An information measure based extended VIKOR method in intuitionistic fuzzy valued neutrosophic value setting for multi-criteria group decision making

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Abstract

The paper presents an extended VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method for solving group decision-making problems. The uncertainties given in the data are handled with the help of the intuitionistic fuzzy valued neutrosophic values (IFVNVs), which allow decision-makers to carry more detailed information while providing their preferences in the imprecise environment. The proposed VIKOR method utilized the features of IFVNVs and computed the distance measures between their pairs using L^p-metric and L^∞-metric. The weights of the different criteria are computed by using the entropy-based measures for the families of IFVNVs. The presented method has been illustrated with a numerical example. A comparative interpretation and the sensitivity analysis of the parameter associated with the technique are achieved to reveal their influences.

Keywords: Intuitionistic fuzzy valued neutrosophic value, decision-making, entropy, distance measure, VIKOR method.

1. Introduction

Uncertainty is an avoidable phenomenon in our day-to-day life situations. In almost every sector, uncertainty plays a dominant role during the process. Thus, dealing with information-based vagueness and uncertainty has always been challenging for researchers. To address this, Zadeh [1] introduced the concept of fuzzy set (FS) theory, which is an extension of the classical set theory. In FS, each element is associated with a membership grade $\mu_A : X \to [0,1]$ which represents the degree of belongingness of the element in the set. After the existence of this set, several models have been reported by the researchers to tackle the vagueness and uncertainty in the data. In this line, Atanassov [2] extended FSs to intuitionistic fuzzy set (IFS) by considering the non-membership grades ν along the membership grades μ, i.e., for a set A, a membership (μ_A) and non-membership (ν_A) functions are associated with the element such that $\mu_A + \nu_A \leq 1$ for $\mu_A, \nu_A \in [0,1]$. Later on, some other extensions of the FSs have been proposed by the researchers such Pythagorean fuzzy sets [3], neutrosophic set [4], q-rung orthopair fuzzy sets [5], spherical fuzzy sets [6, 7]. These extensions have been widely used by the researchers to address the problems such as multi-criteria decision making (MCDM), multi-criteria group decision making (MCGDM), classification and pattern recognition etc. (see, e.g., [8–18]).

Smarandache [4] proposed the neutrosophic set theory from a philosophical perspective as a generalisation of FSs. A neutrosophic set (NS) is characterized by a truth membership function T, an indeterminacy membership function I and a falsity membership function F and each membership degree is a real standard or non-standard subset of the non-standard unit interval. Besides, there is no restraint on the sum of the membership functions. The concept has various generalizations such as single-valued neutrosophic set (SVNS) [19], interval neutrosophic set [20], neutrosophic cubic set [21], single-valued neutrosophic linguistic set [22], simplified neutrosophic set (SNS) [23] etc. nver et al. [24] has defined the concepts of intuitionistic fuzzy valued neutrosophic multi-set (IFVNMS) and intuitionistic fuzzy valued neutrosophic multi-value (IFVNMV).

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They have defined some arithmetic operations for IFVNMVs and have proposed a weighted arithmetic and a weighted geometric aggregation operator. In [24] the authors studying in multi-set setting. In this paper, we take the sequence length equal to 1 for each criterion. Thus, we study with FSs instead of fuzzy multi-sets. We use the term intuitionistic fuzzy valued neutrosophic value (IFVNV) instead of IFVNMV.

The concept of entropy is one of the most important notions of the information theory. Entropy quantifies the amount of uncertainty involved in the value of a random variable or the outcome of a random process. Shannon's entropy that is one of the most important entropy types is frequently used in the literature [25]. Shannon's entropy is a useful measurement method and it is used to measure the uncertainty in randomly distributed data in information analysis. Therefore, this concept is closely related to the FS theory. Zadeh [26] extended Shannon's entropy to the concept of fuzzy entropy. The concept of fuzzy entropy is a fuzzy information measure which is defined by measuring the fuzziness of a fuzzy event or a FS. Various types of entropy have been defined in different FS settings in information theory. For example, De Luca and Termini [27] determined axioms of fuzzy entropy and proposed a fuzzy entropy that is based on Shannons function and Yager [28] proposed a fuzzy entropy by using a distance between a FS and its complement. Ye [29] introduced intuitionistic fuzzy entropy and then proposed a correlation coefficient of interval-valued intuitionistic fuzzy sets by using entropy weights [30]. Then Zhang et al. [31] introduced some new entropy measures based on distance for interval-valued intuitionistic fuzzy sets. Cui and Ye [32] proposed dimension root entropy and exponential entropy measure [33] for simplified neutrosophic sets and developed a sine entropy weight model [34]. Xiao [35] defined a distance measure by using the relative entropy [36].

The VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method [37] was introduced by Serafim Opricovic and it is a MCDM method. The main aim of the VIKOR is to find a ranking for alternatives and a compromise solution that is a feasible solution closest to the ideal solution. VIKOR method has been applied many decision making problems in the literature. For example, Parhizgarsharif et. al. [38] have used the VIKOR method and gray relational analysis to rank facility location in construction site layout for Mehr project in Tehran and shown that both methods yield the same ranking. Lin et. al. [39] have put forward some recent MCDM techniques including VIKOR method to handle MCDM problems with probabilistic linguistic term sets. A Fuzzy VIKOR method [40] which relates to distance of an alternative to the ideal solution was developed to solve MCDM problems in a fuzzy environment. Then, fuzzy VIKOR method has been extended to its several fuzzy versions by using various types of FSs such as intuitionistic fuzzy VIKOR [41, 42], hesitant fuzzy VIKOR [43], linguistic hesitant fuzzy VIKOR [44], spherical fuzzy VIKOR [45], complex q-rung orthopair uncertain linguistic VIKOR [46]. Moreover, Zare et. al. [47] have applied fuzzy group VIKOR method to suitable computerized maintenance management system selection. On the other hand, the concept of entropy is often use in VIKOR method to determine weights of criteria in MCDM process (for more details, we refer to read the articles [48–50]).

This paper proposes an extended VIKOR method by using IFVNVs for MCGDM. In this method, IFVNVs allow reflecting more detailed information while the data is transformed into fuzzy information so decision-makers can assign less strict fuzzy values. Intuitionistic fuzzy values, which are considered fuzzy set's building blocks, help decision-makers perform a more flexible decision-making process. Besides this extended VIKOR method providing a new MCDM technique to the literature, it also presents a more sensitive approach thanks to the decision-making process carried out in a fuzzy environment. The main objectives of this paper are

1) To propose a distance measure and entropy measures for the pairs of IFVNVs.
2) To present an extended VIKOR method for group decision-making process.
3) To validate the method's performance with the aid of some numerical examples.

The rest of the paper is organized as follows. Section 2 deals with the distance and entropy measures for the pairs of IFVNVs and their axioms. Section 3 presents an extended VIKOR method by using the stated L^p-metric. Section 4 gives a numerical example to illustrate the proposed VIKOR method for MCGDM problems. The comparative, as well as sensitivity analysis, is carried out in Section 5. Finally, a conclusion is drawn in Section 6.

2. Proposed distance and entropy measures

In this section, we define a distance measure for IFVNVs and an entropy measure for families of IFVNVs. Let us recall firstly some fundamental concepts that are used in the present paper.

Definition 2.1. [2] Let $\alpha^t, \alpha^f \in [0,1]$ such that $\alpha^t + \alpha^f \leq 1$. Then the pair (α^t, α^f) is called an intuitionistic fuzzy value (IFV).

Definition 2.2. [24] Let $\mathcal{Y} = \{u_1, ..., u_n\}$ be a finite set. An IFVNMS defined on $\mathcal{Y}$ is given with

$$A = \left\{ \left\langle u_i, \left((\mathbb{T}_A^{i,j})_{j=1}^{p_i}, (\mathbb{I}_A^{i,j})_{j=1}^{p_i}, (\mathbb{F}_A^{i,j})_{j=1}^{p_i} \right) \right\rangle : i = 1, ...n \right\}$$

where $(\mathbb{T}_A^{i,j})_{j=1}^{p_i}$, $(\mathbb{I}_A^{i,j})_{j=1}^{p_i}$ and $(\mathbb{F}_A^{i,j})_{j=1}^{p_i}$ are the truth, the indeterminacy and the falsity membership sequences of IFVs, respectively, i.e., for any $i = 1, ..., n,\ j = 1, ..., p_i$

$$\mathbb{T}_A^{i,j} = (T_A^{j,t_i}, T_A^{j,f_i}), \text{with } T_A^{j,t_i}, T_A^{j,f_i} \in [0,1] \text{ such that } 0 \leq T_A^{j,t_i} + T_A^{j,f_i} \leq 1,$$

$$\mathbb{I}_A^{i,j} = (I_A^{j,t_i}, I_A^{j,f_i}), \text{with } I_A^{j,t_i}, I_A^{j,f_i} \in [0,1] \text{ such that } 0 \leq I_A^{j,t_i} + I_A^{j,f_i} \leq 1$$

and

$$\mathbb{F}_A^{i,j} = (F_A^{j,t_i}, F_A^{j,f_i}), \text{with } F_A^{j,t_i}, F_A^{j,f_i} \in [0,1] \text{ such that } 0 \leq F_A^{j,t_i} + F_A^{j,f_i} \leq 1.$$

For a fixed $i = 1, ..., n$ the expression

$$\alpha = \langle (\mathbb{T}_\alpha^j)_{j=1}^p, (\mathbb{I}_\alpha^j)_{j=1}^p, (\mathbb{F}_\alpha^j)_{j=1}^p \rangle := \left\langle u_i, \left((\mathbb{T}_A^{i,j})_{j=1}^{p_i}, (\mathbb{I}_A^{i,j})_{j=1}^{p_i}, (\mathbb{F}_A^{i,j})_{j=1}^{p_i} \right) \right\rangle$$

denotes an IFVNMV.

Taking the sequence lengths equal to 1 we define the notion of IFVNV. An IFVNV has a form

$$A = \langle \mathbb{T}_A, \mathbb{I}_A, \mathbb{F}_A \rangle$$

where $\mathbb{T}_A$, $\mathbb{I}_A$ and $\mathbb{F}_A$ are the intuitionistic fuzzy valued truth, indeterminacy and falsity membership degrees, respectively, i.e.,

$$\mathbb{T}_A = (T_A^t, T_A^f), \text{with } T_A^t, T_A^f \in [0,1] \text{ such that } 0 \leq T_A^t + T_A^f \leq 1,$$

$$\mathbb{I}_A^{i,j} = (I_A^t, I_A^f), \text{with } I_A^t, I_A^f \in [0,1] \text{ such that } 0 \leq I_A^t + I_A^f \leq 1$$

and

$$\mathbb{F}_A^{i,j} = (F_A^t, F_A^f), \text{with } F_A^t, F_A^f \in [0,1] \text{ such that } 0 \leq F_A^t + F_A^f \leq 1.$$

Remark 2.1. Unver et. al. [24] have presented an algebraic weighted arithmetic aggregation operator and an algebraic weighted geometric aggregation operator for IFVNMVs. By simplifying these operators for sequence length 1, we consider the weighted arithmetic operator and weighted geometric operator

$$WA_A - IFVNV(A_1, ..., A_m) = \begin{pmatrix} \left(1 - \prod_{i=1}^m (1 - T_{A_i}^t)^{\omega_i}, \prod_{i=1}^m \left(T_{A_i}^f\right)^{\omega_i}\right), \left(\prod_{i=1}^m (I_{A_i}^t)^{\omega_i}, \right. \\ \left. 1 - \prod_{i=1}^m \left(1 - I_{A_i}^f\right)^{\omega_i}\right), \left(\prod_{i=1}^m (F_{A_i}^t)^{\omega_i}, 1 - \prod_{i=1}^m \left(1 - F_{A_i}^f\right)^{\omega_i}\right) \end{pmatrix} \tag{1}$$

and

$$WG_A - IFVNV(A_1, ..., A_m) = \begin{pmatrix} \left(\prod_{i=1}^m (T_{A_i}^t)^{\omega_i}, 1 - \prod_{i=1}^m \left(1 - T_{A_i}^f\right)^{\omega_i}\right), \left(1 - \prod_{i=1}^m (1 - I_{A_i}^t)^{\omega_i}, \right. \\ \left. \prod_{i=1}^m \left(I_{A_i}^f\right)^{\omega_i}\right), \left(1 - \prod_{i=1}^m (1 - F_{A_i}^t)^{\omega_i}, \prod_{i=1}^m \left(F_{A_i}^f\right)^{\omega_i}\right) \end{pmatrix} \tag{2}$$

respectively, where $\{A_1, ..., A_m\}$ is a family of IFVNVs and $\{\omega_1, ..., \omega_m\}$ is a weight vector such that $0 \leq \omega_i \leq 1$ with $\sum_{i=1}^m \omega_i = 1$.

Now we define a distance measure for IFVNVs. Let $\alpha = (\alpha^t, \alpha^f)$ and $\beta = (\beta^t, \beta^f)$ be two IFVs. Xiao [35] has given the distance measure $d_{\varkappa}$ defined by

$$d_{\varkappa}(\alpha,\beta) = \frac{1}{\sqrt{2}}\left[\begin{array}{l}\alpha^t \ln \dfrac{2\alpha^t}{\alpha^t+\beta^t} + \beta^t \ln \dfrac{2\beta^t}{\alpha^t+\beta^t} + \alpha^f \ln \dfrac{2\alpha^f}{\alpha^f+\beta^f} \\ + \beta^f \ln \dfrac{2\beta^f}{\alpha^f+\beta^f} + \pi(\alpha) \ln \dfrac{2\pi(\alpha)}{\pi(\alpha)+\pi(\beta)} + \pi(\beta) \ln \dfrac{2\pi(\beta)}{\pi(\alpha)+\pi(\beta)}\end{array}\right]^{1/2} \tag{3}$$

Using the distance measure $d_{\varkappa}$ we now propose the promised distance measure.

Definition 2.3. Let $A = \langle \mathbb{T}_A, \mathbb{I}_A, \mathbb{F}_A \rangle$ and $B = \langle \mathbb{T}_B, \mathbb{I}_B, \mathbb{F}_B \rangle$ be two IFVNVs. A distance measure is given by

$$\mathcal{D}(A,B) = \frac{1}{3}\left[d_{\varkappa}(\mathbb{T}_A,\mathbb{T}_B) + d_{\varkappa}(\mathbb{I}_A,\mathbb{I}_B) + d_{\varkappa}(\mathbb{F}_A,\mathbb{F}_B)\right]. \tag{4}$$

Proposition 2.1. For IFVNVs A and B, the proposed distance measure $\mathcal{D}$ satisfies the following properties:

i) $0 \le \mathcal{D}(A,B) \le 1$.
ii) If $A = B$, then $\mathcal{D}(A,B) = 0$.
iii) $\mathcal{D}(A,B) = \mathcal{D}(B,A)$.
iv) $\mathcal{D}(A,B) \le \mathcal{D}(A,C) + \mathcal{D}(B,C)$ for IFVNV C.

Proof. It can be easily obtained from Definition 2.3. □

Now we give an entropy measure for families of IFVNVs.

Definition 2.4. Let $\mathcal{F} = \{A_1, ..., A_m\}$ be a collection of IFVNVs where

$$A_i = \left\langle (T^t_{A_i}, T^f_{A_i}), (I^t_{A_i}, I^f_{A_i}), (F^t_{A_i}, F^f_{A_i}) \right\rangle$$

for any $i = 1, ..., m$. An entropy measure for $\mathcal{F}$ is defined by

$$E(\mathcal{F}) := \frac{1}{m}\sum_{i=1}^{m}(1 - e(A_i))$$

where

$$e(A_i) := \frac{1}{3}\left\{\left|T^t_{A_i} - \frac{1}{3}\right| + \left|T^f_{A_i} - \frac{1}{3}\right| + \left|I^t_{A_i} - \frac{1}{3}\right| + \left|I^f_{A_i} - \frac{1}{3}\right| + \left|F^t_{A_i} - \frac{1}{3}\right| + \left|F^f_{A_i} - \frac{1}{3}\right|\right\}$$

Following theorem gives some properties of function E.

Theorem 2.1. The function E defined in Definition 2.4 satisfies the following properties.

E1) If A_i is a crisp set for any $i = 1, ..., m$, that is $A_i = \langle (1,0), (0,1), (0,1) \rangle$ or $A_i = \langle (0,1), (1,0), (1,0) \rangle$, then $E(\mathcal{F}) = 0$.

E2) If $T^t_{A_i} = T^f_{A_i} = I^t_{A_i} = I^f_{A_i} = F^t_{A_i} = F^f_{A_i} = \frac{1}{3}$, then $E(\mathcal{F}) = 1$.

E3) If $\mathcal{F}$ is crisper than $\mathcal{G}$, that is $T^t_{A_i} \le T^t_{B_i} \le \frac{1}{3}$, $T^f_{A_i} \le T^f_{B_i} \le \frac{1}{3}$, $I^t_{A_i} \le I^t_{B_i} \le \frac{1}{3}$, $I^f_{A_i} \le I^f_{B_i} \le \frac{1}{3}$, $F^t_{A_i} \le F^t_{B_i} \le \frac{1}{3}$, $F^f_{A_i} \le F^f_{B_i} \le \frac{1}{3}$ or $T^t_{A_i} \ge T^t_{B_i} \ge \frac{1}{3}$, $T^f_{A_i} \ge T^f_{B_i} \ge \frac{1}{3}$, $I^t_{A_i} \ge I^t_{B_i} \ge \frac{1}{3}$, $I^f_{A_i} \ge I^f_{B_i} \ge \frac{1}{3}$, $F^t_{A_i} \ge F^t_{B_i} \ge \frac{1}{3}$, $F^f_{A_i} \ge F^f_{B_i} \ge \frac{1}{3}$, then $E(\mathcal{F}) \le E(\mathcal{G})$ where $\mathcal{G} = \{B_1, ..., B_m\}$ is a collection of IFVNVs.

E4) $E(\mathcal{F}) = E(\overline{\mathcal{F}})$ where $\overline{\mathcal{F}} = \{A^c_1, ..., A^c_m\}$ and $A^c = \langle \mathbb{F}_A, \mathbb{I}_A, \mathbb{T}_A \rangle$.

Proof. **E1)** Let $A_i = \langle(1,0),(0,1),(0,1)\rangle$ for any $i = 1,...,m$. Then we have $e(A_i) = 1$ which yields that

$$E(\mathcal{F}) = \frac{1}{m}\sum_{i=1}^{m}(1-1) = 0.$$

Similarly it can be shown that $E(\mathcal{F}) = 0$ whenever $A_i = \langle(0,1),(1,0),(1,0)\rangle$ for any $i = 1,...,m$.
E2) If $T^t_{A_i} = T^f_{A_i} = I^t_{A_i} = I^f_{A_i} = F^t_{A_i} = F^f_{A_i} = \frac{1}{3}$. Then we get $e(A_i) = 0$ which yields that

$$E(\mathcal{F}) = \frac{1}{m}\sum_{i=1}^{m}(1-0) = 1.$$

E3) Assume that $T^t_{A_i} \le T^t_{B_i} \le \frac{1}{3}$, $T^f_{A_i} \le T^f_{B_i} \le \frac{1}{3}$, $I^t_{A_i} \le I^t_{B_i} \le \frac{1}{3}$, $I^f_{A_i} \le I^f_{B_i} \le \frac{1}{3}$, $F^t_{A_i} \le F^t_{B_i} \le \frac{1}{3}$, $F^f_{A_i} \le F^f_{B_i} \le \frac{1}{3}$. Then we have

$$\begin{aligned}
e(A_i) &= \frac{1}{3}\left\{\begin{array}{l} \left|T^t_{A_i} - \frac{1}{3}\right| + \left|T^f_{A_i} - \frac{1}{3}\right| + \left|I^t_{A_i} - \frac{1}{3}\right| \\ + \left|I^f_{A_i} - \frac{1}{3}\right| + \left|F^t_{A_i} - \frac{1}{3}\right| + \left|F^f_{A_i} - \frac{1}{3}\right| \end{array}\right\} \\
&\ge \frac{1}{3}\left\{\begin{array}{l} \left|T^t_{B_i} - \frac{1}{3}\right| + \left|T^f_{B_i} - \frac{1}{3}\right| + \left|I^t_{B_i} - \frac{1}{3}\right| \\ + \left|I^f_{B_i} - \frac{1}{3}\right| + \left|F^t_{B_i} - \frac{1}{3}\right| + \left|F^f_{B_i} - \frac{1}{3}\right| \end{array}\right\} \\
&= e(B_i).
\end{aligned}$$

So we have

$$\begin{aligned}
E(\mathcal{F}) &= \frac{1}{m}\sum_{i=1}^{m}(1 - e(A_i)) \\
&\le \frac{1}{m}\sum_{i=1}^{m}(1 - e(B_i)) \\
&= E(\mathcal{G}).
\end{aligned}$$

Similarly it can be shown that $E(\mathcal{F}) \le E(\mathcal{F})$ whenever $T^t_{A_i} \ge T^t_{B_i} \ge \frac{1}{3}$, $T^f_{A_i} \ge T^f_{B_i} \ge \frac{1}{3}$, $I^t_{A_i} \ge I^t_{B_i} \ge \frac{1}{3}$, $I^f_{A_i} \ge I^f_{B_i} \ge \frac{1}{3}$, $F^t_{A_i} \ge F^t_{B_i} \ge \frac{1}{3}$, $F^f_{A_i} \ge F^f_{B_i} \ge \frac{1}{3}$. □

3. Proposed VIKOR Method

The VIKOR method is a comparison method in which the closest alternative to the ideal solution is preferred. This method is one of the compromising methods in compensatory models, since in this subgroup the alternative that comes closest to the ideal solution is selected. The method ranks the alternatives, chooses an alternative with a number of conflicting criteria and gives a solution that helps the decision maker to achieve the final solution. To find the closest alternative to the best solution the VIKOR method needs distance. In this study while applying the VIKOR method we consider the distance measure given in Definition 2.3 in the fuzzy environment. The following are the steps summarized in the proposed VIKOR method.

- **Step 1:** Form a MCGDM problem with the set of alternatives $\{\mathcal{A}_1,...,\mathcal{A}_m\}$, the set of criteria $\{\gamma_1,...,\gamma_n\}$ with the group of q DMs, DM_1,...,DM_q. Here, the group of the DMs is formed to determine the possible alternative and evaluation criteria for alternatives in the MCGDM.

- **Step 2:** For each DM_k, construct a decision matrix

$$L_k = \begin{pmatrix} l_{11}^k & l_{12}^k & \cdots & l_{1n}^k \\ l_{21}^k & l_{22}^k & \cdots & l_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ l_{m1}^k & l_{m2}^k & \cdots & l_{mn}^k \end{pmatrix} \tag{5}$$

where l_{ij}^k is a IFVNV for each $i = 1, ..., m$ and $j = 1, ..., n$.

- **Step 3**: To express DMs opinions in one decision matrix, aggregate the decision matrices to the decision matrix $L^{agg} = [l_{ij}]$ by using the Algebraic weighted arithmetic aggregation operator $WA_A - IFVNV$ or the Algebraic weighted geometric aggregation operator $WG_A - IFVNV$ recalled in (1) and (2), respectively.

- **Step 4:** Convert the cost type criteria, if any, into the benefit type by taking their complements. Assign a weight to each criterion by using the entropy defined in Definition 2.4. In this step each criterion is considered as a collection of IFVNVs over the set of alternatives. Larger weight is assign to a criterion when the uncertainty of the criterion is less. We calculate the weight ω_j of each criterion with

$$\omega_j = \frac{1 - E(c_j)}{\sum_{i=1}^{n}(1 - E(c_i))} \tag{6}$$

- **Step 5**: Determine the ideal best solution $L^+ = \left\{l_1^+, ..., l_n^+\right\}$ and ideal worst solution $L^- = \left\{l_1^-, ..., l_n^-\right\}$ from the aggregated decision matrix by using the following score function

$$S(A) = \frac{3 + T_A^t - T_A^f - I_A^t + I_A^f - F_A^t + F_A^f}{6} \tag{7}$$

where $A = \Big((T_A^t, T_A^f), (I_A^t, I_A^f), (F_A^t, F_A^f)\Big)$. The ideal best solution L^+ is taken for those IFVNV which has largest score while the worst solution L^- is taken for which score has lowest value.

- **Step 6**: Define the $L^p - metric$ by using the distance measure given in Definition 2.3 with

$$L_i^{p,\mathcal{D}} := \left(\sum_{j=1}^{n}\left(\omega_j \frac{\mathcal{D}(l_j^+, l_{ij})}{\mathcal{D}(l_j^+, l_j^-)}\right)^p\right)^{1/p}, i = 1, ..., m \tag{8}$$

where $1 \leq p < \infty$ and define the L^∞-metric

$$L_i^{\infty,\mathcal{D}} := \max_j \omega_j \frac{\mathcal{D}(l_j^+, l_{ij})}{\mathcal{D}(l_j^+, l_j^-)}, i = 1, ..., m. \tag{9}$$

As in the classical VIKOR method for each $i = 1, ..., m$ calculate the value of group utility $\mathcal{U}_i$ and individual regret $\mathcal{R}_i$ over alternative by $\mathcal{U}_i = L_i^{1,\mathcal{D}}$ and $\mathcal{R}_i = L_i^{\infty,\mathcal{D}}$, respectively. Note that, this step is the defuzzification step.

- **Step 7:** Determine the VIKOR index for each $i = 1, ..., m$ by

$$\mathcal{Q}_i = s\frac{\mathcal{U}_i - \mathcal{U}^+}{\mathcal{U}^- - \mathcal{U}^+} + (1 - s)\frac{\mathcal{R}_i - \mathcal{R}^+}{\mathcal{R}^- - \mathcal{R}^+}, \tag{10}$$

where $\mathcal{U}^+ = \min_i \mathcal{U}_i$, $\mathcal{U}^- = \max_i \mathcal{U}_i$, $\mathcal{R}^+ = \min_i \mathcal{R}_i$, $\mathcal{R}^- = \max_i \mathcal{R}_i$. Here, $0 \leq s \leq 1$ is a weight for the strategy of the maximum group utility and $(1 - s)$ is the weight of the individual regret. Generally, s and $(1 - s)$ are taken as 0.5.

- **Step 8**: Rank the alternatives as descending in values of $\mathcal{U}$, $\mathcal{R}$ and $\mathcal{Q}$. An alternative with a lower score with respect to $\mathcal{Q}$ is a better alternative.

- **Step 9**: Apply the following algorithm to find the compromise solutions:
    i) If $\mathcal{Q}(\mathcal{A}_{(2)}) - \mathcal{Q}(\mathcal{A}_{(1)}) \geq \dfrac{1}{m-1}$ and $\mathcal{A}_{(1)}$ is the best alternative with respect to $\mathcal{U}$ and $\mathcal{R}$, then $\mathcal{A}_{(1)}$ is the compromise solution where $\left\{\mathcal{A}_{(i)}\right\}_{i=1}^{m}$ is the permutation of the alternatives such that $\mathcal{A}_{(1)}$is the best ranked by $\mathcal{Q}$ (minimum).
    ii) If $\mathcal{Q}(\mathcal{A}_{(2)}) - \mathcal{Q}(\mathcal{A}_{(1)}) \geq \dfrac{1}{m-1}$ is not satisfied, then $\mathcal{A}_{(M)}$ is determined from $\mathcal{Q}(\mathcal{A}_{(M)}) - \mathcal{Q}(\mathcal{A}_{(1)}) < \dfrac{1}{m-1}$ for maximum M. So, the alternatives $\mathcal{A}_{(1)}, ..., \mathcal{A}_{(M)}$ are the compromise solutions.
    iii) If $\mathcal{A}_{(1)}$ is not the best alternative with respect to $\mathcal{U}$ or $\mathcal{R}$, then $\mathcal{A}_{(1)}$ and $\mathcal{A}_{(2)}$ are the compromise solutions.

The steps of proposed extended VIKOR method are visualized in the flowchart given in Figure 1.

4. Illustrative Example

In this section, we present a case study which demonstrate the working of the proposed VIKOR method. Consider a group decision-making problem related to the selection of the best candidates for the post of the Lectures in the respective department. For this, an advertisement has been published in the newspaper and after screening the received application, the four candidates (taken as an alternative) are shortlisted for the interview. Denote these candidates as $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$ and $\mathcal{A}_4$. To evaluate these candidates, three different subjects experts denoted by $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3$ are invited to conduct the interview. The major three set of criteria are considered during their selections as γ_1 = subject knowledge, γ_2 = presentation, and γ_3 = academic background. Each expert provides their ratings in terms of IFVNVs $l_{ij}^k = \Big(((T_{A_{ij}}^t)^k, (T_{A_i}^f)^k), ((I_{A_i}^t)^k, (I_{A_i}^f)^k), ((F_{A_i}^t)^k, (F_{A_i}^f)^k)\Big)$, where $i = 1, 2, 3, 4$, $j = 1, 2, 3$ and $k = 1, 2, 3$. To access the best candidate for the post, we implemented the steps of the proposed method as follows.

- **Step 1**: The above formulated problem is considered as a MCGDM with four alternatives $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$ and $\mathcal{A}_4$ under three criteria $\gamma_1, \gamma_2, \gamma_3$. Each alternative is evaluated by three different experts $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3$.
- **Step 2:** The expert or DMs' provide their preferences in terms of IFVNVs, which are recorded as below

$$\mathcal{L}_1 = \begin{array}{c} \\ \mathcal{A}_1 \\ \\ \mathcal{A}_2 \\ \\ \mathcal{A}_3 \\ \\ \mathcal{A}_4 \end{array} \begin{array}{c} \begin{array}{ccc} \gamma_1 & \gamma_2 & \gamma_3 \end{array} \\ \left(\begin{array}{ccc}
\begin{pmatrix} (0.4, 0.4), (0.2, \\ 0.5), (0.4, 0.4) \end{pmatrix} & \begin{pmatrix} (0.8, 0.2), (0.1, \\ 0.5), (0.3, 0.45) \end{pmatrix} & \begin{pmatrix} (0.7, 0.2), (0.15, \\ 0.35), (0.2, 0.7) \end{pmatrix} \\
\begin{pmatrix} (0.7, 0.1), (0.35, \\ 0.6), (0.2, 0.6) \end{pmatrix} & \begin{pmatrix} (0.3, 0.65), (0.4, \\ 0.5), (0.6, 0.25) \end{pmatrix} & \begin{pmatrix} (0.35, 0.6), (0.3, \\ 0.6), (0.7, 0.2) \end{pmatrix} \\
\begin{pmatrix} (0.75, 0.15), (0.2, \\ 0.4), (0.15, 0.7) \end{pmatrix} & \begin{pmatrix} (0.8, 0.1), (0.4, \\ 0.3), (0.2, 0.6) \end{pmatrix} & \begin{pmatrix} (0.4, 0.5), (0.6, \\ 0.4), (0.55, 0.25) \end{pmatrix} \\
\begin{pmatrix} (0.85, 0.1), (0.4, \\ 0.3), (0.15, 0.6) \end{pmatrix} & \begin{pmatrix} (0.2, 0.65), (0.5, \\ 0.5), (0.7, 0.15) \end{pmatrix} & \begin{pmatrix} (0.8, 0.05), (0.3, \\ 0.6), (0.15, 0.7) \end{pmatrix}
\end{array}\right) \end{array}$$

$$\mathcal{L}_2 = \begin{array}{c} \\ \mathcal{A}_1 \\ \\ \mathcal{A}_2 \\ \\ \mathcal{A}_3 \\ \\ \mathcal{A}_4 \end{array} \begin{array}{c} \begin{array}{ccc} \gamma_1 & \gamma_2 & \gamma_3 \end{array} \\ \left(\begin{array}{ccc}
\begin{pmatrix} (0.35, 0.45), (0.2, \\ 0.6), (0.5, 0.4) \end{pmatrix} & \begin{pmatrix} (0.75, 0.25), (0.15, \\ 0.6), (0.4, 0.45) \end{pmatrix} & \begin{pmatrix} (0.75, 0.15), (0.35, \\ 0.25), (0.4, 0.55) \end{pmatrix} \\
\begin{pmatrix} (0.65, 0.05), (0.3, \\ 0.5), (0.35, 0.6) \end{pmatrix} & \begin{pmatrix} (0.25, 0.65), (0.7, \\ 0.2), (0.55, 0.2) \end{pmatrix} & \begin{pmatrix} (0.25, 0.7), (0.6, \\ 0.4), (0.75, 0.2) \end{pmatrix} \\
\begin{pmatrix} (0.85, 0.05), (0.3, \\ 0.5), (0.1, 0.6) \end{pmatrix} & \begin{pmatrix} (0.8, 0.1), (0.5, \\ 0.3), (0.1, 0.65) \end{pmatrix} & \begin{pmatrix} (0.45, 0.55), (0.5, \\ 0.4), (0.75, 0.15) \end{pmatrix} \\
\begin{pmatrix} (0.75, 0.05), (0.4, \\ 0.4), (0.25, 0.55) \end{pmatrix} & \begin{pmatrix} (0.15, 0.75), (0.6, \\ 0.3), (0.75, 0.25) \end{pmatrix} & \begin{pmatrix} (0.85, 0.05), (0.3, \\ 0.7), (0.1, 0.65) \end{pmatrix}
\end{array}\right) \end{array}$$

$$\mathcal{L}_3 = \begin{array}{c} \\ \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \\ \mathcal{A}_4 \end{array} \begin{array}{ccc} \gamma_1 & \gamma_2 & \gamma_3 \end{array}$$

$$\mathcal{L}_3 = \begin{pmatrix} ((0.45,0.45),(0.5,0.4),(0.3,0.6)) & ((0.75,0.2),(0.15,0.65),(0.35,0.5)) & ((0.8,0.15),(0.2,0.55),(0.25,0.65)) \\ ((0.75,0.1),(0.5,0.5),(0.3,0.5)) & ((0.35,0.55),(0.35,0.55),(0.65,0.2)) & ((0.45,0.55),(0.7,0.25),(0.6,0.25)) \\ ((0.75,0.15),(0.25,0.45),(0.35,0.6)) & ((0.85,0.05),(0.4,0.6),(0.1,0.65)) & ((0.45,0.55),(0.5,0.45),(0.65,0.35)) \\ ((0.65,0.15),(0.4,0.4),(0.25,0.6)) & ((0.25,0.7),(0.45,0.5),(0.75,0.15)) & ((0.85,0.05),(0.35,0.6),(0.25,0.75)) \end{pmatrix}$$

- **Step 3:** Utilize $WA_A - IFVNV$ and $WA_G - IFVNV$ operators, as defined in Eqs. (1), (2) respectively, to compute the aggregated decision matrices L_A^{agg} and L_G^{agg}. The obtained matrices are

$$L_A^{agg} = \begin{array}{c} \\ \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \\ \mathcal{A}_4 \end{array} \begin{array}{c} \begin{array}{ccc} \gamma_1 & \gamma_2 & \gamma_3 \end{array} \\ \begin{pmatrix} ((0.4014,0.4327),(0.2714,0.5068),(0.3915,0.4759)) & ((0.7679,0.2154),(0.131,0.5879),(0.3476,0.4672)) & ((0.7534,0.1651),(0.219,0.3969),(0.2714,0.6385)) \\ ((0.7028,0.0794),(0.3744,0.5358),(0.2759,0.5691)) & ((0.3012,0.6148),(0.461,0.4354),(0.5986,0.217)) & ((0.3552,0.6136),(0.5013,0.4354),(0.6804,0.217)) \\ ((0.7891,0.104),(0.2466,0.4515),(0.1738,0.6366)) & ((0.8183,0.0794),(0.4309,0.4191),(0.126,0.6341)) & ((0.4338,0.5328),(0.5313,0.4172),(0.6448,0.2545)) \\ ((0.7641,0.0909),(0.4,0.3684),(0.2109,0.584)) & ((0.201,0.6988),(0.513,0.4407),(0.7329,0.1847)) & ((0.8349,0.05),(0.3158,0.6366),(0.1554,0.7028)) \end{pmatrix} \end{array}$$

and

$$L_G^{agg} = \begin{array}{c} \\ \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \\ \mathcal{A}_4 \end{array} \begin{array}{c} \begin{array}{ccc} \gamma_1 & \gamma_2 & \gamma_3 \end{array} \\ \begin{pmatrix} ((0.3979,0.4338),(0.316,0.4932),(0.4056,0.4579)) & ((0.7663,0.217),(0.1336,0.5799),(0.3513,0.4661)) & ((0.7489,0.167),(0.2383,0.3637),(0.2886,0.6302)) \\ ((0.6988,0.0836),(0.3895,0.5313),(0.286,0.5646)) & ((0.2972,0.6194),(0.5109,0.3803),(0.6021,0.2154)) & ((0.3402,0.622),(0.562,0.3915),(0.6893,0.2154)) \\ ((0.782,0.1179),(0.2511,0.4481),(02078.,0.6316)) & ((0.8163,0.0836),(0.4354,0.378),(0.1347,0.6329)) & ((0.4327,0.5339),(0.5358,0.416),(0.6598,0.2359)) \\ ((0.7455,0.1009),(0.4,0.3634),(0.218,0.5828)) & ((0.1957,0.7028),(0.5209,0.4217),(0.7343,0.1778)) & ((0.833,0.05),(0.3171,0.6316),(0.1691,0.6988)) \end{pmatrix} \end{array}$$

- **Step 4:** By utilizing the rating values of L_A^{agg}, we determine the entropy using Definition 2.4 and hence the weight vector of the criteria are determined with the help of Eq. (6). The obtained weight vector is

$$(0.306, 0.349, 0.345).$$

Similarly, by using rating of L_G^{agg} matrix, we compute entropy values and hence weight vector by Eq. (6) and get the values as

$$(0.299, 0.352, 0.349).$$

- **Step 5:** The ideal best L_A^+ and the ideal worst L_A^- solutions and the ideal best L_G^+ and the ideal worst L_G^- solutions are obtained with respect to $WA_A - IFVNV$ and $WA_G - IFVNV$, respectively, as follows:

$$L_A^+ = \left\{ \begin{pmatrix} (0.7891, 0.104), (0.2466, \\ 0.4515), (0.1738, 0.6366) \end{pmatrix}, \begin{pmatrix} (0.8183, 0.0794), (0.4309, \\ 0.4191), (0.126, 0.6341) \end{pmatrix}, \begin{pmatrix} (0.8349, 0.050), (0.3158, \\ 0.6366), (0.1554, 0.7028) \end{pmatrix} \right\}$$

$$L_A^- = \left\{ \begin{pmatrix} (0.4014, 0.4327), (0.2714, \\ 0.5068), (0.3915, 0.4759) \end{pmatrix}, \begin{pmatrix} (0.201, 0.6988), (0.513, \\ 0.4407), (0.7329, 0.1847) \end{pmatrix}, \begin{pmatrix} (0.3552, 0.6136), (0.5013, \\ 0.4354), (0.6804, 0.217) \end{pmatrix} \right\}$$

$$L_G^+ = \left\{ \begin{pmatrix} (0.782, 0.1179), (0.2511, \\ 0.4481), (0.2078, 0.6316) \end{pmatrix}, \begin{pmatrix} (0.1957, 0.7028), (0.5209, \\ 0.4217), (0.7343, 0.1778) \end{pmatrix}, \begin{pmatrix} (0.3402, 0.622), (0.562, \\ 0.3915), (0.6893, 0.2154) \end{pmatrix} \right\}$$

$$L_G^- = \left\{ \begin{pmatrix} (0.3979, 0.4338), (0.316, \\ 0.4932), (0.4056, 0.4579) \end{pmatrix}, \begin{pmatrix} (0.1957, 0.7028), (0.5209, \\ 0.4217), (0.7343, 0.1778) \end{pmatrix}, \begin{pmatrix} (0.3402, 0.622), (0.562, \\ 0.3915), (0.6893, 0.2154) \end{pmatrix} \right\}$$

- **Step 6:** For each $i = 1, ..., m$, the value of group utilities $\mathcal{U}_i^A$, $\mathcal{U}_i^G$ and individual regret $\mathcal{R}_i^A$, $\mathcal{R}_i^G$ over alternative are calculated with respect to $WA_A - IFVNV$ and $WA_A - IFVNV$, respectively. The values corresponding to these are computed as

$$\mathcal{U}_1^A = 0.0681, \quad \mathcal{U}_2^A = 0.1091, \quad \mathcal{U}_3^A = 0.0455, \quad \mathcal{U}_4^A = 0.0604,$$
$$\mathcal{U}_1^G = 0.0674, \quad \mathcal{U}_2^G = 0.1120, \quad \mathcal{U}_3^G = 0.0454, \quad \mathcal{U}_4^G = 0.0612,$$
$$\mathcal{R}_1^A = 0.0353, \quad \mathcal{R}_2^A = 0.0539, \quad \mathcal{R}_3^A = 0.0455, \quad \mathcal{R}_4^A = 0.0555$$

and

$$\mathcal{R}_1^G = 0.0345, \quad \mathcal{R}_2^G = 0.0548, \quad \mathcal{R}_3^G = 0.0454, \quad \mathcal{R}_4^G = 0.0560.$$

- **Step 7:** By using values of $\mathcal{U}_i$ and $\mathcal{R}_i$, we compute the values of $\mathcal{U}^+$, $\mathcal{U}^-$, $\mathcal{R}^+$, $\mathcal{R}^-$ as follows.

$$\mathcal{U}_A^+ = \min_i \mathcal{U}_i^A = 0.0455, \quad \mathcal{U}_A^- = \max_i \mathcal{U}_i^A = 0.1091, \quad \mathcal{U}_G^+ = \min_i \mathcal{U}_i^G = 0.0454,$$
$$\mathcal{U}_G^- = \max_i \mathcal{U}_i^G = 0.1120, \quad \mathcal{R}_A^+ = \min_i \mathcal{R}_i^A = 0.03253, \quad \mathcal{R}_A^- = \max_i \mathcal{R}_i^A = 0.0555,$$
$$\mathcal{R}_G^+ = \min_i \mathcal{R}_i^G = 0.0345, \quad \mathcal{R}_G^- = \max_i \mathcal{R}_i^G = 0.0560$$

Now, by taking $s = 0.5$ in Eq. (10), we get the following VIKOR indices.

$$\mathcal{Q}_1^A = 0.1777, \quad \mathcal{Q}_2^A = 0.9604, \quad \mathcal{Q}_3^A = 0.2525, \quad \mathcal{Q}_4^A = 0.6171,$$
$$\mathcal{Q}_1^G = 0.1652, \quad \mathcal{Q}_2^G = 0.9721, \quad \mathcal{Q}_3^G = 0.2535, \quad \mathcal{Q}_4^G = 0.6186$$

- **Step 8:** From Steps 6 and 7 we can rank the given alternatives as

i) $\mathcal{U}_3^A < \mathcal{U}_4^A < \mathcal{U}_1^A < \mathcal{U}_2^A$, $\quad \mathcal{R}_1^A < \mathcal{R}_3^A < \mathcal{R}_2^A < \mathcal{R}_4^A$, $\quad \mathcal{Q}_1^A < \mathcal{Q}_3^A < \mathcal{Q}_4^A < \mathcal{Q}_2^A$
ii) $\mathcal{U}_3^G < \mathcal{U}_4^G < \mathcal{U}_1^G < \mathcal{U}_2^G$, $\quad \mathcal{R}_1^G < \mathcal{R}_3^G < \mathcal{R}_2^G < \mathcal{R}_4^G$, $\quad \mathcal{Q}_1^G < \mathcal{Q}_3^G < \mathcal{Q}_4^G < \mathcal{Q}_2^G$.

From these ordering, we conclude that $\mathcal{A}_1$ is the best alternative among the others.

- **Step 9**: Since

$$\mathcal{Q}_3^A - \mathcal{Q}_1^A = 0.0748 < \frac{1}{3}$$

and

$$\mathcal{Q}_4^A - \mathcal{Q}_1^A = 0.4394 \geq \frac{1}{3}$$

Thus, it means that there is a compromise solution consisting of first two alternatives $\mathcal{A}_1$ and $\mathcal{A}_3$ with respect to $WA_A - IFVNV$.

Furthermore, with respect to $WA_G - IFVNV$, we compute

$$\mathcal{Q}_3^G - \mathcal{Q}_1^G = 0.0883 < \frac{1}{3}$$

and

$$\mathcal{Q}_4^G - \mathcal{Q}_1^G = 0.4534 \geq \frac{1}{3}$$

and hence conclude that there is again a compromise solution consisting of first two alternatives $\mathcal{A}_1$ and $\mathcal{A}_3$. From these analysis, it is noted that that there is a certain consistence between the results with respect to both aggregation operations.

5. Comparative and Sensitivity analysis

In this section, we give a comparative as well as sensitivity analysis of the proposed work with some of the existing studies.

5.1. Comparison analysis

We have compared the performance of the stated algorithm with the existing measures such as similarity measure[24] and the score function [51].

A SNS [23] on a universal set $\mathcal{Y} = \{u_1, ..., u_n\}$ is given by

$$A = \{\langle x_i, (T_A(u_i), I_A(u_i), F_A(u_i))\rangle : i = 1, 2, \ldots, n\} \tag{11}$$

where $T_A, I_A, F_A : X \to [0, 1]$ are the truth, indeterminacy and falsity functions. For a fixed $u \in X$

$$\tau = \langle T_\tau, I_\tau, F_\tau\rangle := \langle T_A(u), I_A(u), F_A(u)\rangle \tag{12}$$

is called a simplified neutrosophic value (SNV).

Later on, an improved score function N for SNVs is stated by Nancy and Garg [51], and is given as

$$N(\tau) = \frac{1 + (T_\tau - 2I_\tau - F_\tau)(2 - T_\tau - F_\tau)}{2} \tag{13}$$

For two IFVNVs A and B. By taking the sequence length $p = 1$ in the simplified neutrosophic valued similarity measure CSN [24], we consider the following simplified neutrosophic valued similarity measure CSN_1:

$$CSN_1(A, B) = \left(\begin{array}{c} \dfrac{T_A^t T_B^t + T_A^f T_B^f}{\sqrt{(T_A^t)^2 + \left(T_A^f\right)^2}\sqrt{(T_B^t)^2 + \left(T_B^f\right)^2}}, 1 - \dfrac{I_A^t I_B^t + I_A^f I_B^f}{\sqrt{(I_A^t)^2 + \left(I_A^f\right)^2}\sqrt{(I_B^t)^2 + \left(I_B^f\right)^2}}, \\ 1 - \dfrac{F_A^t F_B^t + F_A^f F_B^f}{\sqrt{(F_A^t)^2 + \left(F_A^f\right)^2}\sqrt{(F_B^t)^2 + \left(F_B^f\right)^2}} \end{array}\right). \tag{14}$$

It is clear that $CSN_1(A, B)$ is a SNV.

The following are the steps implemented by these approaches on the given rating values.

1) First, we calculate the similarities of the alternatives with the ideal best solutions from the decision matrices L_A^{agg} and L_G^{agg} obtained in Step 3 of Section 4 by using CSN_1 for each criterion γ_i, $i = 1, 2, 3$. The results obtained corresponding to this analysis are shown in Tables 1 and 2, respectively.
2) The defuzzified values of these SNVs are obtained and summarized in Tables 3 and 4 by using score function N given in Eq. (13).
3) Finally, we calculate the weighted arithmetic mean of the scores of the criteria for each alternative by using the weights obtained in Step 4 of Section 4. The overall values of each alternative are computed with respect to $WA_A - IFVNV$ and $WA_G - IFVNV$ as $\mathcal{A}_1 = 0.8755, \mathcal{A}_2 = 0.6372, \mathcal{A}_3 = 0.8339$, $\mathcal{A}_4 = 0.7703$ and $\mathcal{A}_1 = 0.8666, \mathcal{A}_2 = 0.6218, \mathcal{A}_3 = 0.832, \mathcal{A}_4 = 0.7718$, respectively. Both approaches suggest that $\mathcal{A}_1 \succeq \mathcal{A}_3 \succeq \mathcal{A}_4 \succeq \mathcal{A}_2$ which is consistent with the proposed extended VIKOR method.

5.2. Sensitivity analysis

To analyze the impact of the parameter s, as mentioned in Step 7 of the proposed VIKOR method, we perform an experiment by varying the value of s from 0 to 1. The final results corresponding to each s along with their rating ordering are summarized in Table 5. From these results, it is concluded that the ranking order of the alternative is consistent $\mathcal{Q}_1^A < \mathcal{Q}_3^A < \mathcal{Q}_4^A < \mathcal{Q}_2^A$ when we varies s from 0.1 to 0.5 with respect to the $WA_A - IFVNV$ operator while $\mathcal{Q}_1^G < \mathcal{Q}_3^G < \mathcal{Q}_4^G < \mathcal{Q}_2^G$ when s varies from 0.1 to 0.6 for $WA_G - IFVNV$ operator. However, when s change from 0.6 to 0.8 then rating observe as $\mathcal{Q}_3^A < \mathcal{Q}_1^A < \mathcal{Q}_4^A < \mathcal{Q}_2^A$ and hence conclude that $\mathcal{A}_3$ is the best alternative, while by using $WA_G - IFVNV$ operator, ordering of the alternatives is $\mathcal{Q}_3^G < \mathcal{Q}_1^G < \mathcal{Q}_4^G < \mathcal{Q}_2^G$. The graphical representation of the behavior of the alternatives with the s variation is show in Figure 2. Finally, if we apply the Borda count [52] on these 18 results, we get the ranking of alternatives as $\mathcal{A}_1 \succeq \mathcal{A}_3 \succeq \mathcal{A}_4 \succeq \mathcal{A}_2$.

6. Conclusion

The main contribution of the study is summarized as follow.

1) Dealing with information-based vagueness and uncertainty has always been a challenging task for the researchers. To address it, in this paper, we consider an intuitionistic fuzzy valued neutrosophic value to handle the uncertainties in the data. These IFVNVs allows the decision maker to carry out more detailed information by utilizing the features of neutrosophic set. The advantages of considering this set is to allow the expert to provide the ratings in terms of independent degress namely truth, indeterminacy and falsity degrees.
2) We propose an extended VIKOR method for solving the group decision-making problems. In this method, we propose the L^p-metric and L^∞-metric spaces and hence used them in the VIKOR method to compute the relative strength between the given alternatives.
3) The weight of each criteria is determined by using an entropy-based approach. For this, an entropy between the elements of IFVNVs is proposed. Some basic properties of this measure is also derived in this paper.
4) The presented MCGDM approach has been illustrated with a numerical examples by considering the multiple experts. The obtained results have been compared with the existing studies. Also, the sensitivity analysis of the proposed approach has been investigated by varying the parameter s in the proposed VIKOR method. The results for this analysis is shown in tabular and graphical representation.

In the future work, we shall extend the proposed measures and approach to solve some other decision-making problems such as classification, medical diagonsis, image processing etc. Furthermore, in the study there is a lack of consideration of the interaction between the pairs of the elements, so we will extend the same in the future work by using some Archimedean norm as well as interaction operations [53]. The future work may also include the development of other extension of the fuzzy set such as soft set-like models with possibility grade for each of their approximate member and their use in decision-making problems.

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Table 1: Similarities with respect to $WA_A - IFVNV$

Similarity	γ_1	γ_2	γ_3
$\mathcal{A}_1$ and L_A^+	$(0.7701, 0, 0.0877)$	$(0.9844, 0.1636, 0.0967)$	$(0.9879, 0.001, 0.0169)$
$\mathcal{A}_2$ and L_A^+	$(0.9998, 0.006, 0.017)$	$(0.5246, 0.0001, 0.4825)$	$(0.5518, 0.0771, 0.4976)$
$\mathcal{A}_3$ and L_A^+	$(1, 0, 0)$	$(1, 0, 0)$	$(0.6766, 0.0972, 0.4407)$
$\mathcal{A}_4$ and L_A^+	$(0.9999, 0.0529, 0.0032)$	$(0.3679, 0.0019, 0.5713)$	$(1, 0, 0)$

Table 2: Similarities with respect to $WA_G - IFVNV$

Similarity	γ_1	γ_2	γ_3
$\mathcal{A}_1$ and L_G^+	$(0.7783, 0.0017, 0.0817)$	$(0.9849, 0.1916, 0.0936)$	$(0.9873, 0.0066, 0.0184)$
$\mathcal{A}_2$ and L_G^+	$(0.9995, 0.0074, 0.0114)$	$(0.5222, 0.0028, 0.4745)$	$(0.5316, 0.121, 0.4856)$
$\mathcal{A}_3$ and L_G^+	$(1, 0, 0)$	$(1, 0, 0)$	$(0.6751, 0.0975, 0.4513)$
$\mathcal{A}_4$ and L_G^+	$(0.9999, 0.0516, 0.0008)$	$(0.365, 0.0006, 0.5675)$	$(1, 0, 0)$

Table 3: Defuzzified values for $WA_A - IFVNV$

Similarity	γ_1	γ_2	γ_3
$\mathcal{A}_1$ and L_G^+	0.8897	0.7575	0.9822
$\mathcal{A}_2$ and L_G^+	0.9772	0.5208	0.4525
$\mathcal{A}_3$ and L_G^+	1	1	0.5183
$\mathcal{A}_4$ and L_G^+	0.9441	0.3901	1

Table 4: Defuzzified values for $WA_G - IFVNV$

Similarity	γ_1	γ_2	γ_3
$\mathcal{A}_1$ and L_G^+	0.8951	0.7341	0.9751
$\mathcal{A}_2$ and L_G^+	0.9813	0.5211	0.4037
$\mathcal{A}_3$ and L_G^+	1	1	0.5126
$\mathcal{A}_4$ and L_G^+	0.9476	0.3913	1

Table 5: Sensitivity analysis with respect to different values of parameter s

s	$WA_A - IFVNV$	$WA_G - IFVNV$
0.1	$\mathcal{Q}_1^A < \mathcal{Q}_3^A < \mathcal{Q}_4^A < \mathcal{Q}_2^A$	$\mathcal{Q}_1^G < \mathcal{Q}_3^G < \mathcal{Q}_4^G < \mathcal{Q}_2^G$
0.2	$\mathcal{Q}_1^A < \mathcal{Q}_3^A < \mathcal{Q}_4^A < \mathcal{Q}_2^A$	$\mathcal{Q}_1^G < \mathcal{Q}_3^G < \mathcal{Q}_4^G < \mathcal{Q}_2^G$
0.3	$\mathcal{Q}_1^A < \mathcal{Q}_3^A < \mathcal{Q}_4^A < \mathcal{Q}_2^A$	$\mathcal{Q}_1^G < \mathcal{Q}_3^G < \mathcal{Q}_4^G < \mathcal{Q}_2^G$
0.4	$\mathcal{Q}_1^A < \mathcal{Q}_3^A < \mathcal{Q}_4^A < \mathcal{Q}_2^A$	$\mathcal{Q}_1^G < \mathcal{Q}_3^G < \mathcal{Q}_4^G < \mathcal{Q}_2^G$
0.5	$\mathcal{Q}_1^A < \mathcal{Q}_3^A < \mathcal{Q}_4^A < \mathcal{Q}_2^A$	$\mathcal{Q}_1^G < \mathcal{Q}_3^G < \mathcal{Q}_4^G < \mathcal{Q}_2^G$
0.6	$\mathcal{Q}_3^A < \mathcal{Q}_1^A < \mathcal{Q}_4^A < \mathcal{Q}_2^A$	$\mathcal{Q}_1^G < \mathcal{Q}_3^G < \mathcal{Q}_4^G < \mathcal{Q}_2^G$
0.7	$\mathcal{Q}_3^A < \mathcal{Q}_1^A < \mathcal{Q}_4^A < \mathcal{Q}_2^A$	$\mathcal{Q}_3^G < \mathcal{Q}_1^G < \mathcal{Q}_4^G < \mathcal{Q}_2^G$
0.8	$\mathcal{Q}_3^A < \mathcal{Q}_1^A < \mathcal{Q}_4^A < \mathcal{Q}_2^A$	$\mathcal{Q}_3^G < \mathcal{Q}_1^G < \mathcal{Q}_4^G < \mathcal{Q}_2^G$
0.9	$\mathcal{Q}_3^A < \mathcal{Q}_4^A < \mathcal{Q}_1^A < \mathcal{Q}_2^A$	$\mathcal{Q}_3^G < \mathcal{Q}_1^G < \mathcal{Q}_4^G < \mathcal{Q}_2^G$

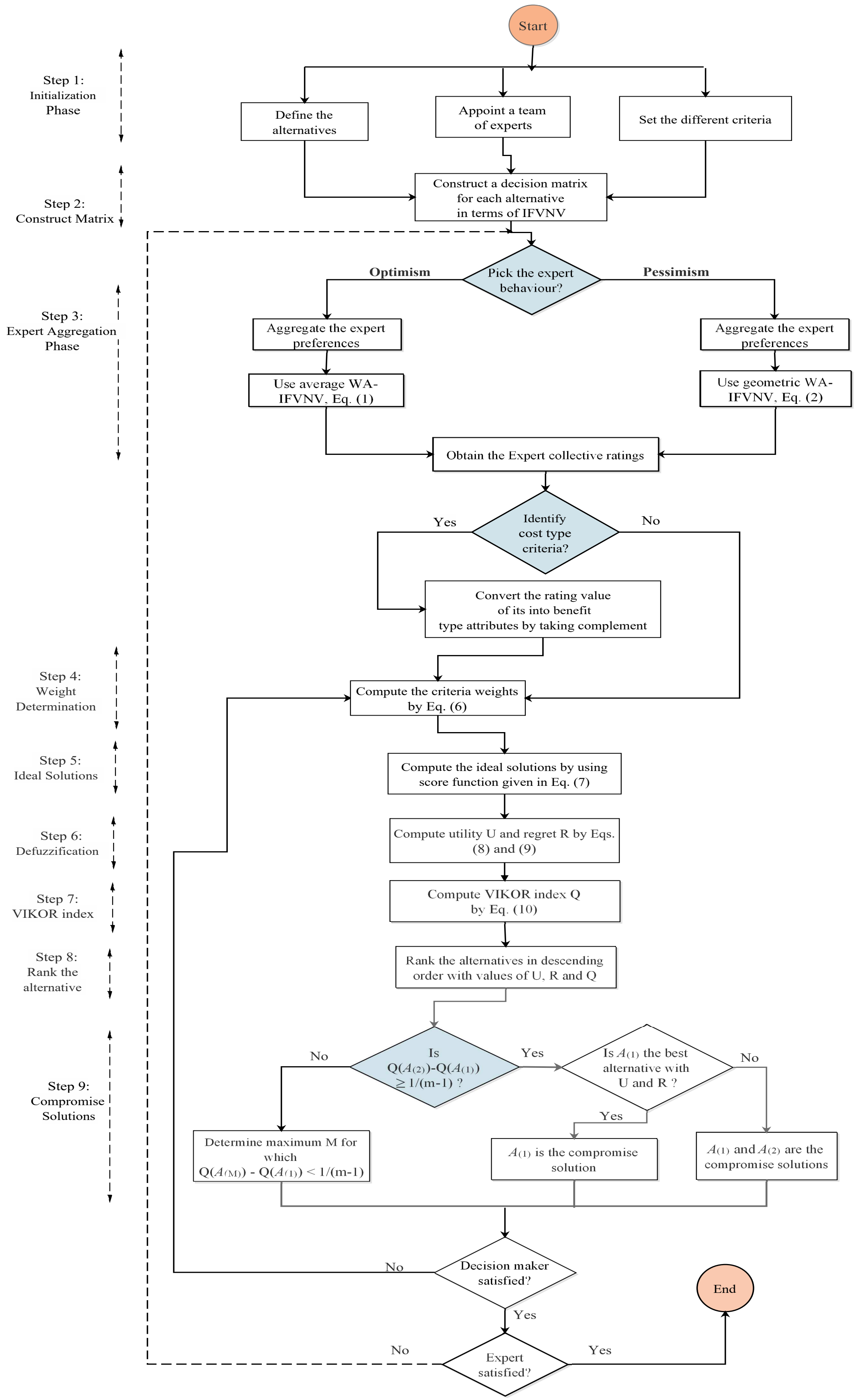

Figure 1: Flowchart of the extended VIKOR method

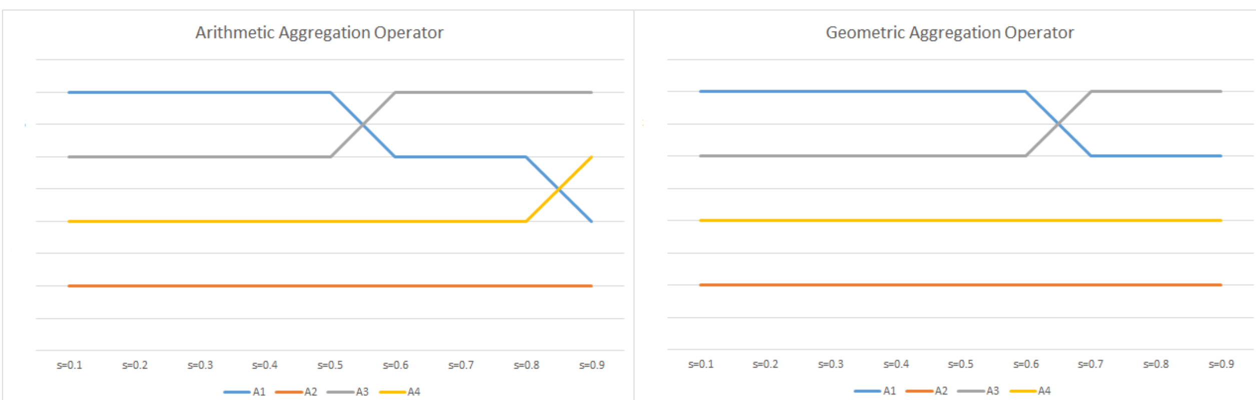

Figure 2: Sensitivity analysis with respect to different values of parameter s

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Journal, Journal of Intelligent & Fuzzy Systems, Kybernetes, Complex and Intelligent Systems, Journal of Industrial & Management Optimization, Technological and Economic Development of Economy, International Journal of Computational Intelligence Systems, CAAI Transactions on Intelligence Technology, Mathematical Problems in Engineering, Complexity, etc.