Chaos Embedded Meta-heuristic Algorithms for Optimal Design of Truss Structures

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Abstract
The success of embedded chaos in meta-heuristic algorithms is mainly due to providing good balance between exploration and exploitation for meta-heuristics. Comparison of optimization results with algorithms in standard mode and embedded of chaos shows a significant improvement in quality of the meta-heuristic algorithms, thus reducing the weight of truss structures. Four chaos meta-heuristic algorithms with logistic, Tenet and Gaussian maps are considered to improve the results. Despite truss optimization being severely nonlinear and non-convex and often having several local optima, the use of different chaos allows the local optima to be escaped and global optimums to be achieved.

Keywords:
Chaos Map, Cross section optimization, Truss structures, Meta-heuristic algorithms

1. Introduction
In the last three decades, the optimal design has been performed for many different types of structures. The main reason for this choice was the limitation of the available resources and attempts to reduce the amount of materials being used. In order to achieve this goal, classical methods including numerical and mathematical methods are proposed. However, each of these solutions had its own limitations and difficulties. For example, in numerical methods, when reaching the local optimal, the computational operation was stopped and no solution was provided to escape from this local optimum. Also, in analytical and mathematical methods, access to derivatives of the objective function was inevitable, while most engineering problems could not provide us with an explicit relation of the objective function. Today, with the complexity of issues and the increase in the number of decision variables, the lack of response of classical methods has become evident. Therefore, to overcome these challenges, a special category of optimization methods called meta-heuristic methods has been developed, which are based on decisions and principles of probabilistic and random search. In these algorithms, the value of the objective function
itself is used instead of derivatives and shows good efficiency in complex problems [1]. The source of inspiration in these algorithms can play an important role in their classification. Some of these sources of inspiration are: algorithms based on physical laws, algorithms based on swarm intelligence, algorithms based on evolution, behavioral algorithms, algorithms based on environment and algorithms based on social and human laws. For each of these groups, we introduce examples, Optimization algorithms such as water Evaporation Optimization (WEO) [2], Charged System Search (CSS) [3], Colliding Bodies Optimization (CBO) [4,5], Vibrating Particles System (VPS) [6], Thermal Exchange Optimization (TEO) [7], Big Bang-Big Crunch (BB-BC) [8], Ray Optimization (RO) [9], Harmony Search (HS) [10] are inspired by physical laws. Cyclical Parthenogenesis Algorithms (CPA) [11], Lion Pride Optimization (LPO) [12], Artificial Coronary Circulation System [13], Ant Colony Optimization (ACO) [14], Whale Optimization Algorithm (WOA) [15], Gray Wolf Optimization (GWO) [16], Particle Swarm Optimization (PSO) [17,18] are inspired by swarming intelligence. Genetic Algorithms (GA) [19], Differential Evolution (DE) [20], Evolutionary Strategy (ES) [21], are inspired by evolution. Shuffled Frog-Leaping Algorithm (SFLA) [22], Biogeography-Based Optimization (BBO) [23], Teaching-Learning-Based Optimization (TLBO) [24], Imperialist Competitive Algorithm (ICA) [25] are also part of the memetic algorithms, based on the environment and based on social and human laws, respectively. In each of these algorithms, a number of random numbers are considered, which we use an alternative chaos system to fix them [26]. Mathematically, chaos refers to the ability of a simple pattern and model, which although this pattern has no signs of random phenomena, but can lead to the emergence of very disordered behaviors in the environment. Today, these dynamic systems have attracted the attention of many scientific communities and are observed in various fields such as engineering, medicine, biology and economics [27]. The salient features of a chaotic system are:

1. They are sensitive to initial conditions.
2. Their alternating rotation is dense.
3. They have quasi-random and non-periodic behavior [28].

The term butterfly effect came from an article by Edward Lorenz. At the 99th Water Summit, he wrote an article entitled: 'Can a butterfly fly in Brazil cause strong winds in Texas? [27] 'Experimental studies show that the use of chaotic signals instead of random signals has yielded very valuable results. In this research, chaos map in optimization algorithms including Cyclical Parthenogenesis Algorithm (CPA), Teaching-Learning-Based Optimization (TLBO), Biogeography-Based Optimization (BBO) and Differential Evolution (DE) are investigated and in most cases the optimization results of the algorithms are improved. In order to access the wide statistical data and increase the diversity of studies, in each modeling, three chaos maps with three different scenarios have been considered.

Rapid convergence, stability and robustness of responses are the main reasons for choosing these algorithms. Reducing the number of iterations and improving the values of standard deviation and coefficient of variation compared to previous research can confirm the superiority of these four algorithms.

2. Formulation of optimization problems

Optimization problems for trusses are generally defined in Eq. (1) that all design limitations must be satisfied and the total weight of the structure must be as low as possible.
Find \( A = \{ A_1, A_2, \ldots, A_n \} \)

to minimize \( W(A) = \sum_{i=1}^{n} \gamma_i A_i L_i \)

subjected to \( g_j(A) \leq 0 \); \( j = 1, 2, \ldots, m \)

\( h_k(A) \leq 0 \); \( k = 1, 2, \ldots, p \)

\( \{ A_L \} \leq \{ A \} \leq \{ A_U \} \)

In this category of Eq. (1), \( A \) is the cross section vector of the members, \( W \) is the total weight of the structure, \( n \) is the number of members of the structure, \( g_i \) and \( h_i \) are design constraints that can include stress, slender constraints and node displacement. Also, \( A_L \) and \( A_U \) are the lower and upper bounds of the decision variables. Despite the selected algorithms are used to optimize unbounded problems. In modeling, the penalty function method is used to convert the bound function to the unbound function. In this method, if there is no violation, the amount of the fine will be zero. Otherwise, and if there is a violation, the value of the penalty function is obtained from Eqs. (2) to (6):

\[
\sigma_i \leq \sigma_{\text{max}} \quad \Rightarrow \quad V_i = \max(0, \frac{\sigma_i}{\sigma_{\text{max}}} - 1); \quad i = 1, 2, \ldots, m
\]

\[
\delta_j \leq \delta_{\text{max}} \quad \Rightarrow \quad V_j = \max(0, \frac{\delta_j}{\delta_{\text{max}}} - 1); \quad j = 1, 2, \ldots, n
\]

\[
\lambda_k \leq \lambda_{\text{max}} \quad \Rightarrow \quad V_k = \max(0, \frac{\lambda_k}{\lambda_{\text{max}}} - 1); \quad k = 1, 2, \ldots, p
\]

\[
F_{\text{penalty}}(A) = 1 + \eta \times \left\{ \sum_{i=1}^{m} V_i + \sum_{j=1}^{n} V_j + \sum_{k=1}^{p} V_k \right\}
\]

to minimize \( \text{Mer}(A) = W(A) \times F_{\text{penalty}}(A) \)

Equations (2), (3) and (4) are related to stress, displacement and slenderness, respectively. The penalty function is presented in Eq. (5) and the objective function is formed after the penalty (merit function) in Eq. (6).

3. Meta-heuristic algorithms and Chaos Map

In each meta-heuristic algorithm, a number of random parameters are selected during the sequential iteration process. Research shows that the type of random distribution function for selecting these parameters has a significant impact on the performance of the algorithms. These parameters play a major role in increasing the speed of convergence and creating a balance between the exploration and exploitation stages. Nowadays, chaotic dynamic series are a good alternative to these stochastic parameters. Chaos systems can provide convergence to global responses and have the potential to prevent them from falling into trap of local optima. Although series created by chaotic map appear to be similar to sentences with random distributions, there are major differences. Some of these differences are: Their values are deterministic, have nonlinear behavior, have a dynamic state, the sentences of the series related
to them are non-repetitive, and finally non-limited to a particular boundary. By applying chaos map, the nonlinear and non-convex behaviors of the object function in truss optimization are easily controlled and adjusted by chaos maps. Three different scenarios have been proposed to apply the chaos maps and improve the optimization results. In the first scenario, the chaos maps replace the random parameters related to the exploration part, for the second scenario, these functions replace the parameters related to the exploitation part, and finally, for the third scenario, the simultaneous application of chaos maps in both the exploration and exploitation parts is considered. By examining structural examples, we will be able to determine the best chaos map and similar scenario for each of the algorithms. In a recent study to investigate the effects of chaos map in improving the results of algorithm optimization, four meta-heuristic algorithms with three chaos map are combined and the results are compared with the standard modes. Due to the wide range of studies and diversity, there is close competition between the optimization results and the possibility of approaching the absolute optimal increases. In this research, the selected meta-heuristic algorithms are: Cyclical Parthenogenesis Algorithm (CPA), Teaching-Learning-Based Optimization (TLBO), Biogeography-Based Optimization (BBO) and Differential Evolution (DE). Chaos maps to combine with these algorithms include logistic, Tent and Gaussian chaos map. In the following, each of these algorithms are introduced.

3.1. Standard cyclical parthenogenesis algorithm (CPA)

This algorithm was proposed by Kaveh and Zolghadr [11]. This algorithm, is inspired by social behavior and wildlife reproduction of certain insects, such as aphids, which can reproduce with and without mating. Also, like most meta-heuristic algorithms, it is population-based. At the following CPA algorithm is organized in five steps, are presented.

3.1.1. Basic steps of cyclical parthenogenesis algorithm

**Step 1.** Formation of the initial population: In the search space, the population of aphid is formed randomly. This number of aphid populations expressed as \( nA \) is formed in colonies with \( nC \) number and each with \( nM \) population. It is clear that \( nM = nA / nC \) and \( nM \) are constant during optimization.

**Step 2.** Reproduction or of aphids: To form the population of offspring in each colony, the number of \( Fr \times nM \) offspring is formed without mating. The parents of these children are female and their selection is done randomly and from the best answers. In MATLAB coding, this selection and formation of the children population is as follows:

\[
rf_i = \text{round}\left(1 + \left(\frac{Fr \times nM}{nM} - 1\right) \times \text{rand}\right) \tag{7}
\]

\[
\text{newCA} = F + \alpha_1 \times \frac{\text{randn}}{\text{NITs}} \times (Ub - Lb) \tag{8}
\]

In Eq. (7), the index related to the female parent is determined and in Eq. (8), the new children aphids related to the state without mating are placed in the new cell array. Now it is time to form offspring by mating. The number of these offsprings in which each male parent \( M \) randomly selects a female parent \( F \) and is placed in the new cell array according to the Eq. (9) new children related to the mating state.

\[
\text{newCA} = M + \alpha_2 \times \text{rand} \times (F - M) \tag{9}
\]

**Step 3.** Flying the best aphids and death of the worst aphids: After the formation of a new generation of offspring the objective function is evaluated and with a probability of \( Pf \) one of the best winged aphids is selected from colony 1 and by reproducing it replaces the worst aphids in colony 2. To keep the colony
population stable, removing the worst colony 2 aphids is compared to the death of the aphids and replacing the best aphids with flying. The probability associated with this step is based on Eq. (10):

\[ Pf = \frac{(NITs - 1)}{(\text{max} NITs - 1)} \]  

**Step 4.** Replacement of the best aphids: In each colony, the population of parents is compared with the children and from them the number of \( nM \) to the best is selected to form the next generation.

**Step 5.** Check the termination conditions and repeat the operation from step 2 if necessary.

### 3.1.2. Chaos embedded cyclical parthenogenesis algorithm (CCPA)

In this algorithm, two modes with and without mating are selected to form the offspring of the new generation. These two modes play the role of exploration and exploitation. By substituting chaos map for random choices in these two steps, we can propose a variety of scenarios. In this research, the following scenarios have been investigated:

**Scenario 1.** Placement of the chaos map in stage without mating stage: In this case, the first chaos map \( \text{CHM1} \) in Eq. (7) replaces random selection, and the result will be Eq. (11):

\[ rf_i = \text{round}(1 + (Fr \times nM) \times \text{CHM1}) \]  

**Scenario 2.** Placement of the chaos map in the mating stage: In this case, the second chaos map \( \text{CHM2} \) in Eq. (9) replaces random selection, and the result will be Eq. (12):

\[ \text{newCA} = M + \alpha_2 \times \text{CHM2} \times (F - M) \]  

**Scenario 3 -** Placing the chaos map in both stages simultaneously: In this case, the two chaotic map simultaneously in Eqs. (7) and (9) replace the random distribution.

### 3.2. Standard teaching-learning-based optimization (TLBO)

The teaching-learning-based optimization was introduced by Rao et al. in 2011 and it is based on the classroom learning process [24]. The algorithm consists of two stages that include the effect of the teacher in the learning process and the students' influence on each other. Similar to other population-based algorithms, it starts with a series of random initial solutions, which are the same students, and the best and smartest of them in each period of repetition is known as the teacher. Students' academic level improves in each repetition, in phase. The first phase is based on the transfer of knowledge from the teacher, also known as the teacher phase, and the second phase is the students' own learning from each other or the students' interactions with each other, also known as the student phase. In the following the TLBO algorithm is organized in four steps, are presented.

#### 3.2.1. Basic steps in Standard teaching-learning-based optimization

**Step 1.** Introduce the parameters of the algorithm and the formation of the initial population of students: The basic parameters include the number of learners \( nL \), the number of decision variables \( nV \), the maximum number of function evaluations NFEs, which is the same parameter for the stopping criterion. Now the initial population is randomly formed based on the search space and their estimation is permormed in the objective function.
**Step 2.** From the introduced population, the best of them is selected as teacher T. Then the average position of students is calculated and based on the teacher phase, the improved academic level of students is determined according to the Eq. (13):

\[ \text{stepsize}_i = T \times TF_i \times \text{MeanL} \]  

\[ \text{newL} = L + \text{rand}_{i,j} \times \text{stepsize} \]

\[ i = 1,2,\ldots,nL \quad \text{and} \quad j = 1,2,\ldots,nV \]  

In Eq. (13), the TF teaching factor is randomly selected to be 1 or 2, indicating how successful the teacher has been in increasing the average level of students. The values obtained are evaluated and if they are better than the previous values, they are replaced.

**Step 3.** Discovering Your Purpose Students' interactions with each other. In this phase, each student exchanges information with another randomly selected student (except himself). It is possible to improve the information when the performance of the selected student is better, otherwise it will change according to their position.

\[ \text{if } \text{PFit}_i < \text{PFit}_{ir} \Rightarrow \text{stepsize}_i = L_i - L_{ir} \]  

\[ \text{if } \text{PFit}_i > \text{PFit}_{ir} \Rightarrow \text{stepsize}_i = L_{ir} - L_i \]

\[ \text{newL} = L + \text{rand}_{ij} \times \text{stepsize} \]

\[ i = 1,2,\ldots,nL \quad \text{and} \quad j = 1,2,\ldots,nV \]  

The values obtained are evaluated and if they are better than the previous values, they are replaced. The best of the populations are introduced at each stage.

**Step 4.** Check the termination conditions and repeat the operation from step 2 if necessary.

### 3.2.2. Chaos embedded teaching-learning-based optimization (CTLBO)

This algorithm consists of two important strategies, including the teacher's effect on the learning process and students' influence on each other, which will play a major role in the exploration and exploitation process. By replacing the chaos map in random selections related to these steps, we will see a significant improvement in the performance of the algorithm. The proposed scenarios for this replacement are as follows:

**Scenario 1.** Placement of the chaos map in the teacher effect stage of the learning process: In this case, the first chaos function \( \text{CHM1} \) is replaced in Eq. (14), and the result will be Eq. (18):

\[ \text{newL} = L + \text{CHM1}_{ij} \times \text{stepsize} \]

**Scenario 2.** Placement of the chaos map in the stage of students interacting with each other: In this case, the second chaos map \( \text{CHM2} \) is replaced in Eq. (17), and the result will be Eq. (19):

\[ \text{newL} = L + \text{CHM2}_{ij} \times \text{stepsize} \]
Scenario 3 - Placing the chaos map in both stages simultaneously: In this case, the two chaotic functions are replaced simultaneously in Eqs. (14) and (17).

3.3. Standard biogeography-based optimization (BBO)

This algorithm was introduced in 2008 by Dan Simon [23]. The main inspiration of the algorithm is based on the distribution of plants and especially animals in different geographical areas. In principle, animal and plant species try to use resources exclusively, but this will not be possible in most cases. Therefore they have to have food, water and . . . Therefore, an ecosystem is formed that becomes the food of other species with the formation of food chains. But this tendency to monopolize has other consequences, and that is that animals insist on migrating to more secluded places. Therefore, a settlement is known as a suitable place when it has a smaller population. Therefore, animal species usually choose a more secluded place from among several options. But where it has richer food resources, it will naturally have more population. Therefore, a factor called the HIS habitat suitable index is introduced, which determines the better location. In optimization problems, this coefficient is the same as the objective function of the problem, and in minimization problems, the lower the coefficient, the better. But the habitats for which this coefficient is higher are in fact very large and because of the competitors, the species want to leave. Therefore, two very common terms of migration are considered: migration to habitat that named Immigration, which Expresses acceptance of immigration, which is expressed as a percentage by the λ, and migration from habitat, that named emigration, which Its percentage state is also expressed by the μ. Habitat location is displayed with SIV that indicating suitability index variable or decision variables in the selected space. Now the variables migrate from the most populated habitats and their migration coefficient is high μ to the habitats with high migration coefficient λ. Also, in order to create variety in the search space, mutations are applied to the components of the variables with a certain probability. The basic steps of this algorithm for standard mode are presented in the following.

3.3.1. Basic steps in biogeography-based optimization:

Step 1. Formation of primary habitats: In the search space, a set of primary habitats is randomly formed and sorted based on the objective function. At this stage we access the initial population. The number of this initial population is represented by npop.

Step 2. The values of the dimensionless coefficients λ and the migration μ are determined for the habitats.

Step 3. For each selected habitat location such as i, repeat steps 4 to 8 to the initial population.

Step 4. For each variable such as r in habitat i, repeat steps 5 to 8 for the number of array variables.

Step 5. With the probability λᵢ in the variable Xᵢᵣ, we make the changes according to steps 6 to 8.

Step 6. The origin of Xₑᵢ migration is determined using the values of μ vectors based on random selection in a discrete distribution (roulette wheel).

Step 7.- The location of the new habitat is done by migrating from Xₑᵢ to Xᵢᵣ as follows:

\[
\text{if } \text{rand} \leq \lambda(i) \Rightarrow X_{\text{new}}^{\text{ir}} = X_{\text{ir}} + \alpha_k \cdot (X_{\text{jr}} - X_{\text{ir}})
\]

\[
\alpha_k = 0.9
\]
**Step 8.** With a certain probability, jump changes with a specific random distribution (preferably 'normal distribution') are performed on the selected variable $X_{ir}$:

\[
\text{if } \text{rand} \leq \text{pmutation} \Rightarrow X_{ir}^{\text{new}} = X_{ir}^{\text{new}} + \sigma \cdot \text{randn}
\]  

(22)

\[
\sigma = 0.02 \times (\text{VarMax} - \text{VarMin})
\]  

(23)

Sigma comprises a percentage of the decision space. In the original version, this value is 2%.

**Step 9.** Migrate the answers from the previous answers and after evaluating and sorting, select the number of npop from the best of them for the next step.

**Step 10.** Check the termination conditions and repeat the operation from step 3 if necessary.

### 3.3.2. Chaos embedded biogeography-based optimization (CBBO)

In this algorithm, the location of the new habitat is based on two important strategies. In the first strategy, migration from $X_{jr}$ to $X_{ir}$ takes place for each location such as $i$ and for each variable such as $K$ in location $i$ with a probability of $\lambda$ corresponding to $i$. In the second strategy, random mutation changes are applied to the variables of the decision variable. These two play the same role in the exploration and exploitation process. Therefore, by replacing the chaos map in random choices related to these steps, we will see a significant improvement in the performance of the algorithm. The proposed scenarios for this replacement are as follows:

**Scenario 1.** Placement of the chaos map in the first stage of the migration strategy of variables: In this case, the first chaos map $CHM1$ is replaced in Eq. (20), and the result will be Eq. (24):

\[
\text{if } CHM1 \leq \lambda(i) \Rightarrow X_{ir}^{\text{new}} = X_{ir}^{\text{new}} + \alpha_k \cdot (X_{jr} - X_{jr})
\]  

(24)

**Scenario 2.** Placing the chaos map in the second strategy phase of random mutation changes: In this case, the second chaos map $CHM2$ is applied in Eq. (22), and the result will be Eq. (25):

\[
\text{if } CHM2 \leq \text{pmutation} \Rightarrow X_{ir}^{\text{mut}} = X_{ir}^{\text{new}} + \sigma \cdot \text{randn}
\]  

(25)

**Scenario 3.** Placing the chaos map in both stages simultaneously: In this case, the two chaotic map are replaced simultaneously in Eqs. (20) and (22).

### 3.4. Standard differential evolution (DE)

This algorithm was proposed by Price and Storn [20]. Differential evolution algorithm is a random and population-based algorithm and is considered as an evolutionary algorithm. In this algorithm, the method of generating new answers is unique. The main difference between differential evolution algorithms and other evolutionary algorithms is in the formation of the offspring population. Also, how to apply the mutation and the select the step length of the mutation according to a specific possible distribution. In contrast, in the differential evolution algorithm, a temporary response is generated first with the mutation operator and then a new response is generated using the cross-over operator. In this algorithm, a known probabilistic distribution is not sampled for the mutation operation, but the distance and difference between the available responses are used for the mutation steps. In fact, the differences between members
of the population contain very useful and valuable information about the objective function and the problem to be solved, the use of which can improve the search process and optimization action. In an optimization algorithm, often, over time and with the passage of different iterations, the responses in the population converge to a single response. Therefore, the distance between the members of the population becomes less and less. Also, the amount of distance in the initial population is quite affected by the number of members of the population. Therefore, the more members of the population, the smaller the distance between the members, and vice versa. The distance between members of the population contains important information about the distribution of members of the population. Based on this, with the distance between the members of the population, we can determine the length of the step and adjust the intensity of the mutation. Now, if this distance is greater, we should be able to reach the ideal point faster by choosing larger steps, and conversely, for smaller distances, smaller steps will be able to search the ideal location with appropriate accuracy. Thus, in this algorithm, the step length and direction of the mutation are determined as follows: - First, two members of the population are randomly selected. - Then the difference between the two selected members is calculated and introduced as the difference vector. Now we consider the coefficient of the difference vector as the mutation vector. The use of difference vectors has advantages, the main of which are: - The information available to members of the population is fully used and will be very suitable for intelligently orienting the optimization and search process. According to the central limit theorem, as the population increases, the length of the jump steps will tend to a random quantity with a normal distribution. To perform the intersection of the proposed relations, a number of arrays will be selected from the initial population and a number from the mutant population. These are shown in Figure 1.

\[
\begin{align*}
X &= \{x_1, x_2, \ldots, x_{n\text{var}}\} \\
Y &= \{y_1, y_2, \ldots, y_{n\text{var}}\} \\
\alpha &= \beta \times (b - c) \\
Z_j &= \begin{cases} 
  y_i & \text{if } j \in J \\
  x_i & \text{if } j \not\in J 
\end{cases}
\end{align*}
\]

Also, the set \( J \) is formed, we first randomly select \( J_0 \) from the numbers 1 to \( n\text{var} \), we add the number \( J_0 \) to the set \( J \), which is initially empty. Now, for all values up to 1 \( n\text{var} \), we repeat the following operations: - We generate a random number such as \( r \in \mathbb{R} \) that has a uniform distribution in the range of zeros and ones:

\[
Z_j = \begin{cases} 
  y_i & r \leq P_c \ or \ j \in j_0 \\
  x_i & \text{otherwise}
\end{cases}
\]

- If \( r \) is less than or equal to \( P_c \) (intersection probability parameter), we add the number \( j \) to the set \( J \). The basic steps in this algorithm in standard mode are discussed below.
3.4.1. Basic steps in differential evolution

**Step 1.** Define the algorithm parameters including: number of \( nvar \) decision variables, top and bottom boundaries of decision variables, maximum number of iterations of the objective function, number of \( npop \) initial population, upper and lower limit of \( betamin \) and \( betamax \) scale factor, which in the initial version were 0.2 and 0.8, respectively. And finally the probability of the percentage of \( Pcr \) crossover process.

**Step 2.** Formation of the initial population: Like most population-based algorithms, the initial population is formed in the search space and randomly.

**Step 3.** Repeat the following steps until the termination conditions are met:

The following steps are performed for each member of the population:
- A temporary response is generated using the jump operator. How to calculate \( \beta \) is as follows:

\[
\text{Beta} = \text{bet}_\text{min} + \text{rand} \times (\text{beta}_\text{max} - \text{beta}_\text{min})
\]  

- Using the intersection operator, a new response is created and evaluated based on the objective function. Now, if the new answer is better than the current answer, it will replace the current answer, otherwise the current answer will be retained.

\[
\text{if } j = j_0 \text{ or rand} \leq \text{Pcr} \\
Z(j) = Y(j) \\
\text{Else} \\
Z(j) = X(j)
\]

The best answer so far is returned as output.

3.4.2. Chaos embedded differential evolution (CDE)

This algorithm, like other evolutionary algorithms, includes two important stages of cross-over and mutation. According to a general view, the main difference between the differential evolution algorithms is the order of these two stages, in which first the mutation and then the cross-over take place. It also exploits the difference between responses as much as possible to achieve convergence and escape local optimization. It is clear that mutation and cross-over play the role of exploration and exploitation stages, respectively. Therefore, by replacing the chaos functions in random choices related to these steps, we will see a significant improvement in the performance of the algorithm, Figure 2. The proposed scenarios for this replacement are as follows:

**Scenario 1.** Placing the chaos map in the step of the mutation operator: In this case, the first turbulence function \( \text{CHM1} \) is replaced in Eq. (29), and the result will be Eq. (31):

\[
\text{Beta} = \text{bet}_\text{min} + \text{CHM1} \times (\text{beta}_\text{max} - \text{beta}_\text{min})
\]

**Scenario 2.** Placing the chaos map in the cross-over operator stage: In this case, the second chaos map \( \text{CHM2} \) is replaced in Eq. (30), and the result will be Eq. (32):
\[
\text{if } j = j_0 \text{ or } \text{CHM2} \leq \text{Pcr} \\
Z(j) = Y(j) \\
\text{Else} \\
Z(j) = X(j)
\]

(32)

Scenario 3. Placing the chaos map in both stages simultaneously: In this case, the two chaotic map are replaced simultaneously in Eqs. (29) and (30).

Figure 2. The flowchart for the chaos Algorithm

4. Introduction of selected chaos map

Chaotic map shown no signs of random behavior, yet they cause very erratic behaviors in the environment. The most important features of these map are sensitivity to initial conditions and non-periodic and ergodic behavior can be introduced [28]. The use of chaos sequences to evolve variables has significant advantages over other methods. In deterministic searches, compared to random search, more speed and convergence towards the general answer is achieved. In this research, the Logistic, Tenet and Gauss maps have been selected as shown in Figure 3. In a Logistic map it is very probable to converge from a local to a global minimum. Therefore, this map is suitable for improving the exploration specifications of algorithm. The Gaussian map is most probable in the local minimum range and is suitable for improving exploitation attribute. Finally, Tenet map simultaneously improves both conditions. Therefore, by selecting these maps, the weakness of algorithms of any kind is improved.

Figure 3. The Chaotic value distribution during 100 iterations

4.1. Logistics map

This map appears in nonlinear dynamic behaviors related to biological populations. The sentences of chaotic sequences in the logistic map are obtained according to the following equations:

\[
\text{CHM}_{k+1} = \alpha \times \text{CHM}_k (1 - \text{CHM}_k)
\]

(33)

\[
\text{CHM}_{k+1} \in (0,1) \text{ ; } \text{CHM}_0 \in (0,1) \text{ ; } \text{CHM}_0 \in (0,0.25,0.5,0.75,1)
\]

(34)

In the performed studies, \( \alpha = 4 \) has been considered.

4.2. Tent map
This map is similar to logistics in some respects and depicts very specific effects of chaotic behavior. The chaotic sequence sentences in this function are expressed by the following equation:

\[
CHM_{k+1} = \begin{cases} 
0.7 & CHM_k < 0.7 \\
10 \times CHM_k (1 - CHM_k) & CHM_k \geq 0.7 
\end{cases}
\] (35)

4.3. Gauss map

The following equation shows between the sentences of chaotic sequences in the Gauss function:

\[
CHM_{k+1} = \begin{cases} 
0 & CHM_k = 0 \\
\frac{1}{CHM_k - \left[\frac{1}{CHM_k}\right]} & CHM_k \neq 0 
\end{cases}
\] (36)

5. Numerical examples of optimal truss design

In this study, the purpose of the optimal design of trusses is to find the optimal value for the cross-section of members that have less weight and in addition to the limitations related to allowable stress, allowable deformation of joints and slenderness of members according to regulations to be. Famous optimization examples have been selected in this regard and the efficiency of algorithms, chaos map and proposed scenarios have been investigated.

5.1. A 47-bar power transmission tower

The topology and nodal numbering of the power transmission tower that consists of 47 members and 22 nodes, are as shown in Figure 4. The density of material is 0.3 lb/in³ and the modulus of elasticity of the members is 30,000 ksi. Both stress and buckling limits must be satisfied for all members. The allowable stress of tensile members is 20 ksi and for compressive members 15 ksi is recommended. Euler buckling compressive stress for members is also calculated from the following equation.

\[
\sigma_i^e = \frac{-KEA_i}{L_i^2} \quad ; \quad i = 1, 2, ..., 47
\] (37)

Figure 4. Schematic of a 47-bar power transmission tower

Figure 5. Optimization results in standard mode and selection of chaos map for the 47-bar power transmission tower
Where $K$ is a constant coefficient that must be selected according to the geometric type of the section, $E$ is the modulus of elasticity of the material, $A_i$ is cross-sectional area of member and $L_i$ is the length of member. Here we consider $k$ to be 3.96. Structural nodes are affected by a combination of triple loading modes. The first group includes loads of 6 kP in the positive direction of the X-axis and 14 kP in the negative direction of the Y-axis, which are acting in nodes 17 and 22. The second group consists of loads of 6 kg in the positive direction of the X-axis and 14 kg in the negative direction of the Y-axis, which acting only at node 17. The third group includes loads of 6 kP in the positive direction of the X-axis and 14 kP in the negative direction of the Y-axis, which acting only in node 22. The first group represents the oblique loads of both power transmission lines and the second and third groups represent the state where one of the two lines breaks. Also, truss members are classified into 27 groups according to geometric symmetry. The cross section areas of the members, are chosen from the 64 discrete values of the AISC code.

In order to ensure the performance of chaos map and algorithms, as well as to increase the accuracy and sensitivity of calculations, each of the modes has been performed independently 30 times and the results related to the best response and the average value of responses are presented in Tables 1. The graph of these results is also shown in Figure 5. The coefficient of variation of the responses, which indicates the stability and robustness of the responses, has been calculated and another criterion for the efficiency of chaos maps and algorithms has been obtained. By examining the optimization results for different combinations of algorithms with chaos map and comparing it with the standard mode, a significant improvement in reducing the weight of the 47-bar power transmission tower is achieved.

Table 1: Statistical results for 47-bar power transmission tower

<table>
<thead>
<tr>
<th>Table 1: Statistical results for 47-bar power transmission tower</th>
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<tbody>
<tr>
<td>These results include: In the cyclical parthenogenesis algorithm, the logistic chaos map with scenario 1 weighing 2365.032 pounds has the optimal response. In the teaching-learning-based optimization, the Gaussian chaos map with scenario 2 weighing 2364.0782 pounds has the optimal answer.</td>
</tr>
</tbody>
</table>

Table 2: Optimal design comparison for the 47-bar power transmission tower

<table>
<thead>
<tr>
<th>Table 2: Optimal design comparison for the 47-bar power transmission tower</th>
</tr>
</thead>
<tbody>
<tr>
<td>In the biogeography-based optimization, the logistic chaos map with scenario 3 and weighs 2364.1827 pounds has the optimal answer, and finally, in the differential evolution, the logistic chaos map with scenario 2 and weighs 2362.6301 pounds has the optimal answer. Table 2 compares the results of this study with a number of previous studies including Harmony Search (HS) [29], Colliding Bodies Optimization (CBO) [5], Enhanced Colliding Bodies Optimization (ECBO) [30] and Vibrating Particles System (VPS) [6].</td>
</tr>
</tbody>
</table>

5.2. A 120-bar dome shaped truss

The topology and nodal numbering of the 120-bar dome shaped truss shown in Figure 6. The material density is 0.288 lb/in³ and the modulus of elasticity is 30,450 lb/in². Permissible members' tensile and compressive stresses are proposed in accordance with AISC ASD code. The maximum deformation for each node in all directions is set at 0.1969 in. All non-abutment nodes are affected by vertical loading, with intensities of -13.49 kips in node one and -6.744 kips in spheres 2 to 14 and 2.248 kips in the other
nodes, respectively. Also, truss members are classified into seven groups according to their shape and geometric symmetry. The minimum cross-sectional area of the members, which is the lower limit of the decision variables, is 0.775 in$^2$, and the maximum cross-sectional area is 20 in$^2$. The allowable stress in tension and pressure is determined based on the following equations.

$$
\sigma_i = \begin{cases} 
0.6F_y & \text{for } \sigma_i \geq 0 \\
\sigma_i^- & \text{for } \sigma_i < 0
\end{cases} \quad (38)
$$

For compressive stresses, we have the following equations:

$$
\sigma_i^- = \begin{cases} 
(1-\frac{\lambda^2}{2C^2})F_y / FS & \text{for } \lambda < C \\
\frac{12\pi^2E}{23\lambda^2} & \text{for } \lambda \geq C
\end{cases} \quad (39)
$$

In this equation we have:

$$
FS = \left(\frac{5}{3} + \frac{3\lambda}{8C} - \frac{\lambda^3}{8C^3}\right); C = \sqrt{\left(\frac{2\pi^2E}{F_y}\right) \frac{kl}{r}} ; r = aA^b ; a = 0.4993 ; b = 0.6777 \quad (40)
$$

The components used include $E$ modulus of elasticity, $F_y$ the yield stress of steel, $C$ the ratio of the limit slenderness which separates the elastic and inelastic buckling region in comparison with the existing slandering $\lambda$. Also, $k$ is the effective length factor of the $l$ and $r$ is the radius of rotation of the limb. In order to ensure the performance of chaos map and algorithms, as well as to increase the accuracy and sensitivity of calculations, each of the modes has been performed independently 30 times and the results related to the best response and the average value of responses are presented in Tables 3. A graph of these results is shown in Figure 7.

Figure 6. Schematic of a 120-bar dome shaped truss

Also, the coefficient of variation of the responses, which indicates the stability and robustness of the responses, has been calculated and another criterion for the efficiency of chaos map and algorithms has been obtained. The minimum and maximum cross-sectional area of all members is 0.775 in$^2$ and 20 in$^2$ respectively. By examining the optimization results for different combinations of algorithms chaos map and comparing it with standard mode, a significant improvement in reducing the weight of the 120-bar dome shaped truss is achieved.
Table 3: Statistical results for the 120-bar dome shaped truss

Table 4: Optimal design comparison for the 120-bar dome shaped truss

In the cyclical parthenogenesis algorithm, the logistic chaos map with scenario 3 weighing 33,242.4819 lb has the optimal response. In the teaching-learning-based optimization, Gaussian chaos map with scenario 2 weighing 33245.5090 lb has the optimal response. In the biogeography-based optimization, The Gaussian chaos map with scenario 3 and weight of 33240.5624 lb has the optimal response and finally in the differential evolution, Gaussian chaos map with scenario 1 and weight of 33241.9660 lb has the optimal response. Table 4 compares the results of this study with a number of previous studies including (HPSACO) [31], Ray Optimization (RO) [9] and Colliding Bodies Optimization (CBO) [4].

5.3. A 200-bar planar truss structure

The topology and nodal numbering of the 200-bar planar truss structure are shown in Figure 8. The material density is 0.283 lb/in³ and the module of elasticity is 30,000 ksi. Allowable stress of members 10 klb has been assumed. No deformation restrictions are intended for optimization. Truss loading is done in three independent groups. The first group is the lateral load of the structure, which includes a load of 1 kip along the positive axis of Xs and is applied in nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62 and 71. The second group covers the gravity load of the structure, which includes a load of 10 kip in the negative direction of the axis of the Ys and in nodes 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19 and . . . 71, 72, 73, 74 and 75 apply. The third group applies both loading groups together.

Truss members are also classified into 29 groups. The minimum cross section of the members, which is the lower limit of the decision variables, is considered 0.1 in² and the maximum cross section is 16 in². In order to ensure the performance of chaotic maps and algorithms, as well as increasing the accuracy and sensitivity of calculations, each state is independently executed 30 times and the results of the best response and the average amount of responses are presented in statistical Tables 5, and shown in Figure 9. Also, the coefficient of changes in responses, which indicates the stability and robustness of the responses, has been calculated and another criterion for the efficiency of chaotic functions and algorithms has been achieved.

These results for each algorithm are: In cyclical parthenogenesis algorithm, Gaussian chaos map with scenario 2 weighing 25117.1081 lb has the optimal response. In the teaching-learning-based optimization, Gaussian chaos map with scenario 1 weighing 25128.7987 lb has the optimal response. In the biogeography-based optimization, the logistic chaos map with scenario 2 and weight of 25125.3835 lb has the optimal response and finally in the differential evolution, the Gaussian chaos map with scenario 3 and weight of 25102.878 lb has the optimal response. Table 6 compares the results of this study with a number of previous studies including (PSO), (PSOPC), (HPSACO) [31], and Ray Optimization (RO) [32].
Figure 8. Schematic of a 200-bar planar truss structure

Figure 9. Optimization results in standard and chaos map for the 200-bar planar truss structure

Table 5: Statistical results for the 200-bar planar truss structure

Table 6: Optimal design comparison for 200-bar planar truss structure

6. Discussion and concluding remarks

One of the most important features to increase the efficiency of meta-heuristic algorithms is to balance the exploration and exploitation stages. Chaos maps have good potentials to establish this balance [33]. By replacing these maps in the exploration, exploitation, or both, different scenarios for optimization are obtained. In this research, by applying chaos maps in several meta-heuristic algorithms, a significant improvement in truss weight optimization has been achieved. Also, in order to form a statistical population and determine the best weight, average weight and coefficient of variation, each structural model has been implemented with 30 independent replications. Other applications can be made to other meta-heuristics [34-36].

Table 7: Final normalized value with the participation of all the examples

To provide the final results, the results of previous tables are normalized and then combined. The Eq. (41) is intended to summarize the information [29]. The results are presented in Table 7 to compare the performance of the algorithms.

\[
NVal^{MV} = \frac{1}{S} \sum_{i=1}^{S} \left( \frac{Val^{MV}}{\text{min}(Val_i)} - 1 \right)
\]  

(41)

In this regard, \( Val^{MV} \) and \( NVal^{MV} \) are the optimal values of the previous tables and the corresponding normalized values, respectively. \( S \) is also the number of structural examples examined and \( \text{min}(Val_i) \) is the lowest value obtained in each study. Based on the results, the first three priorities of chaos functions to improve the optimization results of algorithms will be according to Table 8.

Table 8: Chaos maps effective in improving the results of algorithms

Some of the considerable results in this research will be as follows:

- In most cases, the combination of chaos maps in meta-heuristic algorithms has created a significant improvement. The main factor can be the effect of chaos map in escaping local optimization and preventing premature convergence.

- In Scenarios 1 and 2, the chaos maps are substituted for the exploration and exploitation steps, respectively. Based on this, it can be concluded:
The cyclical parthenogenesis algorithm has a good exploitation and uses the chaos maps for the exploration stage. Other algorithms such as teaching-learning-based optimization, biogeography-based optimization and differential evolution are the opposite of the previous case and have used chaotic functions for the exploitation stage.

- Using the chaos maps, determining the regulatory parameters of algorithms and sensitivity analysis, is significantly removed. In fact, selecting the starting sentence in chaos maps replaces complex settings. It should be noted that in most cases it is more difficult to find the appropriate tuning parameters of each algorithm than self-optimization. Therefore, by using chaos maps, complex engineering problems can be solved without the need to find parameters.

- To investigate the stability and reliability of the answers, the coefficient of variation, which is the standard dimensionless deviation, has been used.

- To select the initiator sentence in a series of chaotic maps, perform several initial iterations before the main iterations and select the appropriate starter sentence to improve the results by leaps and bounds.

- Among the chaos maps studied, the logistic function with scenarios 1 and 2 has provided the best results.

- Chaos maps are predicted to have significant improvements for optimization problems based on frequency constraints.

Biographies

**Hossein Yousefpour** is a PhD student at Tehran University of Science and Research. He also received his BSc and M.Sc. degrees in Civil Engineering from Iran University of Science and Technology, Tehran, Iran. His research interests include artificial intelligence applications in structural design.

**Ali Kaveh** after graduation from University of Tabriz in 1969, he continued his studies on structures at Imperial College of Science and Technology at London University and received his MSc, DIC, and Ph.D. degrees in 1970 and 1974, respectively. He then joined Iran University of Science and Technology. Professor Kaveh is the author of 730 papers published in international journals and 170 papers presented at national and international conferences. He has authored 23 books in Persian and 14 books in English published by Wiley, Research Studies Press, American Mechanical Society, and Springer.

References


**Captions of the Tables**

Table 1: Statistical results for 47-bar power transmission tower

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Best</th>
<th>Mean</th>
<th>C.V(%)</th>
</tr>
</thead>
</table>

Table 2: Optimal design comparison for the 47-bar power transmission tower

Table 3: Statistical results for the 120-bar dome shaped truss

Table 4: Optimal design comparison for the 120-bar dome shaped truss

Table 5: Statistical results for the 200-bar planar truss structure

Table 6: Optimal design comparison for 200-bar planar truss structure

Table 7: Final normalized value with the participation of all examples

Table 8: Chaos maps effective in improving the results of algorithms

Table 1: Statistical results for 47-bar power transmission tower
<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Best Mean C.V(%)</th>
<th>Algorithms</th>
<th>Best Mean C.V(%)</th>
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<tr>
<td>CPA</td>
<td>2383.4062 2406.1394 0.90067</td>
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Table 2: Optimal design comparison for the 47-bar power transmission tower

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Table 3: Statistical results for the 120-bar dome shaped truss
Table 4: Optimal design comparison for the 120-bar dome shaped truss

<table>
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Table 5: Statistical results for the 200-bar planar truss structure

<table>
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<tr>
<th>Algorithms</th>
<th>Best</th>
<th>Mean</th>
<th>C.V(%)</th>
<th>Algorithms</th>
<th>Best</th>
<th>Mean</th>
<th>C.V(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPA</td>
<td>25791.4552</td>
<td>26365.2532</td>
<td>1.9009</td>
<td>TLBO</td>
<td>26040.6007</td>
<td>27062.4480</td>
<td>5.5550</td>
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<td>Logistic-1 → CCPA-21</td>
<td>25118.5905</td>
<td>25348.5381</td>
<td>0.85254</td>
<td>Logistic-1 → CTLBO-21</td>
<td>25150.3705</td>
<td>25434.6628</td>
<td>1.2437</td>
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<tr>
<td>Logistic-2 → CCPA-22</td>
<td>25210.1143</td>
<td>25333.7911</td>
<td>0.40345</td>
<td>Logistic-2 → CTLBO-22</td>
<td>25223.9130</td>
<td>25351.0160</td>
<td>0.47102</td>
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<tr>
<td>Logistic-3 → CCPA-23</td>
<td>25137.3614</td>
<td>25436.2941</td>
<td>1.2308</td>
<td>Logistic-3 → CTLBO-23</td>
<td>25171.1421</td>
<td>25318.9719</td>
<td>0.78233</td>
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<td>Tent-1 → CCPA-31</td>
<td>25202.3896</td>
<td>25481.4435</td>
<td>1.1080</td>
<td>Tent-1 → CTLBO-31</td>
<td>25286.7338</td>
<td>25365.6319</td>
<td>1.6189</td>
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<tr>
<td>Tent-2 → CCPA-32</td>
<td>28181.7681</td>
<td>29695.0024</td>
<td>4.4680</td>
<td>Tent-2 → CTLBO-32</td>
<td>25506.8124</td>
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<tr>
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<td>Tent-3 → CTLBO-33</td>
<td>25387.7182</td>
<td>26223.6360</td>
<td>2.1787</td>
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<tr>
<td>Gauss -1 → CCPA-41</td>
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<td>25441.9059</td>
<td>0.81384</td>
<td>Gauss -1 → CTLBO-41</td>
<td>25128.7987</td>
<td>25838.8707</td>
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<td>Gauss -2 → CTLBO-42</td>
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<td>25580.1194</td>
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<td>25528.9189</td>
<td>1.1622</td>
<td>Gauss -3 → CTLBO-43</td>
<td>25157.8303</td>
<td>25346.7924</td>
<td>0.46121</td>
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<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Best</th>
<th>Mean</th>
<th>C.V(%)</th>
<th>Algorithms</th>
<th>Best</th>
<th>Mean</th>
<th>C.V(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBO</td>
<td>25574.0456</td>
<td>25760.4385</td>
<td>0.68062</td>
<td>DE</td>
<td>25528.1305</td>
<td>25667.6917</td>
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<td>Logistic-1 → CBBO-21</td>
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<td>Logistic-2 → CBBO-22</td>
<td>25125.3835</td>
<td>25373.1812</td>
<td>0.74767</td>
<td>Logistic-2 → CDE-22</td>
<td>25124.3969</td>
<td>25255.6444</td>
<td>0.37892</td>
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<tr>
<td>Logistic-3 → CBBO-23</td>
<td>25174.1990</td>
<td>25519.1921</td>
<td>1.0559</td>
<td>Logistic-3 → CDE-23</td>
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<td>25181.3225</td>
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<td>Tent-1 → CBBO-31</td>
<td>25151.7825</td>
<td>25395.0075</td>
<td>0.65991</td>
<td>Tent-1 → CDE-31</td>
<td>25153.8718</td>
<td>25341.3289</td>
<td>0.46014</td>
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<tr>
<td>Tent-2 → CBBO-32</td>
<td>25137.1304</td>
<td>25242.0480</td>
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<td>Tent-2 → CDE-32</td>
<td>25137.1112</td>
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<td>Tent-3 → CBBO-33</td>
<td>25296.7692</td>
<td>25548.1724</td>
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<td>Tent-3 → CDE-33</td>
<td>25205.8675</td>
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<td>0.29325</td>
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<tr>
<td>Gauss -1 → CBBO-41</td>
<td>25176.0167</td>
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<td>25102.8787</td>
<td>25225.2811</td>
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Table 6: Optimal design comparison for 200-bar planar truss structure

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<tr>
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<td>Mean</td>
<td>C.V</td>
<td>TLBO Best</td>
<td>Mean</td>
<td>C.V</td>
<td>BBO Best</td>
<td>Mean</td>
<td>C.V</td>
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<td>0.00344</td>
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<td>0.00242</td>
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Table 7: Final normalized value with the participation of all the examples
Table 8: Chaos maps effective in improving the results of algorithms

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<tr>
<th>Algorithms</th>
<th>First Priority</th>
<th>Second Priority</th>
<th>Third Priority</th>
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<tr>
<td>CPA</td>
<td>Logistic-1</td>
<td>Gauss-2</td>
<td>Gauss-3</td>
</tr>
<tr>
<td>TLBO</td>
<td>Logistic-2</td>
<td>Logistic-2</td>
<td>Gauss-2</td>
</tr>
<tr>
<td>BBO</td>
<td>Logistic-2</td>
<td>Logistic-1</td>
<td>Standard</td>
</tr>
<tr>
<td>DE</td>
<td>Logistic-2</td>
<td>Logistic-1</td>
<td>Gauss-3</td>
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Figure 1. Mutation process in differential evolution algorithm
Figure 2. The flowchart for the Chaos Algorithm
Figure 3. The Chaotic value distribution during 100 iterations
Figure 4. Schematic of a 47-bar power transmission tower
Figure 5. Optimization results in standard mode and selection of chaos map for the 47-bar power transmission tower
Figure 6. Schematic of a 120-bar dome shaped truss
Figure 7. Optimization results in standard mode and chaos map for the 120-bar dome shaped truss.
Figure 8. Schematic of a 200-bar planar truss structure
Figure 9. Optimization results in standard and chaos map for the 120-bar planar truss structure.