Group buying vs individual buying; a competitive approach for two retailers

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Abstract

Development of technology and expansion of social media emerge group-buying mechanism as a popular strategy. In this paper, two competitive retailers are considered that sell the same product to a customer in a market, group buying and individual buying**.** Three different competition scenarios are considered. In order to implement the first two strategies, the Stackelberg model is used where the group-buying retailer and the individual-buying retailer are assumed to be the leader and follower, respectively. For the third strategy, the Nash equilibrium is applied when they decide separately. We analyse the optimal and equilibrium strategy in each scenario. And also determine the conditions for each retailer to be present in the market in each scenario. Finally, a numerical example is provided, in order to illustrate the effectiveness of the aforementioned three scenarios in models.

Keyword: pricing strategy, group buying, individual buying, competitive, Stackelberg game theory, Nash game theory

1. Introduction

With the rapid development of social media and the facilitate communication between users [1], online group-buying emerge as a common form of selling that encourages individual consumers to join together to purchase products or services at a lower price.

Although in the past the concept of group buying has been used, with the advent and development of e-commerce and online social networks, emerge again in a different way. This led to the emergence of online group buying platforms and many companies sell their products or services through these platforms by group buying mechanism. Groupon, as one of the most well-known of these platforms, earned $2.84 billion in revenue in 2017. This rapid growth has prompted many researchers to study the mechanism of group buying.

This study focuses on group and individual buying mechanism in a competitive market between two retailers. One of the retailers sells the product through the group-buying mechanism and the other retailer goes in the traditional way and sells product individually. We considered three scenarios to examine different modes of competition. At the first two strategies, the Stackelberg model is used where the group-buying retailer and the Individual retailer are assumed to be the leader and follower, respectively. For the third strategy, the Nash equilibrium is applied when they decide separately. In particular, this research addresses the following questions:

- What is the impact of the inconvenience cost through group buying mechanism and Unsociability rate of customer in the market through GB and IB mechanism?
- What is the optimal strategy for competitive retailers in each scenario?

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- What are the conditions for each retailer to be present in the market in each scenario?

These research questions are investigated by developing a mathematical model based on the game theoretical approach, in which two retailers compete against each other. Demand for each retailer is affected by several factors such as inconvenience cost through the group buying mechanism and the unsociability rate of customer in the market.

The purpose of current study is two-fold. First, we will try to investigate the different competition scenario between two retailers and compare the GB and IB mechanisms in the market and second to find out under what conditions, retailers can operate in the market.

The rest of this paper is organized as follows. In Section 2, we provide a summary of the related researches in the literature. In Section 3, we formulate the demand function of each retailer in the market and determining optimal pricing for the two competitive retailers and investigate in three scenarios. in Section 4, the numerical studies are conducted and discussed The conclusion remarks are given in Section 5.

2. Literature Review

Many researchers have investigate the problem of buyers forming a group to increase their power and utilize economies of scale to save costs and obtain quantity discounts from suppliers [2–7] or through Group purchasing organizations (GPOs)[8–16]. However, the problems in these studies are different from ours which focuses on the analysis of competition investigate the different competition scenario between two retailers and compare the GB and IB mechanisms.

The following papers are related to the problem considered here. Anand and Aron [17] investigate over fifty group-buying Web sites, they showed that a GB mechanism outperforms posted pricing under different scenarios related to demand uncertainty and operational economies of scale. Jing and Xie[18] proposes and examines the information-sharing effect of group buying and show that GB sales can be more profitable than other related promotional schemes. Subramanian and Rao [19] investigated the strategic role for displaying sales of small local merchants on GB websites to attract new consumers. Zhou, Dan, Ma, & Zhang [20] analyze the informational advantage of the group buying organization in a supply chain with two manufacturers. they find that each member can reach win–win result under both quantity competition and information sharing. He et al. [21] provides an analytical framework for group buying with fairness concerns in supply chain that include a supplier and two competing retailers. The results show the group buying strategy will be preferred by the supplier, rather than by the retailers. Liang et al. [22] study the group-buying mechanism when a seller offers a product in both GB and spot purchasing channels to end customers. they characterize the customer behavior within a rational expectations framework in the two-period model. The results show that the GB success rate and customer surplus increase with information updates. Tran and Desiraju [23] Consider a market served by retailer and manufacturer and examine market settings where the retailer is privately informed about the degree of market heterogeneity. The result shows that the manufacturer is better off when the retailer is better informed about market size rather than price sensitivity. Deng, Jiang, and Li [24] consider a seller that sells its products through group buying websites and offline channels. they represent an analytical model with finite capacity to optimize GB price and maximum deal size limit. Wu, Li and Li [25] consider a group-buying website and a seller to find an equilibrium between them. The results show that the scale of the seller and the website influence the optimal decisions in the whole system. Zhang, Shang, and Yildirim [26] formulated the demand for online group buying considering two specific types of externalities including the positive effects from buying with others and the negative externality of inconvenience costs. They analyzed the optimal and Nash equilibrium pricing strategies for the three possible cases of competition for a seller: monopoly, duopoly and multiple-firm competition. Our model differs from existing works in that it formulates the demand for group buying and individual buying based on unsociability rate of customer in the market and the inconvenience cost through group buying mechanism and also consider different competitive scenario base game theory approach. **Error! Reference source not found.** shows the compares this study with studies are closely related to the current paper.

Table 1: Compares this study with studies are closely related to the current paper

3. Problem description

In this paper, two competitive retailers are considered that sell the same product to customer in a market. One of the retailers choose to the product through group buying mechanism that we call this retailer as GB retailer the other retailer sells the product as traditional manner and sell to individuals that we name this retailer as IB retailer. Due to the development of social networks and communication of people in these contexts, the degree of penetration of this network among customers in the market was also considered as an involved parameter. In order to model these conditions, Given the strength of each retailer in the market, we have considered three scenarios. In order to implement the first two strategies, the Stackelberg model is used where the GB retailer and the IB retailer are assumed to be the leader and follower, respectively. For the third strategy, the Nash equilibrium is applied when they decide separately. In the following, we describe the subscript, parameters, and decision variable and then determine the demand function for each type of selling scenario and solve the optimization problem to obtain the optimal selling price for both types of selling scenarios.

Subscript

G Denote the group buying retailer

I Denote the individual buying retailer

Parameters

n The potential customer in market

k The inconvenience cost through group buying mechanism

v Valuation of product

λ Unsociability rate of customer

A Highest value of product in market

w The wholesale price

Decision variables

p_G Selling price through group buying mechanism ($)

p_I Selling price through individual buying mechanism ($)

q_G Quantity sold (demand) through group buying mechanism

q_I Quantity sold (demand) through individual buying mechanism

U_G Customer's utility function through group buying mechanism

U_I Customer's utility function through individual buying mechanism

π_{rG} The retailer's profit through group buying mechanism ($)

π_{rI} The retailer's profit through individual buying mechanism ($)

Consider a market in which two retailers compete with different sales policies. Consumers in the market derive a valuation *v* for the product, which is uniformly distributed between 0 and *A*. A indicates the amount of heterogeneity in the market with respect to the value of the product, in which the higher the value of *A*, the higher the level of heterogeneity in the valuation of the product in the market [26]. The market consists of a number of consumers *n*, each of whom is interested in buying a unit of the product from sellers.

Demands of group buying (GB) and individual buying (IB) mechanism are closely related to the customer's utility function, and consumer chooses one of these policies that satisfies his/her expectations more than the other one. The customer's utility function for group and individual buying policies are as Eqs. (1) and (2), respectively.

$$U_G = v - p_G - k \tag{1}$$

$$U_I = \lambda v - p_I \tag{2}$$

In the Eq. (1), Consider a parameter *k* as a cost of inconvenience due to group buying such as signing up, waiting time to group complete, and registration fee. Thus, the amount of utility to buy from retailer G is equal to the value of *v* minus the amount of the price of the product sold by the group (p_G) minus the *k*.

We considered the parameter λ as a coefficient of unsociability rate of customer, The higher value of λ , shows the higher value of the purchase value of *v* for retailer *I*. so the customer's utility function for individual buying obtains as Eq. (2).

Considering that customers decide to buy from two retailers based on the value of utility, So, two conditions should be considered to use the demand function: (1) $U_G \geq U_I$; (2) $U_G < U_I$.

If $U_G \geq U_I$, the consumer chooses to buy with group, so $v - p_G - k \geq \lambda v - p_I$ and we have $v \geq \frac{p_G - p_I + k}{1-\lambda}$ where $v \epsilon U\left[0, A\right]$. Then, the demand for the group buying mechanism can be calculated using Eq. (3)

$$q_G = n \int_{\frac{p_G - p_I + k}{1-\lambda}}^{A} \frac{1}{A} \mathrm{d}v = \frac{n}{A}\left(\frac{A(1-\lambda) - p_G + p_I - k}{(1-\lambda)}\right) \tag{3}$$

If $U_G < U_I$, the consumer chooses the IB mechanism, so $U_I \geq 0$ i.e., $\frac{p_I}{\lambda} < v < \frac{p_G - p_I + k}{1-\lambda}$ and the market demand for IB mechanism can be obtained as Eq. (4).

$$q_I = n \int_{\frac{p_I}{\lambda}}^{\frac{p_G - p_I + k}{1-\lambda}} \frac{1}{A} \mathrm{d}v = \frac{n}{A}\left(\frac{\lambda p_G - p_I + \lambda k}{\lambda(1-\lambda)}\right) \tag{4}$$

The Eqs. (5) and (6) show the profit function for both retailers, respectively. The retailers have the revenue minus the purchase cost.

$$\pi_{r_G} = p_G(q_G) - (q_G)\mathrm{w} \tag{5}$$

$$\pi_{r_I} = p_I(q_I) - (q_I)\mathrm{w} \tag{6}$$

In the following of the section, we consider three strategies. In two of these three strategies, the price is declared with one of retailers who plays the leader role in this Stackelberg game and in the third strategy, they announce their prices separately. In the next three subsection, these three strategies are presented.

3. Group buying retailer as a leader

In this scenario, we consider the GB retailer has more power in the market than IB retailer, in this regard, the term “SG” is used to call this scenario. A Stackelberg game theory is used to model the mentioned scenario. The Stackelberg game is applied between the *G* retailer as a leader and *I* retailer as follower. In the first step, the *G* retailer as a leader determines the Group sale price of the product in order to earn the most profit, and the *I* retailer tries as a follower determines the product price as a traditional selling to increase his own profit in a competitive market. Eq. (7) shows the objective function of this case. This function comprises the profits of selling the product minus the purchase costs for each retailer,

$$\begin{aligned} &max\ \pi_{r_G} = p_G\left(\frac{n}{A}\left(\frac{A(1-\lambda) - p_G + p_I - k}{(1-\lambda)}\right)\right) - \frac{n}{A}\left(\frac{A(1-\lambda) - p_G + p_I - k}{(1-\lambda)}\right)\mathrm{w} \\ &\quad s.t. \\ &\quad max\ \pi_{r_I} = p_I\left(\frac{n}{A}\left(\frac{\lambda p_G - p_I + \lambda k}{\lambda(1-\lambda)}\right)\right) - \frac{n}{A}\left(\frac{\lambda p_G - p_I + \lambda k}{\lambda(1-\lambda)}\right)\mathrm{w} \end{aligned} \tag{7}$$

The first derivatives of individual buying retailer’s profit function with decision variable p_I, shown with $\frac{\partial \pi_{rI}}{\partial p_I} = 0$, provides the optimal values of decision variables.

$$\frac{\partial \pi_{rI}}{\partial p_I} = \frac{n\left(w + k\lambda - 2p_I + \lambda p_G\right)}{A(1-\lambda)\lambda} = 0 \tag{8}$$

The second derivative of profit function for retailer I is $\frac{\partial^2 \pi_{rI}}{\partial p_I^2} = \frac{-2n}{A(1-\lambda)\lambda} \leq 0$, which means the retailer's profit function is concave.

After calculating the optimum value for p_I, now by substituting Eq. (9) into the profit function of retailer G, so p_G can be determined.

$$p_I = \frac{1}{2}\left(w + k\lambda + \lambda p_G\right) \tag{9}$$

After solving equation $\frac{\partial \pi_G}{\partial p_G} = 0$, so p_G and p_I can be calculated as follows.

$$p_G = \frac{1}{2}\left(2A - k + w + \frac{2A - w}{-2 + \lambda}\right) \tag{10}$$

$$p_I = \frac{1}{2}\left(w + k\lambda + \frac{1}{2}\left(2A - k + w + \frac{2A - w}{-2 + \lambda}\right)\lambda\right) \tag{11}$$

Thus, as can be seen from Eqs. (12) and (13), the profit function for each retailer can be determined.

$$\pi_{rG} = \frac{n\left(w + k(2-\lambda) + 2A(-1+\lambda) - w\lambda\right)^2}{8A(-2+\lambda)(-1+\lambda)} \tag{12}$$

$$\pi_{rI} = -\frac{n\left(4w - 2(A+k)\lambda - 5w\lambda + (2A+k+w)\lambda^2\right)^2}{16A(-2+\lambda)^2(-1+\lambda)\lambda} \tag{13}$$

3.ndividual buying retailer as a leader

In the previous section, we considered a situation that the GB retailer plays as a leader in Stackelberg game, while in the present scenario, the IB retailer acts as leader. We use the term "SI" to identify this scenario. The GB retailer plays the follower role in Stackelberg game theory. Now, the objective function of this case is shown in Eq. (14). This function includes the profit of selling the product minus the purchase cost for both retailers.

$$\begin{aligned} & max\ \pi_{r_I} = p_I\left(\frac{n}{A}\left(\frac{\lambda p_G - p_I + \lambda k}{\lambda(1-\lambda)}\right)\right) - \frac{n}{A}\left(\frac{\lambda p_G - p_I + \lambda k}{\lambda(1-\lambda)}\right)\mathrm{w} \\ & s.t. \\ & \quad max\ \pi_{r_G} = p_G\left(\frac{n}{A}\left(\frac{A(1-\lambda) - p_G + p_I - k}{(1-\lambda)}\right)\right) - \frac{n}{A}\left(\frac{A(1-\lambda) - p_G + p_I - k}{(1-\lambda)}\right)\mathrm{w} \end{aligned} \tag{14}$$

First derivatives of group buying retailer's profit function with decision variables p_G, shown with $\frac{\partial \pi_{rG}}{\partial p_G}=0$, provides the optimal values for decision variables.

$$\frac{\partial \pi_{rG}}{\partial p_G}=\frac{n\left(A-k+w-A\lambda+p_I-2p_G\right)}{A(1-\lambda)}=0 \tag{15}$$

The second derivative of profit function for G is $\frac{\partial^2 \pi_{rG}}{\partial p_G^2}=\frac{-2n}{A(1-\lambda)}\leq 0$, which means the retailer's profit function is concave.

After calculating the optimum value for p_G, now by substituting Eq. (16) into the profit function of retailer *I*, so p_I can be calculated.

$$p_G=\frac{1}{2}\left(A-k+w-A\lambda+p_I\right) \tag{16}$$

After solving equation $\frac{\partial \pi_I}{\partial p_I}=0$, so p_G and p_I can be calculated as follows.

$$p_G=\frac{1}{2}\left(A-k+w-A\lambda-\frac{2w+\lambda\left(A+k-A\lambda\right)}{2\left(-2+\lambda\right)}\right) \tag{17}$$

$$p_I=\frac{-2w-\left(k+A(1-\lambda)\right)\lambda}{2\left(-2+\lambda\right)} \tag{18}$$

Thus, as can be seen from Eqs. (19) and (20), the profit function for each retailer can be determined.

$$\pi_{rG}=-\frac{n\left(2w(-1+\lambda)+A(-4+\lambda)(-1+\lambda)+k(-4+3\lambda)\right)^2}{16A(-2+\lambda)^2(-1+\lambda)} \tag{19}$$

$$\pi_{rI}=\frac{n\left(2w(-1+\lambda)+\lambda\left(A+k-A\lambda\right)\right)^2}{8A(-2+\lambda)(-1+\lambda)\lambda} \tag{20}$$

3.2. Nash equilibrium

In the previous two scenarios, we considered a situation that the GB and IB retailers plays as a leader in Stackelberg game, while in the present scenario, they act separately according the Nash equilibrium. We use the term "SN" to call this scenario. Now, the objective function of this case is shown in Eq. (21). This function includes the profit of selling the product minus the purchase cost for both retailers.

$$max\ \pi_{r_G} = p_G\left(\frac{n}{A}\left(\frac{A(1-\lambda)-p_G+p_I-k}{(1-\lambda)}\right)\right)-\frac{n}{A}\left(\frac{A(1-\lambda)-p_G+p_I-k}{(1-\lambda)}\right)\mathrm{w}$$

$$max\ \pi_{r_I} = p_I\left(\frac{n}{A}\left(\frac{\lambda p_G-p_I+\lambda k}{\lambda(1-\lambda)}\right)\right)-\frac{n}{A}\left(\frac{\lambda p_G-p_I+\lambda k}{\lambda(1-\lambda)}\right)\mathrm{w} \tag{21}$$

First derivative of each retailers' profit function with decision variables p_I and p_G, respectively shown with $\frac{\partial \pi_{rI}}{\partial p_I}=0$ and $\frac{\partial \pi_{rG}}{\partial p_G}=0$, provides the optimal values for decision variables.

$$\frac{\partial \pi_{rG}}{\partial p_G}=\frac{n\left(A-k+w-A\lambda+p_{\mathrm{i}}-2p_G\right)}{A-A\lambda}=0 \tag{22}$$

$$\frac{\partial \pi_{rI}}{\partial pI}=\frac{n\left(-w-k\lambda+2p_{\mathrm{i}}-\lambda p_G\right)}{A(-1+\lambda)\lambda}=0 \tag{23}$$

The second derivative of profit function for each retailer is $\frac{\partial^2 \pi_{rG}}{\partial p_G^2}=\frac{-2n}{A(1-\lambda)}\le 0$ and $\frac{\partial^2 \pi_{rI}}{\partial p_I^2}=\frac{-2n}{A(1-\lambda)\lambda}\le 0$ respectively, which mean the retailers' profit functions are concave.

After solving Eqs. (22) and (23), so p_G and p_I can be calculated as follows.

$$p_G=-\frac{2A-2k+3w-2A\lambda+k\lambda}{-4+\lambda} \tag{24}$$

$$p_I=-\frac{2w+A\lambda+k\lambda+w\lambda-A\lambda^2}{-4+\lambda} \tag{25}$$

Thus, as can be seen from Eqs. (26) and (27), the profit function for each retailer can be determined when the retailers choose their selling price independently.

$$\pi_{rG}=-\frac{n\left(2k+w+2A(-1+\lambda)-(k+w)\lambda\right)^2}{A(-4+\lambda)^2(-1+\lambda)} \tag{26}$$

$$\pi_{rI}=-\frac{n\left(2w(-1+\lambda)+\lambda\left(A+k-A\lambda\right)\right)^2}{A(-4+\lambda)^2(-1+\lambda)\lambda} \tag{27}$$

Proposition 1: For each scenario, if $k\le\hat{k}$ or $k\ge\breve{k}$, Then the market is monopolistic, otherwise the market is competitive, the conditions for each scenario shows in **Error! Reference source not found.**.

Table 2: Conditions for the monopolistic or competitive market

In each scenario, in order to obtain the conditions of presence of both retailers in the market, three conditions must be considered: when $q_G \geq 0$ and $q_I \geq 0$, both retilars competete in the market, when $q_G \leq 0$ and $q_I \geq 0$, Only IB retailer is present in the market, and finally when $q_G \geq 0$ and $q_I \leq 0$, Only GB retailer is present in the market. After the solving, the conditions can be obtained based on k values as shown in **Error! Reference source not found.**.

Proposition 1 helps to elucidate when the GB retailer can enter the market base on *k*. when the inconvenience cost through group buying mechanism is high ($k \geq \breve{k}$), the market is monopolistic for IB retailer. When $k \leq \hat{k}$, the market is monopolistic for GB retailer, for other value for *k* ($\hat{k} \leq k \leq \breve{k}$), both of retailers will be compete in the market.

Proposition 2: In the competitive market, for each scenario, the profit of both retailers is equal, when $k = k'$, the value of k for each scenario shows in in **Error! Reference source not found.**.

Table 3: Competitive market conditions when the profits of both retailers are equal

By examining condition $\pi_{rG} = \pi_{rI}$ in each scenario, *k* values can be obtained as **Error! Reference source not found.**.

Proposition 2 clarifies the condition base on k when $k = k'$ the profit of both retailers in the competitive market is equal. For larger value of *k* ($k \geq k'$), the IB Retailer gain more profit and for lower value of $k\left(k \leq k'\right)$ the GB retailer gain more profit.

4. Numerical analysis

In this section, we conduct some numerical examples to examine the impact of the both GB retailer and IB retailers' behaviors, referring to three distinctive scenarios, aimed to outline the conditions that they occur to choose the best strategy. Let $A = 1$, $w = 0.25$, $n = 10$, and the inconvenience cost through group buying mechanism and Unsociability rate of customer be $k = 0.4$ $\lambda = 0.6$.

The k and λ in all scenarios, play an important role in choosing the optimal strategy. Hence, the sensitivity of this variable will be examined further, at the first of this section examine the behavior of each retailer in each scenario separately and then compare the three scenarios together. All the data and computer program code (written by Wolfram Mathematica 11) are available upon request from the authors.

Figures 1 and 2 show the sensitivity of analysis of SG scenario for impact of λ and k parameters. Figure 1 shows the impact of λ on the selling prices, selling quantities and profit functions of each retailer in SG scenario. As it is obvious from Figure 1, with λ changes, variables also show opposite behaviors. In Figure 1, with the increase of λ, the power of IB retailer has increase. Likewise, with increase of λ the selling price, quantity and profit of IB retailer decrease, in accordance with scenario SG. As can be seen in Figure 1(b), for $\lambda \gtrsim 0.8$ the market is monopolized by the IB retailer and for $\lambda \lesssim 0.32$ the market is monopolized by the GB retailer and we have competitive market for values between these two number for λ .

Figure 1. The impact of λ on the first scenario (SG)

Figure 2. The impact of $\boldsymbol{k}$ on the first scenario (SG)

Figure 2 shows the impact of k on the selling prices, selling quantities and profit functions of each retailer in SG scenario. As it is obvious from Figure 2, with k changes, variables also show opposite behaviors like λ in Figure 1. In Figure 2, with the increase of k, noticeable is, the power of IB retailer has increase. Likewise, with increase of k the selling price, demands and profit of GB retailer decrease, in accordance with scenario SG. As can be seen in Figure 2 (b), for $k \gtrsim 0.5$ the market is monopolized by the IB retailer and for $k \lesssim -0.13$ the market is monopolized by the GB retailer and we have competitive market for values between these two number for k. It can also be seen that with k changes, selling prices and demands for both retailers change as linear, but not in the case of the profit functions.

Figures 3 and 4 show the sensitivity of analysis of SI scenario for impact of λ and k parameters. We have a similar behavior for selling prices, demands and profit functions for change of both of λ and k parameters. in Figure 3(b), for $\lambda \gtrsim 0.75$ the market is monopolized by the IB retailer and for $\lambda \lesssim 0.32$ the market is monopolized by the GB retailer and we have competitive market for values between these two number for k and also in Figure 4(b), for $k \gtrsim 0.51$ the market is monopolized by the IB retailer and for $k \lesssim -0.08$ the market is monopolized by the GB retailer and we have competitive market for values between these two number for k .

Figure 3. The impact of λ on the second scenario (SI)

Figure 4. The impact of $\boldsymbol{k}$ on the second scenario (SI)

Figures 5 and 6 show the sensitivity of analysis of SN scenario for impact of λ and k parameters. In this scenario We have a similar behavior for decision variable like the previous scenario. in Figure 5(b), for $\lambda \gtrsim 0.70$ the market is monopolized by the GB retailer and for $\lambda \lesssim 0.31$ the market is monopolized by the IB retailer and we have competitive market for values between these two number for k and also in Figure 6(b), for $k \gtrsim 0.58$ the market is monopolized by the GB retailer and for $k \lesssim 0$ the market is monopolized by the IB retailer and we have competitive market for values between these two number for k .

Figure 5. The impact of λ on the third scenario (SN)

Figure 6. The impact of $\boldsymbol{k}$ on the third scenario (SN)

In here, we provide several new numerical examples to better illustrate the comparison of the scenarios, we assume $A=1.14$, $w=0.03$, $n=10$, and $k=0.35$ $\lambda=0.9$. Figures 7 and 8 show the sensitivity of analysis of selling prices and retailers profit in all scenarios for impact of λ and k parameters sequence. Figure 7(a) depicts the impact of λ on group buying prices in three scenarios. As mentioned earlier, it is observed that with the increase of λ, the group buying price will decrease. For different amounts of λ, group selling prices also vary in different scenarios. Figure 7(b) shows the impact of λ on individual buying prices in three scenarios for IB retailer. As seen in this example the individual buying prices for SI scenario is larger than the other of individual buying prices. Figure 7(c) shows the impact of λ on group retailer profit in all scenarios. We are witnessing a downward trend with the increase of λ and also the amount of GB retailer profit in scenario SI is more than the other scenarios. For $\lambda \gtrsim 0.85$, the presence of a GB retailer in the market for all three scenarios is not justified. Figure 7(d) shows the impact of λ on individual retailer profit in all scenarios. The upward trend can be seen with the increase of λ.
Figure 8 shows the sensitivity of analysis of selling prices and retailers' profit in all scenarios for impact of k parameters. Figures 8 (a, b) show the linear downtrend and uptrend of group buying prices and individual buying prices in three scenarios with the increase of the k parameter, respectively. Figures 8(c, d) show the impact of k on GB retailer profit and IB retailer profit in all scenarios. With the increase of k In Figure 8(c), we see a decrease in the amount of GB retailer profit, and for $k \lesssim -0.1$, the presence of the IB retailer in the market is not justified. As can be seen for $\boldsymbol{k}$ values close to minus one, we have $\pi_G^{SG} > \pi_G^{SN} > \pi_G^{SI}$. In Figure 8 (d), with the increase of k ,we see an increase in the amount of IB retailer profit in all scenarios, and for $k \gtrsim 0.2$, the presence of the GB retailer in the market is not justified. As can be seen for $\boldsymbol{k}$ values close to one, we have $\pi_I^{SI} > \pi_I^{SN} > \pi_I^{SG}$.

Figure 7. The impact of λ on the three scenarios

Figure 8. The impact of $\boldsymbol{k}$ on the three scenarios

Figure 9 depicts the impact of k and λ on the presence of retailers in the market in scenario SG, as shown in this figure, based on the values obtained by k and λ, the presence of each retailer in the market can be change. according to the proposition 1, for $-\frac{(-1+\lambda)(-4w+2A\lambda+w\lambda)}{(-2+\lambda)\lambda} \le k \le \frac{(2A-w)(-1+\lambda)}{-2+\lambda}$, both retailers' presence and compete at the market. When both k and λ have high values, the market is monopolized by the IB retailer, also for small quantities of both k and λ, the market is monopolized by the GB retailer. As shown in Figure 9 for $\frac{(2A-w)(-1+\lambda)}{-2+\lambda} \le k \le -\frac{(-1+\lambda)(-4w+2A\lambda+w\lambda)}{(-2+\lambda)\lambda}$ none of the retailers are present in the market.

Figure 9. The impact of k and λ on the presence of retailers in the market in scenario SG

Figure 10 shows The impact of k and λ on the equality of retailer's profit in the market in scenario SG, when both retailers are present and compete to each other in the market according to the proposition 2, for $k'=\frac{\left(w(8-3\lambda)+2A(-4+\lambda)\right)(-2+\lambda)(-1+\lambda)\lambda+4\sqrt{2}\sqrt{-(-2+\lambda)^3(-1+\lambda)^2\lambda(w-A\lambda)^2}}{(-2+\lambda)^2\lambda(-4+3\lambda)}$, profit of both retailer are equal, As shown in this figure, for greater than k' amounts, the IB retailer's profit is higher than the GB retailer and vice versa.

Figure 10. The impact of $\boldsymbol{k}$ and λ on the equality of retailer's profit in the market in scenario SG

5. Conclusions

Group buying is a growing business model that companies use to encourage people to integrate and shop with the group. Currently, the popularity of group buying in e-commerce is also increasing. Thus, for the purpose of investigating this matter, we have explored a model, we consider two competitive retailers in a market. One of these retailers is trying to sell their product through the group buying mechanism, and the other is using the same traditional method that sell to individuals. Due to the development of social networks and communication of people in these contexts, the degree of penetration of this network among customers in the market was also considered as an involved parameter. In order to model these conditions, Given the strength of each retailer in the market, we have considered three scenarios. In order to implement the first two scenarios, the Stackelberg model is used where the GB retailer and the IB retailer are assumed to be the leader and follower, respectively. For the third scenario, the Nash equilibrium is applied when they decide separately.in all three scenarios we find the equilibrium strategy for both competitive retailers.

From a managerial perspective, this study suggests the following useful insights. For the presence in market, GB retailer must reduce the cost of inconvenience (k) such as signing up, waiting time to group complete, and registration fee, it is recommended that GB retailer should reduce this cost and time as much as possible to keep his/her portion in the market. Also, regarding the retailers' profit, it can be said that any retailer who has a leading role in the market can earn more profit from the market.

This research can be extended in several ways. In the present paper, we discussed the group buying and individual buying retailers in three scenarios that consider the diverse mode of competition between retailers. Correspondingly, it would be of interest to consider a manufacturer that sells the product to these two retailers and examine the case of vertical and horizontal competition between this manufacturer and retailers. Furthermore, we laid focus on the competition between GB and IB retailers when the demand is deterministic. Prospectively, development and focus could be dedicated to the impact of stochastic demand on the structure of behavior. Finally, considering that GB are done through the Internet platform and the consumers may not be satisfied with the product, future research may consider the return policy for GB retailer.

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Biographies

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Table 4: Compares this study with studies are closely related to the current paper

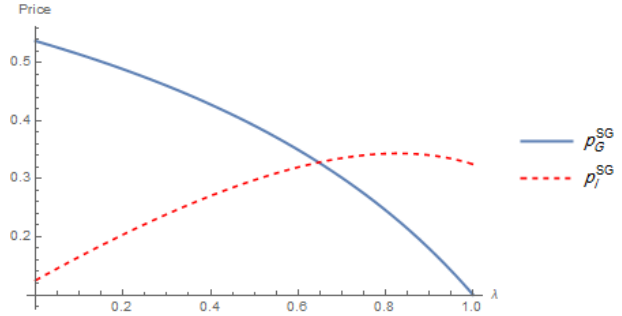
Research	Utility Function	Supply chain: Retailer	Supply chain: Manufacturer	Competition	Cooperation	Game theory: Nash	Game theory: Stackelberg	Individual Buying	Group Buying
Liang et al. [22]			1					✓	✓
Deng, Jiang, and Li [24]	$D_g(p_g) = n\varphi_g(p_g) + n\alpha\varphi_n(p_g)$	1						✓	✓
Tran and Desiraju [23]	$D_H^G(g';\eta,\beta) = \eta\alpha - \beta g'$ $D_L^G(g;\eta,\gamma) = (1-\eta)\alpha - \gamma(g+k_L)$	1	1		✓			✓	✓
Zhang, Shang, and Yildirim [26]	$U_G(v_i) = v + v_i + \lambda q_G v_i - c q_G - p_G$	n		✓		✓		✓	✓
Current study	$U_G = v - p_G - k$ $U_I = \lambda v - p_I$	**2**		✓		✓	✓	✓	✓

Table 5: Conditions for the monopolistic or competitive market

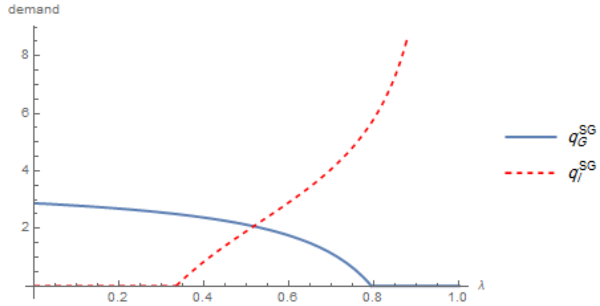
Scenario	Both retailers in the market When $\hat{k} \le k \le \breve{k}$	Exclusive market for IB retailer When $k \ge \breve{k}$	Exclusive market for GB retailer When $k \le \hat{k}$
SG	$-\frac{(-1+\lambda)(-4w+2A\lambda+w\lambda)}{(-2+\lambda)\lambda} \le k \le \frac{(2A-w)(-1+\lambda)}{-2+\lambda}$	$k \ge \frac{(2A-w)(-1+\lambda)}{-2+\lambda}$	$k \le -\frac{(-1+\lambda)(-4w+2A\lambda+w\lambda)}{(-2+\lambda)\lambda}$
SI	$\frac{(-1+\lambda)(-2w+A\lambda)}{\lambda} \le k \le -\frac{(-1+\lambda)(-4A+2w+A\lambda)}{-4+3\lambda}$	$k \ge -\frac{(-1+\lambda)(-4A+2w+A\lambda)}{-4+3\lambda}$	$k \le \frac{(-1+\lambda)(-2w+A\lambda)}{\lambda}$
SN	$\frac{2-7\lambda+5\lambda^2}{5\lambda} \le k \le \frac{9(-1+\lambda)}{5(-2+\lambda)}$	$k \ge \frac{9(-1+\lambda)}{5(-2+\lambda)}$	$k \le \frac{2-7\lambda+5\lambda^2}{5\lambda}$

Table 6: Competitive market conditions when the profits of both retailers are equal

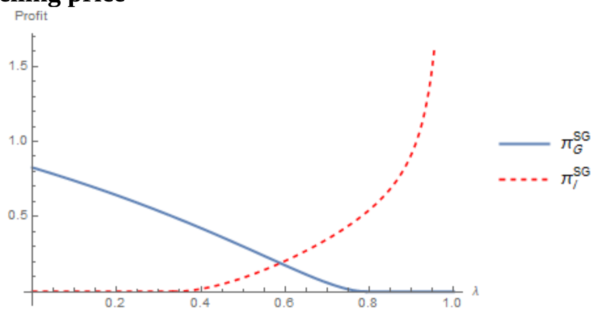
Scenario	$\boldsymbol{k = k'}$ **value for each scenario when** $\boldsymbol{\pi_{rG} = \pi_{rI}}$
SG	$k = \dfrac{\left(w(8-3\lambda)+2A(-4+\lambda)\right)(-2+\lambda)(-1+\lambda)\lambda+4\sqrt{2}\sqrt{-(-2+\lambda)^3(-1+\lambda)^2\lambda(w-A\lambda)^2}}{(-2+\lambda)^2\lambda(-4+3\lambda)}$
SI	$k=-\dfrac{A\lambda\left(-16+28\lambda-13\lambda^2+\lambda^3\right)+2\left(2\sqrt{2}\sqrt{-(-2+\lambda)^3(-1+\lambda)^2\lambda(w-A\lambda)^2}+w\lambda\left(8-13\lambda+5\lambda^2\right)\right)}{\lambda\left(16-28\lambda+11\lambda^2\right)}$
SN	$k = \dfrac{A\lambda - w\lambda + \sqrt{w^2\lambda - 2Aw\lambda^2 + A^2\lambda^3}}{\lambda}$



(a) Impact on selling price

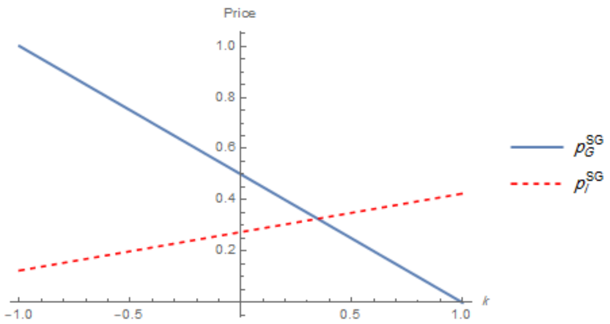


(b) Impact on selling quantity

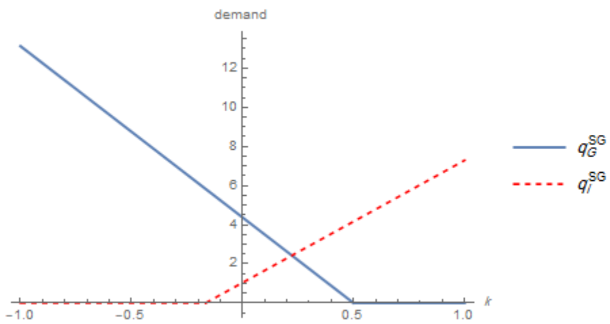


(c) Impact on retailers' profit

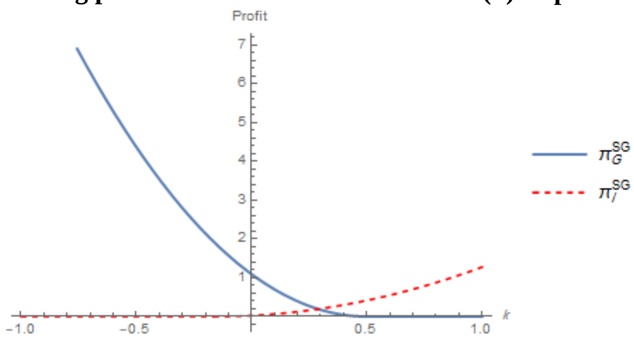
Figure 11. The impact of λ on the first scenario (SG)



(a) Impact on selling price

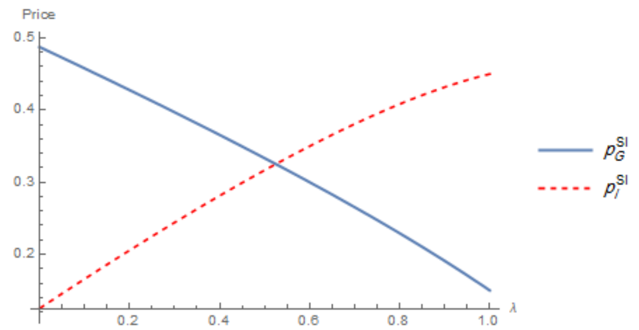


(b) Impact on selling quantity

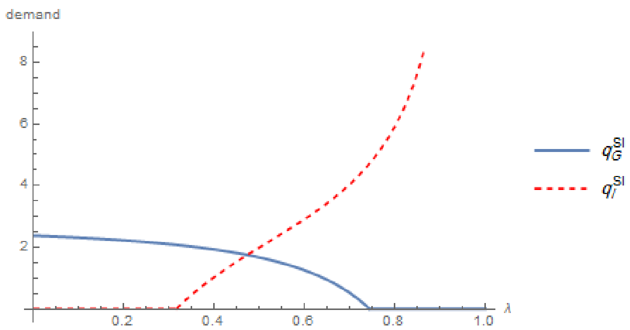


(c) Impact on retailers' profit

Figure 12. The impact of $\boldsymbol{k}$ on the first scenario (SG)



(a) Impact on selling price



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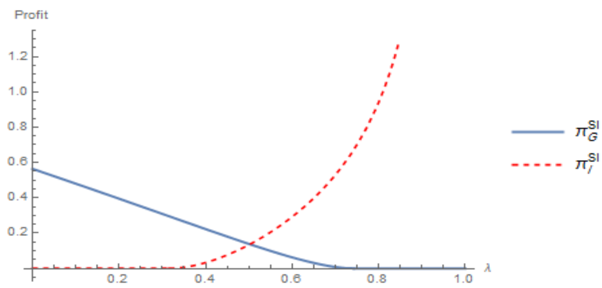
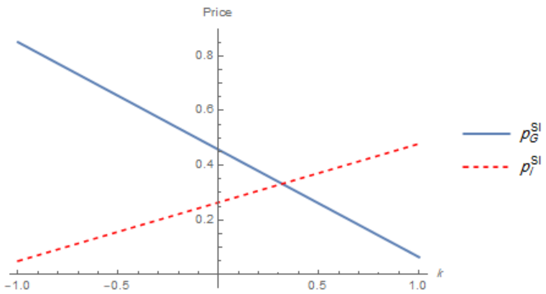


(c) Impact on retailers' profit

Figure 13. The impact of λ on the second scenario (SI)



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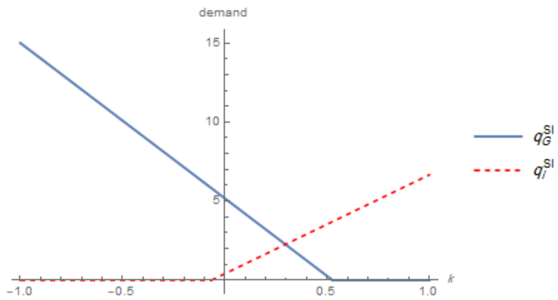

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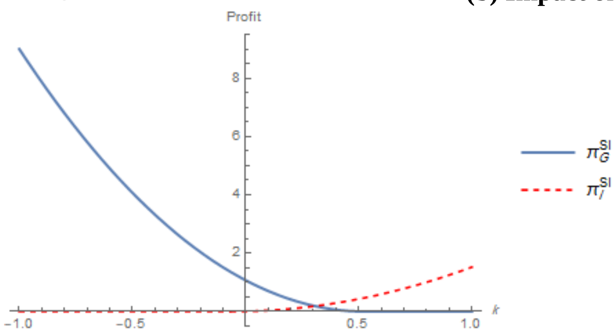
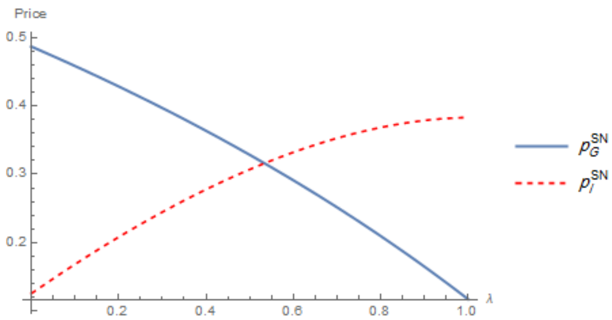
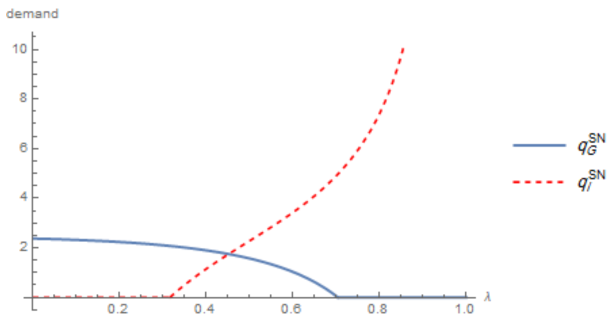


(c) Impact on retailers' profit

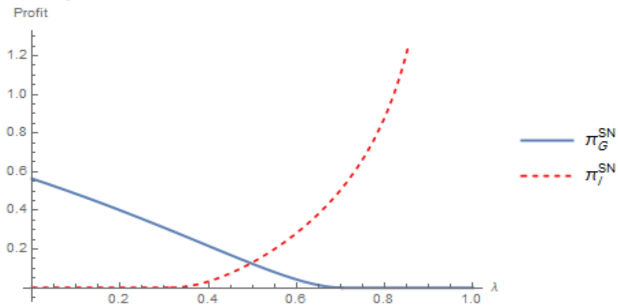
Figure 14. The impact of $\boldsymbol{k}$ on the second scenario (SI)



(a) Impact on selling price

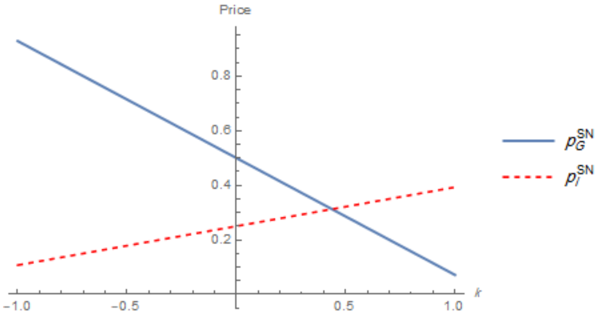


(b) Impact on selling quantity

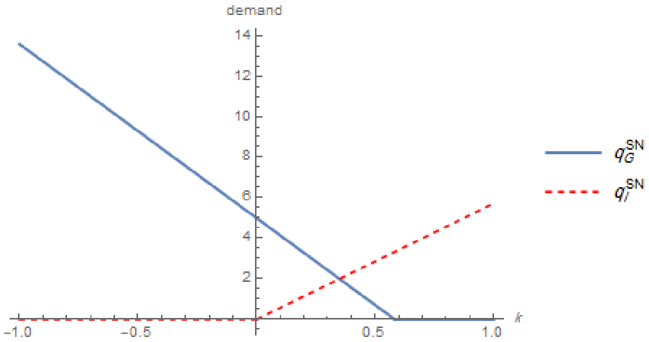


(c) Impact on retailers' profit

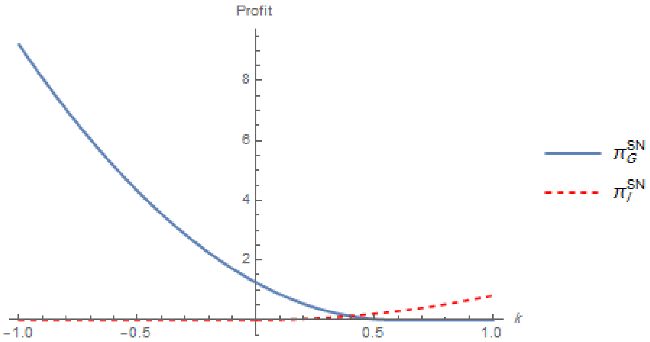
Figure 15. The impact of λ on the third scenario (SN)



(a) Impact on selling price

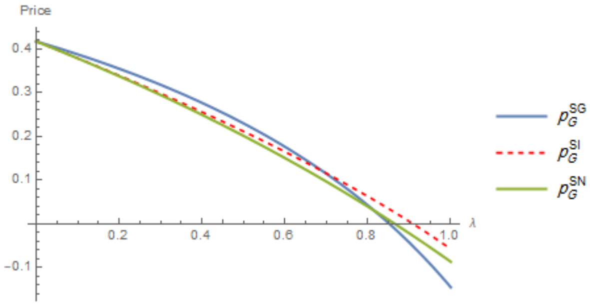


(b) Impact on selling quantity

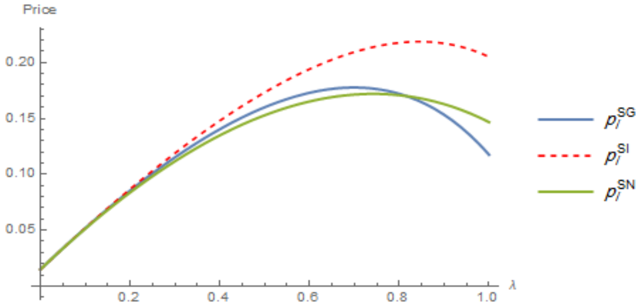


(c) Impact on retailers' profit

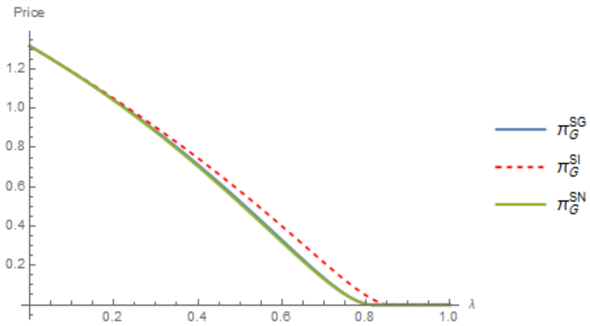
Figure 16. The impact of $\boldsymbol{k}$ on the third scenario (SN)



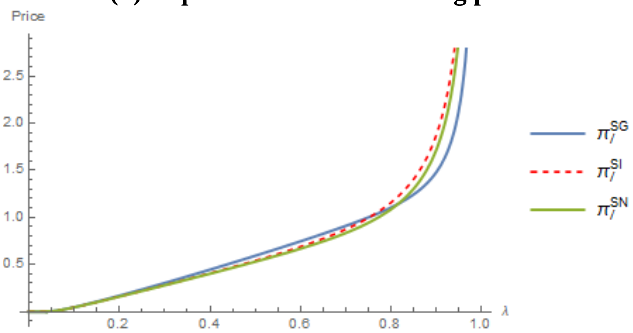
(a) Impact on group selling price



(b) Impact on individual selling price

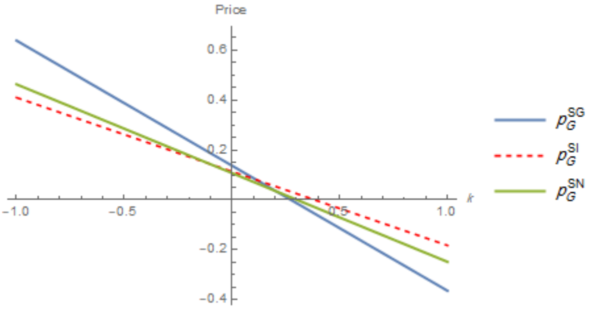


(c) Impact on GB retailer profit

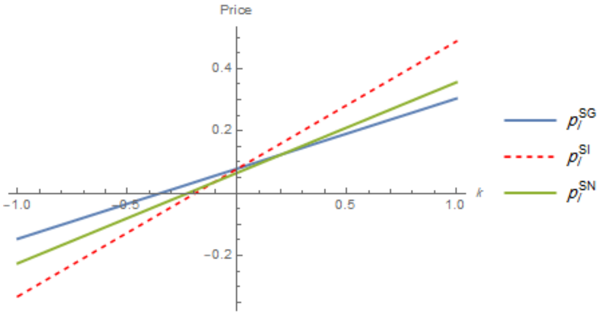


(d) Impact on IB retailer profit

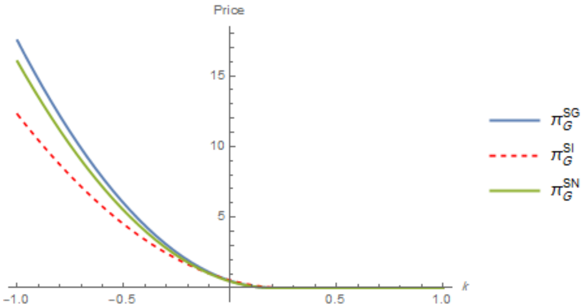
Figure 17. The impact of λ on the three scenarios



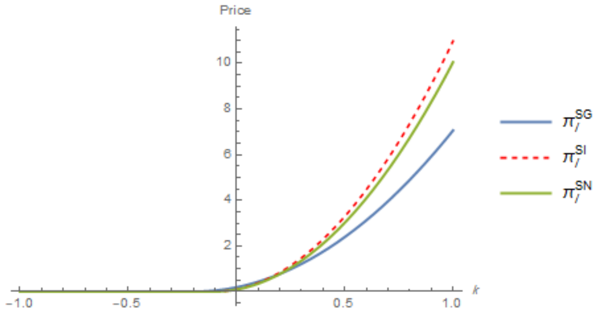
(a) Impact on group selling price



(b) Impact on individual selling quantity



(c) Impact on GB retailer profit



(d) Impact on IB retailer profit

Figure 18. The impact of $\boldsymbol{k}$ on the three scenarios

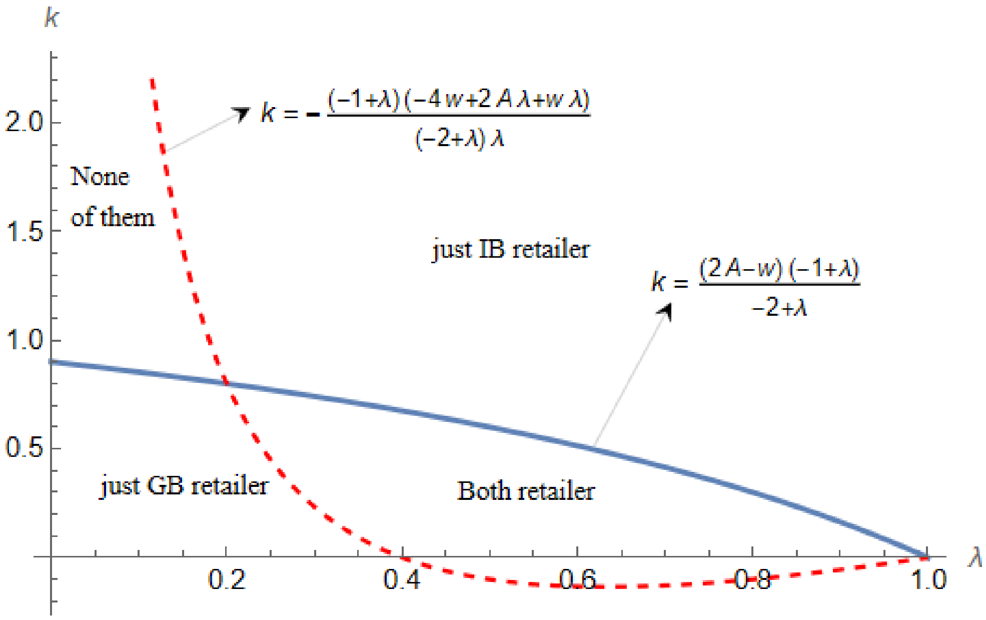


Figure 19. The impact of k and λ on the presence of retailers in the market in scenario SG

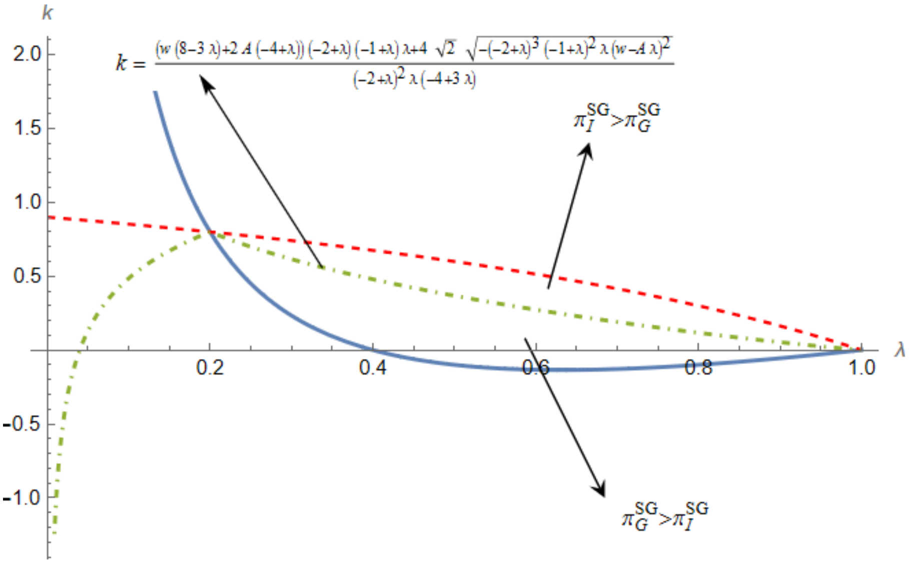


Figure 20. The impact of ***k*** and $\boldsymbol{\lambda}$ on the equality of retailer's profit in the market in scenario SG