 0 \).

![Fig. 15b](image-url)

**Table 1.** Comparison between exact [10] and numerical results when \( x = 0, k = 3, d = 0.6, \eta = 2, Br = 2 \) and \( y = 0 \) (center of channel).

<table>
<thead>
<tr>
<th>Temperature &quot;( \theta )&quot;</th>
<th>Exact solution</th>
<th>Numerical solution</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curvature ( k = 3 )</td>
<td>2.1417356921458826</td>
<td>2.141735716954361</td>
<td>( 2.48084783 \times 10^{-8} )</td>
</tr>
<tr>
<td>4</td>
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<td>1.848984325697747</td>
<td>( 4.19750658 \times 10^{-8} )</td>
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<tr>
<td>5</td>
<td>1.7201816733646638</td>
<td>1.7201815944967997</td>
<td>( 7.88678640 \times 10^{-8} )</td>
</tr>
<tr>
<td>( Br = 2 )</td>
<td>2.141735692146792</td>
<td>2.141735716954361</td>
<td>( 2.48075688 \times 10^{-8} )</td>
</tr>
</tbody>
</table>
### Table 2. Comparison between exact and numeric solutions for various values of curvature parameter.

<table>
<thead>
<tr>
<th>Curvature parameter</th>
<th>Exact solution</th>
<th>Numerical solution</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 2$</td>
<td>$-0.4503693110499591$</td>
<td>$-0.4503693138950907$</td>
<td>$2.84513 \times 10^{-9}$</td>
</tr>
<tr>
<td>3</td>
<td>$-0.4719977215329828$</td>
<td>$-0.471997721882102$</td>
<td>$3.49227 \times 10^{-10}$</td>
</tr>
<tr>
<td>4</td>
<td>$-0.4820714866459177$</td>
<td>$-0.4820714863988828$</td>
<td>$2.47034 \times 10^{-10}$</td>
</tr>
</tbody>
</table>