# Coordination in the supply chain considering total lead times and delivery times

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## *Abstract*

This study considers a two-echelon supply chain (SC) consisting of a single vendor and a single buyer by reducing delivery time. This paper examines delivery time optimization as an essential component of lead times. The length of delivery time and production time are studied simultaneously. The delivery time as a decision variable is considered in the proposed model. Reducing delivery time is considered a vital incentive factor in encouraging the buyer to participate in the coordinated model to guarantee profitability. A suggested mathematical model consisting of the profit functions of both participants (i.e., vendor and buyer) are investigated under two decision-making scenarios: the decentralized decision structure and coordinated decision structure. The analyses show that our proposed model ensures better performance for both participants and makes the whole process more profitable by an adequate sharing of risks between two participants. In other words, under the coordinated model, decreasing the delivery time and buyer's shortage costs and increasing the order quantity leads to an increase in the profit of the vendor and buyer.

***Keywords:*** Supply chain coordination, Total lead times, Delivery time, Production time, Reducing delivery time, Lead times reduction.

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# 1. Introduction

Supply chain (SC) planning is concerned with coordinating several activities of different functions and different SC's actors from the very beginning (Vosooghidizaji et al. [1]). Hence, SC coordination can be achieved in various ways, such as using contracts, combining contracts, or incentive schemes. Therefore, many studies have used a contract or combinations of contracts or incentive strategies based on SC's type (Heydari [2], Sarathi et al. [3], Yang et al. [4]). In recent decades, researchers have sought to maximize the profitability of SC members and the whole SC. Considering the broad collaboration among SC members and the effects of each member's decision on others, coordinated decision-making increases the profitability of the partners and the entire SC. SC coordination models can play as stimuli for SC members so that practitioners would become motivated to participate more in optimal decision-making from the entire SC's sight (Heydari [2]). Furthermore, one way to motivate the buyer to participate more involved in the SC is to use incentive schemes. For instance, reducing lead times regarding the transportation modes is considered to stimulate SC members to participate in coordination (Heydari et al. [5]). On the other hand, the vendors can better respond to customer demands by improving their production and transportation plans. Usually, the cost of creating such incentives is paid by the vendor or supplier (Karampour et al. [6]).

This research considers a two-echelon SC consisting of a single vendor and a single buyer by reducing delivery time. Additionally, this paper proposes a mathematical model that includes an incentive scheme to satisfy the buyer to participate under a coordinated structure. Hence, the production time as a parameter and the delivery time as a variable is considered. The research gaps in this paper are included in two sections. First, components of lead times, such as delivery time and production time, are examined. Second, the reduced delivery time can encourage the buyer is encouraged to participate in the SC.

In other words, the highlights of this research are reducing the delivery time as an incentive scheme for the buyer. While reducing the delivery time, the buyer's profitability also increases. The problem-solving approach in this research is based on the two decision-making structures decentralized and coordinated. Also, the proposed mathematical model shows that partners' profit by reducing delivery time under coordinated decision-making compared with decentralized decision-making is increased. Finally, SC’s main sections (from the beginning to the moment the product is delivered to the buyer) are indicated in Figure. 1. Moreover, Figure. 1 shows that production time and delivery time are two important and vital items in the production or service cycles. Therefore, reducing delivery time creates a competitive advantage for different industries.

The primary contributions of this paper can be summarized as follows. First, this paper is among the first studies on the length of delivery and production time as two important components of lead times. Second, reducing the delivery time as a motivating factor for buyer participation is used. In other words, reducing the length of delivery time makes it possible to reduce the lead times to be used as an incentive scheme for the buyer to cooperate in the SC under coordinated decision-making. Finally, increased profitability of SC members under coordinated decision-making is indicated.

The remainder of this work is organized as follows. Section 2 reviews relevant literature in this paper. Section 3 presents the problem and notations and suggests a new mathematical model under the decentralized structure. Section 4 proposes and investigates the new coordination model. Numerical results and a thorough sensitivity analysis regarding resources in the research are presented in Section 5. Section 6 provides management insights. Conclusions and future research are presented in Section 7.

## 2. Literature review

SC networks designing, SC management, and SC coordination in different fields and industries have been significant for researchers. The SC coordination is done to motivate the partners to participate and maximize the profit of the SC members and the whole SC compared to the decentralized decision structure. Hence, different types of contracts are widely used to coordinate SC members. Furthermore, some previous papers have addressed incentive schemes that can lead SC members toward a coordinated decision structure. Generally, having a plan for SC coordination is good and increases the efficiency of systems (Jiang et al. [7]).

### 2.1. SC networks considering lead time or delivery time

In SC networks, pricing is a fundamental aspect of the economic modeling, which affects the obtained revenue and profit. Integrating pricing with facility location and inventory control decisions helps the companies to gain the appropriate insight for competing with their rivals

(Vahdani and Ahmadzadeh [8]). Many companies face challenges in reducing their SC costs while increasing sustainability and customer service levels. Therefore, the decision to reduce the cost of a SC is very important and necessary. A comprehensive framework for a sustainable closed-loop SC network is a practical solution to these challenges (Tavana et al. [9]).

On the other hand, using a closed-loop SC in various fields has applications and efficiency. For example, a sustainable closed-loop SC network is used for an integrated water supply and wastewater collection system (Fathollahi-Fard et al. [10]). Moreover, SC network managers face challenges. One of the SC managers' major challenges is selecting the best suppliers among all possible ones for their business (Fallahpour et al. [11]). Another issue considered about the characteristics of a SC is flexibility against various risks and its sustainability (Mojtahedi et al. [12], Ali et al. [13]).

Today, the advancement of technology in industries and the development of SCs have led vendors to decrease the delivery time of their goods to augment buyers' willingness to participate under the coordinated structure. On the other hand, the buyers must pay the other costs (e.g., inventory holding costs, ordering costs, and the expenditures of the shortage). As a result, the buyer is unwilling to pay more to participate in the SC under a coordinated structure. So, payment of the costs of such incentive schemes is the vendor's responsibility.

We have reviewed papers that focus on decreasing or controlling the lead times in the SC. Moreover, decreasing or controlling the lead times occurs through SC coordination. So, some tools are used for SC coordination. Hence, pricing and advertising are considered effective tools for coordination, especially in a competitive environment (Ghashghaei and Mozafari [14], Mokhlesian and Zegordi [15]). In some papers, motivation schemes such as reducing product or service delivery time are used to persuade the buyer to participate in a coordinated SC. Lead times reduction is one factor that creates motivation for a retailer (buyer) (Ye and Xu [16], Hayya et al. [17]). Reducing the length of lead times fluctuation by using a secure transportation system is an effective way to motivate the retailer (buyer) to participate in the coordinated decision-making (Heydari [2], Heydari et al. [5]). Setting a price discount mechanism can be considered as a possible way to control the length of lead times (Li et al. [18]). Sometimes, it is better to reduce the length of lead times by optimizing additional costs (Li et al. [19]). Manufacturers or vendors often emphasize controlling delivery lead times and minimizing costs to efficiently handle a SC (Vijayashree and Uthayakumar [20], Giri and Roy [21]). Length of lead times includes production time, startup time, and shipping time, which can be crashed in a total length of lead times (Glock [22]).

### 2.2. SC coordination

Many papers have generally studied SC coordination (Li et al. [23], Zissis et al. [24], Chan et al. [25]). Some papers related to SC coordination are implemented using coordination contracts. These contracts are used to motivate SC members to participate and obtain more profit in the SC. There are different methods to motivate all SC members to participate in the SC. One of the typical incentive contracts is revenue sharing (Raza [26]). Under such a contract, the supplier (vendor) will reduce the product's wholesale price for the retailer (buyer). Also, the retailer (buyer) guarantees that it will pay some parts of its revenue at the end of the sale period to the supplier (vendor). Another contract provided for the coordination between supplier (vendor) and retailer (buyer) is an incentive scheme called quantity discount (Venegas and Ventura [27]). Other coordination contracts include the return policy (Xu et al. [28], Xu et al. [29]), sales rebates (Genc and Giovanni [30]), sales effort sharing contracts (Saha et al. [31]), option contracts (Hu et al. [32], Hua et al. [33]), etc (Wee et al. [34], Zia and Taleizadeh [35], Bicer and Hagspiel [36], Aljazzar et al. [37], Fadaei et al. [38]). Designing incentive contracts under asymmetric information of demand is another way of stimulating the buyer to take part in the SC (Ha et al. [39], Asfaw and Venkataraman [40]). Several types of contracts are used by researchers and specialists, aiming to obtain SC coordination. Among the different contracts, the wholesale price and cost-sharing contracts are the two most popular ones (Xu et al. [41], Ji et al. [42]). In some papers, SC coordination through wholesale price and delivery cost-sharing contracts have been discussed. In this regard, the optimal wholesale price of the manufacturer, the optimal retail price, and the delivery time of the retailer have been examined (Xu et al. [43]). Another interesting topic for conducting research has been vertical coordination contracts used extensively. One of the important vertical contracts, called a two-part tariff, has been proved more effective than a wholesale price contract (Cachon and Kok [44], Ozer and Raz [45], Feng and Lu [46]).

### 2.3. Research gaps and contributions

We found a few papers close to this field of study by reviewing the recent works. In fact, in the previous studies, components of the lead times such as delivery time and production time have been less studied. So, previous works have focused more on coordination models by considering lead times and have less studied the components of lead times and their impact on the model. While paying attention to the components of lead times is very important for the vendors, suppliers, and buyers. Because reducing each of them (such as the delivery time or production time) creates a competitive advantage for vendors or suppliers. Therefore, examining the components of total lead times is considered a research gap for this study.

Hence, it is attempted to fill the research gaps by considering the components of the lead times, controlling each of them, such as length of delivery, length of production time, and reducing delivery time in the two-level SC. In other words, reduced delivery time results in reduced lead times, allowing buyers to send their orders more confidently to vendors or suppliers of products. In this case, buyers do not have to worry about losing customers and their market. Besides, reducing the length of delivery time makes it possible to reduce the lead times to be used as an incentive scheme for the buyer to participate in the SC under coordinated decision-making. As a result, this paper aims to present a proposed coordination model in the SC by reducing delivery time. Therefore, SC members agree to participate in the SC to make more profit under the coordinated decision-making.

Table 1 indicates a summary of some relevant literature by using SC coordination. Hence, the important features of several papers similar to this research are examined and compared in Table 1.

## 3. Proposed Model

This paper assumes that SC consists of a single vendor and a single buyer. Furthermore, the length of delivery time as a decision variable and length of production time as a parameter in a mathematical model are investigated. Also, the production time is fixed. In other words, we decided to adopt a new approach to the participation of SC members. So, this research is among the first studies on the delivery and production time simultaneously. In addition, the reduction of delivery time as an incentive for buyer participation in the SC is considered. Also, reducing a product or service delivery time leads the buyer to attract more customers in a competitive market and ultimately be more profitable.

On the other hand, a reduction in the length of delivery time will also enable the vendor to meet the buyer's needs in a short time, resulting in more revenue for the vendor. Under these new conditions, the vendor and the buyer enter into a partnership with each other with better and more accurate sight for greater profitability. Finally, increasing the profitability of a vendor and a buyer under a coordinated decision structure compared to a decentralized decision structure makes partners willing to participate in the SC.

The proposed mathematical model has developed some new terms to extend the coordination model. Some other basic assumptions regarding the proposed mathematical model are as follows: Based on Heydari et al. [5], the buyer adopts a continuous review inventory system, and demand is uncertain. Also, the vendor is a distributor. The notations used in this research are presented as:

The decision variables are:

| | |
|---|---|
| $Q$ | The buyer's order quantity; |
| $L_d$ | Length of delivery time; |

The other related parameters are:

| | |
|---|---|
| $D$ | Demand; |
| $p$ | Retail price per unit; |
| $w$ | Wholesale price per unit; |
| $r$ | Raw material price per unit of product; |
| $H_b$ | Buyer's holding costs per unit of product; |
| $H_v$ | Vendor's holding costs per unit of product; |
| $O_b$ | Buyer's ordering costs per order; |
| $O_v$ | Vendor's ordering costs per order; |
| $S_b$ | Buyer's shortage costs per unit; |
| $C_v$ | Vendor's delivery time reduction costs; |
| $L_p$ | Length of production time (controlled by the vendor); |
| $\delta$ | The standard deviation of demand; |
| $k$ | Inventory safety factor; |

| | |
|---|---|
| $TR$ | Vendor's transportation costs; |
| $s$ | Vendor's set-up costs per set-up; |
| $n$ | Reproduction coefficient by the vendor (vendor's reproduction size is $n$ times greater than the buyer's order quantity); |
| $LR$ | Lead times reduction; |
| $LRC$ | Lead times reduction costs; |

### 3.1. Decentralized

In a decentralized decision structure, the buyer only intends to maximize its profit from the presence in the SC. Generally, each SC member seeks to maximize its profits. Hence, we present two buyer and vendor profit functions under the decentralized decision structure. Based on Heydari et al. [5], the profit function of the buyer is formulated as follows:

$$\pi_b(Q, L_d) = (p - w)D - O_b \frac{D}{Q} - H_b\left[\frac{Q}{2} + k\delta\sqrt{L_d}\right] - (S_b + p - w)\frac{D}{Q}.L_d \tag{1}$$

Equation (1) shows the buyer's revenue and costs. The first term denotes the buyer's revenue from sold products, the second term indicates the buyer's ordering costs, the third term shows the buyer's holding costs, and the last term demonstrates the buyer's shortage costs. We have Proposition 1.

**Proposition 1.** The buyer’s profit function $\pi_b(Q)$ is concave in $Q$. By optimizing the profit function with respect to $Q$, the optimal order quantity $Q^*$ under the decentralized structure can be calculated as follows:

$$Q^* = \sqrt{\frac{2D\left[O_b + L_d(S_b + p - w)\right]}{H_b}} \tag{2}$$

**Proof.** It is necessary to derive the profit function of the buyer from the variable $Q$, to obtain the optimal buyer's order quantity in the decentralized model. Please see Appendix A.

$$\frac{d\pi_b\left(Q,L_d\right)}{dQ} = O_b\cdot\left(\frac{D}{Q^2}\right) - \frac{H_b}{2} + \left(S_b + p - w\right).\left(\frac{D}{Q^2}\right).L_d = 0 \tag{3}$$

Now, it is indicated that the second derivative of the buyer's profit function is negative $\frac{\partial^2\pi_b}{\partial Q^2} = -2\left[O_b.\left(\frac{D}{Q^3}\right)\right] - 2\left[\left(S_b + p - w\right).\left(\frac{D}{Q^3}\right).\left(L_d\right)\right] < 0$. Therefore, the profit function of the buyer is concave in $Q$.

**Proposition 2.** The buyer’s profit function $\pi_b(L_d)$ is concave in $L_d$. By optimizing the profit function with respect to $L_d$, the optimal length of delivery time $L_d^*$ under the decentralized structure is calculated as follows:

$$L_d^* = \frac{\frac{1}{4}H_b^2.k^2.\delta^2}{\left[S_b + p - w\right]^2.\frac{D^2}{Q^2}} \tag{4}$$

**Proof.** Derived from the buyer's profit function with respect to $L_d$. Please see Appendix A.

$$\frac{d\pi_b\left(Q,L_d\right)}{dL_d} = \frac{1}{2}H_b.k.\delta.\left(L_d\right)^{-\frac{1}{2}} - \left(S_b + p - w\right).\frac{D}{Q} = 0 \tag{5}$$

Since the second-order derivative of buyer's profit function with respect to $L_d$ is less than zero $\frac{\partial^2\pi_b}{\partial L_d^2} = -\frac{1}{4}H_b.k.\delta.L_d^{\frac{-3}{2}} < 0$, thus the profit function of the buyer is concave in $L_d$.

Based on Heydari et al. [5], the profit function of the vendor under the decentralized structure can be formulated as:

$$\pi_v\left(n\,|\,Q^*,L_d^*\right) = \left(w - r\right)D - O_v\left(\frac{D}{nQ^*}\right) - \left(s + TR\right).\left(\frac{D}{Q^*}\right) - C_v.L_d^* - H_v\frac{(n-1)Q^*}{2} \tag{6}$$

Where the first term shows the vendor's revenue from sold products or services, the second term denotes the vendor's ordering costs, the third term indicates the vendor's set-up costs and transportation costs, the fourth term demonstrates the vendor's delivery time reduction costs, and the last term shows the vendor's holding costs. Eventually, after determining $Q$ and $L_d$ by the buyer, the vendor's reproduction size can be calculated as:

$$Q_v^* = nQ_b^* \tag{7}$$

**Proposition 3.** Given $Q$ and $L_d$, the optimal reproduction coefficient under the decentralized structure is calculated as follows:

$$n^* = \sqrt{\frac{2O_v D}{H_v Q^{*2}}} \tag{8}$$

**Proof.** We know that to obtain the optimal reproduction coefficient, it is enough to optimize the respective vendor's profit function with respect to $n$. Please see Appendix A.

$$\frac{d\pi_v\left(n \mid Q^*, L_d^*\right)}{dn} = \frac{O_v D}{n^2 Q^*} - \frac{H_v Q^*}{2} = 0 \tag{9}$$

Since the second-order derivative of the vendor's profit function $(\frac{-2O_v D}{n^3 Q^*}) < 0$ is negative. So, the vendor profit function $\pi_v (n \mid Q^*, L_d^*)$ is concave in $n$.

## 4. Coordination model

A new coordination model is presented in this study, and we decided to have a new approach based on new conditions and assumptions. Since to encourage the buyer to participate in the SC, it is necessary to use an incentive scheme. Hence, to induce the buyer to take part in the coordinated structure $(Q^*,\ L_d^*)$ is changed to $(Q^{**},\ L_d^{**})$. In other words, by reducing the length of delivery time, the buyer ensures that the product or service is delivered in less time to the customer (compared to the decentralized structure). Furthermore, in a coordinated model, the length of production time controlled by the vendor is investigated. It is worth noting that we have made changes based on our assumptions to expand the coordinated model. We intend to provide a new approach for the buyer and the vendor profit functions. The buyer's profit function under the coordinated structure can be written as follows:

$$\pi_b\left(Q^{**},L_d^{**},L_p\right)= \tag{10}$$
$$\left(p-w\right)D-O_b\frac{D}{Q^{**}}-H_b\left[\frac{Q^{**}}{2}+k\,\delta\sqrt{L_p+L_d^{**}}\right]-\left(S_b+p-w\right).\frac{D}{Q^{**}}.\left(L_p+L_d^{**}\right)$$

**Proposition 4.** The optimal order quantity $Q^{**}$ under the coordinated structure can be calculated from the buyer's profit function as follows:

$$Q^{**}=\sqrt{\frac{2D.(O_b+S_b+p-w).\left(L_p+L_d\right)}{H_b}} \tag{11}$$

**Proof.** Derived from the buyer's profit function with respect to $Q$. Please see Appendix A.

$$\frac{d\pi_b}{dQ}=O_b.\left(\frac{D}{Q^2}\right)-\frac{1}{2}H_b+\left(S_b+p-w\right).\left(\frac{D}{Q^2}\right)\left(L_p+L_d\right)=0 \tag{12}$$

Since $\frac{\partial^2\pi_b}{\partial Q^2}=-2O_b.(\frac{D}{Q^3})-2(S_b+p-w).\frac{D}{Q^3}.(L_p+L_d)<0$ is less than zero, so the profit function of the buyer is concave in $Q$.

**Proposition 5.** The optimum length of delivery time under the coordinated structure can be formulated from the buyer's profit function as follows:

$$L_d^{**}=\frac{\left(\frac{1}{2}H_b+k.\delta\right)^2}{\left(S_b+p-w\right)^2.\frac{D^2}{Q^2}}-L_p \tag{13}$$

**Proof.** Derived from the buyer's profit function with respect to $L_d$. Please see Appendix A.

$$\frac{d\pi_b}{dL_d}=H_b.k.\delta.\left(L_p+L_d\right)^{-\frac{1}{2}}-\left(S_b+p-w\right).\frac{D}{Q}=0 \tag{14}$$

In the coordinated mathematical model, the delivery time and production time are incorporated, and according to Equation (13), the value of $L_p$ can be calculated as:

$$L_p = \frac{\left(\frac{1}{2}H_b.k.\delta\right)^2}{\left(S_b + p - w\right)^2.\frac{D^2}{Q^2}} - L_d \tag{15}$$

According to the second-order derivative of the buyer's profit function with respect to $L_d$ is less than zero $\frac{\partial^2 \pi_b}{\partial L_d^2} = -\frac{1}{4}H_b.k.\delta.\left(L_p + L_d\right)^{-\frac{3}{2}} < 0$. Therefore the profit function of the buyer is concave in $L_d$.

The profit function of the vendor under the coordinated structure must be written via the following equation:

$$\pi_v(Q^{**}, L_d^{**}, n^{**}, L_p) = \tag{16}$$

$$\left(w - r\right)D - O_v\left(\frac{D\left[\delta.k\sqrt{L_p + L_d^{**}}\right]}{n^{**}(Q^{**})^2}\right) - \left(s + TR\right).\left(\frac{D}{Q^{**}}\right) - C_v.\left(L_p + L_d^{**}\right) - \left(LRC\right)\frac{D}{Q^{**}}$$

The lead times reduction costs can be calculated as:

$$LRC = C_v.L_d^{**} \tag{17}$$

## 4.1. Buyer's terms for participating in the SC

The buyer does not participate in the SC under the coordinated structure unless its profit function is greater than the decentralized structure. Therefore, from the buyer's sight, the following condition is always required and calculated as:

$$\pi_b\left(Q^{**}, L_d^{**}, L_p\right) \geq \pi_b\left(Q^{*}, L_d^{*}\right) \tag{18}$$

According to Equation (18), if the minimum reduction of lead times under the coordinated structure is considered, the buyer tends to participate in the SC. Hence, the minimum acceptable value of $LR$ can be formulated as:

$$LR \geq \frac{\frac{1}{4}H_b^2.k^2.\delta^2}{\left[S_b+p-w\right]^2.\frac{D^2}{Q^2}} - L_p = LR_{min} \tag{19}$$

### 4.2. Vendor's conditions for taking part in the SC

If the following equation holds, the vendor is convinced to participate in the SC under the coordinated structure. So, from the vendor's sight, the following condition is always required and calculated as:

$$\pi_v\left(Q^{**}, L_d^{**}, n^{**}, L_p\right) \geq \pi_v\left(Q^{*}, L_d^{*}\right) \tag{20}$$

Since Equation (20) guarantees the vendor's profit under coordinated decision-making, therefore, the maximum acceptable value of *LR* can be formulated as:

$$LR \leq \frac{\frac{1}{4}H_b^2.k^2.\delta^2}{\left[S_b+p-w\right]^2.\frac{D^2}{Q^2}} + L_p = LR_{max} \tag{21}$$

## 5. Numerical experiment

In this section, the validation of the mathematical model is verified by a sensitivity analysis of important parameters. Therefore, we aim to investigate the impact of some important parameters on the decentralized model and the coordinated one. Hence, to examine the performance of the proposed model, five experiments are implemented. Table 2 provides data for five investigated test problems. The test problems are executed under different strategies (decentralized and coordinated decision-making models). As demonstrated in Table 3, the decision variables and the profit functions' behavior change under the two decision-making strategies by changing parameters.

Since decision-making under the coordinated structure causes an increase in SC members' profit compared to the decentralized model. Hence, the results of solving five different test problems in Table 3 show that as the delivery time decreases, the profitability of the SC members under the coordinated structure is more than in the decentralized structure. Therefore, the veracity of the mathematical model is confirmed according to the results in Table 3. Additionally, Section 5 is indicated that rational and optimal decisions based on the sensitivities analysis are made to motivate SC members to be more profitable. Eventually, all of these result in forming an efficient, effective, and profitable SC.

In the remainder of this section, the effect of the wholesale price and the standard deviation of demand on $LR_{min}$ and $LR_{max}$ are analyzed. Afterward, the impact of the length of delivery time on the order quantity under the decentralized and coordinated structures is compared and examined. Finally, the effect of the buyer's shortage costs on the order quantity is investigated.

### 5.1. The impact of the wholesale price and the standard deviation of demand on $LR_{min}$ and $LR_{max}$:

Figure. 2 indicates the effect of the wholesale price on $LR_{min}$ and $LR_{max}$. A rise in the wholesale price leads to an increase in the vendor's profitability. Hence, the percentage of $LR_{max}$ is faced with a downward trend, which is very suitable from the vendor's sight because the vendor wants to reduce the value of $LR$ from the upper limit ($LR_{max}$). On the other hand, the continuous increase in wholesale prices practically reduces the effect of lead times reduction as a motivating factor for the buyer's participation in the SC. So, the buyer is not willing to participate in the SC under coordinated decisions.

In other words, the proximity of $LR_{min}$ and $LR_{max}$ causes the model to practically lose its efficiency and the lead times reduction (agreed between the buyer and the vendor) not to be realized. As a result, the vendor must be very careful in increasing the wholesale price because its continuous increase can have a negative impact on buyer participation.

Figure. 3 indicates that the model is enforceable when $LR_{max}$ is greater than $LR_{min}$. As the value of the standard deviation of demand increases, the range between $LR_{max}$ and $LR_{min}$ becomes closer. In other words, if the standard deviation of demand increases uncontrollably, the model practically loses its efficiency. For instance, Figure. 3 shows that if the rate of deviation of demand exceeds 800 units of product and this trend continues, $LR_{max}$ will be less than $LR_{min}$. As described earlier, increasing the value of the standard deviation of demand results in the proximity of $LR_{min}$ and $LR_{max}$. Besides, for values over 800, the mathematical model fails to solve the problem since the inequality $LR_{max} \geq LR_{min}$ should hold.

Nevertheless, the buyer makes a mistake in predicting the demand of its customers and can not have a proper estimate of future demand (the amount of standard deviation of demand is constantly increasing). In that case, the order quantity is less or more than the actual demand of customers most of the time. If the order quantity is less than the actual amount, the buyer decides to re-order at short intervals to prevent lost sales. Hence, the vendor can not deliver the product or service to the buyer at the expected lead times. Therefore, Figure. 3 demonstrates that the $\delta$ is highly sensitive to $LR_{min}$ and $LR_{max}$ and its upward trend connotes that the buyer does not accurately predict demand, which in turn will have a detrimental impact on $LR$. Consequently, the buyer must control the amount of standard deviation of demand. Also, the buyer must strive to better and more accurately estimate its future demands because determining the exact amount of standard deviation of demand to reduce lead times and timely delivery of the product to the buyer is effective.

### 5.2. The impact of the length of delivery time and the buyer's shortage costs on the order quantity:

Figure. 4 shows the value of $L_d$ over $Q$ changes. According to the mathematical model and the logical dependency between $L_d$ and $Q$, it is evident that the behavior of the two decision variables can be analyzed. Increasing the delivery time of a product causes the buyer to increase the order quantity value because the buyer has no desire to face a shortage of products. Therefore, for the buyer to be more profitable, the amount of $Q$ under the coordinated decision structure must be more than the decentralized decision structure. In other words, the buyer is only willing to participate in the SC if its profitability increases under the coordinated structure compared to the decentralized structure. Moreover, reducing the delivery time makes it possible to reduce the lead times. Hence, the buyer's motivation is raised for participation in the coordinated structure.

It should be noted that the buyer tends to reduce the delivery time and get the products in a short time to maintain its market share in competition with other competitors. Hence, the buyer prefers to change its order quantity and increase it to remain in the competitive market, expand its market, and attract more customers.

On the other hand, in a shortage of products, the buyer incurs shortage costs and loses the market and its customers. Hence, the buyer prefers to increase the order quantity to avoid this condition. In Figure. 5, raising the buyer's shortage costs by increasing the order quantity is denoted.

The buyer should pay attention to adjusting the order quantity so as not to face a shortage and not incur payment for the shortage costs. Therefore, in Figure. 5, the increase in shortage costs induces the buyer to pursue an order quantity raising policy. As a result of this behavioral policy, the buyer's markets remain, and the shortage costs are avoided. At last, the figures in Section 5 help the buyer and vendor adjust the various parameters and their impact on the decision variables to adopt a precise policy for participation in the SC to gain benefits under the coordinated structure.

## 6. Managerial insight

In this section, we decide to express the sensitivities analysis in the form of managerial insight. The delivery time of a product or service is always a significant subject for SC managers. The delivery time is crucial for vendors because it creates a competitive advantage compared to competitors. However, it can act as a defect causing a vendor to lose the market and buyers. It should be noted that SC members in competitive and exclusive markets have different approaches to the delivery time. This difference in SC partners' policies to gain greater profitability has increased the need for SC coordination. Hence, managers must have the ability to make decisions in both competitive and exclusive markets.

Figure. 5 shows the managerial concept that in an exclusive market, the vendor or manufacturer determines the policies of SC, and buyers are forced to follow them. For example, a service or product that is only offered by one or more limited companies in the market forces the buyers to comply with the vendors' policies. In an exclusive market, prolonging the lead times by the vendor or manufacturer causes the buyer to issue more order quantity each time so that he does not face a shortage. Hence, the buyer has to order more than it needs to avoid losing customers.

Nonetheless, in a competitive market, the buyer has the power and the choice to decide. Since the buyers and vendors have a conflict of interest, each tends to make more profit. Therefore, SC coordination is considered a solution to maximize the profits of SC members and the entire SC for the decision of managers.

In the remainder of this section, other research implications are discussed. As mentioned earlier, creating a coordination mechanism requires that SC members participate in a coordinated SC. Hence, considering the logical conditions and constraints, the mathematical model of this research helps the manager of a SC make an optimal decision for SC to ensure the profitability of all members and the whole SC while maintaining the market and its buyers. The lead times reduction motivates the buyer to participate in the SC, but it also incurs costs to the vendor. Therefore, in the mathematical model, logical equations (logical constraints), such as Equation (18), Equation (19), Equation (20), and Equation (21) encourage SC members to make coordinated decisions.

Figure. 2 provides the management insight for the SC managers that if the wholesale price increases continuously, the defined range $LR_{min} \leq LR \leq LR_{max}$ practically disappears, and the mathematical model loses its efficiency. As a result, it must be planned to maintain the efficiency and performance of the mathematical model in the optimal mode by setting a reasonable wholesale price and determining the lead times reduction (within the defined range). In other words, SC managers must plan for the profitability of SC members so that the lead times reduction is such that both the vendor and the buyer are motivated to participate under a coordinated decision structure.

## 7. Conclusion

This paper considered the components of lead times (i.e., production time and delivery time) in a mathematical model. The buyer was stimulated to participate in the SC under the coordinated decision structure by reducing the delivery time as an incentive scheme and increasing the profit function. This research provided a more practical approach to coordination in SC. Hence, the decentralized and coordinated structures were considered in the mathematical model. Besides, components of total lead times include the length of production time as a parameter and the length of delivery time as a decision variable were studied. Also, the impacts of the wholesale price and the standard deviation of demand on $LR_{min}$ and $LR_{max}$ were taken into account. Furthermore, the effect of the delivery time and the buyer's shortage costs on the order quantity were examined.

The computational results revealed that by fine-tuning the parameters and sensitivity analysis of the parameters' effect on the decision variables and the lead times, the buyer and vendor tend to participate in the SC. Moreover, the decentralized or coordinated structures had similar performances concerning the order quantity. Nonetheless, the changes showed a more ameliorated trend in the coordinated model than in the decentralized one.

This paper reduced delivery time as an incentive scheme for buyer participation in the SC. For future studies, this research stream can be further developed by considering other components of the lead times, such as startup time, shipping time, and so on. In such cases, the cost of lead times reduction (the vendor pays for it) should be such that the vendor also wants to be present in the SC under a coordinated decision structure. Finally, combining incentive schemes with coordination contracts is an interesting and challenging task for future studies.

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## Figures and Tables Captions' List

**Table 2.** The five examined test problems

**Table 3.** Earned results from the five investigated test problems

**Figure 2.** The effect of the wholesale price on $LR_{min}$ and $LR_{max}$

**Figure 3.** The effect of the standard deviation of demand on $LR_{min}$ and $LR_{max}$

**Figure 4.** Value of $Q$ for changes in $L_d$

**Figure 5.** Value of $Q$ for changes in $S_b$

## Figures and Tables

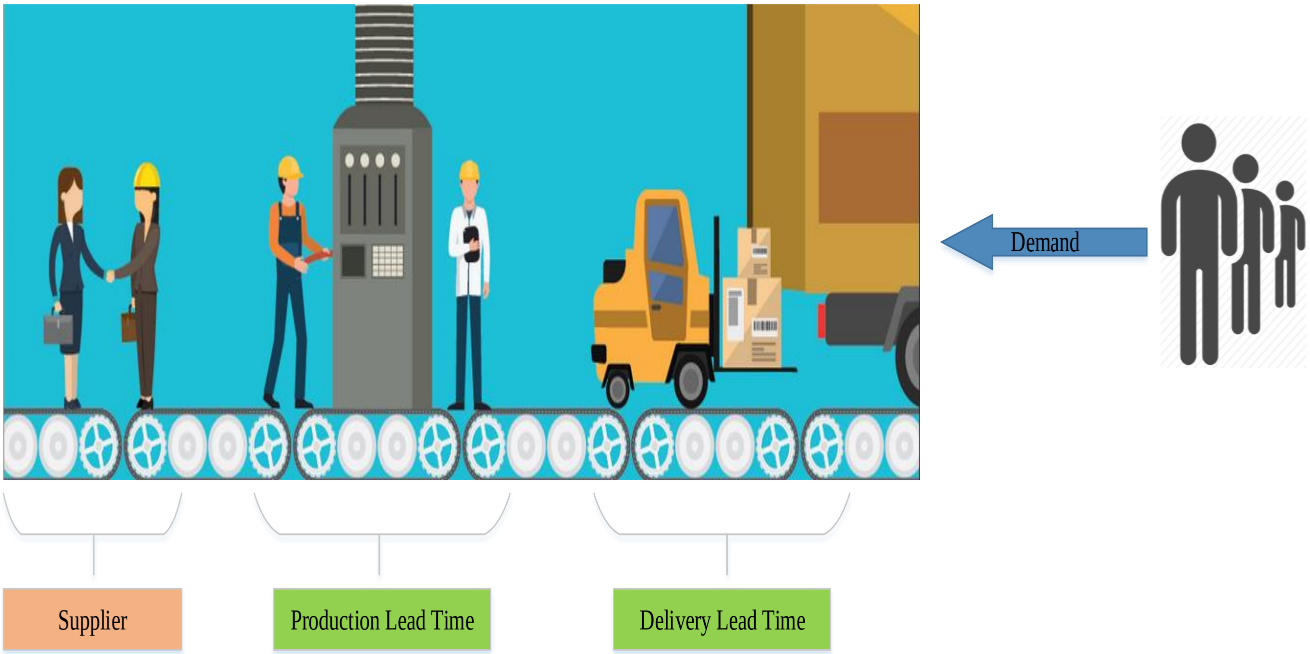

**Figure. 1**

**Table 1.**

| Reference | SC system | | Number of periods | | Decision-making strategy | | | Demand depends on | | | | | | Practical area | | | | Plan for coordination | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Two echelon | Three echelon | Single- period | Multi-periods | Decentralized | Centralized | Coordinated | Time | Price | Lead time | Order quantity | Delivery time | Production time | Different industries | Seasonal products | Financing | environmental impact of industries | Contract | Incentive mechanism |
| [5] | ✓ | | ✓ | | ✓ | ✓ | ✓ | | | ✓ | | | | ✓ | | | | | ✓ |
| [21] | ✓ | | ✓ | | ✓ | ✓ | ✓ | | ✓ | ✓ | | | | ✓ | | | | | ✓ |
| [24] | ✓ | | ✓ | | ✓ | | ✓ | | | | ✓ | | | ✓ | | | ✓ | ✓ | |
| [27] | ✓ | | ✓ | | ✓ | ✓ | ✓ | | ✓ | | | | | ✓ | | | | ✓ | |
| [28] | ✓ | | ✓ | | ✓ | | ✓ | | | | ✓ | | | | ✓ | | | ✓ | |
| [33] | ✓ | | | ✓ | ✓ | | ✓ | ✓ | ✓ | | | | | ✓ | | ✓ | | ✓ | |
| [36] | ✓ | | ✓ | | | | ✓ | ✓ | | | | | | ✓ | | | | ✓ | |
| [37] | ✓ | | ✓ | | | | ✓ | | | | ✓ | | | | | | ✓ | ✓ | |
| **This paper** | ✓ | | ✓ | | ✓ | | ✓ | | | ✓ | | ✓ | ✓ | ✓ | | | | | ✓ |

**Table 2.**

| Parameter | Problem 1 | Problem 2 | Problem 3 | Problem 4 | Problem 5 |
|---|---|---|---|---|---|
| $D$ | 25,000 | 10,000 | 15,000 | 22,000 | 25,000 |
| $p$ | 35 | 20 | 22 | 24 | 25 |
| $w$ | 22 | 14 | 15 | 20 | 19 |
| $r$ | 12 | 10 | 10 | 14 | 15 |
| $H_b$ | 10 | 3 | 5 | 8 | 9 |
| $H_v$ | 5 | 2 | 3 | 4 | 5 |
| $O_b$ | 300 | 200 | 250 | 160 | 80 |
| $O_v$ | 80 | 40 | 190 | 100 | 50 |
| $S_b$ | 3 | 1 | 2 | 3 | 1 |
| $C_v$ | 15 | 10 | 8 | 5 | 3 |
| $L_p$ | 14 | 12 | 13 | 14 | 14 |
| $\delta$ | 500 | 300 | 300 | 400 | 500 |
| $k$ | 3 | 2 | 1.5 | 2.8 | 2.5 |
| $TR$ | 100 | 90 | 80 | 70 | 65 |
| $s$ | 60 | 50 | 40 | 30 | 35 |

**Table 3.**

| | **Problem 1** | **Problem 2** | **Problem 3** | **Problem 4** | **Problem 5** |
|---|---|---|---|---|---|
| **Decentralized decision structure** | | | | | |
| $\boldsymbol{Q}^*$ | 2110 | 1876 | 1650 | 1820 | 2050 |
| $\boldsymbol{L}_{\boldsymbol{d}}^*$ | 35 | 25 | 20 | 25 | 30 |
| $\boldsymbol{n}^*$ | 1 | 1 | 1 | 1 | 1 |
| $\boldsymbol{\pi}_{\boldsymbol{b}}$ | 60,345 | 52,254 | 49,456 | 55,140 | 51,750 |
| $\boldsymbol{\pi}_{\boldsymbol{v}}$ | 105,788 | 98,456 | 88,650 | 93,288 | 91,855 |
| **Coordinated decision structure** | | | | | |
| $\boldsymbol{Q}^{**}$ | 2240 | 1950 | 1955 | 2064 | 2185 |
| $\boldsymbol{L}_{\boldsymbol{d}}^{**}$ | 29 | 22 | 17 | 19 | 23 |
| $\boldsymbol{n}^{**}$ | 1 | 1 | 1 | 1 | 1 |
| $\boldsymbol{\pi}_{\boldsymbol{b}}$ | 61,578 | 52,925 | 49,956 | 55,870 | 52,477 |
| $\boldsymbol{\pi}_{\boldsymbol{v}}$ | 106,956 | 100,345 | 89,340 | 94,032 | 92,625 |
| $\boldsymbol{LR}_{\boldsymbol{min}}$ | 48% | 43% | 40.3% | 38% | 24.5% |
| $\boldsymbol{LR}_{\boldsymbol{max}}$ | 61% | 54% | 63% | 55% | 30% |
| $\boldsymbol{LR}$ | 50% | 50% | 48% | 43% | 28% |

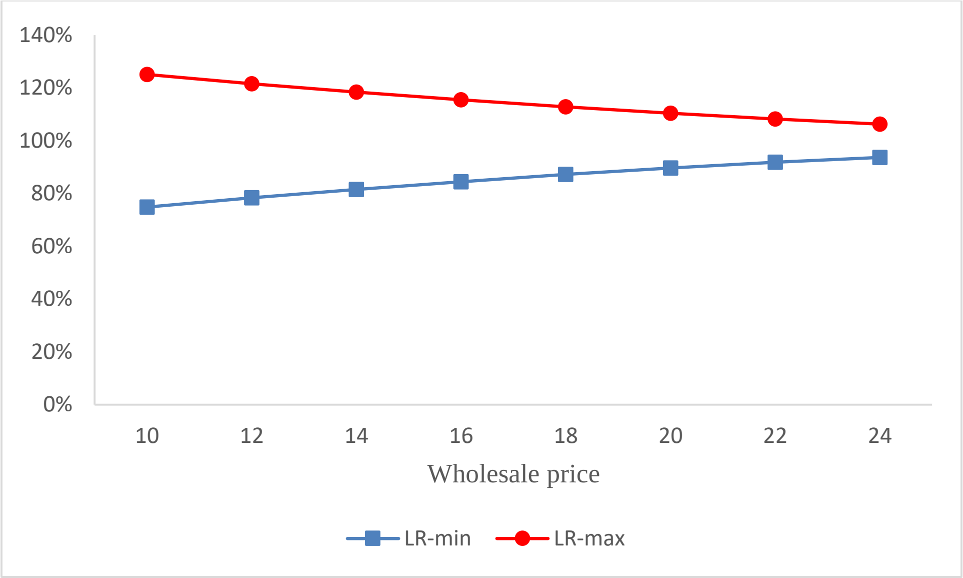

**Figure. 2**

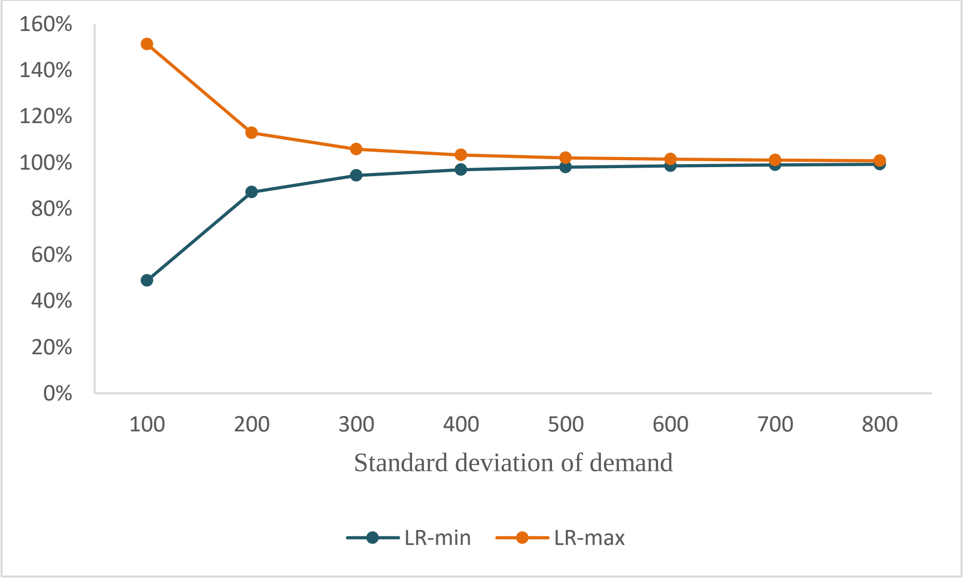

**Figure. 3**

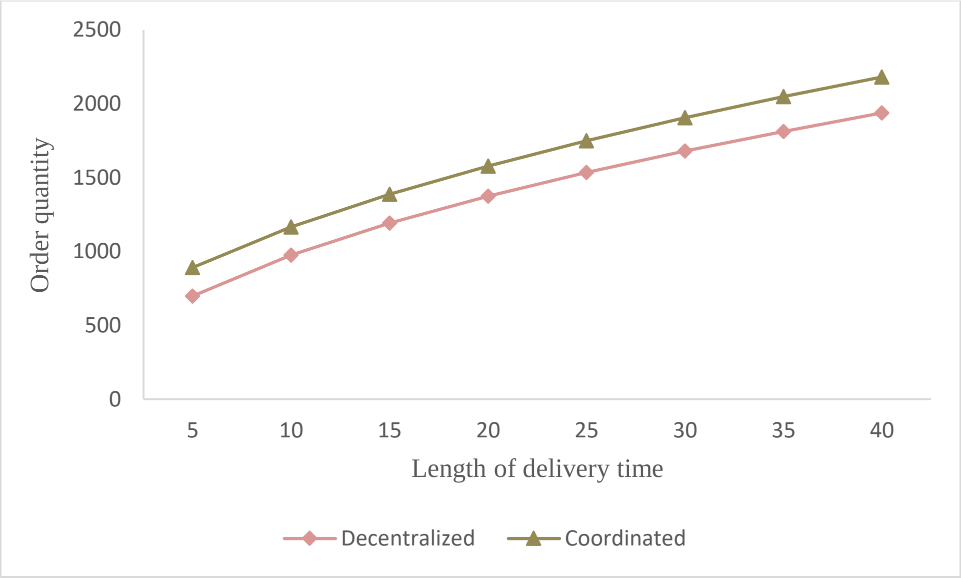

**Figure. 4**

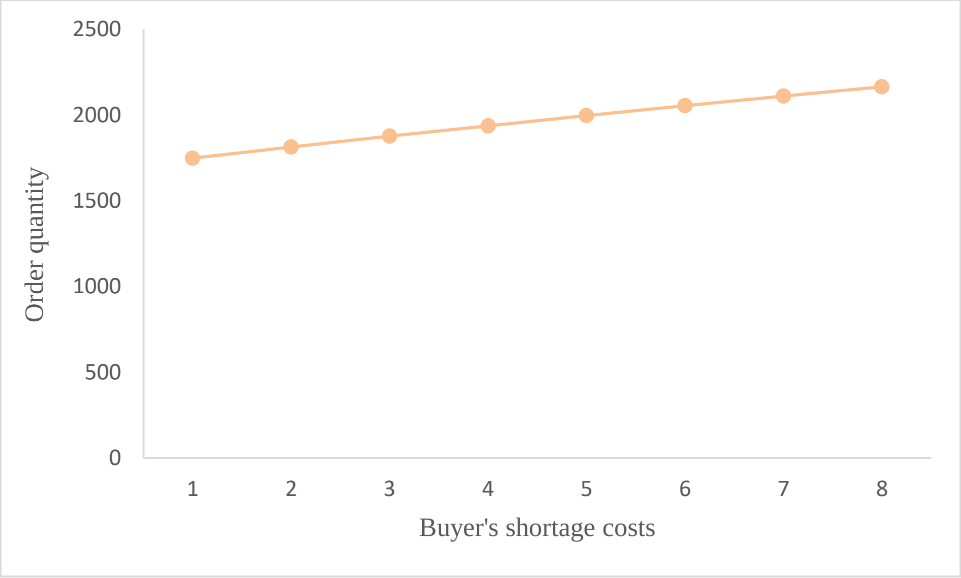

**Figure. 5**

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# Appendix A

## Proof of proposition 1

From Equation (1), we have:

$$\frac{d\pi_b(Q,L_d)}{dQ} = O_b.\left(\frac{D}{Q^2}\right) - \frac{H_b}{2} + \left(S_b + p - w\right).\left(\frac{D}{Q^2}\right).L_d = 0 \tag{A.1}$$

Then, derivative calculations are continued:

$$\frac{D}{Q^2} = \frac{H_b}{2\left[O_b + L_d.\left(S_b + p - w\right)\right]} \tag{A.2}$$

$$Q^2.H_b = 2D.[O_b + L_d.(S_b + p - w)] \tag{A.3}$$

$$Q^2 = \frac{2D.\left[O_b + L_d.\left(S_b + p - w\right)\right]}{H_b} \tag{A.4}$$

So, we obtained optimal order quantity under the decentralized structure $(Q^*)$. This proof is complete.

## Proof of proposition 2

From Equation (1), we obtain:

$$\frac{d\pi_b(Q,L_d)}{dL_d} = \frac{1}{2}H_b.k.\delta.(L_d)^{-\frac{1}{2}} - (S_b + p - w).\frac{D}{Q} = 0 \tag{A.5}$$

Then derivative calculations are continued:

$$\frac{1}{\sqrt{L_d}}.\left(-\frac{1}{2}H_b.k.\delta\right) = \left(S_b + p - w\right).\frac{D}{Q} \tag{A.6}$$

$$\frac{1}{\sqrt{L_d}} = \frac{\left(S_b + p - w\right).\frac{D}{Q}}{-\frac{1}{2}H_b.k.\delta} \tag{A.7}$$

$$\sqrt{L_d}.[S_b + p - w]\frac{D}{Q} = -\frac{1}{2}H_b.k.\delta \tag{A.8}$$

$$L_d.\left[S_b + p - w\right]^2.\frac{D^2}{Q^2} = \frac{1}{4}H_b^2.k^2.\delta^2 \tag{A.9}$$

Therefore, we calculate the optimal length of delivery time under the decentralized structure $(L_d^*)$. This proof is complete.

**Proof of proposition 3**

From Equation (6), we have:

$$\frac{d\pi_v(n\,|Q^*, L_d^*)}{dn} = \frac{O_v D}{n^2 Q^*} - \frac{H_v Q^*}{2} = 0 \tag{A.10}$$

$$2O_v D = n^2 H_v Q^{*2} \tag{A.11}$$

$$n^2 = \frac{2O_v D}{H_v Q^{*2}} \tag{A.12}$$

Thus we compute the optimal reproduction coefficient under the decentralized structure $(n^*)$. This proof is complete.

**Proof of proposition 4**

From Equation (10), we have:

$$\frac{d\pi_b}{dQ} = O_b.\left(\frac{D}{Q^2}\right) - \frac{1}{2}H_b + \left(S_b + p - w\right).\left(\frac{D}{Q^2}\right)\left(L_p + L_d\right) = 0 \tag{A.13}$$

$$\left(\frac{D}{Q^2}\right) = \frac{\frac{1}{2}H_b}{(O_b + S_b + p - w)\left(L_p + L_d\right)} \tag{A.14}$$

$$\frac{1}{2}H_b.Q^2 = D.(O_b + S_b + p - w)\left(L_p + L_d\right) \tag{A.15}$$

$$Q^2 = \frac{2D.(O_b + S_b + p - w)\left(L_p + L_d\right)}{H_b} \tag{A.16}$$

As a result, we can get the optimal order quantity under the coordinated structure $(Q^{**})$. This proof is complete.

**Proof of proposition 5**

From Equation (10), we have:

$$\frac{d\pi_b}{dL_d} = H_b.k.\delta.\left(L_p + L_d\right)^{-\frac{1}{2}} - \left(S_b + p - w\right).\frac{D}{Q} = 0 \tag{A.17}$$

$$\frac{\frac{1}{2}H_b.k.\delta}{\sqrt{\left(L_p + L_d\right)}} = \left(S_b + p - w\right).\frac{D}{Q} \tag{A.18}$$

$$\sqrt{L_p + L_d} = \frac{\frac{1}{2}H_b.k.\delta}{\left(S_b + p - w\right).\frac{D}{Q}} \tag{A.19}$$

$$\left(L_p + L_d\right) = \frac{\left(\frac{1}{2}H_b.k.\delta\right)^2}{\left(S_b + p - w\right)^2.\frac{D^2}{Q^2}} \tag{A.20}$$

Therefore, we calculate the optimal length of delivery time under the coordinated structure $(L_d^{**})$. This proof is complete.